Laser Dynamics from a Membrane Computing Perspective

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Summary. Modelling real systems and processes is a task classically performed through the approach of differential equation systems, defining the evolution of different variables as the different components of the system. A bad feature that differential equations have is that if some new parameter has to be introduced in the system, then often the previous equations are not useful, and the whole system has to be remodeled again. Bio-inspired computational models are abstractions of reality into a mathematical system that works with specific semantics and can perform some tasks, such as solving problems or demonstrating the universality of themselves or other models. An interesting application of these models is the modelling of real-life processes, where some of them as the so-called *P systems* have demonstrated previously that their performance is remarkable. This is not only for the similarity of the results with the experimental ones, but for its adaptability and modularity of the system, that is, if a new component of the real system is taken into account, not the whole system but a small part of it has to be changed in order to simulate the changed scenario. In this work, a first look at the dynamics of a laser physical system is given, reproducing the behavior of a first model with a PDP system.

Key words: Membrane Computing, Laser Dynamics, PDP system, Modelling.

1 Introduction

Lasers are devices that emit light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. In fact, its name comes from "Light Amplification by Stimulated Emission of Radiation". In [21, 22], a

wide explanation of them can be found. While the principles of such devices were introduced by A. Einstein in [9], they were not applied until the 1950s. Nowadays, there exists a vast amount of applications based on lasers, for example, communications, optical memories and medicine, between other ones. For a good coverage of the history of laser early development, the reader should take a look at [14].

Cellular automata were discovered in the 1940s by Stanislaw Ulam and John von Neumann as a computational paradigm based on a regular grid of cells (that is why it is called *cellular* automata) in a given topology, each of them in one of a finite number of states. Classical models, like *Rule 110* [23] and *Conway's Game of Life* [11] use 1 and 2 dimensions, respectively. However, these devices are not restricted to 2 dimensions, but they can be defined by a grid of $n \in \mathbb{N}$ dimensions. In the beginning, Stephen Wolfram developed a systematic theoretical study of one-dimensional cellular automata, called *elementary* cellular automata. Later on, in [23] he claimed the usefulness of them not only as a theoretical model, but as a framework to simulate natural processes, applicable to cryptography and biology, as some patterns generated by them can be found in the nature [24], and several more applications. In [12, 13], a cellular automata model simulating the behavior of lasers is defined, giving some hints about how laser dynamics can be simulated by a computational simulation paradigm, and how good are they treating the topology of the real process.

Membrane Computing is a bio-inspired paradigm based on the structure and behavior of living cells. It was first introduced in [16], trying to give an alternative perspective to fields such as formal language theory and computability theory. The main devices within this framework are the so-called P systems. Several kinds of these systems have been defined, some of them are explained in [17, 18]. Apart from theoretical results, such as computational completeness and efficiency, a wide range of applications have been found by specific types of P systems. As some of them can be found in [10, 19, 26], we want to stress the impact of probabilistic-like systems, called *PDP systems*, in the field of ecosystems. From the first successful implementation of a model for the endangered species *Gypaetus barbatus*, or bearded vultures, in the Pyrenean and Prepyrenean mountains of Catalonia [6], passing through the Pyrenean chamois [4] and the zebra mussel [7] in the fluvial reservoir of Riba-roja, to the Giant Panda conservation in China [26], it has been proved that this framework is plausible for the simulation of real-life processes. In fact, in [1, 2], two simple models are defined to simulate two classical physics problems such as the Stern-Gerlach experiment and the Uranium 238 decay. Some tools have been developed in order to simulate and validate these models, such as `P-Lingua` [27], `MeCoSim` [28] as well as some GPU based simulators in the `PMCGPU` project [29]. In this work, we want to prove the usefulness of these kinds of systems in the simulation of the dynamics of lasers, as a different approach to the simulation of these devices. The paper is organized as follows: in the next sections, some definitions are given to make the work self-contained. Section 3 will be devoted to a brief explanation of laser dynamics. In Secion 4, cellular automata will be defined and a glimpse of the model of laser dynamics from [12, 13] will be given. Section 5

will introduce the framework of PDP systems and in the next one a first model to model laser dynamics within the framework of Membrane Computing will be explained. The work will be closed with some conclusions and open research lines.

2 Preliminaries

Some basic concepts are going to be introduced in this section. The reader is supposed to have a basic knowledge about alphabets, multisets, graphs and trees. For a broader definition, the reader is invited to take a look at [17, 18].

2.1 Lattices and topology

A *lattice* in $\mathbb{R}^n$ is a subgroup of the additive group $\mathbb{R}^n$ which is isomorphic to the additive group $\mathbb{Z}^n$, and which spans with the real vector space $\mathbb{R}^n$. Given a lattice Λ in an n-dimensional space, each element in Λ can be labelled by a natural number i. If Λ is a finite 2-dimensional square lattice, we can define the label of an element as $i = (x, y)$, being (x, y) the *position* of the element in the lattice, defined in a natural way. Given two elements of a lattice, we define the operator $+ : \mathcal{L}^2 \to \mathcal{L}$ as a sum of elements in the lattice into another one. In the previous example, $i + j = (x_i + x_j, y_i + y_j)$. The size of the lattice Λ is denoted by $|\Lambda|$.

The lattice can be a finite or an infinite set of elements. If the lattice is finite, its bounds can act as real bounds, that is, they can act as actual elements with static states, or they can be joint with other sides of the lattice, giving the lattice a different topology. Usually, 2-dimensional square cellular automata use cylinder or torus topology.

3 Laser dynamics

The basic components of a laser system [21, 20, 25] are a *laser medium*, where particles interact with each other, the *pumping process*, through which electrons get artificially excited into higher quantum-mechanical energy levels and *optical feedback elements*, that reflect repeatedly the radiation beam into the laser medium, building a resonant cavity in laser oscillators, which we will refer to generically as "laser devices" in this work, as in [12, 13].

In Figure 1, a description of these components can be visualized in a graphical way.

Lasers are based on the principle of *stimulated emission*, that is, an electron can decay to a lower energy state while it is stimulated by the presence of a photon, with the restriction that the latter has to have energy equal to the difference between the two energy levels, and as a result of this process, a new photon with the same wavelength, phase and polarization as the first one is emitted. In order to have a simplified model, only two energy levels for electrons, E_2 (upper level) and

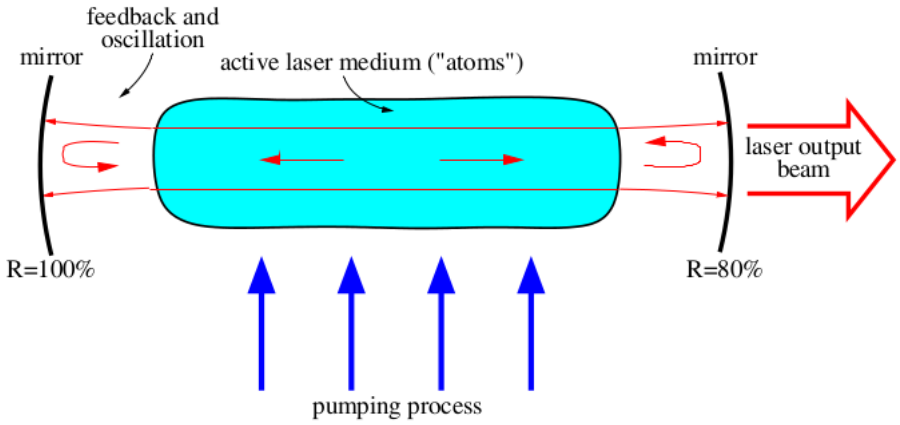

Fig. 1. Components and operation of a typical laser. R stands for reflectivity.

E_0 (lower level) are considered. The pass of an electron from the upper to the lower level is called *population inversion*. Under the assumption of this simplification, the dynamics of the laser can be modeled by the following coupled differential equations [21, 22]:

$$\frac{dn(t)}{dt} = KN(t)n(t) - \frac{n(t)}{\tau_c} \tag{1}$$

$$\frac{dN(t)}{dt} = R - \frac{N(t)}{\tau_a} - KN(t)n(t) \tag{2}$$

being:

- $n(t)$ the number of laser photons;
- $N(t)$ the population inversion (the number of electrons in the upper level);
- τ_c the decay time of photons in the cavity;
- τ_a the decay time of the upper laser level (E_2);
- R the pumping rate; and
- K the coupling constant.

First equation denotes the variation of laser photons through time, while Equation 2 give the temporal variation of the population inversion. The term $KN(t)n(t)$ accounts for the stimulated emission, $\frac{n(t)}{\tau_c}$ and $\frac{N(t)}{\tau_a}$ for the decay of photons and electrons in the upper level, respectively, and R introduces the pumping of electrons with a pumping rate R to the upper laser level E_2.

4 Cellular automata

Cellular automata are computational models defined informally as a grid of cells with states like *on* and *off* that will evolve through a set of rules.

4.1 Formal definition

Definition 1. *A cellular automaton can be defined as a tuple* $C = (\mathcal{L}, \mathcal{S}, S_0, \mathcal{N}, g)$ *where:*

- $\mathcal{L} \in \mathbb{R}^n$ *is a lattice;*
- $\mathcal{S}$ *is a finite set of states;*
- $S_0 \in \mathcal{S}^m$*, being* m *the number of cells of the automaton;*
- $\mathcal{N} = \{r_1, r_2, \ldots, r_\sigma\}$*,* $r_i \in \mathcal{L}$ *(*$1 \leq i \leq \sigma$*) and* σ *is the neighborhood size; and*
- $g : S^\sigma \to S$ *is a map from a set of states to a single one.*

A cellular automaton can be seen as a grid of m cells arranged with the cellular space given by $\mathcal{L}$ such that: (a) S_0 represents the initial state of the grid; (b) the neighborhood of the cell $i \in \mathcal{L}$ is given by the set of cells $\{r_1, r_2, \ldots, r_\sigma\}$, where r_i can depend on the cell i, even being itself.

A *configuration* at any instant of such kind of cellular automaton is described by the state of each cell at the moment. The initial configuration of $C = (\mathcal{L}, \mathcal{S}, S_0, \mathcal{N}, g)$ is $(s_1, s_2, \ldots, s_m)$, being $s_i \in \mathcal{S}$ $(1 \leq i \leq m)$.

A transition from a configuration $\mathcal{C}_t$ to another configuration $\mathcal{C}_{t+1}$ is obtained by applying g to all the cells in the cellular space $\mathcal{L}$. Thus, the state of each cell at any time step is determined by the state of the neighbouring cells at the previous time step. A *computation* of the system is a sequence of transitions starting from the initial configuration, where any term of the sequence other than the first, is obtained from the previous configuration in one transition step, and it is denoted by $\mathcal{C}_t \Rightarrow_C \mathcal{C}_{t+1}$.

4.2 Cellular automata model of laser dynamics

In [12, 13], a cellular automaton modelling the behavior of a laser is defined as a tuple $C = (\mathcal{L}, \mathcal{S}, S_0, \mathcal{N}, g)$, where:

- $\mathcal{L}$ is a 2-dimensional square lattice (a subset $\mathcal{L} \in \mathbb{Z}^2$) containing $N_c = L \times L$ sites, with periodic boundary conditions, that is, it follows the topology of a torus. Each site or element of the lattice is a cell, which is labelled by its position $\mathbf{r} = (i, j) \in \mathcal{L}$, where i and j are the row and the column indices, respectively;
- Four variables are associated at each cell, being $a_{\mathbf{r}}(t) \in \{0, 1\}$ and $c_{\mathbf{r}}(t) \in \{0, 1, \ldots, M\}$ being the state of the electron (being 1 when the electron is in the upper state) and the number of photons, respectively; and $\widetilde{a}_{\mathbf{r}}(t) \in \{0, 1, \ldots, \tau_a\}$ and $\widetilde{c}^k_{\mathbf{r}}(t) \in \{0, 1, \ldots, \tau_c\}$ the number of time steps that an electron has been in its upper state and that a photon $k \in \{1, 2, \ldots, M\}$ has been in a cell, respectively;
- In the initial configuration, all electrons are in their lower state and there are no photons in the system;

- $\mathcal{N}$ is the *Moore neighbourhood*, that is, given a cell $i = (x, y)$, $\mathcal{N} = \{(x-1, y-1), (x-1, y), (x-1, y+1), (x, y-1), (x, y), (x, y+1), (x+1, y-1), (x+1, y), (x+1, y+1)\}$
- g is a set of rules based on the value of the variables of each cell, which are not going to be reproduced here. For the definition of these rules, we invite the reader to view paper [13] or the thesis [12] for a wider explanation and meaning.

To get an idea, rules from g reproduce the different processes of the system, that is, *pumping*, *stimulated emission*, photon and electron decay and the introduction of noise photons. Three variables are taken into account when trying to reproduce the behavior of laser systems: the pumping rate, that makes an electron to promote to an upper level with probability λ, the electron decay time τ_a and the photon decay time τ_c.

First successful model works with $\lambda = 0.192$, $\tau_a = 30$ and $\tau_c = 10$, where λ is adimensional and τ_a and τ_c are measured in time steps. The evolution of the system simulates the behavior of a laser with such characteristics, as graphics of the Figure 3 demonstrate.

Some other results can be found in the reference, but only this example is going to be used for our purpose.

5 PDP systems

PDP systems are a variant of P systems inspired by the functioning of cells. Cells are able to run multiple processes in parallel in a perfectly synchronized manner, making them good candidates to be imitated for modeling complex problems. A PDP system can be viewed as a cellular tissue in which each cell is within a special compartment called environment. The cells have a particular structure hierarchy in which there is a skin membrane that defines and distinguishes the inside from the outside. In turn, inside a cell there are a number of hierarchically arranged membranes, where organelles or chemical substances capable of evolving according to specific reactions of the membrane may appear. PDP systems are probabilistic P systems, that is, the applications of their rules is commanded by a predefined probability on them. For a more exhaustive explanation of this model, see [5].

The key of software implementations of these systems are conflicts. If two or more rules compete for a resource, the algorithm has to take a strategy. The resolution of conflicts depends on the algorithm used to simulate the system. Some algorithms as the *Binomial Block Based simulation algorithm* (`BBB`) [3], the *Direct Non Deterministic distribution with Probabilities algorithm* (`DNDP`) [4] and the *Direct distribution based on Consistent Blocks Algorithm* (`DCBA`) [15] have been developed, each of them treating these conflicts in a different way. Thus, the state of the system at any time step is determined by the state of the system at the previous time step.

6 Laser dynamics modelling through a PDP system

In this first approach, to model the system, we use a PDP system of degree $(1,1)$ $\Pi = (\Sigma, G, R_E, \Gamma, R, T, \{f_r : r \in R_\Pi\}, M_{11})$, where:

- $\Gamma = \{a, z\} \cup \{p_i | 1 \leq i \leq \tau_c\} \cup \{e_i | 1 \leq i \leq \tau_a\}$, and $\Sigma = \emptyset$;
- $G = (V, E)$, where $V = \{e_1\}$ and $E = \emptyset$;
- $R_E = \emptyset$;
- R is the set of the following rules:

  6.1 Rules to simulate the *pumping process*:

  $[a]_1 \xrightarrow{\lambda} [e_1]_1$

  $[a]_1 \xrightarrow{1-\lambda} [a]_1$

  6.2 Rules to simulate the laser action:

  $$\left.\begin{array}{l} [e_i\, p_j\, p_k]_1 \xrightarrow{0.5} [a\, p_1\, p_{j+1}\, p_{k+1}]_1 \\ [e_i\, p_j\, p_k]_1 \xrightarrow{0.5} [e_{i+1}\, p_{j+1}\, p_{k+1}]_1 \end{array}\right\} \text{for } 1 \leq i \leq \tau_a, 1 \leq j \leq k \leq \tau_c$$

  6.3 Rules to reproduce the photon decay:

  $[p_i]_1 \longrightarrow [p_{i+1}]_1$ for $1 \leq i \leq \tau_c - 1$

  $[p_{\tau_c}]_1 \longrightarrow [z]_1$

  6.4 Rules to reproduce the electron decay:

  $[e_i]_1 \longrightarrow [e_{i+1}]_1$ for $1 \leq i \leq \tau_a - 1$

  $[e_{\tau_a}]_1 \longrightarrow [a]_1$

  6.5 Rules to introduce noise photons:

  $[z]_1 \xrightarrow{0.0003} [z\, p_1]_1$

  $[z]_1 \xrightarrow{0.9997} [z]_1$
- $T = 1000$; and
- $M_{11} = \{a^{160000}, z^{160000}\}$.

The inner probabilistic P system has a single membrane labelled by 1, so the initial configuration can be represented as in Figure 2.

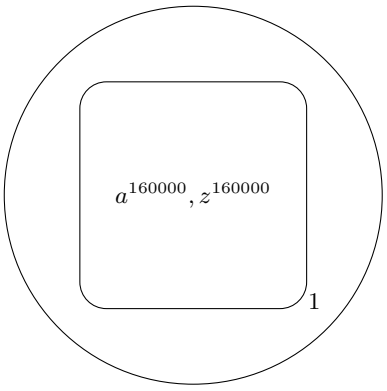

Fig. 2. Initial configuration of the PDP system Π.

Objects a represent electrons in the lower energy state E_0, while e_i represent electrons that have been i time steps in population inversion. Analogously, objects z represent photons that are not in the system and p_i represent photons that have been i time steps in the system.

Basically, the system works as said: electrons (respectively, photons) are excited (resp., introduced in the system) by rules from **6.1** (resp., **6.5**), as well as rules from **6.4** (resp., **6.3**) represent their evolution through time until reaching τ_a (resp., τ_c), while they become an object a (resp,. z) again. Rules from **6.2** simulate the interaction between electrons in population inversion and photons.

In the first case, we use the following constants: $\lambda = 0.192$, $\tau_c = 10$ and $\tau_a = 30$.

For the simulations, we have used the `DCBA algorithm` implemented in CUDA [29], using the cluster of the RGNC group [30] to increase their speed. The results seem pretty interesting since they reproduce almost exactly the obtained in the cellular automata approach and the experimental results. In Figure 4 it can be seen that the graphics obtained with this model are very similar with the ones of Figure 3.

7 Conclusions and future work

In this work, a new approach to the modelling of laser dynamics has been developed. In [12, 13], a cellular automaton modelling this process is defined, taking advantage their inherent parallelism to implement the algorithm in distributed systems. In this case, we use the parallelism of P systems and its implementation in CUDA to obtain similar results understanding the principles of the phenomenon. The use of probabilities seems reasonable since physical processes are usually probabilistic. In this first version, we only use a single membrane and a single environment to simulate the whole system, but we could try to use more membranes to simulate different parts of the laser, as well as to simulate the interaction of particles from the outside of the laser and so on. It can be used in a second version since the results with parameters $\lambda = 0.0125$, $\tau_c = 10$ and $\tau_a = 180$, the results are not similar to the experimental ones. Another interesting research line could be the use of *stochastic* P systems instead of *probabilistic* ones, since the equations defining the dynamics come with two constants K and R that could be introduced directly in the system.

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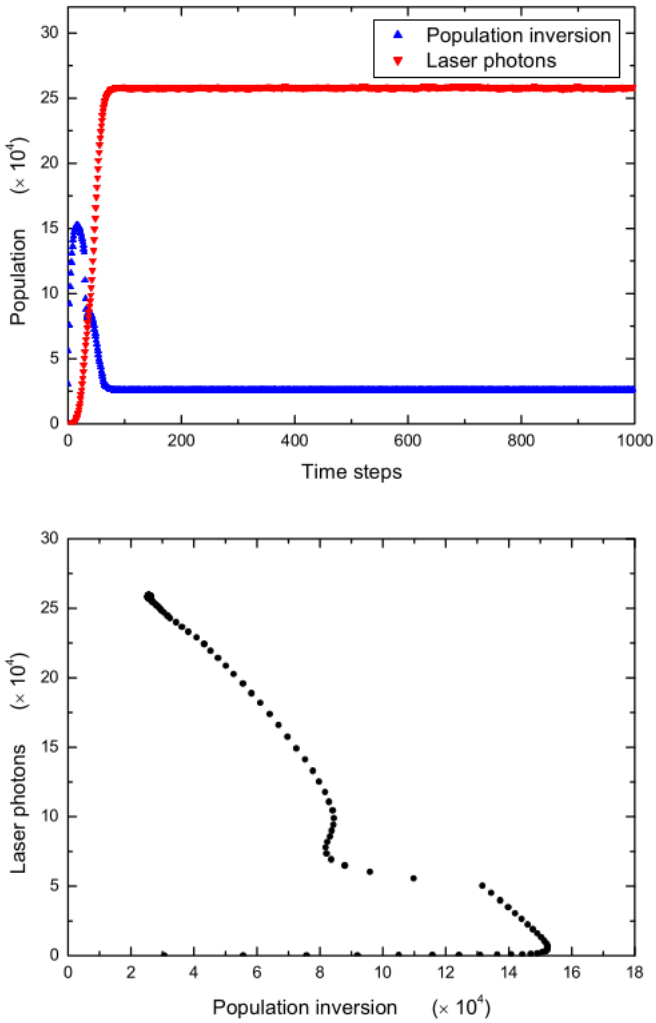

Fig. 3. Evolution of the cellular automaton for $\lambda = 0.192$, $\tau_c = 10$ and $\tau_a = 30$. Up: number of laser photons and population inversion versus time. Down: evolution in a phase space with the number of laser photons versus the population inversion.

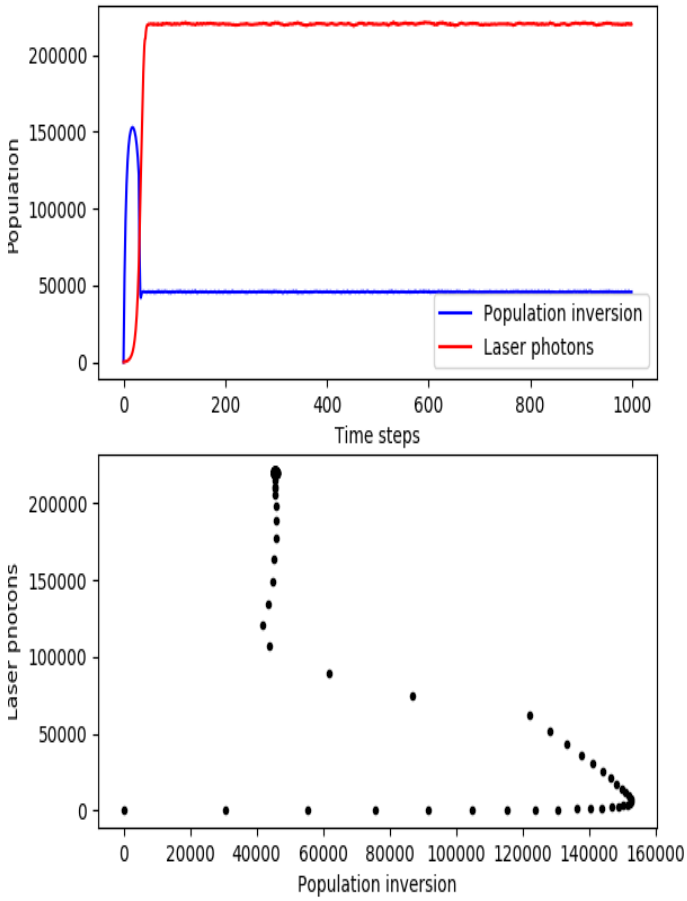

Fig. 4. Evolution of the PDP system for $\lambda = 0.192$, $\tau_c = 10$ and $\tau_a = 30$. Up: number of laser photons and population inversion versus time. Down: evolution in a phase space with the number of laser photons versus the population inversion.

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