Analysis of the Propagation of Leaky Magnetostatic Modes in Normally Magnetized Microstrip and Slot Lines

Ricardo Marqués, *Member, IEEE*, Rachid Rafii-El-Idrissi, Francisco Mesa, *Member, IEEE*, and Francisco Medina, *Senior Member, IEEE*

***Abstract*—The propagation of leaky forward magnetostatic (MS) volume waves along ferrite-loaded microstrip and slot lines is analyzed. This phenomenon is studied by means of a numerical approach based on the residue calculus technique because of its good numerical convergence and stability. The proposed method allows for a quick and accurate computation of the phase and attenuation constants of the leaky MS modes. A comparative analysis between both microstrip lines and slot lines is carried out, and some new physical effects, such as MS resonances in the radiation loss, are reported. The advantages of the proposed method of analysis over other numerical methods, such as Galerkin's or moment methods, are also discussed.**

***Index Terms*—Leakage, magnetostatic waves, printed-circuit lines.**

I. Introduction

MAGNETOSTATIC (MS) wave propagation in epitaxial low-loss yttrium–iron–garnet (YIG) films finds wide application in the design of ferrite microwave devices [1]. The applications include magnetostatic wave (MSW) transducers, filters, and delay lines. Recently, an increasing interest has arisen on the soliton behavior [2] and other nonlinear effects in YIG film MS waveguides because of the reduced MSW wavelengths, which allows for soliton observation in samples of small size. Many of these devices and experiments have been designed using planar structures of (practically) infinite width. More recently, stripline [3]–[6], slot lines [6], [7], and other finite MS waveguides [8] have been proposed and analyzed. Propagation along finite-width strips or slots has the advantage of field confinement near the strip/slot. A new MSW delay line using a microstrip line has been recently demonstrated [4]. The soliton behavior in this kind of MS waveguides has been also investigated [9].

In two recent papers [5] and [6], some of the authors have analyzed the propagation of bound MSW in striplines and slot lines by applying Galerkin's method in the spectral domain. Bound magnetostatic surface waves (MSSWs) can propagate in striplines [3], [5], [6] and bound backward volume magnetostatic waves (BVMSWs) can propagate in slot lines [6], [7]. Propagation of forward volume magnetostatic waves (FVMSWs) in these type of waveguides is expected to be leaky [6]. In [9], it has been reported that the propagation of FVMSWs along microstrip lines always involves some amount of radiation loss. Although, in some cases, the phase constant of the FVMSWs propagating in microstrip lines can be adequately computed by means of a simple parallel-plate model [4], [9], it is clear that this simple model cannot provide any information about radiation losses. Appropriate characterization of losses is essential both in the design of MS devices such as delay lines or filters and in the analysis of soliton behavior [9], [10] and other nonlinear effects. Since radiation may be an important source of losses in microstrip FVMS waveguides, the computation of these leakage losses becomes unavoidable for the adequate description of microstrip and stripline MS waveguides. Propagation of FVMSWs along slot waveguides has been less investigated. Since slot lines may be an alternative to microstrip lines in the design of MSW microwave devices, a comparative analysis of these waveguides becomes of interest.

In this paper, the propagation of FVMSWs along microstrip lines and slot lines is analyzed. Following the qualitative method used in [6], the physical mechanism of guidance and power leakage is first identified on the basis of the analysis of the propagation along infinite parallel-plate waveguides. Once this mechanism has been established, the phase and attenuation constants of the FVMSWs guided in striplines and slot lines are computed. For this computation, a method based on the residue calculus technique (RCT) [11] has been adapted to the present problem. This method makes use of a field expansion in modes of the different parallel-plate waveguides that the microstrip line or slot line can be subdivided into. This field expansion is very appropriate since it is better adapted to the physics of the problem than a field expansion into basis functions of general purpose. This latter fact explains why Galerkin's method, which was successfully applied to the analysis of propagation of bound MSWs in [5] and [6], does not provide accurate results when applied to the analysis of FVMSWs in microstrip and slot lines. The usefulness of our proposed method, however, strongly depends on the availability of a straightforward technique for computing the poles and zeros of the corresponding spectral Green's function. Fortunately, this requirement is easily fulfilled in planar structures with normal magnetization.

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R. Marqués, R. Rafii-El-Idrissi, and F. Medina are with the Department of Electronics and Electromagnetism, University of Seville, 41012 Seville, Spain (e-mail: marques@us.es; rafii;@us.es; medina@us.es).

F. Mesa is with the Department of Applied Physics I, University of Seville, 41012 Seville, Spain (e-mail: mesa@us.es).

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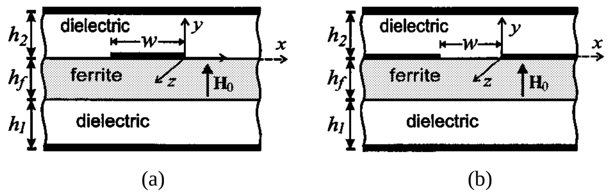

Fig. 1. Structures under analysis. (a) Microstrip line. (b) Slot line.

II. ANALYSIS

Fig. 1(a) and (b) shows the structures under analysis, i.e., a microstrip line and slot line MS waveguides printed on a normally magnetized ferrite slab, which may incorporate upper and/or lower ground planes. An implicit field dependence of the kind $\exp j(\omega t - k_z z)$ is assumed. Previous to the study of MS modes in microstrip lines and slot lines, the propagation of FVMSWs in the normally magnetized ferrite slab waveguides outside the strip/slot will be analyzed. These slab waveguides correspond respectively to the nonmetallized [$w \rightarrow \infty$ in Fig. 1(b)] and the metallized [$w \rightarrow \infty$ in Fig. 1(a)] ferrite slabs. Both structures can support guided FVMSWs in the frequency range $f_0 < f < f_1$, where $f_1 = \sqrt{f_0(f_0 + f_M)}$, f_0 is the ferrite resonant frequency ($2\pi f_0 = \gamma H_0$), $2\pi f_M = \gamma 4\pi M_s$, γ is the gyromagnetic ratio, M_s is the magnetization of saturation of the ferrite, and H_0 is the internal magnetizing field [12], [13]. Since propagation is isotropic in the x–z-plane, the x- and z-components of the wave vectors of these slab FVMS modes, i.e., (k_x, k_z), must satisfy

$$k_x^2 + k_z^2 = \gamma_n^2, \qquad n = 1, 2, \ldots \tag{1}$$

with $\{\gamma_n^2\}$ being an infinite and numerable set of *real* eigenvalues (for lossless structures), which represent the square wavenumbers of the different FVMS modes of these waveguides. As is discussed in [12] and [13], $\gamma_n^2 \rightarrow 0$ at the lower cutoff frequency f_0 and $\gamma_n^2 \rightarrow \infty$ at the upper cutoff frequency f_1, $\gamma_n(f)$ showing an increasing monotone dependence with n for any value of f.

Following the method in [6], it is assumed that MS modes propagating along the strip or the slot of Fig. 1 are formed by the consecutive reflections of MS waves at the strip/slot edges. These reflections would also excite propagative MS waves in the slab waveguides outside the strip/slot region, thus causing the strip/slot mode to be leaky. In view of (1), the *leakage condition* [14] for the excitation of a given nth slab mode propagating in the region outside the strip/slot can be written as

$$\Re(k_z) < \gamma_n \tag{2}$$

where k_z is the propagation constant of the strip/slot mode and γ_n^2 is the eigenvalue of the excited MS mode. Since the strip/slot mode will be bound only if (2) is *not* fulfilled for all n, the increasing nature of the $\{\gamma_n\}$ set suggests that FVMS modes guided by strip/slot lines will be leaky in the whole frequency range of excitation of these waves, namely, for $f_0 < f < f_1$ [6]. The eigenvalues for the slabs, which will be denoted as

$$\gamma_n^2 \equiv \begin{cases} K_n^2, & \text{for metallized slabs} \\ k_n^2, & \text{for nonmetallized slabs} \end{cases} \tag{3}$$

satisfy that $K_n^2 < k_n^2$ [12], [13] for any value of n. Consequently, it is expected that the entire spectrum of slab FVMS modes be excited by the first strip-guided FVMS mode. Conversely, it is not expected that the first FVMS mode of the metallized slab be excited by the first FVMS mode of the slot line, as far as the corresponding eigenvalue K_1 is smaller than the phase constant of the first slot-guided FVMS mode (at least for wide slots).

The numerical analysis of the strip/slot lines is developed starting from the spectral-domain expression that relates the spectral counterparts of the normal magnetic flux density at the strip/slot interface B_y and the current function $I(x)$ at this interface, i.e.,

$$\tilde{B}_y(k_x) = \tilde{G}(k_x;\, k_z,\, \omega)\tilde{I}(k_x) \tag{4}$$

where

$$I(x) = \int_{-\infty}^{x} J_{s,\,z}(x')\, dx' \tag{5}$$

with $J_{s,\,z}(x)$ being the z-component of the surface current density and $\tilde{G}(k_x;\, k_z,\, \omega)$ being the corresponding spectral-domain MS Green's function. $\tilde{G}$ is a meromorphic function (without branch points) of k_x, whose explicit expression for a general magnetic bias field can be found, for example, in [6, Appendix]. From (4), it would be possible to carry out an integral-equation analysis similar to that proposed in [5] and [6]. In the microstrip-line case, the integral equation for the strip current (5) can be expressed as

$$B_y(x) = \int_{-w}^{0} G(x-x')I(x')\, dx' = 0, \qquad -w < x < 0 \tag{6}$$

where $G(x - x')$ stands for the inverse Fourier transform of $\tilde{G}(k_x;\, k_z,\, \omega)$

$$\begin{aligned} G(x - x') &= \mathcal{F}^{-1}\left\{\tilde{G}\right\} \\ &= \frac{1}{2\pi}\int_C \tilde{G}(k_x;\, k_z,\, \omega)\exp(-jk_x x)\, dk_x. \end{aligned} \tag{7}$$

For leaky modes, (7) has to be defined along a proper inversion contour C in order to enclose all the poles of the spectral Green's function associated with the radiating slab modes [15] (leakage into free space does not occur for MS modes). Since the entire spectrum of slab FVMS modes is expected to be excited by the microstrip leaky mode, this contour should be as that shown in Fig. 2. The integral equation in (6) can be solved using Galerkin's method in the spectral domain [6]. A similar procedure can be followed for the computation of slot-line modes.

As it will be shown in Section III, the use of Galerkin's method with general-purpose basis functions gives rise to a very poor convergence, leading to results that do not completely agree with the expected physical properties of FVMS modes in the analyzed structures. Thus, a different approach has been chosen that takes advantage of the physical meaning of the zeros and poles of the spectral Green's function. The poles of $\tilde{G}(k_x;\, k_z,\, \omega)$ are found to be $\pm k_{x,\,n}$, where $k_{x,\,n}$ is taken as that pole with positive real part, i.e.,

$$k_{x,\,n} = \sqrt{k_n^2 - k_z^2}, \qquad \Re(k_{x,\,n}) > 0. \tag{8}$$

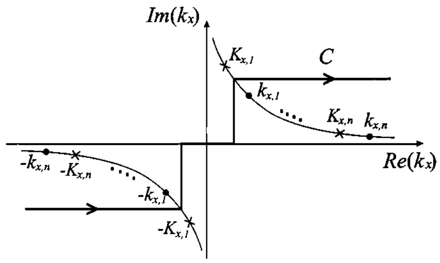

Fig. 2. Representation of the integration path in (7). The curves represent the hyperbolas defined in (10).

The $\pm k_{x,n}$ poles can be viewed as the x-component of the $(\pm k_{x,n}, k_z)$ wave vector of a FVMS mode propagating in the nonmetallized slab. In a similar way, the zeros of $G(k_x; k_z, \omega)$ are $\pm K_{x,n}$ with

$$K_{x,n} = \pm\sqrt{K_n^2 - k_z^2}, \qquad \Re(K_{x,n}) > 0 \tag{9}$$

which can be related to the x-component of the wave vector of a FVMS mode propagating in the metallized slab.

Taking into account that physical leaky modes with $\Re(k_z) > 0$ must have $\Im(k_z) < 0$ (i.e., the mode is attenuated as it propagates) and that $\Im(k_n^2) = 0$ for *lossless* ferrite slabs, it is found that the poles and zeros of $\tilde{G}(k_x; k_z, \omega)$ lie on the hyperbolae defined by

$$\Re(k_z)\Im(k_z) + \Re(k_x)\Im(k_x) = 0 \tag{10}$$

which are shown in Fig. 2.

The total magnetic flux density B_y at the strip interface can be expressed as

$$B_y(x) = B_y^+(x) \pm B_y^+(-x - w) \tag{11}$$

where $B_y^+(x)$ is the magnetic flux density on the right-hand side of the strip ($x > 0$) and the upper (lower) sign applies for even (odd) modes (FVMS modes in the analyzed structure must be even and odd, as shown in [4]). Since all the slab MS modes are expected to be excited by the first strip-guided FVMS mode, $B_y^+(x)$ can be expressed as

$$B_y^+(x) = \begin{cases} \sum_{n=1}^{\infty} A_n \exp(-jk_{x,n}x), & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

where $k_{x,n}$ are given by (8). It should be noticed that the above definition of $B_y^+(x)$ guarantees, after substitution in (11), that $B_y(x) = 0$ for $-w < x < 0$, namely, that (6) is satisfied. Expression (12) also guarantees that B_y is built with slab waveguide modes increasing and outgoing from the strip. Since the MS eigenvalues k_n^2 increase with n, the field expansion in (11) and (12) comprises strongly oscillating terms.

As is expected for leaky modes, (11) and (12) give an unbounded field. However, a generalized Fourier transform of $B_y^+(x)$ can still be defined [15], [16], which turns out to be

$$\tilde{B}_y^+(k_x) = j\sum_{n=1}^{\infty} \frac{A_n}{k_x - k_{x,n}}. \tag{13}$$

It is easily recognized that (12) is recovered by taking the generalized inverse transformation

$$B_y^+(x) = \mathcal{F}^{-1}\left\{\tilde{B}_y^+\right\} \tag{14}$$

provided that the inversion contour is defined as that shown in Fig. 2. From (11), the generalized Fourier transform of $B_y(x)$ is then given by

$$\tilde{B}_y(k_x) = \tilde{B}_y^+(k_x) \pm \exp(-jk_x w)\tilde{B}_y^+(-k_x) \tag{15}$$

where the upper (lower) sign applies for even (odd) modes.

Since the total current function on the strip $I(x)$ is a bound function defined over a finite interval $x \in [-w, 0]$, its corresponding Fourier transform $\tilde{I}(k_x)$ must have no poles in the entire k_x domain [16]. From (4), this implies that $\tilde{B}_y(k_x) = 0$, $\forall\, k_x = \pm K_{x,n}$, where $K_{x,n}$ is given in (9). The above condition leads to the following set of equations:

$$\tilde{B}_y(\pm K_{x,m}) = 0, \qquad m = 1, 2, \ldots \tag{16}$$

which, after substitution of (13) and (15), can be expressed as the following matrix equation:

$$[\Gamma_{mn}][A_n] = 0, \qquad m, n = 1, 2, \ldots \tag{17}$$

with

$$\Gamma_{mn} = \frac{1}{K_{x,m} + k_{x,n}} \pm \frac{\exp(jK_{x,m}w)}{-K_{x,m} + k_{x,n}} \tag{18}$$

where the upper (lower) sign holds for the even (odd) modes and only the minus sign in (16) has been considered (the plus sign would have led to the same set of equations).

The set of equations (17) is very similar to that found in the first steps of the application of the modified residue calculus technique (MRCT) to the quasi-TEM analysis of striplines [11]. However, two main differences appear. First of all, (17) is an homogeneous set of equations for coefficients A_n, whose determinant must vanish for the appropriate complex wavenumber k_z, thus, giving rise to the following implicit dispersion equation of the structure:

$$\det\left[\Gamma_{mn}(k_z, \omega)\right] = 0. \tag{19}$$

Since the imaginary parts of $k_{x,n}$ and $K_{x,n}$ decrease with n (see Fig. 2), no advantage can be taken from the application of the sophisticated MRCT or Wiener–Hopf procedures to solve (17). Nevertheless, this disadvantage is compensated by the straightforward computation of $K_{x,n}$ and $k_{x,n}$, as it will be shown later. It is also important to note that the $[\Gamma_{mn}]$ matrix is well conditioned with the maximum of each row or column lying in the diagonal of the matrix. Thus, after truncation to a given $n = m = N$, $\det[\Gamma_{mn}]$ can be computed on a Pentium PC with very short computation time, even for $N \gtrsim 100$. The dispersion equation given in (19) is then very suitable for the numerical analysis of the analyzed structures.

The question of the computation of the poles $k_{x,n}$ and zeros $K_{x,n}$ still remains. As is obvious in the way the dispersion relation has been posed, an easy computation of the poles and zeros is a key condition for the efficiency of the proposed method.

From their definitions, i.e., (8) and (9), it is clear that the numerical effort to obtain $k_{x,n}$ and $K_{x,n}$ lies entirely on the computation of k_n and K_n. For $h_1,\ h_2 \rightarrow \infty$, this computation is carried out by means of the analytical formulas in [12, eqs. (4.75a) and (4.138)]. For finite values of h_1 and/or h_2, the above values are used as convenient seeds in a Muller's method based automatic routine that searches for the zeros of the corresponding implicit dispersion equations, which can be found in [13]. In any case, the computation of both k_n and K_n turns out to be very quick and accurate.

The analysis of the slot-line structure of Fig. 1(b) follows the same steps. It must be taken into account, however, that since K_1 is always smaller than k_1 [12], [13], it is not expected that the first FVMS mode be excited in the metallized slab as a propagating wave. This fact slightly modifies the equations, giving the following final result:

$$A_1\left\{\frac{1}{K_{x,1}-k_{x,m}} \pm \frac{\exp(jk_{x,m}w)}{K_{x,1}+k_{x,m}}\right\} - \sum_{n=2}^{\infty} A_n\left\{\frac{1}{K_{x,n}+k_{x,m}} \pm \frac{\exp(jK_{x,m}w)}{K_{x,n}-K_{x,m}}\right\} = 0 \quad (20)$$

with $m = 1, 2, \ldots$ and the upper (lower) sign holding for even (odd) modes.

At this point, it is interesting to point out that the use of a method of moments (MoM) approach would have been involved to perform spectral integrals along integration paths such as that shown in Fig. 2. These integrals are not easy to deal with because the integration path never runs along the real axis. The considerable computational effort required to get accurate values of these integrals can be easily realized. This drawback is completely overcome by the proposed method, without introducing other sources of numerical problems.

III. Numerical Results

First, the convergence of the proposed numerical method with the number N of equations in (19) is shown in Fig. 3(a). Results are for a microstrip line and $N > 10$. It can be observed that a very good convergence of more than five and three digits for the phase and attenuation constants, respectively, is achieved with reasonable values of N. Similar results are obtained for the slot line, although they will not be explicitly shown. In the remaining numerical results, a value of $N = 100$ has been systematically used to ensure very good accuracy in all cases for both the phase and attenuation constants. A typical computation time for each determinant with $N = 100$ takes less than 0.1 s in a PC Pentium III at 500 MHz. This results in a total computation time typically below 1 s for each point in the plots (using $N = 10$, which ensures enough accuracy for most applications, and would have resulted in computation times of approximately 10 ms for each point). In order to compare the results obtained by the proposed method with those obtained when using Galerkin's method, we have developed two codes based on this latter technique: one employing subsectional triangular basis functions [6] and the other using weighted Chebyshev polynomials of the second kind [18] as basis functions. In this sense,

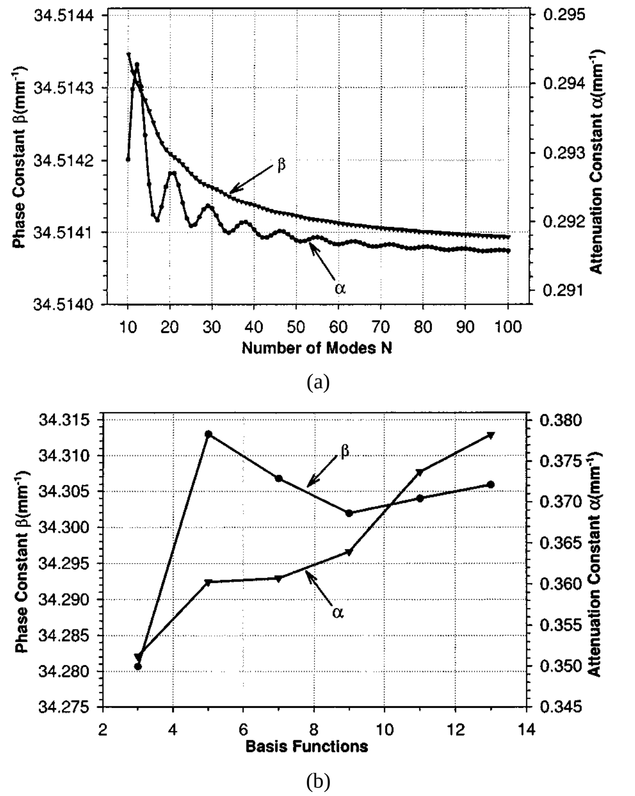

Fig. 3. Convergence of the computed values for the phase (β) and attenuation (α) constants of the first FVMS mode in a microstrip line. (a) Our proposed method. (b) Galerkin's method with weighted Chebyshev polynomials as basis functions. Structure parameters: $h_1 = 400\ \mu$m, $h_f = 20\ \mu$m, $h_2 \rightarrow \infty$, $w = 300\ \mu$m, $4\pi M_s = 1750$ G, $H_0 = 570$ Oe, $f = 2.5$ GHz.

Fig. 3(b) shows the poor convergence achieved by Galerkin's method using Chebyshev polynomials in the analysis of the structure considered in Fig. 3(a). Moreover, since the calculated quantity is the complex propagation constant $k_z = \beta - j\alpha$ and the attenuation constant α is about two orders of magnitude smaller than the phase constant β, this lack of convergence in k_z leads to meaningless results for α. Convergence for both the phase and attenuation constants is even worse if piecewise basis functions are used. In addition, the overall computational effort is found to be 100–1000 times larger due to the intensive numerical integration. Therefore, spectral-domain analysis (SDA)/MoM does not seem to be appropriate for the problem under study, even though it has been proven to be suitable for the analysis of bound modes [5], [6]. Some reasons explaining this lack of numerical efficiency will become apparent later.

The phase and attenuation constants of a microstrip line and slot line on an epitaxial YIG substrate are shown in Figs. 4 and 5, respectively. The first three FVMS modes are shown in each figure. The phase constant of the first FVMS mode of an infinite metallized ferrite slab [$w \rightarrow \infty$ in Fig. 1(a)] is also shown in Fig. 4, as well as data for the phase constant obtained following the magnetic wall model proposed in [4, eq. (7), with $b_0 = 0$ and $n = 1, 2, 3$]. The agreement with the results computed following [4] is excellent. It should be noted, however, that the magnetic wall model of [4] does not provide any information on the mode attenuation due to leakage losses. In both microstrip and slotline, the first and third modes are even modes—for $I(x)$ and $B_y(x)$, respectively—whereas the second mode is an odd

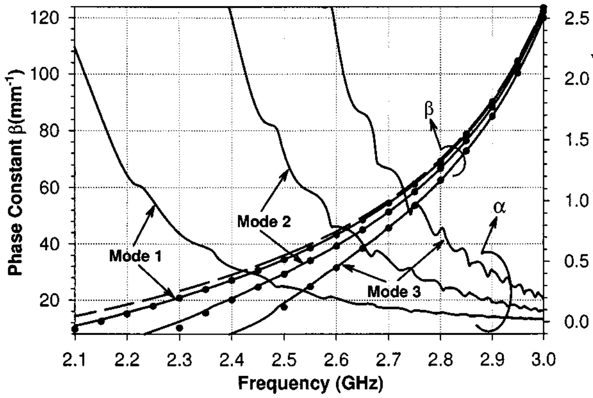

Fig. 4. Dispersion relations for the three first FVMS modes in a normally magnetized microstrip line. The first FVMS mode for $w \to \infty$ is also shown (dashed line), as well as the results obtained for the phase constant using [4, eq. (7)] (dots). Structure parameters: $h_1 = 400$ μm, $h_f = 20$ μm, $h_2 \to \infty$, $w = 300$ μm, $4\pi M_s = 1750$ G, $H_0 = 570$ Oe.

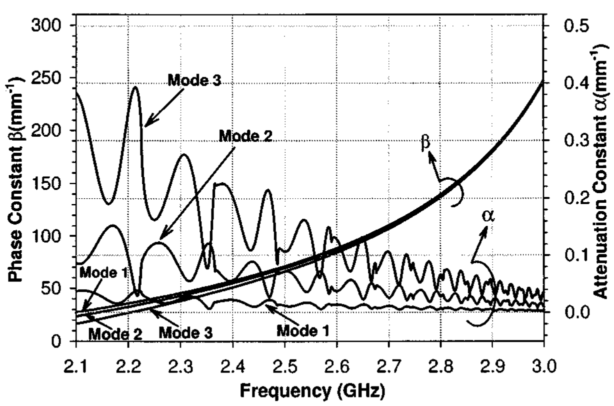

Fig. 5. Dispersion relations for the three first FVMS modes in a normally magnetized slot line. Structure parameters: $h_1 = 400$ μm, $h_f = 20$ μm, $h_2 \to \infty$, $w = 300$ μm, $4\pi M_s = 1750$ G, $H_0 = 570$ Oe.

mode. Higher order modes (with increasing attenuation constants) have been also computed, although they are not explicitly shown. In both structures, losses decrease and phase constants increase as frequency increases. It can also be observed that radiation losses in a slot line are much less significant than in a microstrip line having the same dimensions and substrate. This result can be understood from the fact, mentioned above, that radiation losses in slot lines are caused by the excitation of the second and higher order FVMS modes in the surrounding metallized slab, whereas the excitation of the first FVMS mode in the surrounding slab waveguide also contributes to radiation losses in the microstrip line. Therefore, the first slab FVMS mode inside the slot is reflected at the slot edges without almost exciting radiating modes outside the slot, while the reflections of the first metallized slab FVMS mode at the strip edges will always excite the first slab FVMS mode outside the strip. Another important feature of both figures is that they describe multimode waveguides. In fact, there is always a multiplicity of modes with similar phase and attenuation constants over a (relatively) wide frequency range. Ripples in the dispersion curves of the attenuation constants can be related to the resonances of the higher order FVMS modes between the microstrip or slot edges. We will turn to this fact in more detail further.

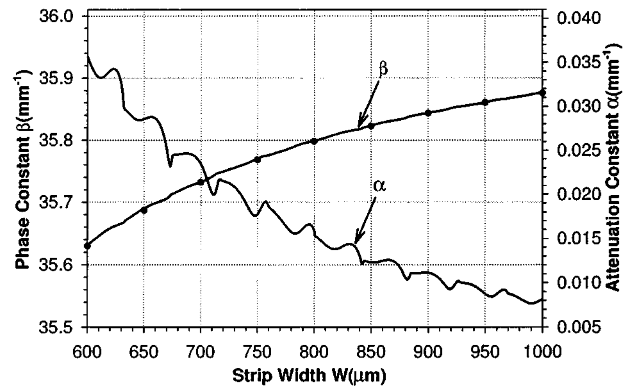

Fig. 6. Behavior of the phase and attenuation constants with respect to the strip width for the first FVMS mode in a normally magnetized microstrip line. Structure parameters: $h_1 = 400$ μm, $h_f = 20$ μm, $h_2 \to \infty$, $4\pi M_s = 1750$ G, $H_0 = 570$ Oe, $f = 2.5$ GHz. The dots show the result obtained using the model in [4].

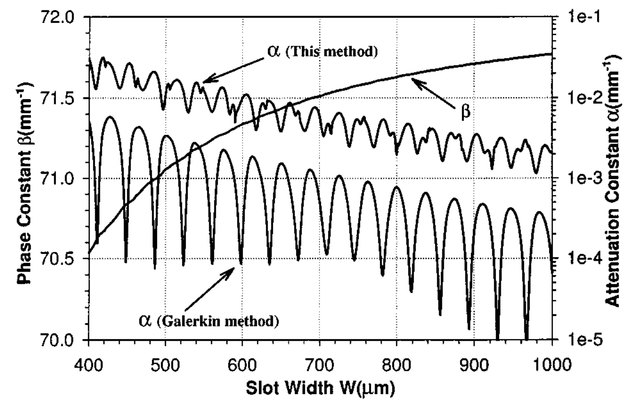

Fig. 7. Behavior of the phase and attenuation constants with respect to the slot width for the first FVMS mode in a normally magnetized slot line. Structure parameters: $h_1 = 400$ μm, $h_f = 20$ μm, $h_2 \to \infty$, $4\pi M_s = 1750$ G, $H_0 = 570$ Oe, $f = 2.5$ GHz. The results obtained using Galerkin's method with weighted Chebyshev polynomials as basis functions are also shown (dotted lines).

The dependence of microstrip/slot-line phase constant and losses with respect to the width of the strip/slot is shown in Figs. 6 and 7 (in Fig. 6, the results for the phase constant obtained by using the magnetic wall model of [4] are also shown). It can be seen that losses substantially decrease for relatively wide strips or slots, as was reported in [9]. Ripples in the attenuation constant, similar to those shown in Figs. 4 and 5, also appear in Figs. 6 and 7, being of relatively higher amplitude for the slot line. These ripples are similar to those reported in [17] for dielectric leaky waveguides, also having the same explanation. For instance, in the slot-line example of Fig. 7, a quasi-periodicity with a period of $\Delta w \approx 30$ μm in the ripples may be observed. This quasi-period coincides with the theoretical value obtained from the following equation (similar to [17, eq. (2)]):

$$\Delta w \simeq \frac{2\pi}{\Re(k_{x,2})} = 30\ \mu\text{m} \tag{21}$$

where $k_{x,2}$ is the x-component of the wave vector corresponding to the second FVMS mode *inside* the slot. Following the theory in [17], this mode will be "bouncing back and forth above cutoff" inside the slot and, at some values of w [which are recurrent with the period given by (21)], the leakage is almost cancelled by the destructive interference of this resonant wave at the slot edges. However, unlike the structures analyzed in [17], the resonant cancellation of the leakage is not reduced here to the second FVMS mode inside the slot. The third and higher order FVMS modes (with higher values of $k_{x,n}$) also produces resonant cancellation of the leakage. This effect appears in Fig. 7 as additional ripples (superimposed to the main sequence) in the plot of the attenuation constant. For the microstrip line, the resonant cancellation of the attenuation constant can be observed in Fig. 6. In this case, the period Δw is approximately given by

$$\Delta w \simeq \frac{2\pi}{\Re(K_{x,2})} = 34.5\ \mu\text{m} \tag{22}$$

where $K_{x,2}$ is the x-component of the wave vector corresponding to the second FVMS mode *inside* the strip. However, since the main part of the radiation losses in the microstrip line is carried out by the first FVMS mode, which does not resonate inside the strip, the amplitude of the attenuation constant ripples is less significant in microstrip lines than slot lines.

A general conclusion that may be deduced from (21) and (22), as well as from Figs. 4–7, is that radiation losses in microstrip/slot lines decrease, in average, with the dimensional parameter $\gamma_2 w$, where $\gamma_2 = K_2(k_2)$ is the eigenvalue of the second FVMS mode inside the strip (slot).

Another important conclusion of our numerical analysis is that the widely used Galerkin's method (at least in its standard versions) is not found to be very adequate for analyzing the proposed structures. The results obtained for the slot-line attenuation constant using Galerkin's method in the spectral domain with 13 weighted Chebyshev polynomials as basis functions are shown in Fig. 7. The results for the attenuation constant neither agree quantitatively, nor show the qualitative behavior expected from the theory of leaky waves [17]. They show a quasi-periodical behavior, but the measured quasi-period $\Delta w \approx 37\ \mu$m does not correspond to any resonant mode in the slot. Similar results are obtained when subsectional basis functions (∼50) are used. The explanation of this failure lies on the resonant behavior of fields inside the slot. Since the real parts of both the $\{k_{x,n}\}$ and $\{K_{x,n}\}$ sets sharply increase with n, the fields inside the strip or the slot are strongly oscillatory functions of x. Therefore, these fields (or any related magnitude) are not expected to be properly fitted by using a reasonable number of basis functions of general purpose, such as the subsectional or the Chebyshev polynomial expansions (indeed, since these basis functions form a complete set, fitting would be achieved if a large enough number of functions were used, but this number would be so large that computations would become endless or unpractical). Notice that this situation never happens in the analysis of the propagation of electromagnetic (not MS) waves along microstrip or slot lines. In this case, the field expansion inside the strip/slot seldom includes more than one or two resonant modes. The same is true for the bound MS modes analyzed in [5] and [6], where the use of the spectral-domain Galerkin's method was appropriate.

IV. Conclusions

The propagation of FVMS modes along microstrip and slot lines with normal magnetization has been analyzed. The method of analysis is based on the RCT and modal decomposition of the fields inside and outside the strip/slot region. It has been shown that both the microstrip line and slot line do not support bound FVMS modes, but a multiplicity of *leaky* FVMS modes having values of the phase constant that increase with frequency. The numerical analysis of the structures has shown that radiation losses are significantly smaller in slot lines than in microstrip lines. It has also been found that radiation losses decrease, in average, with the strip/slot width, in agreement with previous results. Radiation losses also decrease, in average, as frequency increases. In addition, the attenuation constant shows quasi-periodical oscillations as a result of the transverse resonances of the metallized/unmetallized ferrite-slab modes *inside* the strip/slot. These resonances make the attempts of expanding the fields in basis functions of general purpose, such as subsectional triangles or Chebyshev polynomials, inaccurate, thus precluding the application of Galerkin's method or MoMs to the analysis of this type of structures.

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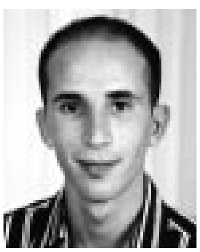
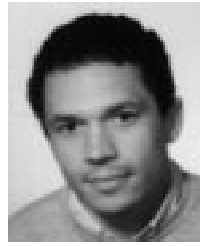
Ricardo Marqués (M’95) was born in San Fernando, Cádiz, Spain. He received the Licenciado and Doctor degrees from the University of Seville, Seville, Spain, in 1983 and 1987, respectively, both in physics.

Since 1984, he has been with the Department of Electronics and Electromagnetism, University of Seville, Spain, where he is currently an Associate Professor. His main fields of interest include computer-aided design (CAD) for microwave integrated-circuit (MIC) devices, wave propagation in ferrites, and other complex anisotropic and/or bi-isotropic media and field theory.



Rachid Rafii-El-Idrissi was born in Meknes, Morocco, in July 1971. He received the Licenciado degree in physics from the University of Meknes, Meknes, Morocco, in 1995, and is currently working toward the Ph.D. degree in physics at the University of Seville, Seville, Spain.

He is currently with the Microwave Group, University of Seville, His research interests involves wave propagation in MIC ferrite devices.



Francisco Mesa (M’94) was born in Cádiz, Spain, on April 1965. He received the Licenciado and Doctor degrees from the University of Seville, Seville, Spain, in 1989 and 1991, respectively, both in physics.

He is currently an Associate Professor in the Department of Applied Physics I, University of Seville. His research interest focuses on electromagnetic propagation/radiation in planar lines with general anisotropic materials.

Francisco Medina (M’90–SM’01) was born in Puerto Real, Cádiz, Spain, in November 1960. He received the Licenciado and Doctor degrees from the University of Seville, Seville, Spain, in 1983 and 1987, respectively, both in physics.

From 1986 to 1987, he spent the academic year with the Laboratoire de Microondes de l’ENSEEIHT, Toulouse, France. From 1985 to 1989, he was a Profesor Ayudante (Assistant Professor) with the Department of Electronics and Electromagnetism, University of Seville, and since 1990, he has been a Profesor Titular (Associate Professor) of electromagnetism. He is also currently Head of the Microwaves Group, University of Seville. His research interest includes analytical and numerical methods for guidance, resonant and radiating structures, passive planar circuits, and the influence on these circuits of anisotropic materials.

Dr. Medina was a member of both the Technical Program Committee (TPC) of the 23rd European Microwave Conference, Madrid, Spain, 1993, and the TPC of ISRAMT’99, Malaga, Spain. He is on the Editorial Board of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES. He has been a reviewer for other IEEE and Institution of Electrical Engineers (IEE), U.K., publications.