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IFAC-PapersOnLine 49-28 (2016) 250–255

Case Study based on Inequality Indices for the Assessments of Industrial Fleets

V. González-Prida*, M. Orchard, C. Martín***, A. Guillén*, J. Shambhu****, S. Shariff*******

** University of Seville, Spain (vicente.gonzalezprida@gdels.com; ajguillen@us.es)*
*** University of Santiago de Chile, Chile (morchard@u.uchile.cl)*
**** University of Toulouse, France (Carmen.Martin@enit.fr)*
***** University of Stavanger, Norway (shambucet@gmail.com)*
****** University of Taibah, Saudi Arabia (samshariff@yahoo.com)*

Abstract: This contribution illustrates the advantages of measures and indicators based on the notions of Shannon entropy which is widely implemented in thermodynamics, information theory, econometrics, and biology. The presented case study applies these indicators to a fleet of industrial assets, introducing innovatively the entropy concept to the field of availability. There is no reason to deal the effect of performance variables in assets independently without taking into account disparities in terms of reliability, operation hours, applied maintenance, operating conditions, and usage profiles, etc. Therefore, the characteristic of the method and their independence from the heterogeneity of the sample that compares is intended to be promising for applications in availability analysis. In other words, this research presents a derivation from well-known concepts such as the Gini, Hoover and Theil indices illustrating their application by the support of an example where attributes for different groups of assets are compared. To illustrate the application we are using two relevant and innovative scenarios as case study examples.

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Keywords: Asset and maintenance management; Reliability, Statistical Approaches; Diagnostics, Prognostics, Reasoning, Decision Support.

1. INTRODUCTION

From the notion of Shannon entropy, this contribution presents an application of indices for inequality measures (Gini, Hoover, and Theil index) for the analysis of the availability in fleets of industrial assets. This is in order to make possible comparisons among fleets operating under different boundary conditions and usage profiles. The obtained indices refer to that portion of the total group of assets that has to be redistributed in order to obtain a 100% equal fleet, in terms of availability and performance. The given approach and application of the entropy concept to the field of availability is innovative. Particularly the characteristics of the method and their independence from the heterogeneity of the sample that compares, is intended to be promising for applications in availability analysis. The application, proposed with the help of two scenarios, is also relevant in this new area of study and tries to illustrate this development.

It is assumed that components operate individually. However their availability has certain dependency with the associated components since they belong to the same system. This aspect is clarified throughout this paper. With that aim, the paper presents how these indicators can be implemented for the analysis of the life cycle of an industrial asset. The theoretical development of such methodology (including Shannon entropy, application of in-dices, and all these keywords) is developed in reference to Gonzalez-Prida et al 2016. A summary of formulae are shown here below:

Shannon Entropy:

$$H=\sum_{i=1}^{n}\left(-\frac{(UA_i/p_i)N_i}{\sum_{i=1}^{n}((UA_i/p_i)N_i)}\ln\left[\frac{(UA_i/p_i)N_i}{\sum_{i=1}^{n}((UA_i/p_i)N_i)}\right]\right)$$

Gini Index:

$$G=1-\frac{(\sum_{i=1}^{n}G_i)/(\sum_{i=1}^{n}N_i)}{\sum_{i=1}^{n}(UA_i/p_i)}$$

Hoover Index:

$$Hv=(1/2)\sum_{i=1}^{n}\left|\frac{(UA_i/p_i)N_i}{\sum_{i=1}^{n}((UA_i/p_i)N_i)}-N_i/(\sum_{i=1}^{n}N_i)\right|$$

Theil Index:

$$T_{sym}=(1/2)\sum_{i=1}^{n}\left[\left(\frac{(UA_i/p_i)N_i}{\sum_{i=1}^{n}((UA_i/p_i)N_i)}-N_i/(\sum_{i=1}^{n}N_i)\right)\ln\left(\left(\frac{UA_i/p_i}{\sum_{i=1}^{n}(UA_i/p_i)}\right)/(N_i/(\sum_{i=1}^{n}N_i))\right)\right]$$

These concepts are implemented in a practical case study, referring to the comparison of two scenarios (A and B). Nevertheless, this situation can be used in various applications including different phases of wind farms where the severity of the environment is different from one phase to

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.11.043

another; exportations of products from emerging countries to developed countries competing with the local manufacturing, etc. After the case study, the paper concludes with some interesting future research possibilities.

2. CASE STUDY

Nobel Prize winner Akerlof G. indicated that, in a second-hand market, customers do not really know exactly if the relationship between quality and price is fair when they are buying a used car (Akerlof G, 1970). Uncertainty due to this asymmetric information, causes the sellers of high quality cars to be positioned out of the market, since owners of cars with lower quality have the same opportunities to sell their vehicles at a high enough price. The proposed approach is intended to com-pare different fleets of industrial assets, where scenarios have different attributes (e.g., productivity and unavailability). Nevertheless, it could be useful to compare second-hand car markets. Comparing inequality indices enables to detect what scenario present more equality in the items' behaviour of its population. The higher is the equality, easier the foreseeing of how will be the remaining useful life of the physical assets in operation and, as a consequence, the worthwhileness of one fleet over the other one. In order to provide a practical application of the above mentioned theoretical subjects, we proceed now to synthesize an example that shows the utility of the described inequality indices, with a case study related to a fleet of complex industrial assets with different behaviour and degradation along their lifetime.

2.1 Study Scenario

Let us consider two different scenarios (A and B). These scenarios can relate to, for example, specific geographic areas or different fleets from the same company. Each scenario will include a fleet of vehicles with specific settings and configuration, which are assumed to be equal for all of them. However, the age of the vehicles will be ranging from 1 to 10 years old. Vehicles from a specific age (1 to 10) and scenario (A or B), will present: different population (Ni); different productivity (pi); different unavailability (UAi). As commented, both pi and UAi, are variables that represent characteristics of a population Ni of vehicles of the same age (i).

Table 1. Amount, unavailability and productivity per year and scenario

i (age)	Ni (A)	Ni (B)	UAi (A)	UAi (B)	pi (A)	pi (B)
1	800	3.500	0000000480	0000000400	24.0	20.0
2	850	4.250	0000006010	0000005893	17.0	16.8
3	975	4.750	0000015988	0000014677	14.6	11.5
4	1.000	5.195	0000025971	0000023356	13.4	10.0
5	1.033	6.000	0000034030	0000031018	12.6	9.0
6	1.050	6.100	0000041912	0000037601	12.0	8.0
7	1.075	7.000	0000047880	0000043243	11.6	7.5
8	1.100	8.000	0000053999	0000048107	11.3	7.0
9	1.125	8.500	0000058487	0000052335	11.2	6.6
10	1.125	9.000	0000062424	0000056044	11.1	6.3

2.2 Conditions and objective

Table 1 gathers data from the two scenarios (A and B). Scenario A, in comparison to Scenario B, assumes vehicles with productivity that remains higher as they age, although they become less available. It is assumed that the degradation trend for each vehicle, for same age and scenario, is similar year after year. The question here is how to choose the group of assets with the best availability-productivity relationship. In particular, we can observe that Scenario A has a lower truck population than Scenario B (Fig. 1).

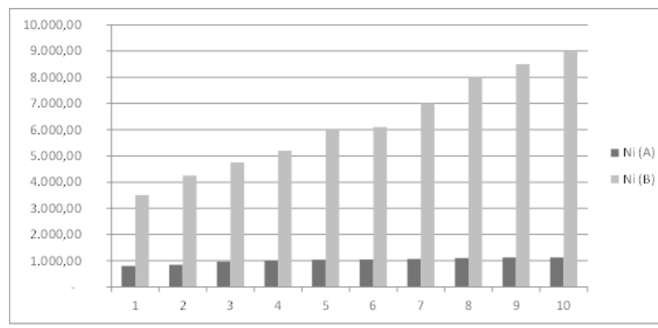

Fig 1. Amount of assets in operation per year for both scenarios

Nevertheless, due to the properties of Shannon entropy, the model is also applicable even when the scale changes. In other words, it is applicable even with different sizes of fleets. Therefore, the indices we are going to obtain are comparable. In figure 1, the abscissa refers to the year and the ordinate refers to the amount of trucks in use. In addition to this, we observe that the asset operation is different from one scenario to another; since the unavailability follows different trends (Fig. 2).

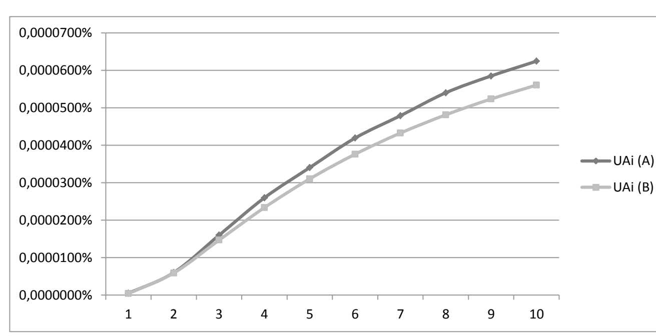

Fig. 2. Unavailability per age and scenario (%)

These differences provide us the notion of diverse usage profile or maintenance policies in both scenarios. In other words, over time, if the usage profile is similar in both contexts, then Scenario A applies a worse maintenance policy than Scenario B. Finally, comparing productivity, such characteristic in Scenario A is higher than in Scenario B and, in both cases, it rapidly decreases (Fig. 3).

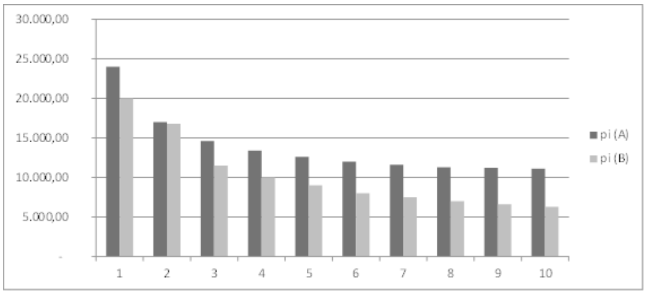

Fig. 3. Productivity per assets age and scenario (€)

2.3 Diversity indices applied to the practical case ns

Applying the parameters defined in Section 1 to the data from figures 1 to 3, it is possible to obtain the following values for each scenario (see Table 2 and Table 3):

Table 2. Relative and accumulated values for parameter of scenario A

#	Ni (relative)	Ni (accumulated)	UAi / pi	UAi / pi (relative)	UAi / pi (accumulated)	(UAi / pi) Ni (relative)	(UAi / pi) Ni (accumulated)
1	7.89%	7.89%	1.6E-10	0.05%	0.05%	0.07%	0.07%
2	8.39%	16.28%	3.0052E-09	0.95%	1.00%	1.20%	1.27%
3	9.62%	25.91%	1.0677E-08	3.37%	4.37%	3.73%	5.00%
4	9.87%	35.77%	1.9381E-08	6.12%	10.49%	6.60%	11.61%
5	10.19%	45.97%	2.78993E-08	8.81%	19.30%	9.20%	20.81%
6	10.36%	56.33%	3.66734E-08	11.58%	30.87%	11.90%	32.71%
7	10.61%	66.94%	4.43712E-08	14.01%	44.88%	14.06%	46.77%
8	10.86%	77.80%	5.25655E-08	16.60%	61.48%	16.28%	63.05%
9	11.10%	88.90%	5.87481E-08	18.55%	80.03%	17.79%	80.84%
10	11.10%	100.0%	6.32673E-08	19.97%	100.00%	19.16%	100.00%

Table 3. Relative and accumulated values for parameter of scenario B.

A-ge	Ni (relative)	Ni (accumulated)	UAi / pi	UAi / pi (relative)	UAi / pi (accumulated)	(UAi / pi) Ni (relative)	(UAi / pi) Ni (accumulated)
1	5.62%	5.62%	7E-10	0.02%	0.02%	0.05%	0.05%
2	6.82%	12.44%	1.49068E-08	0.48%	0.50%	0.84%	0.89%
3	7.63%	20.07%	6.06236E-08	1.94%	2.44%	3.07%	3.96%
4	8.34%	28.41%	1.21335E-07	3.89%	6.33%	5.62%	9.58%
5	9.63%	38.04%	2.06788E-07	6.63%	12.96%	8.29%	17.86%
6	9.79%	47.83%	2.86705E-07	9.19%	22.16%	11.30%	29.16%
7	11.24%	59.07%	4.03597E-07	12.94%	35.10%	13.86%	43.02%
8	12.84%	71.91%	5.4979E-07	17.63%	52.72%	16.52%	59.55%
9	13.64%	85.55%	6.74014E-07	21.61%	74.33%	19.06%	78.61%
10	14.45%	100.00%	8.00627E-07	25.67%	100.0%	21.39%	100.00%

Hence, we can obtain the different indices applying the corresponding formula, already expressed in Section 1. Particularly, indices for scenario A are here below (see Table 4):

Table 4. Diversity indices for scenario A.

Age	Gini	Hoover	Theil (sym)	Tt	Tl	Shannon H
1	1.6000E-10	7.84%	39.63%	-0.0026	0.3988	0.38%
2	3.3452E-09	7.44%	16.21%	-0.0207	0.1828	4.42%
3	1.7961E-08	6.25%	6.56%	-0.0354	0.1009	11.43%
4	4.8754E-08	3.75%	1.79%	-0.0292	0.0472	17.09%
5	9.8282E-08	1.39%	0.20%	-0.0129	0.0149	21.40%
6	1.6493E-07	1.22%	0.13%	0.0128	-0.0115	24.96%
7	2.5078E-07	3.40%	0.94%	0.0389	-0.0295	27.53%
8	3.5458E-07	5.74%	2.44%	0.0704	-0.0461	29.81%
9	4.7514E-07	7.44%	3.82%	0.0952	-0.0570	31.25%
10	5.9716E-07	8.87%	5.21%	0.1173	-0.0652	32.17%

Similarly for Scenario B (see Table 5):

Table 5. Diversity indices for scenario B.

Age	Gini	Hoover	Theil (sym)	Tt	Tl	Shannon H
1	7.0000E-10	5.60%	30.91%	-0.0012	0.3103	0.19%
2	1.6607E-08	6.34%	16.58%	-0.0127	0.1814	2.55%
3	9.5845E-08	5.68%	6.46%	-0.0266	0.1042	7.66%
4	2.9246E-07	4.45%	1.22%	-0.0297	0.0636	12.63%
5	6.8471E-07	3.00%	-0.89%	-0.0248	0.0360	17.99%
6	1.1931E-06	0.60%	-0.49%	-0.0058	0.0062	21.94%
7	2.1017E-06	1.70%	1.94%	0.0183	-0.0159	26.46%
8	3.4130E-06	4.78%	6.76%	0.0558	-0.0407	30.60%
9	4.8844E-06	7.96%	13.50%	0.0994	-0.0627	33.11%
10	6.6861E-06	11.22%	21.71%	0.1475	-0.0830	34.91%

Regarding the Gini index, the value of this indicator is between zero and one, increasing inequality as the index approaches the upper bound. This measure also allows a geometric interpretation in terms of the Lorenz concentration curve (is twice the area between the concentration curve and the line of equal distribution) (Gastwirth J.L. 1972).

Regarding the Hoover index, its value refers to the proportion of a particular characteristic (UAi / pi), which would have to be redistributed taken from the richest part of the population to the poorest part, in order to ensure uniformity in the fleet.

Regarding the Theil index, if there is a greater disparity between the considered measure (UAi / pi), the Theil index value increases, but in any case appears limited by the covariance between relative measure and its logarithm. The introduction of a compensating logarithm function provides a better view of the problem in intuitive terms.

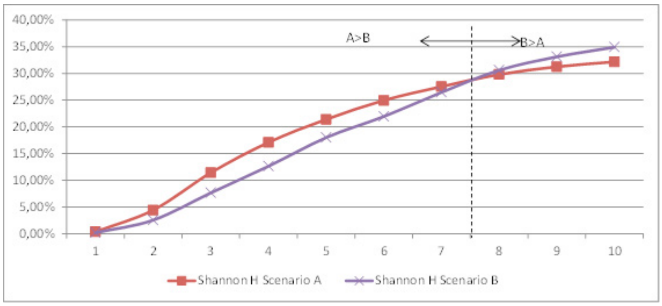

Fig. 4. Shannon Entropy for Scenarios A and B.

Regarding the Shannon entropy, it measures the specific diversity (or inequality). This entropy index (H) is expressed as a positive number, so that higher the value of H, higher the diversity (or inequality). In above graph (Fig. 4), the curve that has higher entropy index presents a higher inequality degree for assets at that specific age. Therefore, the relationship availability-productivity is generally worse respecting the other fleet of assets.

2.4 Results discussion

In the example, when comparing two fleets, during the specific period of time in which the calculation is made, the fleet A has a higher Hoover index than fleet B (26.67%> 25.67%). This means, that the application of standard maintenance policies into more homogeneous fleets (i.e., lower Hoover index) will be more efficient than when they are applied into more diverse fleets (higher Hoover index). The ratio indicates which scenario is more homogeneous in general terms throughout the whole period. On the other hand, illustrating the Shannon Entropy results in a graph, two curves are obtained as it is observed in Figure 4. The figure shows a change point for the trend of Shannon Entropy comparing the two fleets. It may help identifying an improvement in the maintenance policy on one fleet compared to the other one. Further studies may define maintenance strategies for both fleet (i.e. systematic preventive maintenance vs. a conditional or predictive maintenance for instance). The best choice considering availability and productivity per vehicle and age is precisely in the lowest point of the curves. In other words, when comparing two fleets according to their Shannon Entropy, scenario A is more diverse (has more inequalities) than scenario B (which is more homogeneous) until the seventh year when the situation is reversed. That means, that the application of standard maintenance policies into more homogeneous fleets (i.e., with lower Shannon Entropy) will be more efficient than when applied in most diverse fleets (higher Shannon Entropy). The ratio indicates which scenario is more homogeneous at a given time. Of course, in a real case, the degradation degree can be very different considering various usage profile throughout the same fleet. Therefore, there may be groups of assets with accelerated degradation compared to the rest of the fleet. Other factors apart from the aging (e.g., the mentioned usage profile) shall be taken into ac-count in further research as additional reasons for vehicle degradation and as an additional indicator to estimate the remaining life of the physical asset.

3. ADVANTAGES AND DISADVANTAGES

The applied set of indicators allows describing the evolution of some asset attributes, from the point of view of grouping management. For analysis of availability, there are various indices/measures other than Shannon entropy, like as per SMRP and also by European Maintenance Assessment Committee KPIs. Nevertheless, in this case, the assessment by inequality indices enables the comparison of groups according to established criteria as availability and productivity, but also other possible specific features as reliability, costs, performance etc. (Goepel K.D. 2013). This aspect makes Shannon entropy a better than other established KPIs that do not consider this grouped point of view. The use of these kind of indices related to fleets reduces subjectivity and provide reasonable values, easy to be manipulated. The methodology allows considering different factors for each asset group, according to the characteristics of the fleet and its operation. It is also very useful to represent these results in a graphical way, with the measurement for each group and its evolution over time. This simple process may help to observe the gradual change along different stages of an item/fleet life cycle. For this purpose, the Theil index provides an interesting advantage (compared to Gini index) as it can explain relative differences be-tween groups of the same fleet. As it is developed in references like Fields G.S. 1980 or Holsinger & Jacob 2010, one of the advantages of the Theil index is the fact that it is a weighted average of inequality within fleets, plus inequality among these fleets. In our case study, it is possible to try with other parameters to examine whether similar conclusions can be obtained. With approach of our case study, one can develop a hybrid measure to directly combine the productivity with the unavailability without the need to use information indices. Therefore, a problem-based (rather than method-based) approach would be preferred for each specific case study to solve. According to this, it is important to emphasize that any indicator could be prioritized as a random variable and, therefore, the concept of entropy could be applied at a broad sense in many other situations (for example ergodicity cases). In other words, one of the theoretical problems here is that entropy seeks to characterize the information of a random variable depending on its distribution, but it cannot prove that there are realizations of the same distribution in every case. This is the disadvantage of applying it to a statistical problem, a method designed to work in a probabilistic environment. In short, it must be understood that the presented case assumes that one can get samples of UAi/pi indicator, which comes from an independent, identically distributed process, when actually this is a non-testable fact. For a given age, it is a crucial issue how to specify UAi. This issue needs particular attention as far as an appropriate quality measure and the availability of its information may be the key of the problem (Jiang R. 2013).UAi/pi would be meaningful to use in this cases. In addition to this and according to the applied indices, variance is similar to the entropy, but actually, variance as a simple measure may be better if it can provide a reasonable and intuitive interpretation (Jiang R. & Murthy DNP. 2013).

4. FUTURE RESEARCH

This case study is just an approach with very few parameters, because we are considering an ex-ample where all assets tend to degrade over time in the same way. In other words, trucks have similar use-profile and run in similar contexts over time. However, different versions of the same problem can be analysed, just changing three conditions in reference to the age: the amount of units, the unavailability, and the productivity. It could be possible to consider different operating scenarios for different models of assets throughout their life cycle. Another application can be considering complex industrial assets as wind turbines (instead of vehicles), connected in farms for energy production, where the different farms (as our fleets in the case study) work under different boundary conditions with different degradations over time. In addition to this, other diversity indices can be applied in maintainability assessment or in Prognostics and Health Management (PHM). There are other indices with their own strengths and weaknesses. Therefore, scientists from other areas (biology, physics or econometrics) often use a combination of several indices to take advantages of the strengths of each one of them and develop a more complete understanding of a community structure. In the context of a case based reasoning, it is possible to consider other cross-related areas like the maintainability assessment in the scope (for example) of CRM or AHP. It is important here to add that PHM is a field with plenty of possibilities on further related research lines (Scanff E. et Al. 2007; Izraeli, O. 2003; Roemer M.J. et Al. 2001), as far as indicators and attributes assessment developed for maintenance (Moreu et. Al 2012) can be applied in a similar way as an aid tool for predicting the RUL of critical industrial assets. Apart from the above future researches, other contributions can add a deeper description of the well-known indices (Gini, Hoover, Theil), specifically when they are focused to other areas, illustrating the concept behind the formulas and explaining how they can be applied in different areas than the usual ones.

5. CONCLUSIONS

We have briefly depicted the Shannon entropy, as well as inequality indices such as the Gini, Hoover, and Theil index. They are successfully applied in other subject areas as thermodynamics, statistics, biology or econometrics. This paper has intended just to approach such concepts to the physical as-sets management, in order to find practical and useful applications. Particularly, one of the possible applications has been the comparison of two fleets of complex assets with different behaviour despite the fleets being constituted by the same asset configuration. The unavailability-productivity measure adopted in this case study has been (UAi / pi). This measure decreases if the unavailability is lower and the productivity is higher, and vice-versa. That means that the operation and maintenance of one specific group of assets is more profitable and efficient. This comparison allows us to determine:

- A better maintenance policy for the rest of the remaining useful life of the items,
- A better maintenance policy for those components that are foreseen to operate under the same environment and/or usage profile.
- A more efficient use of assets from one fleet in comparison to another, etc.

These indices are recommended to be used together with others already known in PHM as a complement for a better judgement. As future re-search, it is proposed to develop more elaborate in-dices that simplify the choice of preferences from individual or fleet attributes. These new indices can be translated from other fields or by the application of a conjoint analysis.

REFERENCES

Akerlof G.A. (1970). The Market for 'Lemons: Quality Un-certainty and the Market Mechanism. *Quarterly Journal of Economics*. The MIT Press, 84 (3): 488–500

Fields G.S. (1980). *Poverty, Inequality, and Development*. Cambridge University Press. ISBN: 0 521 22572 8, 0 521 29852 0.

Gastwirth J.L. (1972). The Estimation of the Lorenz Curve and Gini Index. *The Review of Economics and Statistics*. MIT Press. 54 (3): 306-316.

Gini, C. (1936). *On the Measure of Concentration with Special Reference to Income and Statistics*. Colorado College Publication, General Series. 208:73–79.

Goepel K.D. (2013). *Business Performance Management Singapore*, http://bpmsg.com.

Gonzalez-Prida V., Orchard M., Martin C., Crespo A., Soto A. (2016). Inequality Indices Based on the Notion of Shannon-Entropy for the Assessments of Industrial Fleets. *Proceedings of the 10th World Congress on Engineering Asset Management* (WCEAM 2015). DOI 10.1007/978-3-319-27064-7_18

Holsinger D.B., Jacob W.J. (2010). *Inequality in Education: Comparative and International Perspectives*. Springer. ISBN 9048185076, 9789048185078

Izraeli, O., Murphy, K.J. (2003) The effect of industrial diversity on state unemployment rate and per capita income. *The Annals of Regional Science* 37, pp. 1-14

Jiang R. (2013). Two Methods to Estimate the Change Points of a Bathtub Curve. *International Journal of Performability Engineering*. 9 (5): 569-579.

Jiang R., Murthy DNP. (2009). Impact of quality variations on product reliability. *Reliability Engineering & System Safety*. 94(2): 490-496

Moreu P., Gonzalez-Prida V., Barbera L., Crespo A. (2012). A practical method for the maintainability assessment in industrial devices using indicators and specific attributes. *Reliability Engineering and System Safety* 100 (2012) 84–92.

Roemer M.J., Kacprzynski G.J., Orsagh R. F. (2001). Assessment of Data and Knowledge Fusion Strategies for Prognostics and Health Management. *Proceedings of IEEE* 2001

Scanff E., Feldman K.L., Ghelam S., Sandborn P., Glade M., Foucher B. (2007). Life cycle cost impact of using

prognostic health management (PHM) for helicopter avionics. *Microelectronics Reliability* 47: 1857–1864
Shannon C.E (1948). A Mathematical Theory of Communication. *Bell System Technical Journal*. Vol. 27.
Theil H. (1967). *Economics and Information Theory*. North Holland
Theil H. (1972). *Statistical Decomposition Analysis*. North Holland.