Extraordinary Transmission Through Arrays of Electrically Small Holes From a Circuit Theory Perspective

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***Abstract*—Extraordinary optical transmission of light or electromagnetic waves through metal plates periodically perforated with subwavelength holes has been exhaustively analyzed in the last ten years. The study of this phenomenon has attracted the attention of many scientists working in the fields of optics and condensed matter physics. This confluence of scientists has given rise to different theories, some of them controversial. The first theoretical explanation was based on the excitation of surface plasmons along the metal–air interfaces. However, since periodically perforated dielectric (and perfect conductor) slabs also exhibit extraordinary transmission, diffraction by a periodic array of scatterers was later considered as the underlying physical phenomenon. From a microwave engineering point of view, periodic structures exhibiting extraordinary optical transmission are very closely related to frequency-selective surfaces. In this paper, we use simple concepts from the theory of frequency-selective surfaces, waveguides, and transmission lines to explain extraordinary transmission for both thin and thick periodically perforated perfect conductor screens. It will be shown that a simple transmission-line equivalent circuit satisfactorily accounts for extraordinary transmission, explaining all of the details of the observed transmission spectra, and easily gives predictions on many features of the phenomenon. Although the equivalent circuit is developed for perfect conductor screens, its extension to dielectric perforated slabs and/or penetrable conductors at optical frequencies is almost straightforward. Our circuit model also predicts extraordinary transmission in nonperiodic systems for which this phenomenon has not yet been reported.**

***Index Terms*—Extraordinary transmission, frequency-selective surfaces (FSSs), surface plasmon polaritons.**

I. Introduction

PARTIAL transparency of opaque slabs (metal slabs) periodically perforated with electrically small holes was reported some years ago by Ebbesen *et al.* [1] (see also the popular article [2] in the same issue). This phenomenon, in apparent contradiction with Bethe's theory for small apertures [3], was called *extraordinary optical transmission*. Since this seminal work, hundreds of scientific papers have been published giving explanations and reporting details about this (or related) phenomenon. The phenomenon refers to the appearance, around a certain frequency, of a narrow and strong peak of transmission through an opaque screen perforated with small holes. The surprising fact was that the diameters of the holes were significantly smaller than the corresponding wavelength (Bethe's theory for small holes predicted much less transmitted power than observed). The cylindrical holes of the original experimental device were arranged into a two-dimensional (2-D) periodic square lattice whose unit cell had dimensions close to the wavelength of the "extraordinary" transmitted beam. This key feature strongly suggests that *periodicity* should play a crucial role in the phenomenon. However, the first theoretical explanations relied basically on the behavior of metals at optical frequencies. At those high frequencies, metals are described by a complex permittivity with a large negative real part (plasma behavior). Metals are thus penetrable materials that can support a special kind of surface waves, the so-called surface plasmons. The excitation of such waves due to the scattering of the impinging planar transverse electromagnetic (TEM) wave by the periodic structure was then assumed to be the physical fact behind extraordinary transmission (see [4]–[6], among others). In this interpretation, apart from periodicity, the behavior of the metal as an imperfect conductor (more precisely, as a *lossy solid plasma*) seems to be essential to the phenomenon. Nevertheless, extraordinary transmission has also been found in metal structures at millimeter-wave frequencies (see, for instance, the papers by Beruete *et al.* [7], [8]). At these frequencies, metals are described by a real conductivity and penetration of electromagnetic fields (skin effect) is marginal. In this situation, surface plasmons are not supported by the metal–air interfaces. Moreover, enhanced transmission of electromagnetic waves has also been reported in periodically perforated perfect dielectric slabs [9], [10]. Genuine surface plasmons (i.e., surface waves supported by a uniform metal–air interface at optical frequencies) do not appear to be always required to explain extraordinary transmission phenomena, although they can still play some role in modifying the frequency value at which the transmission peak is expected to occur.

Fortunately, all of the above facts can be explained by means of full-wave diffraction models, which account for both propagating and evanescent fields around the periodic structure. The diffraction model was first used for one-dimensional (1-D) periodic arrays of infinitely long slits [11], [12] (diffraction gratings). However, the slits problem is slightly different from the 2-D array of holes. TEM modes without cutoff frequency

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are possible in the slit structure but not in the 2-D array of holes. Thus, the experimental situation treated in [1] is better accounted for by the diffraction models specifically developed for 2-D arrays of subwavelength holes [6], [9], [10], [13], [14]. In these latter models, periodicity, diffraction, and interference are the relevant concepts while surface plasmons would play a secondary role modulating the main physical reason behind extraordinary optical transmission. Nevertheless, the concept of surface plasmon was rescued in this context by Pendry *et al.* in [15]. Although the model in [15] is not accurate (see, for instance, [16]), its underlying qualitative idea is worthy: periodically structured perfect conductor surfaces can support surface waves that mimic surface plasmons (some researchers call these waves *spoof* plasmons). A comprehensive review of this type of wave for the 1-D periodic case (slits on a metal surface) was given previously in [17]. Nevertheless, to the best of our knowledge, these are the same type of surface waves supported by perfectly conducting periodic structures that are well known by the microwave community since the 1950s or even before. In order to give proper credit to pioneering works, it should be mentioned that surface modes guided by open corrugated surfaces were reported in a classified memorandum by Cutler in the 1940s [18]; see also the historical review paper by the same author in [19]. (A corrugated surface is basically the same type of electromagnetic system as that involved in extraordinary optical transmission.) A thorough analysis of corrugated surfaces based on the Floquet–Lucke method was reported as early as 1954 [20], and an in-depth experimental study of various periodic open waveguides was carried out in [21]. The similarity between the model proposed in the recent paper by Pendry *et al.* [15] for 2-D structured surfaces and the model suggested in a note published almost 50 years ago by Goldstone and Oliner [22] for 1-D structured surfaces is notorious. Many more antecedents could be given, although it is enough to mention that the well-known classical textbook by Collin [23] includes the topic in the chapter devoted to surface waves.

Despite being a case of "rediscovering," the analysis of extraordinary optical transmission in terms of the coupling of the impinging TEM wave to *spoof* plasmons is actually appealing (see the excellent review papers by Genet *et al.* [24] and García de Abajo [25]). Nevertheless, in our opinion, the theory based on surface plasmons is not easy to use since its predictions are basically attained in the form of numerical solutions to very intensive computational problems. Moreover, there are still some unclear points as well as some situations that this model cannot explain (for instance, situations where extraordinary transmission is possible and plasmons—including spoof plasmons—are not present). These drawbacks have been the motivation of the present work. Our proposal in this paper is to provide a much simpler perspective and theory, at least for those familiar with microwave field and circuit theories, founded on *waveguide and impedance matching concepts*. A preliminary work based on these ideas was reported by the authors in [26]. Our present paper will extend considerably the above work and will present detailed explanations for more general problems. In particular, we will show how relatively simple "textbook" waveguide theory concepts provide a complete account of the observed extraordinary transmission phenomena in a wide variety of situations. Moreover, some new systems exhibiting extraordinary transmission (which, to the authors' knowledge, have not yet been reported) will be briefly discussed. This paper will be organized as follows. Section II will show our proposed model to study extraordinary transmission. Section III will present the basics of our theory through the analysis of the simplest 2-D periodic structure exhibiting extraordinary transmission (a periodically perforated zero thickness screen). Section IV will introduce a modification of the model to account for finite thickness screens, and Section V will describe a more general model explaining some additional details of the dependence of the transmission spectrum with respect some geometrical dimensions. In Section VI, we will give some insight and qualitative explanations about the details of the transmission spectrum at frequencies above the onset of the first grating lobes. Also, we will discuss other possible structures exhibiting some kind of extraordinary transmission. Finally, some concluding remarks will be summarized in Section VII.

II. Modeling of Extraordinary Transmission

The starting point of our modeling of extraordinary transmission at optical and lower frequencies will be the key role played by *periodicity*, rather than any other consideration about the material properties. (In this sense, the diffraction model standpoint is very close to our point of view on the phenomenon.) Any microwave or antenna practitioner can readily appreciate the similarity between the periodic structures exhibiting extraordinary optical transmission and the frequency-selective surfaces (FSSs). As it is well known, FSSs are 2-D arrays of planar metallic scatterers (stopband FSS) or slots practiced in a metal plate (passband FSS). The shape and size of the planar scatterers/slots are tailored to control the frequency dependence of the transmission and/or reflection coefficients [27]. The slots, usually with complex shapes, are designed to resonate at a certain frequency to give a *total* transmission peak at this frequency, provided that material losses are neglected. In the theoretical and experimental works on extraordinary optical transmission, the geometry of the holes is commonly very simple: circular/cylindrical or rectangular/prism. Thus, one question that is immediately raised is why extraordinary transmission was not previously reported by FSS practitioners. Before giving a possible explanation to this fact, it should be pointed out that extraordinary transmission always appears at frequencies very close to the onset frequency of the first grating lobe and that, at this onset frequency, slot-like FSS always exhibit a zero transmission point known as Wood–Rayleigh anomaly [28]–[30]. For FSS practitioners, the range of frequencies close to the Wood–Rayleigh anomaly is not of practical interest because of the presence of undesired grating lobes, and, actually, it has only been explored as a limitation factor of the operation of FSS. Moreover, for the very thin metal screens commonly employed in FSS, the transmission peak is extremely narrow—as it will become apparent later—and typical ohmic losses might seriously mask the phenomenon. On the contrary, extraordinary transmission experiments at optical frequencies were carried out with electrically thick screens,

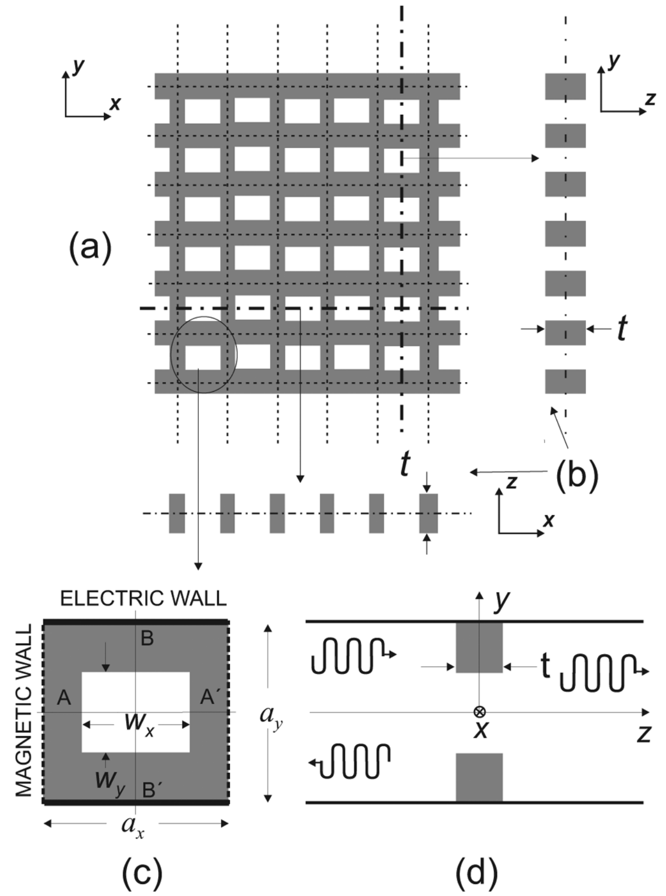

Fig. 1. Perfect conductor screen perforated with rectangular holes: (a) front view and (b) two lateral cuts and (c) front and (d) lateral views of the structure unit cell (parallel-plate transmission line with diaphragm discontinuity with thickness t).

giving place to wider transmission peaks that could be detected by the human eye (the always-present appropriate "detector").

Thus, assuming that FSSs and extraordinary transmission structures are the same thing, we will apply the FSS analysis methodology to the study of structures exhibiting extraordinary transmission. As is well known, the analysis of an infinite FSS can be reduced to the analysis of a single unit cell. This concept is illustrated in Fig. 1, where the original periodic structure and the equivalent unit cell are depicted. For normal incidence and linear polarization (along the y-direction, in our case), the unit cell is a parallel-plate transmission line with a thin or thick (depending on the value of t) rectangular iris diaphragm placed transversely to the axis of the transmission line [see Fig. 1(c) and (d)]. This way of thinking is not new for FSS practitioners [27] and microwave field theory researchers. For instance, a thorough analysis of 1-D zero-thickness periodic structures was carried out more than 40 years ago using equivalent network analysis [31]. Two comprehensive papers about the diffraction by an array of strips printed on a dielectric substrate using circuit modeling were published almost 20 years ago [32], [33]. More recently, and in close connection with the topic treated in this paper, circuit models have been used in the analysis of stacked perforated screens (each of them exhibiting extraordinary transmission) for left-handed electromagnetic

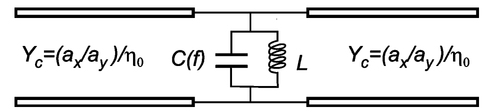

Fig. 2. Circuit model for zero-thickness diaphragm in parallel-plate waveguide.

wave propagation [34], [35]. Our contribution in the present paper is to show how extraordinary transmission can be explained in all its details by means of equivalent circuit models. The main advantage of this approach is that qualitative and semiquantitative predictions can be done without performing heavy numerical computations (or limiting such computations to a few frequency points). This will allow us to give easy explanations for most of the observed features of the phenomenon and even to predict novel situations exhibiting extraordinary transmission. Detailed derivation of increasingly complex circuit models will be given in the forthcoming sections.

III. Basic Theory for Extraordinary Transmission Through Zero-Thickness Screens

Let us consider the periodically perforated perfect conductor screen in Fig. 1 for the particular case of infinitesimal thickness ($t \rightarrow 0$). Since the underlying physics is not affected by the shape of the slots, we will consider rectangular holes in order to keep the computations as easy as possible. Using a detailed analytical/numerical approach [13], it has been established that this structure exhibits *a single peak* of extraordinary transmission (it was carried out for circular holes but the shape is not relevant). The unit cell under consideration is a parallel-plate transmission line formed by two vertical magnetic walls separated by a distance a_x and two horizontal perfect electric walls separated by a distance a_y. More precisely, due to symmetries of the structure and the excitation, the plane AA' in Fig. 1(c) is an electric wall and the plane BB' is a magnetic wall. Thus, apart from the TEM mode (TEM to z) representing the incident, reflected, and transmitted waves in the periodic structure, the transmission line can support $\mathrm{TE}_{2n,2m}$ (to z) and $\mathrm{TM}_{2n,2m}$ (to z) modes (n, m are integer numbers). If the structure is used as an FSS, TE and TM modes are always at cutoff. In common FSS applications, the size of the slots is chosen in such a way that they resonate well below the frequency of the first Wood–Rayleigh anomaly. For the structure under study, this frequency is given by $f_{\mathrm{WR}} = c/a_y$; note that this frequency is also the cutoff frequency of the TM_{02} mode of the waveguide, $f_c^{\mathrm{TM}_{02}}$ (the first subindex 0 corresponds to variations along the x-direction and the second subindex, 2, corresponds to variations along the y-direction).

The original problem is then reduced to the scattering of the incident TEM mode by a rectangular iris diaphragm practiced in a transverse metal sheet of zero thickness. This is a classical problem of discontinuities in waveguide theory and, as is well known [36], a simple equivalent circuit can account for the most important features of such discontinuity (see Fig. 2). The resonance of the *LC* tank circuit obviously corresponds to a peak of *total transmission*. From a physical point of view, C is related to the electrical energy in excess associated with below-cutoff TM modes excited at the discontinuity plane. Equivalently, L is associated with the excess magnetic energy of the below-cutoff TE modes excited at this same plane. The values of L and C can be estimated from the geometrical dimensions of the structure using some approximations reported in [36]. These values are considered to be weakly dependent on frequency in normal FSS operation. The resonance (total transmission) frequency can be obtained by considering the rectangular hole as a short-circuited section of a slot line. In order to obtain very accurate results, end effects should be added, especially for short slits (small w_x). From this perspective, w_x would be close to a half wavelength of the slot mode at the desired total transmission frequency (w_y should satisfy such a condition for x-polarized electric field). Higher order resonances are also possible, but they appear above the onset frequency of the first grating lobe $f_{\rm WR}$. Roughly speaking, if the resonance (transmission) frequency has to be lower than $f_{\rm WR} \equiv f_c^{\rm TM_{02}}$, w_x should be chosen larger than $a_y/2$. (It is worth mentioning here that w_x and not w_y is the relevant dimension because of the polarization of the impinging electric field.) This is the typical scenario for regular FSS operation. However, extraordinary optical transmission in zero-thickness perfect conductor screens has been reported for very small apertures [13]. From a naive perspective, very small apertures should resonate at frequencies well above $f_{\rm WR}$ and no transmission peaks would then be obtained below $f_{\rm WR}$. The main fault of the above reasoning is to forget that $C(f)$ in Fig. 2 is not a smooth function of frequency for frequencies near $f_{\rm WR}$. As a matter of fact, it is well known that the input impedance corresponding to a TM mode below cutoff excited in an infinitely long waveguide is given by

$$\mathrm{Z}_{\rm TM} = -\mathrm{j}\eta_0 \sqrt{\left(\frac{f_c^{\rm TM}}{f}\right)^2 - 1} \tag{1}$$

where η_0 is the characteristic impedance of vacuum, $f_c^{\rm TM}$ is the cutoff frequency of the TM mode, and f is the operation frequency. It is then clear that the equivalent capacitance associated with this mode is

$$C_{\rm TM}(f) = \frac{A_{\rm TM}}{2\pi f \eta_0 \sqrt{\left(\frac{f_c^{\rm TM}}{f}\right)^2 - 1}} \tag{2}$$

where $A_{\rm TM}$ is a coefficient accounting for the relative degree of excitation of this particular TM mode (in comparison with the other higher order TM modes). For simplicity in the forthcoming discussion, the frequency dependence of $A_{\rm TM}$ will be ignored. (Although it has been verified that this coefficient shows a significant increase around the extraordinary transmission frequency, this fact is hardly relevant for the following qualitative considerations). The overall capacitance in the circuit model in Fig. 2 is the result of the parallel connection of an infinite number of elementary contributions such as that in (2). For a working frequency below $f_{\rm WR} = f_c^{\rm TM_{02}}$, the contribution of higher order TM modes different from $\rm TM_{02}$ to $C(f)$ is a weakly depending function on frequency (because f is well below their corresponding cutoff frequencies). Hence, for our purposes, this contribution to the total capacitance can be considered to be constant and will be denoted as C_0. The frequency-dependent contribution is assumed to be given by the TM mode with the smallest cutoff frequency ($\rm TM_{02}$ in our case). The overall capacitance can then be written as follows:

$$C(f) = C_0 + C_{\rm TM_{02}}(f). \tag{3}$$

The important point here is that, since $C_{\rm TM_{02}}(f) \to \infty$ as $f \to f_c^{\rm TM_{02}}$, our model predicts that one transmission peak *is always present below the first Wood–Rayleigh anomaly* for any value of L. For small apertures, L is also small, and then $C(f)$ has to reach very high values to fulfill the resonance condition. This is the reason to find extraordinary transmission only close (but below) to the Wood–Rayleigh anomaly for this situation. An interesting test of this point of view is the comparison of the extraordinary transmission frequencies for two identical rectangular slots but with perpendicular orientations. In this case, numerical simulations say that the total transmission peak corresponding to the horizontal orientation appears at a lower frequency. The equivalent circuit theory gives an easy explanation for this fact. The horizontally oriented rectangular slot will perturb more strongly the surface currents (with respect to the nonperforated screen case) than the vertically oriented one. The corresponding higher value of the inductance for the horizontally oriented slot requires a lower capacitance to satisfy the resonance condition, which will then occur at a lower frequency.

The simplicity of the geometry under study has allowed us to implement a relatively easy computer code based on the mode matching technique [37] to accurately compute the transmission and reflection coefficients. (In our mode-matching computations for the zero-thickness case, we introduce a very small screen thickness and use many modes to reach convergence. A more efficient numerical procedure for this case would have been the solution of an integral equation for the unknown equivalent magnetic current in the aperture [38], but this numerical fact is not relevant for the purposes of the present paper). The unknown parameters of the circuit model in Fig. 2, C_0, L, and $A_{\rm TM}$, can be easily computed from a few low-frequency values of the transmission coefficient (S_{12}) and from the value of the total transmission frequency. This information is generated using the mode-matching code. For instance, for the equivalent circuit in Fig. 2, S_{12} can be obtained from

$$\frac{\mathrm{j}2\omega}{S_{12}} = -\left(\frac{C}{Y_c}\right)\omega^2 + \mathrm{j}2\omega + \frac{1}{LY_c} \tag{4}$$

where ω is the angular frequency and $Y_c = (a_x/a_y)/\eta_0$ is the characteristic admittance of the input and output transmission lines. For low frequencies (in comparison with $f_{\rm WR}$), L and C are constants and (4) is just a polynomial whose coefficients can be properly fitted from a few low-frequency data. The parameter $A_{\rm TM}$ is obtained by adding the information of the resonance frequency. With this reduced set of parameters, we should be able to reproduce the whole transmission spectrum. It is worth mentioning that the equivalent circuit with the single-mode frequency-dependent capacitance contribution in (2) perfectly accounts for the Wood–Rayleigh anomaly. Certainly, at $f = f_{\rm WR} = f_c^{\rm TM_{02}}$, the capacitance is infinity and the diaphragm will behave as a short circuit. This has been numerically checked verifying total reflection and that the phase of the reflected wave at that frequency is π.

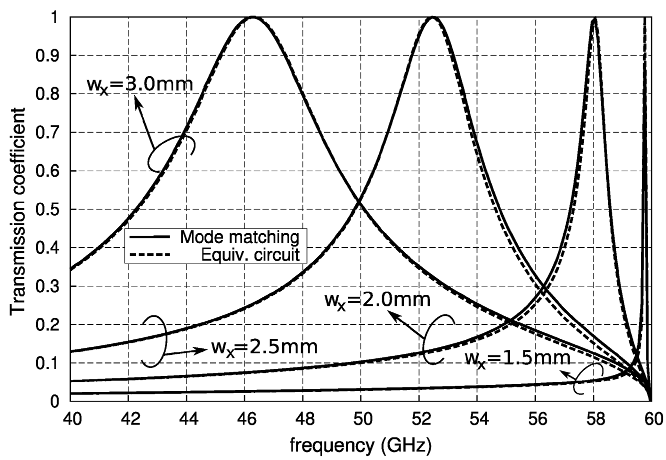

Fig. 3. Transmission coefficient $|S_{12}|$ through a zero-thickness perfect conductor screen with rectangular holes of various sizes. The unit cell is square: $a_x = a_y = 5$ mm, which corresponds to a Wood–Rayleigh anomaly frequency of $f_{\rm WR} = 59.9585$ GHz. For large slot size ($w_x = 3$ mm, $w_x = 2.5$ mm; $w_y = 0.5$ mm), we have normal FSS operation, and the passband is wide and well below the Wood–Rayleigh anomaly. For small slots ($w_x = 2$ mm, $w_x = 1.5$ mm; $w_y = 0.5$ mm), with intrinsic resonance frequency above $f_{\rm WR}$, the periodicity of the structure is responsible for extraordinary transmission near but below $f_{\rm WR}$ with narrow bandwidths.

In order to validate the above proposals, we have computed the transmission coefficient ($|S_{12}|$) for various slots keeping the same spatial periodicity. The "exact" mode-matching results (the number of modes has been increased until reach good convergence) and the predictions of the circuit model are given in Fig. 3. The first apparent conclusion is that the equivalent circuit model matches very well the computed full-wave results in the whole frequency range. This is a significant hint of the validity of our model. In Fig. 3, it can also be observed that large slots yield wideband resonances far away from and below $f_{\rm WR}$, as it is qualitatively expected since this corresponds to normal FSS operation. For very small slots of subwavelength size, the resonance (transmission) peaks move up to the proximities of $f_{\rm WR}$ and their corresponding bandwidths become smaller and smaller as the slot size is reduced. This is consistent with the qualitatively expected behavior of our equivalent circuit. When the holes are very small and the inductances are correspondingly small, resonance must be very close to the singularity of the capacitance at $f_{\rm WR}$. Note that the bandwidth of resonators with high C and small L is small. Moreover, due to the fast variation of C near $f_{\rm WR}$, a small variation of frequency around the resonance frequency makes the circuit far from resonance conditions. Thus, very narrow bandwidths are qualitatively expected if the transmission peak is near $f_{\rm WR}$ (which is clear from the results reported in Fig. 3).

Previously it has been shown that total transmission is predicted by simple waveguide theory arguments in the case of periodically perforated perfect conductor zero-thickness screens with arbitrarily sized holes. Some additional interesting theoretical conclusions can also be deduced from the equivalent circuit model. For example, it is the behavior of the $\rm TM_{02}$ mode near cutoff that is more relevant to the extraordinary transmission phenomenon. Note that the $\rm TE_{20}$ mode is also near cutoff ($f_c^{\rm TM_{02}} = f_c^{\rm TE_{20}}$ for the considered square lattice). This $\rm TE_{20}$ mode also contributes with a singular inductance near its cutoff frequency (which could reach very high values). However, this fact does not affect our previous conclusions about the dominant role of the $\rm TM_{02}$ mode in the extraordinary transmission occurrence. It is only the shunt-connected large capacitance that yields noticeable variations in the transmission coefficient, since large L in parallel with C and other L's coming from other TE modes will not affect the resonance condition and frequency response. In summary, only the *overall* capacitance (and not the overall inductance) becomes singular at $f_{\rm WR}$. Incidentally, this also explains that it is *the periodicity along the direction of the polarized electric field* (y-direction) that actually determines the value of the extraordinary transmission frequency. The x periodicity *is not relevant at all*. Indeed, periodicity along the x-direction *is not required* for the observation of enhanced transmission peaks. For instance, in [39], it has been demonstrated that a single row of holes (1-D periodicity) exhibits extraordinary transmission peaks. In this case, extraordinary transmission refers to transmitting much more power than the power impinging on the area of each individual hole (total transmission has no sense in this case, obviously). This qualitative prediction of our model is an additional validation of its physical soundness as well as its predictive potential.

Ohmic losses were neglected in our previous discussion, but it is expected that their presence leads to a reduction of transmitted power at the critical frequencies of otherwise perfect transmission systems. Following our equivalent circuit model, we can qualitatively advance that losses will be more significant for the case of small holes (extraordinary transmission) than for the case of large holes (regular FSS operation). In the circuit model, losses would be modeled as a resistance connected in series with L. Taking into account that the values of L involved in extraordinary transmission peaks are typically much smaller than those involved in common FSS transmission peaks, the effect of the losses on the quality factor of the peaks would be more pronounced for extraordinary transmission operation than for usual FSS operation. Thus, practical application of narrowband spatial filters and polarizers based on extraordinary transmission could be seriously affected by ohmic losses at microwave and millimeter-wave frequencies. In optical applications, metals are not characterized by a real conductivity but rather by a frequency-dependent complex permittivity with a large and negative real part. Maybe, in this case, the phenomenon of extraordinary optical transmission could be fruitfully exploited in the design of new devices such as those reported in [40].

IV. Extending the Model to Thick Screens

Most of the experiments and numerical simulations of extraordinary transmission systems have been carried out with relatively thick screens. Numerical simulations and some simplified analytical models predict that two instead of just one transmission peaks should be observed; see, for instance [7, Fig. 2], [6, Figs. 3 and 4], [25, Fig. 8], [41, Fig. 2], and [42, Fig. 2]. Nevertheless, losses and experimental limitations can make difficult the observation of two separate peaks. It is clear that the equivalent circuit in Fig. 2 cannot account for such a pair of peaks. The physical reason is that the reactive energy stored inside the

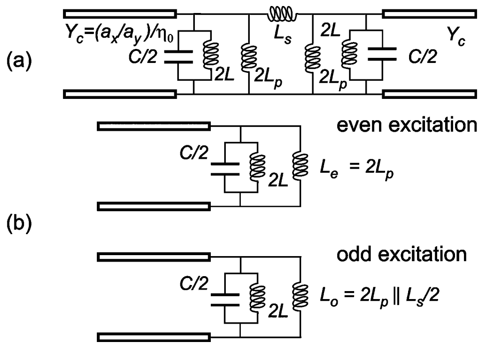

Fig. 4. (a) Lumped-elements circuit model for thick diaphragm in parallel-plate waveguide. (b) Even- and odd-mode circuits used to obtain scattering parameters and critical transmission frequencies. The equivalence between parameters of the model in (a) and the even-/odd-mode model in (b) is included in the figure.

hole was neglected in that simple equivalent circuit (only valid for zero-thickness screens).

For subwavelength holes, the dominant evanescent mode inside the hole is the TE_{10} mode. The hole can then be viewed as a rectangular waveguide section operating well below the cutoff of the first propagating mode. From a microwave engineering point of view, the problem corresponds to the scattering by a finite thickness diaphragm. For diaphragms that are not too thick, the introduction of a series inductance [L_s in the circuit model depicted in Fig. 4(a)] is an appropriate manner of accounting for the modification introduced by the nonnegligible thickness of the metal screen [36]. For thin screens, it is relatively obvious that L_s should be proportional to the thickness t. The inclusion of just this single series inductor will already provide two resonance peaks. Unfortunately, this simple model (commonly used to model practical thin diaphragms in closed waveguides [36]) would not account for subtle details of the transmission spectrum obtained from numerical simulations. Due to this, we propose the slightly more sophisticated π-network of inductances shown in Fig. 4(a). The idea underlying this π-circuit is that the equivalent circuit for a TE mode below cutoff is a ladder network whose elements are infinitesimal inductances. The distributed network is here replaced by a single cell with finite inductance values. Alternatively, the above π-circuit can be reached by noting the presence of a vertical symmetry plane at the middle of the hole. For even excitation from the two sides of the screen, this plane is a magnetic wall, and, for odd excitation, it is an electric wall. The corresponding equivalent circuits for these two situations are depicted in Fig. 4(b), where the inductances L_e and L_o are related to the magnetic energy stored inside the hole under even (e) and odd (o) mode conditions. The relationship between L_e and L_o with the inductances of the π-circuit is obvious and has been explicitly shown in Fig. 4(b).

Keeping in the circuit model only L_s (i.e., taking $L_p = \infty$) is equivalent to neglecting the magnetic energy stored inside the hole by the below-cutoff TE modes in the case of even excitation. If we want to accurately account for nonzero thickness effects, this latter contribution must be taken into account, and a finite value of L_p has to be used in the model. For very thin screens, it is found that $L_p \gg L$ (note that L accounts for TE modes in the external waveguides, not in the hole). In this case, the effect of L is predominant since L_p and L are shunt-connected. For infinitesimally thin screens, it is additionally found that $L_s \to 0$, giving place to a short circuit in such a way that the results of the zero-thickness screens of previous section are recovered. However, for appreciably thick screens, L_s is not small and L_p can be of the same order of magnitude as L. The circuit model in Fig. 4(a) predicts two total transmission peaks at frequencies below f_{WR} and, moreover, it can give some qualitative insight about the evolution of those peaks as a function of the screen thickness (t). A simple even–odd excitation analysis of this circuit leads to the following transmission coefficient:

$$S_{12} = \frac{1}{2}\left[S_{11}^{e} - S_{11}^{o}\right] \tag{5}$$

where the reflection coefficients for even and odd excitations are given by

$$S_{11}^{e,o} = \frac{\mathrm{j}2\omega L_{\mathrm{eq}}^{e,o} Y_c - [1-\omega^2 L_{\mathrm{eq}}^{e,o} C]}{\mathrm{j}2\omega L_{\mathrm{eq}}^{e,o} Y_c + [1-\omega^2 L_{\mathrm{eq}}^{e,o} C]} \tag{6}$$

with the following values of the equivalent inductances for even and odd excitations:

$$2L_{\mathrm{eq}}^{e} = \frac{2LL_p}{L+L_p} \quad 2L_{\mathrm{eq}}^{o} = \frac{4LL_pL_s}{4LL_p + L_s(L+L_p)}. \tag{7}$$

These are the equivalent inductances of the shunt associations of inductances loading the transmission lines in Fig. 4(b).

The inductances L_s and L_p depend on the screen thickness but, provided that the electrical thickness of the screen is not too large, they only slightly depend on frequency. The values of C and L are related to the fields outside the hole, and they are almost independent of t. Equation (5)–(7), together with the frequency dependence of $C(f)$ in (2), predict the following two *total transmission* frequencies:

$$f_e = \frac{1}{\sqrt{C(f_e)L_{\mathrm{eq}}^{e}}} \quad f_o = \frac{1}{\sqrt{C(f_o)L_{\mathrm{eq}}^{o}}} \tag{8}$$

provided the following condition is fulfilled: $(2Y_c/C)^2 \ll (L_{\mathrm{eq}}^{e} + L_{\mathrm{eq}}^{o})/(L_{\mathrm{eq}}^{e}L_{\mathrm{eq}}^{o}C)$. Fortunately, this last condition is always satisfied. The equations in (8) are a set of implicit equations that determine the resonance (total transmission) frequencies using the capacitance in (3) and the inductances in (7). Equivalently, if the resonance frequencies are known from a mode-matching analysis, (8) would provide a method to obtain the inductances in (7). The frequencies in (8) are always below f_{WR} (onset of the first grating lobe) due to the singular behavior of $C(f)$ at f_{WR}. Note that each of the transmission peaks can be related to the resonance of the reactive load associated with each of the excitation modes [even or odd, see Fig. 4(b)]. Since $L_{\mathrm{eq}}^{o} \le L_{\mathrm{eq}}^{e}$, this implies that $f_o \ge f_e$ and, therefore, the bandwidth around f_o will be smaller than that around f_e. The above features coincide with the reported behavior for thick screens in many previous papers based on purely numerical approaches or cumbersome analytical developments; see, for instance, [6, Fig. 3] or [25, Fig. 8].

A different situation is found if the slot width (w_x) is sufficiently large (typically when $w_x > a_y/2$) to allow transmission of the TE_{10} mode inside the hole at frequencies below f_{WR}. While the equivalent circuit in Fig. 4 is still valid below the onset of this mode, $f < f_c^{\mathrm{TE}_{10}} = c/(2w_x)$, a different circuit model should be used for the even excitation mode in Fig. 4(b) for frequencies $f > f_c^{\mathrm{TE}_{10}}$. In this latter case, the contribution of the TE_{10} mode fields inside the hole is a capacitance that should be added to $C/2$, while the relevant inductance is only the external inductance L. This is because a short section of waveguide above cutoff terminated with a magnetic wall (open circuit) is equivalent to a capacitance. The resonance associated with L and the sum of this capacitance plus $C/2$ in Fig. 4(b) yields the typical wide transmission peak observed in common FSS operation. In the case of the odd mode, the circuit model in Fig. 4(b) is still valid above $f_c^{\mathrm{TE}_{10}}$ because a short section of waveguide above the cutoff terminated with an electric wall (short circuit) is equivalent to a lumped inductance (L_o). This lumped inductance is expected to be much lower than L, and its effect will prevail over the effect of L. This small value of the overall inductance together with the frequency behavior of C will lead to a *second narrow resonance* very close to f_{WR}. This resonance has been unnoticed in the past because L_o is extremely small for the electrically very thin screens used in FSS applications. Its associated transmission peak would then be extremely narrow and, in practice, possibly masked by ohmic losses.

Let us now discuss in detail the two different situations previously mentioned using some examples.

A. Screens With Large Holes

For relatively large slots (say $w_x > a_y/2$), the lower frequency in (8), associated with the case of even excitation resonance (f_e), corresponds to conventional FSS operation. (Note that the value of w_y is not relevant because of the orientation of the impinging electric field). This frequency f_e is mainly controlled by $C/2$ and $2L$ in Fig. 4, although $C/2$ should be slightly increased with the value of the capacitance coming from the contribution of the TE_{10} mode under even excitation conditions for frequencies above $c/(2w_x)$. As a consequence, the position of this transmission peak (f_e) mainly depends on w_x, and nothing "extraordinary" happens in such a case (regular FSS operation). This transmission peak has been called elsewhere *localized waveguide resonance*; see, for instance, [41] among others. With this terminology, the authors of [41] seem to refer to a situation where the transmission frequency roughly matches the cutoff frequency of the first mode launched above cutoff in the hole (TE_{10} in our case). However, this terminology is misleading because the propagation of a waveguide mode inside the hole does not necessarily imply strong transmission. In fact, strong transmission is only observed around a specific frequency, although the TE_{10} mode propagates for all of the frequencies above $c/(2w_x)$. The circumstance actually required for total transmission is *impedance matching*, and this condition is only reached when average electric and magnetic energies stored around the holes are identical. This condition only tangentially might be related to the onset of a waveguide mode inside the hole. Surprisingly, this point seems to be systematically ignored in most of the physical explanations reported in many (if not all) of the published papers on the topic. In Fig. 5, we have plotted transmittance $\left(|S_{12}|^2\right)$ results for several slots with different values of w_x. These results have been obtained

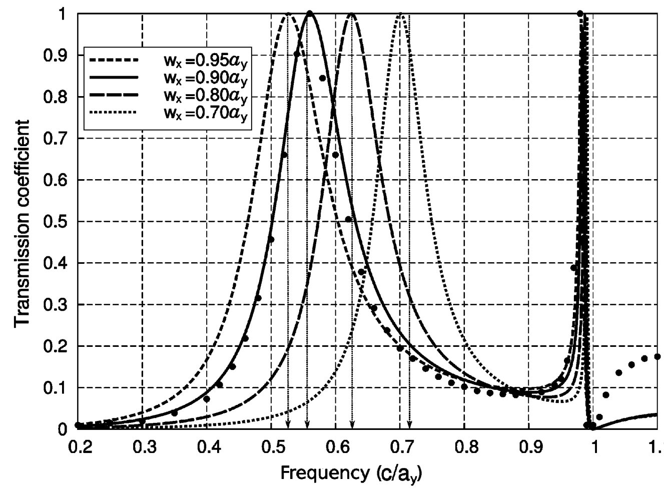

Fig. 5. Transmission spectrum $|S_{12}|^2$ of a perforated perfect conductor finite-thickness plate for several widths of the rectangular holes. Data from [41] (black circles) have been included for comparison purposes. Vertical arrows mark the positions of the first maximum derived from a simple reasoning based on short-circuited slot resonance. Slight shifts of these theoretical resonances with respect to numerical data is due to the neglected end effects. Dimensions: $a_x = a_y$, $w_y = 0.2a_y$, and $t = 0.2a_y$.

using our mode-matching code. The finite-difference time-domain (FDTD) results reported in [41] for $w_x = 0.9a_y$ are included for comparison purposes (our data reproduced accurately all of the results in [41]). Actually, the accurate mode-matching computation of the lower transmission peak frequency reveals that the maximum transmission occurs at frequencies clearly below the onset frequency of the TE_{10} mode (see, for instance, the case corresponding to $w_x = 0.7a_y$ in Fig. 5). This fact can be explained in terms of the apparent larger length of the slot resonator due to end effects. As stated in the analysis of the zero-thickness case, the ordinary transmission peak can be interpreted in terms of the resonance of the fundamental mode of the short-circuited finite-length section of slot line, in our case, slot width w_y, slot length w_x, metallization thickness t, and with the resonance condition given by $\beta_{\mathrm{slot}} w_x = \pi$, where β_{slot} is the propagation constant of the slot mode. In Fig. 5 some arrows have been included to mark the above "theoretical" total transmission frequencies derived from the resonant slot model neglecting end effects and taking β_{slot} as the vacuum wavenumber $k_0 = \omega/c$. Note that, in this way, the above arrows also account for the cutoff frequency of the TE_{10} mode inside the hole. Since $\beta_{\mathrm{slot}} \approx k_0$, the shift to lower frequencies of the transmission peak (when compared with the onset frequency of the TE_{10} mode in the hole) that can be appreciated in Fig. 5 is mainly due to end effects at the two short-circuited ends of the slot resonator. This shift to lower frequencies is more evident for shorter slots because of the larger relative weight of the end effect. The shift is also more pronounced when the screen thickness is small because β_{slot} is no longer so close to k_0 (see, for instance, the case $w_x =$ 3 mm shown in Fig. 3).

After the discussion in the previous paragraph, it is clear that the first transmission peak cannot be considered "extraordinary" in any sense. The important observation concerning extraordinary transmission through nonzero-thickness screens is that, apart from that FSS-like peak, a second narrow transmission

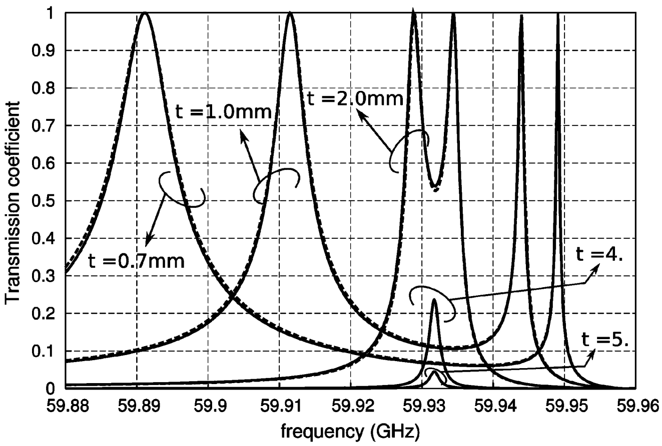

Fig. 6. Transmission coefficient $|S_{12}|$ through small holes practiced in several nonzero thickness screens (solid lines). The peaks approach to each other as the screen thickness increases. For large thickness, the two peaks collapse into a single peak, and, finally, transmission disappears for very thick screens. For those cases where the two peaks are clearly noticeable, the circuit model prediction is included (dashed lines). The dimensions are $a_x = a_y = 5$ mm; $w_x = 1.5$ mm, and $w_y = 0.5$ mm.

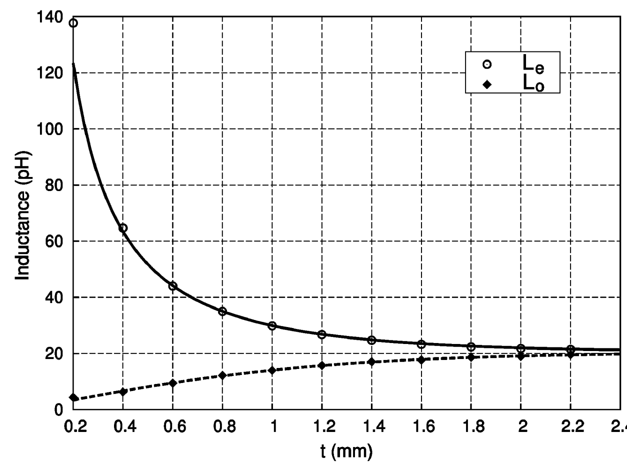

Fig. 7. Inductances L_e and L_o accounting for the reactive fields inside the rectangular hole as a function of screen thickness (t). These inductances are the inductances in parallel with L and C in Fig. 4(b). (⊙, ◆): Data extracted from the numerical mode-matching analysis. Solid/dashed lines: theoretical data following the discussion in Section V. The dimensions are the same as in Fig. 6.

peak *always appears below and close to the onset frequency of the first grating lobe* [41]. This is the transmission peak that can be properly called "extraordinary" because, to the best of the authors' knowledge, it has not been reported and discussed before the paper by Ebbesen [1]. From Fig. 5, it is apparent that the ordinary peak position (i.e., the FSS-like peak with large bandwidth) depends on the size of the hole along the x-direction (w_x), but the extraordinary peak is always close to f_{WR}. This observation is in perfect agreement with our previous theoretical discussion. The extraordinary transmission peak has been related to surface plasmons in previous literature (genuine plasmons, spoof plasmons, or both). However, in our theory, ordinary and extraordinary transmission peaks are both related to *impedance matching* due to cancellation of reactive energy (imaginary part of the Poynting vector flux) by proper balance of electric and magnetic energy stored in the cutoff modes involved in the discontinuity problem. This condition can be fulfilled in the proximity of the onset of spoof plasmons but, in our opinion, it is the impedance matching that should be considered as the relevant cause. As will be briefly discussed in Section VI, this kind of impedance matching can also be attained in *closed* waveguide systems where surface plasmons are absent or simply have no sense.

B. Screens With Small Holes

Next, the case of a thick screen with small (subwavelength) holes (true extraordinary transmission situation) will be considered. In Fig. 6, we plot $|S_{12}|$ for the same small rectangular holes practiced on various screens with different thickness. Since the holes are small, the two perfect transmission peaks are close to f_{WR}, as expected from our theory. Also, we can see that the larger the thickness of the screen is, the closer the two peaks are located. How does our model account for this fact? If L_p is ignored in the model in Fig. 4(a) (as is typical in the modeling of thin diaphragms in waveguides), it would be possible to account for the thickness dependence of the odd-excitation peak closer to f_{WR} but not for the thickness dependence of the even-excitation peak. In the latter case, the f_e peak frequency would be controlled exclusively by L and C, which are not dependent on the thickness of the screen. Thus, L_p must be included in the model to account for the displacement of the lower frequency peak when the hole thickness t varies. Qualitatively, it is expected an increase of L_s with t (for small values of t, L_s should be proportional to t) and that L_p decreases monotonically with t starting from ∞ at $t = 0$. From our circuit model and the assumed frequency dependence of the various involved parameters (L, L_s, L_p, C_0, and A_{TM} are frequency-independent, while $C_{\mathrm{TM}_{02}}$ in (2) is the only frequency-dependent parameter), it is possible to extract the values of all of the parameters of the equivalent circuit from a few data computed with mode matching. Using reasonable approximations, the function $\mathrm{j}2\omega/S_{12}(\omega)$ can be approximated by a third-order polynomial function (something similar to (4) for zero-thickness screens) at low frequencies. The coefficients of this polynomial and the values of the resonance (transmission) frequencies determine all of the parameters of the circuit model in Fig. 4(a). Using this simple fitting scheme, we have obtained the values of the inductances appearing in parallel with L and C in Fig. 4(b), i.e., $L_e = 2L_p$ and $L_o = 2L_pL_s/(L_s + 4L_p)$, for several thicknesses of the screen. These results have been plotted in Fig. 7 (discrete circles and rhombuses). Our previous qualitative discussion about the dependence of inductances with respect to t is clearly supported by these results. As we can see from Fig. 7, L_p is large (and goes to infinity) for very thin screens while L_s becomes small and proportional to t for thin screens. However, as the thickness of the screen increases, L_e and L_o tend to approach each other, as they are identical for electrically very thick screens. The evolution of the pair of peaks in Fig. 6 with respect to the screen thickness can be easily explained in terms of the results for L_e and L_o in Fig. 7. As the values of L_e and L_o approach each other, the transmission peaks are closer and closer. Note that the predictions of our equivalent circuit have also been plotted in Fig. 6 (dashed lines). These predictions are very accurate for the two-peak cases. However, it can be seen that *the two transmission peaks collapse into a single peak when the hole thickness is sufficiently large*, i.e., when L_e and L_o are almost identical. (For this single-peak case, the circuit model in Fig. 4 does not provide good results.) It is expected that the transmission peaks disappear for very thick screens due to the evanescent behavior of the fields inside the hole (the fields of the below-cutoff modes excited at the left side of the screen cannot reach the second interface due to strong reactive attenuation). This would be in perfect agreement with the numerical results reported in [25, Fig. 8] for circular shaped holes in thick screens.

The single peak curves in Fig. 6 reveal a limitation of the lumped equivalent circuit shown in Fig. 4. When the two resonance frequencies collapse into a single one, it is found that $L_e = L_o$, and then the equivalent circuit in Fig. 4 predicts no transmission at all. Therefore, in spite of the success of our model up to this point to explain extraordinary transmission results, it would be convenient to have a more sophisticated model of the diaphragm discontinuity that can account more accurately for the actual dependence of L_e and L_o with t. With this modification, the equivalent circuit model is expected to give appropriate results even when the two peaks collapse. As we will show in Section V, this can be done, but the lumped-element circuit model should be abandoned in favor of a distributed model.

V. Simplified Distributed Model for Extraordinary Transmission

The complex dependence of the transmission peak frequencies in terms of the screen thickness has been considered in the literature using a model that uses electric and magnetic dipoles to account for the effects of small holes [25]. A sophisticated explanation of this behavior based on the formation of a "surface plasmon molecule" was provided years ago in [6]. It is then an interesting challenge for our equivalent circuit model to account for this complex dependence on the screen thickness using simple arguments. We have found a relatively simple solution based again on well-known waveguide concepts. The key point is that changing the thickness of the screen (t) does not appreciably affect what happens outside the hole (i.e., L and C are not dependent on t). Therefore, the parameter t affects transmission frequencies through the values of L_s and L_p (or, equivalently, L_e and L_o). Although we already have a qualitative idea about the dependence of these parameters on t from the discussions in connection with Fig. 7, a much more accurate estimation of the dependence of L_e and L_o with t can be achieved after considering that, for thick screens, the distance t can be comparable to the longitudinal variation of electromagnetic fields inside the hole. This means that electromagnetic fields inside the hole depend on *z in the specific manner given by waveguide theory*, and this specific variation rate can be easily included in the model. In particular, since the dominant mode (below cutoff for extraordinary transmission conditions) inside the hole is the TE_{10} mode, we propose the approximate *distributed* equivalent circuit shown in Fig. 8. In this equivalent circuit, the hole is substituted by a section of an evanescent transmission line of length t. This transmission line is characterized by the known imaginary characteristic impedance $\mathrm{j}Z_{\mathrm{TE}_{10}}$ and the attenuation factor $\alpha_{\mathrm{TE}_{10}}$ corresponding to the TE_{10} mode of the small rectangular waveguide of dimensions w_x and w_y. The parameter n in Fig. 8(a) corresponds to the excitation factor of the mode, which will be obtained here from mode-matching simulation (although it can be roughly estimated from the geometries of the large and small waveguides involved in the problem). A simple even/odd excitation analysis of the structure in Fig. 8 yields

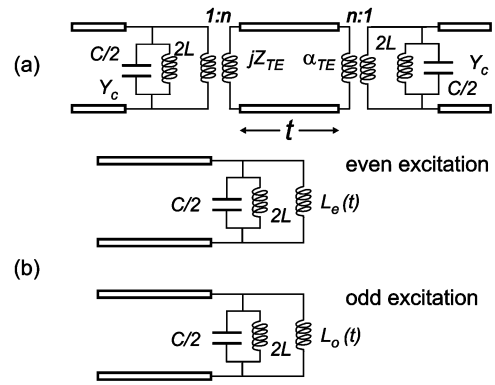

Fig. 8. (a) New circuit model accounting for distributed effects inside the hole for nonzero-thickness screens. (b) Even- and odd-mode equivalent circuits. The values of L_e and L_o are given in the text.

$$L_e = \frac{Z_{\mathrm{TE}_{10}}}{2\pi n f_{\mathrm{WR}}} \coth\left(\frac{\alpha_{\mathrm{TE}_{10}} t}{2}\right) \tag{9}$$

$$L_o = \frac{Z_{\mathrm{TE}_{10}}}{2\pi n f_{\mathrm{WR}}} \tanh\left(\frac{\alpha_{\mathrm{TE}_{10}} t}{2}\right). \tag{10}$$

Note that the distributed model provides the explicit dependence on t of the parameters of the model in Fig. 4(b).

The circuit parameters of Fig. 8 (C, L, and n) can be obtained from a few mode-matching simulations, as was done with previous equivalent circuit models. Using (9) and (10), we have obtained the data corresponding to the solid and dashed lines plotted in Fig. 7. As the dependence of L_e and L_o with t is explicitly known *a priori*, the values of L_e and L_o have to be computed only for a single value of t. If we choose a large value of t so that $L_e = L_o \equiv L_\infty$, it is found that

$$L_e(t) = L_\infty \coth\left(\frac{\alpha_{\mathrm{TE}_{10}} t}{2}\right) \tag{11}$$

$$L_o(t) = L_\infty \tanh\left(\frac{\alpha_{\mathrm{TE}_{10}} t}{2}\right). \tag{12}$$

This means that the frequency response for any screen thickness can be known without performing mode-matching simulations for different thicknesses. It should be highlighted that the perfect matching of the curves in Fig. 7 with the discrete points (circles and rhombuses) confirms all of our assumptions (for instance, that the only relevant mode inside the hole is the TE_{10} mode).

The use of the distributed model adds another significant advantage when compared with the lumped model. The availability of the explicit expressions in (9) and (10) allows the equivalent-circuit model to account for what happens when the two peaks of extraordinary transmission collapse. Thus, in Fig. 9, we compare the full-wave mode-matching results (lines) with the distributed equivalent circuit predictions (circles) for several cases of thick screens. Now, our equivalent circuit model shows an excellent agreement with the numerical results even when a single peak occurs and only partial transmission

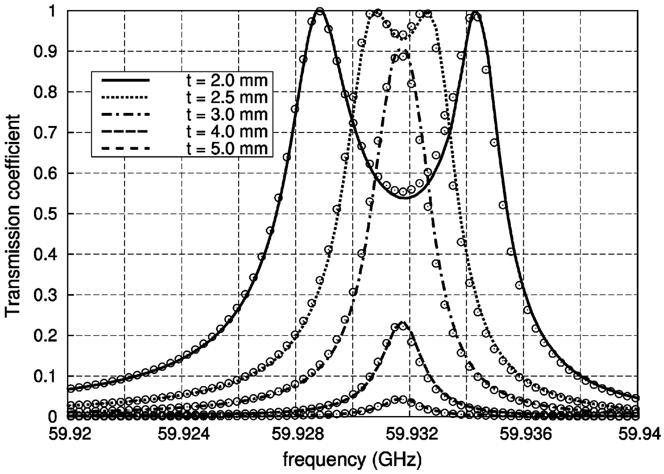

Fig. 9. Detail of the transmission frequency region for a thick perforated screen with sub-wavelength holes. For progressive increasing of thickness the two peaks collapse into a single one whose height decreases to zero for very thick screens. Mode-matching numerical results of $|S_{12}|$ (lines) are compared with circuit model predictions (circles) using the model in Fig. 8 with the parameters in (9) and (10). The dimensions are the same as in Fig. 6.

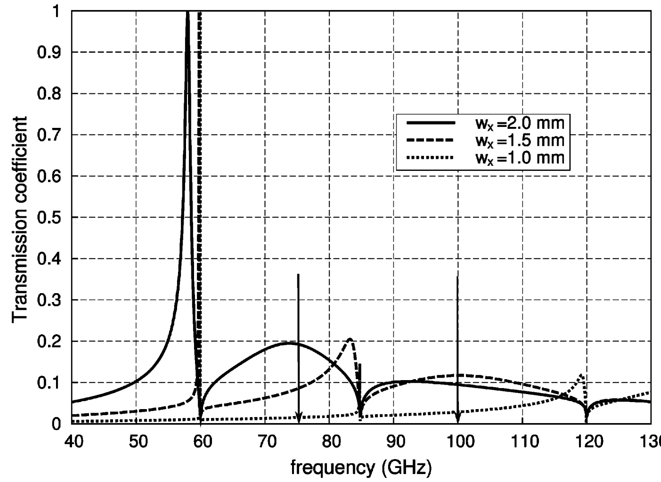

Fig. 10. Wideband transmission spectra $|S_{12}|$ of thin screens with rectangular holes of three different sizes. The "theoretical" resonance frequencies are, respectively, 75 GHz $(w_x = 2.0$ mm), 100 GHz $(w_x = 1.5$ mm), and 150 GHz $(w_x = 1.0$ mm). Note that enhanced transmission peaks appear near the Wood-Rayleigh frequencies occurring below theoretical resonances but not above. Dimensions: $a_x = a_y = 5$ mm, $w_y = 0.5$ mm, $t = 0.2$ mm.

is possible. This new validation is an additional hint of the suitability of the point of view on extraordinary transmission proposed in this paper.

VI. Additional Features of the Proposed Models

The waveguide and circuit models proposed in previous sections have been used to explain the extraordinary transmission phenomenon up to the onset of the first grating lobe (first Wood–Rayleigh anomaly). However, it is evident that the full-wave numerical scheme based on mode matching must still be valid for higher frequencies. The implemented computer code allows for the computation of, for instance, the reflected and transmitted power traveling in each of the secondary spots arising from diffraction. The angles of each grating lobe can be easily obtained from the knowledge of the propagation constants of each above-cutoff higher order mode of the parallel-plate waveguide in Fig. 1. However, the equivalent circuit models have been developed to be used up to the frequency of the first grating lobe. These models do not account for multiple mode operation (frequencies above $f_{\rm WR}$). In order to account for that regime, more sophisticated models similar to the ones reported in [32] and [33] should be adapted to the case of 2-D periodic structures having nonzero-thickness screens. Nevertheless, the physics behind our circuit models can still explain some qualitative details of the transmission spectrum at higher frequencies.

Our models can predict, for instance, if a peak of enhanced transmission will appear or not near a particular Wood–Rayleigh anomaly of order higher than one. Let us consider a structure having a very thin screen. It has been shown that the bottom line of extraordinary transmission is the increase of the value of the equivalent capacitance of TM modes just below their cutoff frequencies. This increase makes it possible to cancel the reactive part of the input impedance seen from the input transmission line before the hole array. For single-mode operation, that means total transmission due to perfect matching. However, once a second mode is launched into the parallel-plate transmission lines (operation frequency above $f_{\rm WR}$), no perfect matching is possible for the impinging TEM mode. However, cancellation of reactive energy is still possible, and this would yield a peak of enhanced transmission near and below some of the Wood–Rayleigh anomalies (the amplitude of such peak would be of course far from unity). In order to illustrate these comments, wideband transmission spectra for three structures having the same periodicity $(a_x = a_y = 5$ mm) but different hole sizes are shown in Fig. 10. The solid line corresponds to a rectangular hole with $w_x = 2$ mm and $w_y = 0.5$ mm. It can be seen that a total transmission peak appears below the first Wood–Rayleigh anomaly (about 60 GHz) but no peaks are visible near the second (at about 85 GHz, the cutoff frequency of the $\rm TM_{22}$ mode) and third (about 120 GHz, the cutoff frequency of the $\rm TM_{04}$ mode) anomalies. The reason is that the considered hole has a "theoretical" slot-line resonance frequency around 75 GHz (see the wideband maximum of the transmission curve around that frequency). Adding more capacitance coming from terms such as that in (2) cannot produce additional resonances.

However, if the "theoretical" resonance of the slot is above 85 GHz but below 120 GHz, the singular behavior of the capacitance associated with Wood–Rayleigh anomalies should produce total-transmission resonance peaks near 60 GHz $(\rm TM_{02})$ and partial-transmission resonance peaks around 85 GHz $(\rm TM_{22})$. This is what happens if $w_x = 1.5$ mm (theoretical slot-line resonance frequency near 100 GHz). The dashed line in Fig. 10 confirms this prediction. The dotted line in Fig. 10 corresponds to $w_x = 1.0$ mm (theoretical resonance frequency around 150 GHz). In this case, our reasoning predicts transmission peaks before the first, second, and third Wood–Rayleigh anomalies, and this is what we obtain with mode-matching computations in Fig. 10.

Let us finally comment on some of the advantages of our models and point of view. As has been previously mentioned,

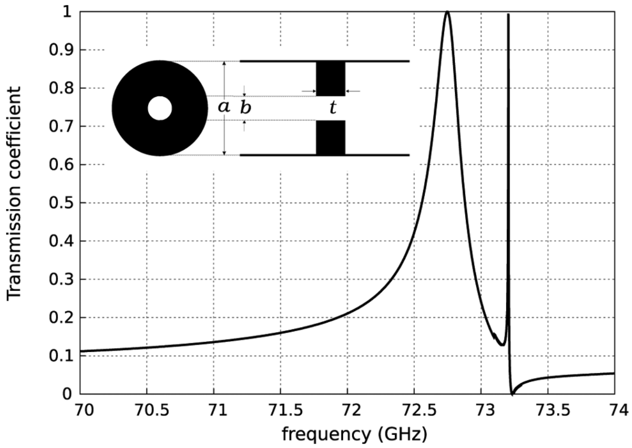

Fig. 11. Enhanced transmission peaks, $|S_{12}|$, through a subwavelength diaphragm in a circular waveguide. Dimensions: $a = 5$ mm, $b = 1.4$ mm, $t = 0.3$ mm. The impinging mode is the fundamental TE_{11} mode and the higher order mode involved in extraordinary transmission is TM_{11}.

currently accepted theories about extraordinary optical transmission are based on the existence of surface waves that can be excited in the periodically structured surface by an impinging planar TEM wave. Our equivalent-circuit interpretation of the phenomenon also accounts for the same experiments and simulated results. It is plausible to say that our approach, based on the impedance-matching concept, is in some extent equivalent to the theory founded on surface plasmons. Indeed, using the transverse resonance concept as in [22], our impedance-matching condition could be close to the condition for the onset of spoof plasmons in the periodic structure. However, it should not be inferred that both approaches are completely equivalent. This can be corroborated by noting, for example, that impedance-matching conditions can be achieved in structures where transverse surface waves are not possible. In this way, our proposed theory anticipates the possibility of having extraordinary transmission in structures without periodicity and without the possibility of existence of surface plasmons. As it has been exhaustively discussed in this paper, extraordinary transmission is here basically associated with the excitation of TM modes near cutoff. Certainly, this situation is also possible if the diaphragm is practiced in a simple metallic rectangular or cylindrical waveguide. (The parallel-plate transmission line in Fig. 1 associated with the periodic structure is just a particular example.) Our model states that a total transmission peak can be observed, independently of the size of the hole, near the onset frequency of the first TM mode launched in the structure (provided the thickness of the diaphragm is not too large and material losses are neglected). This has been verified by using a commercial software (CST Microwave Studio). In Fig. 11, we show a typical extraordinary transmission situation in a cylindrical waveguide with a diaphragm. In this situation, the impinging mode is the fundamental TE_{11} mode with a cutoff frequency of approximately 35.14 GHz. The first higher order TM mode involved in the extraordinary transmission is the TM_{11} mode with a cutoff frequency of approximately 73.14 GHz (the discrepancy observed in Fig. 11 between the zero-transmission frequency or Wood's anomaly and this cutoff frequency is due to the polygonal nature of the simulated circular waveguide). The TM_{01} mode, having a lower cutoff frequency, does not play any role in this structure because it is not excited due to the symmetries of both excitation and geometry. Note that the lowest modal cutoff frequency in the cylindrical hole is 125.5 GHz, which corresponds to the TE_{11} mode.

It is obvious that the above closed structure cannot support any kind of surface waves, in particular plasmons. Clearly, the cylindrical waveguide cannot be associated with any sort of equivalent periodic structure. That means that, although surface plasmons could be an alternative explanation to that offered in this paper to extraordinary transmission in periodic planar structures, the concept of a surface plasmon hardly can be considered as a possible explanation in the case of diaphragms in closed waveguides of arbitrary transverse cross section.

Additionally, it is worth mentioning that our model would provide a complementary physical explanation of the tunneling of electromagnetic energy through subwavelength channels and bends reported in [43], but this is beyond of the scope of this paper and will be the object of a forthcoming work.

VII. Conclusion

The phenomenon of extraordinary transmission of electromagnetic waves through metal screens periodically perforated with small holes has been interpreted in terms of the scattering by a thin or thick diaphragm of a TEM mode traveling along a parallel-plate transmission line. This is a classical problem in the microwaves area where powerful numerical methods and suitable equivalent circuits are available. In this paper, we have implemented the mode-matching method to solve the corresponding electromagnetic problem in an efficient way. The numerical results obtained with this method match perfectly those given by other cumbersome techniques previously reported in the literature. However, a more important advantage of our approach is that the problem has been posed in such a way that it is amenable to simple equivalent circuit models with just a few parameters to be determined. Since the proposed circuit models perfectly capture the physics of the problem, we can account for all of the fine details of extraordinary transmission phenomena. The equivalent circuit models allows us to give an interpretation of the extraordinary transmission problem in terms of impedance matching. This point of view could be, in some cases, equivalent to the currently accepted theory founded on the excitation of *spoof* surface plasmons. However, it has the advantage of being simpler (at least for microwave engineers) and allows for the easy predictions of many features of the expected behavior of the analyzed system. An additional and relevant advantage of our point of view is the anticipation of new systems exhibiting extraordinary transmission without the presence of surface plasmons (or any other class of surface waves). The proposed equivalent circuit models could also be used as easy computer-aided design tools for designing practical devices based on extraordinary transmission.

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