Low-Terahertz Transmissivity with a Graphene-Dielectric Micro-Structure

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***Abstract*— In this paper, we report on the analysis of transmissivity of electromagnetic waves through a stack of dielectric slabs loaded with atomically thin graphene sheets at low-terahertz frequencies. It is observed that the structure supports a series of bandpass regions separated by bandgap regions, similar to the case of stacked metallic meshes separated by dielectric slabs at microwave/THz frequencies or metal-dielectric stack at optical frequencies. The transmission resonances in the bandpass region are identified as coupled Fabry-Pérot resonances associated with the individual dielectric slabs loaded with graphene sheets. The study is carried out using a simple circuit theory model, with the results verified against the numerical simulations.**

***Index Terms*— Electromagnetic propagation, conductivity, passband, Fabry-Pérot, analytical models.**

I. INTRODUCTION

Controlling the electromagnetic wave propagation using periodic structures has been a subject of research for many decades. Relatively recent examples of this kind of research can be found in [1], [2], where high optical transmission through a thin-metal-dielectric stack has been reported, in spite of extremely weak transmission through each individual isolated thin metal layer. The spectra for such a structure consist of a series of bandpass and bandstop regions. However, mimicking these properties in the microwave regime is quite a difficult task due to the quasi-perfect conductor behaviour of metal at microwave frequencies. To overcome this problem, the replacement of the thin metallic sheets of the optical system with metallic mesh grids has been recently proposed [3], wherein the transmission spectrum at microwaves includes several passband regions of high transmissivity associated with coupled Fabry-Pérot resonances of individual cavities.

With the recent developments in the fabrication technology of graphene [4], there have been numerous applications developed with the use of graphene at optical, infrared, and terahertz frequencies. In particular, the low-terahertz band has been of interest, with graphene applications as frequency multiplication [5], plasmon oscillators [6], and cloaking [7], among others. With high prospects of graphene at low-terahertz frequencies, in this paper, we report the transmissivity of electromagnetic waves through a stack of monolayer graphene sheets separated by dielectric slabs (with the geometry shown in Fig. 1). It is observed that at low-terahertz frequencies (several THz) the resonances of high transmission occur, and the number of transmission peaks corresponds to the number of dielectric layers. These transmission resonances lie within a characteristic frequency band independent of the number of layers, and correspond to the passband regime of an infinite periodic structure. This is exactly the same behavior which has been observed with a stack of metallic meshes separated by dielectric layers at microwaves [3] (and, in general, THz frequencies) and with a thin-metal-dielectric stack at optical frequencies [2]. However, graphene sheets used in a stack shown in Fig. 1 are atomically thin monolayers, which due to extraordinary properties of graphene behave as reactive inductive surfaces (with low real part and negative imaginary part of the surface conductivity of graphene at low-terahertz frequencies). In this regard, a graphene monolayer at low-terahertz frequencies mimics the properties of a reactive inductive surface at microwave/THz frequencies (for example, fishnet surface) and that of a thin solid metallic

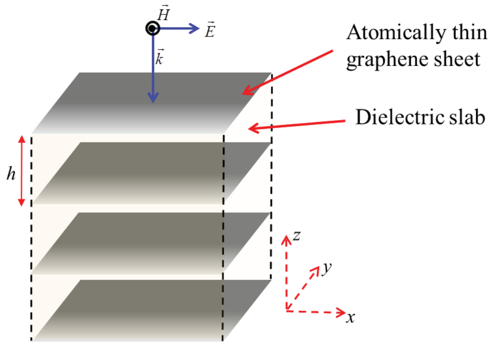

Fig. 1. Geometry of a stack of atomically thin graphene sheets separated by dielectric slabs with a plane-wave incidence.

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surface at optical frequencies.

II. THEORY

In the analysis to follow, graphene is characterized by the surface conductivity $\sigma(\omega,\mu_c,\Gamma,T)$ model based on the Kubo formula [8]

$$\sigma(\omega,\mu_c,\Gamma,T)=\frac{je^2\left(\omega-j2\Gamma\right)}{\pi\hbar^2}\times\left[\frac{1}{\left(\omega-j2\Gamma\right)^2}\int_0^\infty\varepsilon\left(\frac{\partial f_d(\varepsilon)}{\partial\varepsilon}-\frac{\partial f_d(-\varepsilon)}{\partial\varepsilon}\right)\mathrm{d}\varepsilon-\int_0^\infty\frac{f_d(-\varepsilon)-f_d(\varepsilon)}{\left(\omega-j2\Gamma\right)^2-4\left(\varepsilon/\hbar\right)^2}\mathrm{d}\varepsilon\right]\quad(1)$$

where $-e$ is the charge of an electron, ω is the radian frequency, $\hbar = h/2\pi$ is the reduced Planck's constant, $f_d(\varepsilon) = \left(e^{(\varepsilon-\mu_c)/k_BT}+1\right)^{-1}$ is the Fermi-Dirac distribution, k_B is Boltzmann's constant, T is temperature, ε is the energy, μ_c is the chemical potential, and Γ is the phenomenological scattering rate. The first term in (1) is due to intraband contributions, which can be evaluated in closed form [8],

$$\sigma_{intra}=-j\frac{e^2k_BT}{\pi\hbar^2\left(\omega-j2\Gamma\right)}\times\left(\frac{\mu_c}{k_BT}+2\ln\left(e^{-\mu_c/k_BT}+1\right)\right)\quad(2)$$

and the second term is due to interband contributions approximated for $k_BT \ll |\mu_c|, \hbar\omega$ as [8],

$$\sigma_{inter}=\frac{-je^2}{4\pi\hbar}\ln\left(\frac{2|\mu_c|-\left(\omega-j2\Gamma\right)\hbar}{2|\mu_c|+\left(\omega-j2\Gamma\right)\hbar}\right).\quad(3)$$

From the above two equations it is found that in far-infrared region, the contribution due to the interband electron transition is negligible [8], and the surface conductivity of graphene depends predominantly on intraband transitions (given by (2)), and is complex-valued with a negative imaginary part. This in turn corresponds to the surface impedance of graphene monolayer $Z_s = 1/\sigma$, which at low terahertz frequencies behaves as a low-loss inductive surface due to small values of $\mathrm{Re}\{\sigma\}$ and $\mathrm{Im}\{\sigma\} < 0$.

With the graphene sheet characterized by a complex surface conductivity, and since the interaction in a graphene dielectric stack is by plane-wave reflection and transmission (no higher-order modes are excited), the transmissivity, $|T|^2$, of the graphene-dielectric stack can be obtained by applying the two-sided impedance boundary condition at the graphene-dielectric interfaces [8] with the use of a transfer matrix approach for dielectric layers. In this analysis it is assumed that the lateral dimensions of the graphene are greater than few tens of μm (i.e., much greater than the mean-free path of electrons).

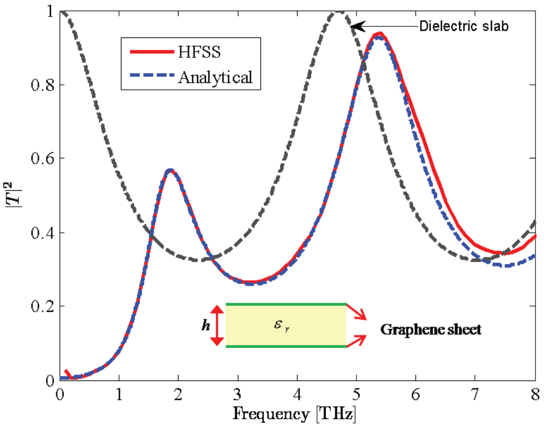

Fig. 2. Comparison of analytical and full-wave HFSS results of the transmissivity, $|T|^2$, for a two-sided graphene structure with a plane wave incident at normal incidence. Structural parameters used: $h = 10$ μm, $\varepsilon_r = 10.2$, and $\mu_c = 0.5$ eV.

III. RESULTS

First we consider the case of a dielectric layer (with $h = 10$ μm and $\varepsilon_r = 10.2$) sandwiched between graphene sheets (two-sided graphene structure). The analytical results of the transmissivity are depicted in Fig. 2, showing very good agreement with the full-wave results obtained with HFSS [9]. It can be seen that a transmission resonance appears at low frequencies (when compared to the typical Fabry-Pérot (FP) resonance of the dielectric slab), and is associated with the FP type resonance of the dielectric slab loaded with graphene sheets. It is observed that with a further increase in the number of identical layers, there are as many peaks (transmission resonances) as the number of the dielectric slabs, and all the peaks lie in a characteristic frequency band. The analytical results of the transmissivity of four- and eight-layer graphene structure are depicted in Fig. 3, showing the observed phenomena. It should be noted that the characteristic frequency band is dependent on the geometrical parameters of the unit cell (dielectric slab and graphene sheet), but not on the overall length of the structure.

As an example, we consider the case of the four-layered graphene structure, with the same parameters of the dielectric layer used in the calculations of Fig. 2, but with different values of chemical potentials μ_c (electrostatic bias) for the graphene sheets. The analytical calculations for the transmission response of the structure are depicted in Fig. 4, showing a very good agreement with the numerical results. It can be observed that there is no significant change in the frequency corresponding to the upper band edge for $\mu_c = 0.5$ eV and $\mu_c = 1$ eV, however, there is a considerable shift in the frequency corresponding to the lower-band edge. Also, it is noticed that the upper

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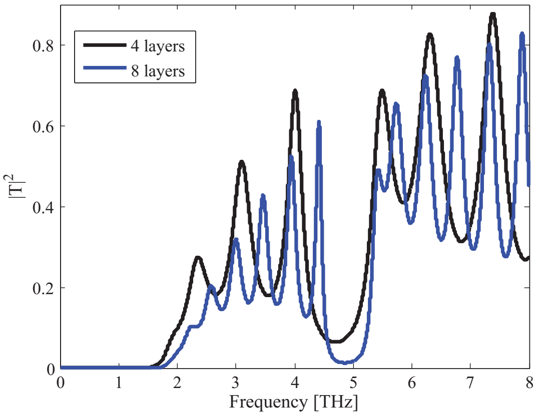

Fig. 3. Transmissivity, $|T|^2$, of four-layer and eight-layer graphene-dielectric stack structures. Structural parameters used: $h = 10$ μm, $\varepsilon_r = 10.2$, and $\mu_c = 1$ eV.

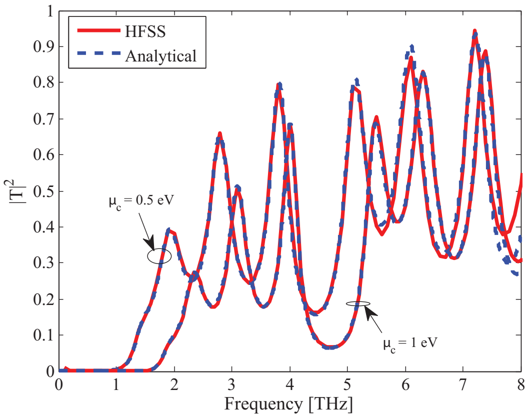

Fig. 4. Comparison of analytical and full-wave HFSS results of the transmissivity, $|T|^2$, for a four-layer graphene-dielectric stack for $\mu_c = 0.5$ eV and $\mu_c = 1$ eV. Structural parameters used: $h = 10$ μm, and $\varepsilon_r = 10.2$.

frequency band edge is the FP limit of the single dielectric layer, and the lower-band edge depends largely on the graphene impedance. This observation is consistent with the theory reported in [3] for mesh grid-dielectric stack at microwaves. Thus, by varying the chemical potential of the graphene sheets (without changing the structural parameters), the transmission band (pass-band) of the structure can be controlled.

However, in order to grow graphene a thicker dielectric substrate is needed for mechanical handling. With a motive to have an experimental verification of the observed phenomena, we choose the parameters of the dielectric layers that are feasible. The analytical calculations of the transmission response of the four-layered graphene structure formed by thick dielectric slabs (with $h = 250$ μm and $\varepsilon_r = 2.2$) are shown in Fig. 5. It can be observed that the structure exhibits a series of bandpass regions separated by the bandgaps, i.e., series of pass-band and stop-band type characteristics.

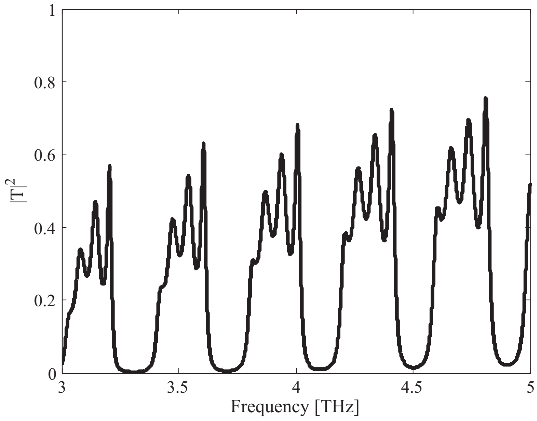

Fig. 5. Transmissivity, $|T|^2$, of a four-layer graphene-dielectric stack. Structural parameters used: $h = 250$ μm, $\varepsilon_r = 2.2$, and $\mu_c = 1$ eV.

IV. CONCLUSION

Transmission properties of various graphene-dielectric stacks have been analyzed at low-terahertz frequencies. The characteristics of the bandpass region (consisting of transmission peaks) of the transmission spectra are explained in terms of Fabry-Pérot resonances by correlating it to the bandpass regions of mesh grid-dielectric stack (studied at microwaves). The study has been carried using an analytical model, and the results are verified using the numerical simulations.

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