Generalized Additional Boundary Conditions and Analytical Model for Multilayered Mushroom-Type Wideband Absorbers

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***Abstract*—We present an analytical model to study the reflection properties of a multilayered wire media loaded with 2-D arrays of thin imperfect conductors. Based on charge conservation, generalized additional boundary conditions (ABCs) for the interface of two uniaxial wire mediums loaded with thin arbitrary imperfect conductors at the junction are derived. It is observed that by proper selection of the structural parameters, the mushroom structure acts as a wideband absorber for an obliquely incident TM-polarized plane wave. The presented model along with the new ABCs are validated using the full-wave numerical simulations.**

I. Introduction

In recent years, characterization of metamaterial structures, which constitute wire media has attracted special attention due to their ability in enabling anomalous phenomena such as negative refraction and sub-wavelength imaging, among others. It has been shown in [1] that wire media exhibits strong spatial dispersion at microwave frequencies, and that the constitutive relations between the macroscopic fields and the electric dipole moment are non-local. In [2]–[5] the role of spatial dispersion has been discussed and it was demonstrated that nonlocal homogenized models with additional boundary conditions (ABCs) become essential in solving electromagnetic problems associated with wire media.

In this work we study the reflection properties of a multilayered mushroom structure with thin (resistive) patches using an analytical model, which is an extension of the analysis presented in [5] for a single-layered mushroom. Based on charge conservation two-sided ABCs are derived at the interface of two uniaxial wire mediums with thin imperfect conductors at the junction. To illustrate the validity of the ABCs, we characterize the reflection properties of the multilayer structure, demonstrating that such a configuration with proper choice of the geometrical parameters acts as an absorber. Interestingly, it is noticed that the presence of vias results in the enhancement of the absorption bandwidth and an improvement in the absorptivity performance for increasing angles of the obliquely incident TM-polarized plane wave.

II. Analytical Model

The geometry of a multilayered mushroom structure is shown in Fig. 1 (a is the period of the patches and vias). It is assumed that each dielectric layer, perforated with thin lossless metallic vias of radius $r_l \ll a$, is homogeneous and isotropic of thickness h_l, characterized by relative permittivity $\varepsilon_{r,l}$ and permeability of free space, and loaded with 2-D periodic thin conductive patches of conductivity $\sigma_{2\mathrm{D},l}$ at the interface $d_l, l = 1, 2, \ldots, m$. Consider a time-harmonic plane wave incident on

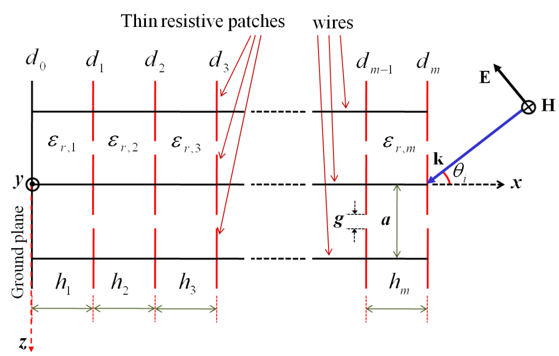

Fig. 1. Schematic of a multilayer mushroom structure formed by periodically loading grounded wire medium with thin metallic square patches.

the multilayer mushroom structure as shown in Fig. 1. Each wire-medium slab is characterized by the nonlocal dielectric function [2] $\varepsilon_{\mathrm{eff},l} = \varepsilon_0 \varepsilon_{r,l} \left[\varepsilon_{xx,l}(\omega, k_x)\hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z}\right]$, where $\varepsilon_{xx,l}(\omega, k_x) = 1 - k_{p,l}^2/(k_{h,l}^2 - k_x^2), k_{h,l} = k_0\sqrt{\varepsilon_{r,l}}$ is the wavenumber in the host material, k_0 is the wavenumber in free space, $k_{p,l}$ is the plasma wavenumber which depends on the period and radius of the vias: $k_{p,l}^2 = (2\pi/a^2)/[\ln(a/2\pi r_l) + 0.5275]$, and k_x is the x-component of the wave vector $\mathbf{k} = (k_x, 0, k_z)$. Let $J_{w,l}$ be the currents induced on the metallic wires. It is known that for a TM plane-wave incidence, the wire medium excites both transverse electromagnetic (TEM) and TM^x modes, and thus, following [2], the total magnetic fields in the air region ($x > d_m$) are given by:

$$\eta_0 H_y = \mathrm{e}^{\gamma_0 (x - d_m)} + R \mathrm{e}^{-\gamma_0 (x - d_m)} \quad (1)$$

and in the wire-medium slab ($d_{l-1} < x < d_l$) are written as,

$$\eta_0 H_y^{(l)} = A_{\mathrm{TM},l}^{+} \mathrm{e}^{\gamma_{\mathrm{TM},l}(x - d_{l-1})} + A_{\mathrm{TM},l}^{-} \mathrm{e}^{-\gamma_{\mathrm{TM},l}(x - d_{l-1})}$$

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$$+ B^{+}_{\text{TEM},l} e^{\gamma_{\text{TEM},l}(x-d_{l-1})} + B^{-}_{\text{TEM},l} e^{-\gamma_{\text{TEM},l}(x-d_{l-1})} \quad (2)$$

where $d_l = h_1 + h_2 + \ldots + h_l$, $l = 1, 2, \ldots, m$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the free-space impedance, R is the reflection coefficient, $A^{\pm}_{\text{TM},l}$, $B^{\pm}_{\text{TEM},l}$ are the amplitude coefficients of the TM and TEM fields, $\gamma_0 = \sqrt{k_z^2 - k_0^2}$, $\varepsilon^{\text{TM}}_{xx,l} = 1 - k_{p,l}^2/(k_z^2 + k_{p,l}^2)$, $\gamma_{\text{TEM},l} = \mathrm{j}k_0\sqrt{\epsilon_{r,l}}$, $\gamma_{\text{TM},l} = \sqrt{k_{p,l}^2 + k_z^2 - k_{h,l}^2}$, and $k_z = k_0 \sin\theta_i$.

To calculate the unknown coefficients, R, $A^{\pm}_{\text{TM},l}$, $B^{\pm}_{\text{TEM},l}$, we impose the following boundary conditions at $x = 0, d_1, d_2, \ldots, d_m$: 1. At the thin metal patch interfaces ($x = d_l^{\pm}$, $l = 1, 2, \ldots, m$), the macroscopic two-sided impedance boundary conditions establish that the tangential electric ($E_z^{(l)}$) and magnetic fields ($H_y^{(l)}$), can be related via sheet impedance [4]. 2. At the ground plane interface ($x = 0^+$), we have two boundary conditions [2]: i) tangential macroscopic total electric field vanishes ($E_z^{(1)}|_{x=0^+} = 0$) and ii) derivative of current is zero ($\mathrm{d}J_{w,1}(x)/\mathrm{d}x|_{x=0^+} = 0$), [3]. 3. At the top patch interface $x = d_m^-$, following [5], we have $J_{w,m}(d_m^-) + (\sigma_{2d,m}/\mathrm{j}\omega\varepsilon_0\varepsilon_{r,m})(\mathrm{d}J_{w,m}(x)/\mathrm{d}x)|_{d_m^-} = 0$. However, these conditions are not sufficient to calculate the unknown coefficients. Hence, new ABCs for the microscopic current are derived at the interface of two uniaxial wire mediums with thin metallic patches at the junction resulting in:

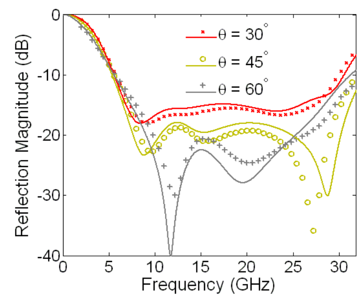
$$\frac{\sigma_{2d,l}}{2\mathrm{j}\omega\varepsilon_0}\left[\frac{1}{\varepsilon_{r,l}}\frac{\mathrm{d}J_{w,l}(x)}{\mathrm{d}x}\bigg|_{d_l^-} + \frac{1}{\varepsilon_{r,l+1}}\frac{\mathrm{d}J_{w,l+1}(x)}{\mathrm{d}x}\bigg|_{d_l^+}\right] + \left[J_{w,l}(d_l^-) - J_{w,l+1}(d_l^+)\right] = 0 \quad (3)$$

$$\frac{1}{\varepsilon_{r,l}}\frac{\mathrm{d}J_{w,l}(x)}{\mathrm{d}x}\bigg|_{d_l^-} - \frac{1}{\varepsilon_{r,l+1}}\frac{\mathrm{d}J_{w,l+1}(x)}{\mathrm{d}x}\bigg|_{d_l^+} = 0\,. \quad (4)$$

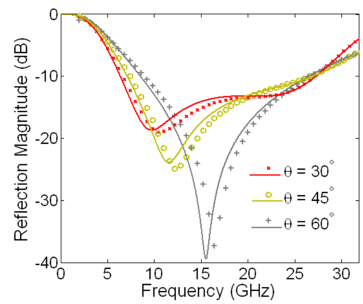
Using these boundary conditions (3) and (4), and the ones discussed above, we can easily obtain the reflection coefficient R.

III. Numerical Results and Discussions

In order to validate the new ABCs, here we study the reflection properties of the two-layer mushroom structure using the analytical model discussed in Section II. The absorber consists of two resistive patch arrays separated by dielectric slabs perforated with metallic vias, with a ground plane at the bottom. The parameters of the dielectric slabs used in the design together with the dimensions and sheet resistivity values (R_{s1} and R_{s2}) of the square patches are given in the caption of Fig. 2. Here R_s is the sheet resistance, which depends on the bulk conductivity ($\sigma_{3\text{D}}$, S/m) of the material, i.e., $R_s = 1/\sigma_{2\text{D}} = 1/(\sigma_{3\text{D}}t)$, with t being the material thickness. The analytical results obtained using the new ABCs (3) and (4) agree well with the full-wave HFSS [6] results shown in Fig. 2(a). In Fig. 2(b) we present the behavior of the reflection coefficient for a similar structure without vias. It is observed that for large angles of incidence there is an enhancement in the absorption bandwidth and an increase in the absorption with respect to the structure without vias.



(a)



(b)

Fig. 2. Comparison of analytical (solid lines) and full-wave HFSS results (crosses, circles, and plus signs) of the reflection coefficient for the two-layer structure excited by a TM-polarized plane wave at oblique angles of incidence θ_i: (a) with vias and (b) without vias. Structural parameters used in this work (with a general case shown in Fig. 1): $h_1 = h_2 = 3.2\,\text{mm}$, $\varepsilon_{r,1} = 2.2$, $\varepsilon_{r,2} = 1.33$, $r_1 = r_2 = 0.05\,\text{mm}$, $a_1 = a_2 = 5\,\text{mm}$, $g_1 = g_2 = 0.1\,\text{mm}$, $R_{s1} = 196\,\Omega$, $R_{s2} = 1078\,\Omega$.

IV. Conclusion

A simple analytical model has been presented to analyze the reflection properties of multilayered mushroom HIS structures with thin resistive patches. Additional boundary conditions for the double-sided junctions of wire media with thin resistive patches at the interface have been obtained. The new boundary conditions have been applied and verified for the two-layer mushroom-type structure using full-wave simulations.

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