Analytical Circuit Model for Dipole Frequency-Selective Surfaces

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***Abstract*— This contribution provides a fully analytical transmission-line circuit model for the transmission/reflection of an obliquely incident plane wave by a periodic array of printed dipoles. The topology of the equivalent circuit is rigorously derived in the analysis. In contrast with previously reported circuit models, the proposed approach accounts for dynamical effects that enable the application of the model in a very wide frequency range.**

***Index Terms*— Equivalent circuits, analytical models, frequency selective surfaces.**

I. Introduction

The study of the properties of periodic distributions of planar metallic scatterers embedded in layered dielectric media is a classical topic in the microwave and antenna literature. In fact, this is the description of a very usual type of frequency-selective surfaces (FSS) [1]. Physicists have also focused their attention on this type of problems due to their interest on extraordinary transmission, metasurfaces, and related problems [2]. The analysis and design of this class of structures is commonly carried out by means of general-purpose commercial electromagnetic solvers or in-house specific-purpose numerical codes. However, the availability of analytical solutions adds physical insight and provides apparent advantages for analysis/design purposes (for instance, when optimization algorithms need to be applied for designing specific structures). Thus, a lot of effort has been devoted to the obtaining of approximate analytical solutions for a variety of geometries [3], [4]. Analytical solutions leading to equivalent circuit models are very convenient and useful for extraordinary transmission periodic structures [5], planar metallic gratings [6] or FSS-like systems [7], [8]. In [5], [6], [8] very accurate and wideband circuit models were reported to account for dynamic effects that had not been incorporated in previous circuit-like models. However, in these papers and in many other on this topic, some of the essential parameters of the circuit models had to be retrieved from full-wave numerical simulations at certain frequency points. In the present contribution we propose a fully analytical model that avoids the need for numerical simulators. It will be shown that our fully analytical solution completely captures the physics of the problem over a very wide frequency region, even inside the grating lobe regime.

II. Analysis

The structure under analysis in this work is a periodic dipole FSS printed on a dielectric substrate [see Fig. 1(a)].

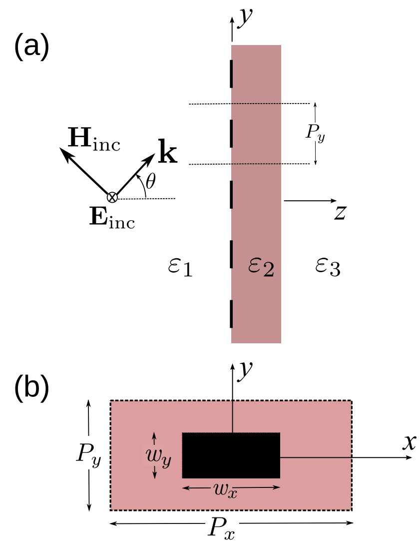

Fig. 1. (a) Structure under study. (b) Unit cell of the considered 2-D periodic structure.

The obliquely incident field is assumed to be TE polarized (the amplitude of the electric field is assumed to be unity). Other layered environments and field polarizations can be treated in a similar way to that described in this work. The presence of a ground plane can also be incorporated in the analysis in a straightforward way.

The periodic nature of the problem here considered allows for the following Floquet expansion of the electric field in the plane containing the printed dipoles ($z = 0$):

$$\mathbf{E}(x,y) = (1+R)\mathrm{e}^{-\mathrm{j}k_{y0}y}\hat{\mathbf{x}} + \sum_{(n,m)\neq(0,0)} \mathbf{E}_{nm}^{\rm TE}\mathrm{e}^{-\mathrm{j}(k_n x + k_{ym} y)} + \sum_{nm,n\neq 0} \mathbf{E}_{nm}^{\rm TM}\mathrm{e}^{-\mathrm{j}(k_n x + k_{yn} y)} \quad (1)$$

where the indexes TE/TM stand for the field polarization, and

$$k_n = \frac{2\pi n}{P_x} \quad (2)$$

$$k_{ym} = k_0 \sin\theta + \frac{2\pi m}{P_y}\ . \quad (3)$$

[Note that the incident field can be identified as the TE_{00} harmonic.] The electric field is continous in this plane, but the magnetic field must be expanded at both sides of the $z = 0$ plane as follows

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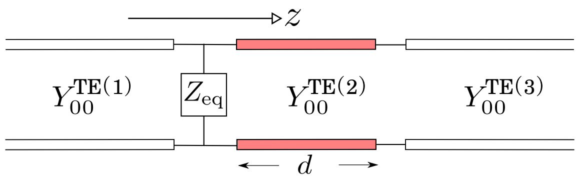

Fig. 2. Equivalent transmission-line circuit of the scattering problem under analysis.

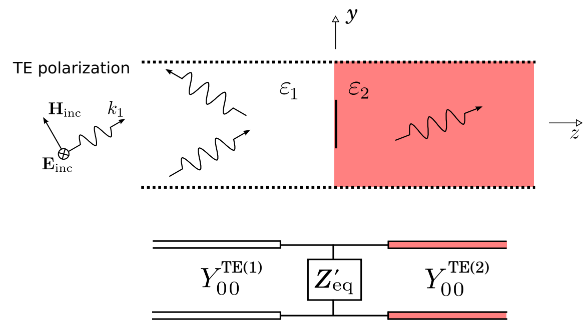

Fig. 3. FSS embedded between two semiinfinite media.

$$\mathbf{H}^{(1)}(x,y) = Y_{00}^{\mathrm{TM}(1)}(1-R)\mathrm{e}^{-\mathrm{j}k_{y0}y}\hat{\mathbf{y}} + \sum_{(n,m)\neq(0,0)} \mathbf{H}_{nm}^{\mathrm{TE}(1)}\mathrm{e}^{-\mathrm{j}(k_n x + k_{ym} y)} + \sum_{nm,n\neq 0} \mathbf{H}_{nm}^{\mathrm{TM}(1)}\mathrm{e}^{-\mathrm{j}(k_n x + k_{yn} y)} \tag{4}$$

$$\mathbf{H}^{(2)}(x,y) = Y_{00}^{\mathrm{TM}(1)}(1+R)\mathrm{e}^{-\mathrm{j}k_{y0}y}\hat{\mathbf{y}} + \sum_{(n,m)\neq(0,0)} \mathbf{H}_{nm}^{\mathrm{TE}(2)}\mathrm{e}^{-\mathrm{j}(k_n x + k_{ym} y)} + \sum_{nm,n\neq 0} \mathbf{H}_{nm}^{\mathrm{TM}(2)}\mathrm{e}^{-\mathrm{j}(k_n x + k_{yn} y)} \tag{5}$$

where

$$\mathbf{H}_{nm}^{(i)} = \pm Y_{nm}^{(i)}(\hat{\mathbf{z}} \times \mathbf{E}_{nm}) \tag{6}$$

with the sign $+/-$ corresponding to medium (2)/(1). The modal admittances are given by

$$Y_{nm}^{(i)} = \begin{cases} \dfrac{\beta_{nm}^{(i)}}{\omega\mu_0} & \text{TE harmonics} \\[2ex] \dfrac{\omega\varepsilon_i}{\beta_{nm}^{(i)}} & \text{TM harmonics} \end{cases} \tag{7}$$

with

$$\beta_{nm}^{(i)} = \sqrt{\varepsilon_{r,i}k_0^2 - k_n^2 - k_{ym}^2} \; . \tag{8}$$

The original scattering problem in Fig. 1(a) can then be reduced to the analysis of the scattering in the unit cell shown in Fig. 1(b); namely, an equivalent waveguide problem characterized by horizontal walls of Floquet type and vertical ones imposing perfect electric conductor conditions. In this waveguide the incident fundamental TE_{00} mode is scattered by a metallic patch printed on a dielectric of thickness d. The equivalent transmission-line model for the scattering of this incident mode is shown in Fig. 2. Our problem is then to find an expression for Z_{eq} and also to derive the topology of the network that corresponds to this equivalent impedance.

For that purpose we first consider the problem shown in Fig. 3, where the FSS is considered to be embedded between two semi-infinite media. We propose the following key approximation for the surface current density on the metallic patch:

$$\mathbf{J}_{\mathrm{p}}(x,y;\omega) = A(\omega)\mathbf{j}_{\mathrm{p}}(x,y) \; . \tag{9}$$

This approximation implies that the spatial profile of the patch current is independent of the frequency. Although this may appear rather restrictive, it is found to be suficiently accurate for single-resonant elements in a wide frequency range (which usually extends even well inside the grating-lobe regime). In the present case, and due to the rectangular geometry of the patches under study and the considered polarization, we have chosen the following spatial profile:

$$\mathbf{j}_{\mathrm{p}}(x,y) = \sqrt{\frac{1-(2x/w_x)^2}{1-(2y/w_y)^2}}\;\hat{\mathbf{x}} \; . \tag{10}$$

Now, using (10) and the expansions (4) and (5) in the magnetic-field jump condition at the metallic patch,

$$\mathbf{J}_{\mathrm{p}} = \hat{\mathbf{z}} \times (\mathbf{H}^{(2)} - \mathbf{H}^{(1)}) \; , \tag{11}$$

we can find expressions of the $\mathbf{E}_{nm}$ coefficients in terms of the reflection coefficient R [the details of the derivation are omitted here]. These expressions can be used to eliminate the $\mathbf{E}_{nm}$ coefficients from the electric field expansion in (1). Next, we project the boundary condition for $\mathbf{E}$ onto the surface current density to reach the integral equation

$$\int_{\mathrm{patch}} \mathbf{J}_{\mathrm{p}}^* \cdot \mathbf{E}\,\mathrm{d}S = 0 \; , \tag{12}$$

from which we can finally obtain the following analytic expression for the equivalent impedance, Z'_{eq}, of the problem considered in Fig. 3:

$$Z'_{\mathrm{eq}} = \sum_{(n,m)\neq(0,0)} \frac{F_{nm}^{\mathrm{TE}}}{Y_{nm}^{\mathrm{TE}(1)} + Y_{nm}^{\mathrm{TE}(2)}} + \sum_{nm,n\neq 0} \frac{F_{nm}^{\mathrm{TM}}}{Y_{nm}^{\mathrm{TM}(1)} + Y_{nm}^{\mathrm{TM}(2)}} \tag{13}$$

where

$$F_{nm}^{\mathrm{TE}} = 4\left[\frac{J_1(k_n\frac{w_x}{2})J_0(k_{ym}\frac{w_y}{2})}{k_n\frac{w_x}{2}J_0(k_{y0}\frac{w_y}{2})}\right]^2 \frac{k_{ym}^2}{k_n^2 + k_{ym}^2} \tag{14}$$

$$F_{nm}^{\mathrm{TM}} = \frac{k_n^2}{k_{ym}^2}F_{nm}^{\mathrm{TE}} \; . \tag{15}$$

In order to obtain the equivalent impedance, Z_{eq}, of the problem in Fig. 2 (namely, to take into account the presence of a layered environment as well as the eventual existence of a ground plane) we follow a procedure very similar to that

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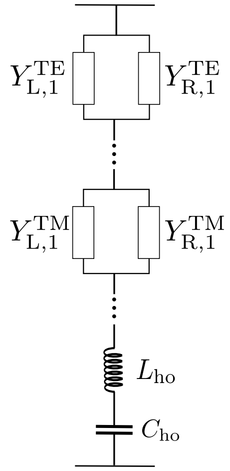

Fig. 4. Topology of the equivalent impedance.

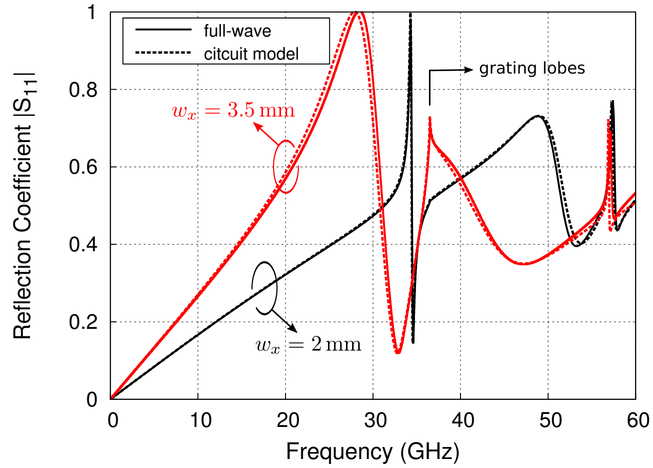

Fig. 5. Magnitude of the reflection coefficient under 40° incidence for the structure of Fig. 1 with $\varepsilon_{r1} = \varepsilon_{r3} = 1$, $\varepsilon_{r2} = 3$, $d = 0.5\,\mathrm{mm}$, $P_x = P_y = 5\,\mathrm{mm}$ and $w_y = 0.5\,\mathrm{mm}$.

explained in [9]. In this procedure we should consider that the admittance of the TE/TM harmonics appearing in (13) is no longer that provided in (7) but the *input* admittance seen by the corresponding harmonic to the right/left of the discontinuity. In [9] it is also suggested to split out the infinite series so that we can finally write the equivalent impedance as (see Fig. 4)

$$Z_{\mathrm{eq}} = \sum_{i=1}^{N_{\mathrm{TE}}} \frac{1}{Y_{\mathrm{L},i}^{\mathrm{TE}} + Y_{\mathrm{R},i}^{\mathrm{TE}}} + \sum_{i=1}^{N_{\mathrm{TM}}} \frac{1}{Y_{\mathrm{L},i}^{\mathrm{TM}} + Y_{\mathrm{R},i}^{\mathrm{TM}}} + \mathrm{j}\omega L_{\mathrm{ho}} + 1/\mathrm{j}\omega C_{\mathrm{ho}} \quad (16)$$

where C_{ho} and L_{ho} are frequency-independent capacitance and inductance that account for the reactive energy stored in the very higher-order TM and TE modes. It should be pointd out that these parameters should not be directly identified with the quasi-static parameters defined in other works [3], [7]. In the present case, these parameters incorporate the influence of the layered environment (or the eventual presence of a ground plane) in a rigurous way. The number of higher-order TE and TM harmonics with lowest cutoff frequency whose complete frequency-dependent contribution is considered (N_{TE} and N_{TM}, respectively) can be taken as one plus the number of propagative harmonics inside the dielectric layer at the highest frequency of interest.

III. Results

In this section we compare our analytical equivalent-circuit results with numerical data provided by a commercial electromagnetic simulator. It should be noted that, in our approach, we have to compute two double infinite series with poor convergence. However, due to the decomposition shown in (16), these series have to be computed only once. Even without the use of any numerical treatment to speed up the convergence of the series, the CPU time required by a standard laptop to compute our 1000 frequency data shown in each curve of Fig. 5 is below 1 second. It can be observed a very good agreement between our analytical results and the numerical data, even inside the grating lobe regime. Our approach has been checked in other situations and a similar good agreement has been obtained.

IV. Conclusion

A fully analytical solution for the scattering problem posed by a periodic 2D array of metal dipoles printed on a dielectric slab has been provided. The solution is expressed in the form of an equivalent circuit whose parameters are known analytically. The model considers that the spatial current profile over the printed dipoles does not change as frequency varies, which is found to provide accurate values of the scattering parameters in a very wide frequency band. In contrast with usual circuit models, our model incorporates dynamic effects associated with the frequency-dependent contributions of the first few higher-order modes with the lowest cutoff frequencies.

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