A COMPARISON OF RESIDUAL DEMAND MODELS FOR OLIGOPOLISTIC MARKETS

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Abstract - In many pool-based electricity markets two types of generating companies coexist, namely price-taking and leader companies. In this context, the optimal bidding of a leader company, and consequently the market clearing price, is mainly determined by its residual demand curve, which can be modeled in different ways. In this work, three different residual demand curves have been adopted to simulate the optimal strategy of an oligopolistic company. The influence of these models on the gap between the expected and actual market prices, as well as on the resulting leader company profit is analyzed.

Keywords - competitive electricity markets, nonlinear programming, residual demand.

1 INTRODUCTION

IN oligopolistic markets one or very few generating companies have the ability to manipulate the market-clearing prices. For this purpose, they resort to the so-called residual demand curves, relating the market-clearing price, for a given hour, with their own power production [1]. A generating company exercises market power when it reduces its output or raises the price at which it is willing to sell its energy in order to change the market-clearing price. Different models have been adopted to materialize this kind of curves, such as: Taylor series expansion [2], piecewise-linear approximation [3, 4] and multi-step curves [1]. However, as those curves are necessarily built from incomplete and aggregated information, where relevant individual constraints affecting market agents are ignored, there will always exist a gap between the optimal prices expected by the leader company and actual market-clearing prices determined by the market operator on the basis of all bids submitted to the pool. The motivation for this work is to show how the model adopted to represent the residual demand curves influences the market-clearing prices, when the leader company is willing to exercise its market power.

The reminder of the paper is organized as follows, Section 2 summarizes the mathematical model used to simulated the leader generating company, presented as a non-linear optimization problem, and the Market Operator problem, presented as a complex auction optimization problem based on net social profit. Section 3 shows the methodology used to compare the optimal prices and the market-clearing prices. In Section 4 a numerical test and the results obtained are presented. Finally, Section 5 presents several conclusion derived from this work.

2 OPTIMIZATION MODELS

In this section, the optimization problems respectively faced by the leader generating company and market operator are briefly summarized.

2.1 Leader Generating Company Model

The profit function is defined as the total company income minus the total costs. The goal is then to determine the production that maximizes the total benefit

$$\max \quad \sum_{h=1}^{24} \Big\{ \lambda^h(\phi^h)\cdot\phi^h - \sum_{j=1}^{n_j}\left(CP_j^h + CU_j\cdot a_j^h + CS_j\cdot s_j^h\right)\Big\} \quad (1)$$

where $\lambda^h(\phi^h)$ represents the residual demand function, ϕ^h is the total energy produced at hour h, n_j represents the number of generating units belonging to the leader company, CP_j^h is the variable production cost of unit j at hour h, CU_j and CS_j are the start-up and shut-down costs of unit j, respectively. Finally, a_j^h and s_j^h are binary variables which are null if unit j is not starting up or shutting down at the beginning of hour h, respectively.

Apart from the expressions characterizing the residual demand curves, the above maximization problem is subject to a set of constraints, which can be defined in terms of piecewise-linear equations [5][1]:

- Total generation, ϕ^h, is the sum of individual productions,

$$\phi^h = \sum_{j=1}^{n_j} P_j^h \quad (2)$$

  where P_j^h is the average output of unit j at hour h.

- Upper and lower generation limits for each unit,

$$P_j^m \cdot x_j^h \le P_j^h \le P_j^M \cdot x_j^h \quad (3)$$

  where P_j^m and P_j^M are the minimum and maximum power output of unit j, and x_j^h is binary variable which is null if unit j is not committed at hour h.

- Logical relationships among status changes,

$$x_j^h - x_j^{h-1} = a_j^h - s_j^h \quad (4)$$

$$a_j^h + s_j^h \le 1 \quad (5)$$

- Maximum up and down ramps,

$$-RD_j \leq P_{j,h} - P_{j,h-1} \leq RU_j \tag{6}$$

where RU_j and RD_j are the ramp-up and ramp-down limit of unit j, respectively.

- Variable production cost,

$$Cp_j^h(P_j^h) = fc_j \cdot x_j^h + \sum_{k=1}^{n_c} m_{j,k} \cdot \sigma_{j,k}^h \tag{7}$$

$$P_j^h = P_j^m \cdot x_j^h + \sum_{k=1}^{n_c} \sigma_{j,k}^h \tag{8}$$

$$0 \leq \sigma_{j,k}^h \leq p_{j,k}^{max} \tag{9}$$

where $Cp_j^h(P_j^h)$ is the piecewise linear cost function of unit j at hour h, fc_j is the fixed cost of unit j, n_c represents the number of segments of the piecewise linear cost function, $\sigma_{j,k}^h$ is a positive variable that represents the power of segment k at hour h, $m_{j,k}$ represents the slope of the segment k, and $p_{j,k}^{max}$ is the upper limit of segment k of the cost function of unit j.

- Hydraulic power output versus water discharge,

$$P_j^h(Q_j^h) = P_j^m x_j^h + \sum_{l=1}^{n_l} mq_{j,l} \cdot \alpha_{j,l}^h \tag{10}$$

$$Q_j^h = q_j^m \cdot x_j^h + \sum_{l=1}^{n_l} \alpha_{j,l}^h \tag{11}$$

$$0 \leq \alpha_{j,l}^h \leq q_{j,l}^{max} \tag{12}$$

where $P_j^h(Q_j^h)$ is the piecewise linear output function of hydro unit j at hour h, Q_j^h is the water discharge, q_j^m is the minimum water discharge, n_l represents the number of segments of the piecewise linear output function, $\alpha_{j,l}^h$ is a positive variable that represents the water discharge of segment l at hour h, $mc_{j,l}$ is the slope of the segment l of the piecewise linear function, and $q_{j,l}^{max}$ is the upper limit of segment l of the piecewise linear output function of hydro plant j.

- Reservoir spill-out as a function of the reservoir volume,

$$S_j^h(V_j^h) = \sum_{m=1}^{n_m} ms_{j,m} \cdot \psi_{j,m}^h \tag{13}$$

$$V_j^h = \sum_{m=1}^{n_m} \psi_{j,m}^h \tag{14}$$

$$0 \leq \psi_{j,m}^h \leq v_{j,m}^{max} \tag{15}$$

where $S_j^h(V_j^h)$ is the piecewise linear spill function of reservoir j at hour h, V_j^h is the water volume of reservoir j at the end of hour h, $ms_{j,m}$ is the slope of the segment m of the piecewise linear spill function, $\psi_{j,m}^h$ is a positive variable that represents the spillage rate of segment m, and $v_{j,m}^{max}$ represents the upper limit of segment v of the piecewise linear spill function of reservoir j.

- Water dynamic balance with travel delay,

$$V_j^h = V_j^{h-1} + r_j^h + Q_{j-1}^{h-\tau_j} + \\ + S_{j-1}^{h-\tau_j} - Q_j^h - S_j^h \tag{16}$$

$$V_j^m \leq V_j^h \leq V_j^M \tag{17}$$

where r_j^h in the net inflow to the reservoir j during hour h, τ_j is the water delay time in hours between reservoir j and the next reservoir downstream, V_j^m and V_j^M are the minimum and maximum reservoir volumen limits, respectively.

The above nonlinear maximization problem is solved using the commercial package DICOPT [7] under GAMS [8].

2.2 *Market Operator Model*

In a market where network losses and constraints can be neglected, the market price is determined by maximizing the total surplus of generators and consumers. The objective function is defined as

$$\max_{\varphi,\delta} \sum_{h=1}^{24} \left\{ \sum_{i\in I}\sum_{d\in D} \varphi_{i,d}^h \cdot PC_{i,d}^h - \sum_{j\in J}\sum_{b\in B} \delta_{j,b}^h \cdot PG_{j,b}^h \right\} \tag{18}$$

where J is the set of generating units, I is the set of consumers, D is the set of energy blocks offered by consumers, B is the set of energy blocks offered by producers, $\varphi_{i,d}^h$ is the amount of energy corresponding to block d of consumer i at hour h, $PC_{i,d}^h$ is the associated energy price, $\delta_{j,b}^h$ is the amount of energy corresponding to block b of producer j at hour h and $PG_{j,b}^h$ is the corresponding energy price.

The objective function is subject to the following constraints ($h = 1, \ldots, 24$) [6]:

- At any time, the total energy sold has to be equal to the total energy bought. Also, each block of cleared energy must not exceed the amount of energy offered by each producer or consumer for that block.

$$\sum_{i\in I}\sum_{d\in D} \varphi_{i,d}^h = \sum_{j\in J}\sum_{b\in B} \delta_{j,b}^h \tag{19}$$

$$\varphi_{i,d}^h \leq Qc_{i,d}^h \tag{20}$$

$$\delta_{i,d}^h \leq Qg_{i,d}^h \tag{21}$$

where $Qc_{i,h,d}$ is the energy that consumer i is willing to buy at hour h of block d, and $Qg_{j,h,b}$ is the energy that unit j is willing to produce at hour h of block b.

- Upper and lower limits for each unit and consumer,

$$P_j^m \cdot x_j^h \le P_j^h = \sum_{b=1}^{B} \delta_{j,b}^h \le P_j^M \cdot x_j^h \qquad (22)$$

$$C_i^m \le C_i^h = \sum_{d=1}^{D} \varphi_{i,b}^h \le C_i^M \qquad (23)$$

where $P_{j,h}$ is the energy of unit j at hour h, and $C_{i,h}$ is the energy served to consumer j at hour h.

- Logical relationships among status changes of units,

$$x_j^h - x_j^{h-1} = a_j^h - s_j^h \qquad (24)$$
$$a_j^h + s_j^h \le 1 \qquad (25)$$

where a_j^h, and s_j^h are binary variables which are equal to one if unit j is started-up or shut-down at the beginning of hour h, respectively.

- Maximum up and down ramps of units,

$$-RD_j \le P_{j,h} - P_{j,h-1} \le RU_j \qquad (26)$$

where RU_j and RD_j are the ramp-up and ramp-down limit of unit j, respectively.

The above optimization problem has been implemented and solved by the linear programming solver CPLEX [9], under GAMS commercial software [8].

3 ASSESSMENT METHODOLOGY

In this section, the methodology adopted to assess the differences between the optimal price and the market-clearing price is described. The optimal price is defined as the price determined by the leader company when it solves its own optimization problem, assuming the remaining agents are not able to exercise market power. The market-clearing price, on the other hand, represents the pool price obtained through the auction mechanism, when the leader company has submitted its strategic bid to exercise market power. It is assumed, like in the Spanish pool, that all agents pay/get the same price for the cleared energy they demand/offer, irrespective of their own bids for each block energy.

Figure 1 shows a flow diagram representing the major steps involved in the comparison process. The information about the rivals' bidding curves and the aggregated demand are assumed to be known. Consequently, it is possible to obtain the residual demand curves of the leader company for each hour of the bidding period. The residual demand is obtained as the difference between the aggregated demand and the rivals' bidding curves for each level of prices.

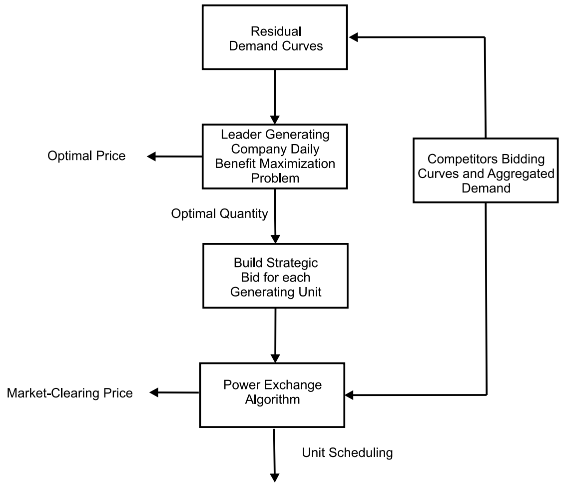

Figure 1: Methodology adopted to compare optimal and market-clearing prices.

Obtaining in practice the residual demand model for a given hour is an involved stochastic process which, based on historical data, leads to a set of sample points. In order to use this information within a deterministic optimization problem, the set of points is replaced or approximated by a certain analytical function. Usually, in simulation environments, we come up with a step-wise curve, whose step size is related to the energy block sizes, but this does not mean that the step-wise model is the true or best substitute in real life for the original cloud of points. The oligopolistic company will be interested in using the model for which the resulting market prices better match its expectations.

In order to find the optimal price and the optimal amount of energy that the leader company has to offer at each period, the daily benefit function described in Section 2.1 is maximized, where residual demand curves play a critical role. To exercise its potential market power the leader company must establish the prices for each of its energy blocks, in accordance with the total quantity of energy determined previously in the optimization process.

Then, knowing all generators' bids, including that of the leader company, and the aggregated demand, the complex auction algorithm described in Section 2.2 is used to obtain the energy blocks assigned to each generating unit. Afterwards, the market-clearing price, determined by the most expensive block of accepted energy, is compared with the optimal price, in order to assess the gap between both prices.

Although both prices should be rather similar, it is evident that they will not be necessarily equal, as a consequence of the different data handled by each optimization model. While the market operator knows all of the details associated with each individual bid, the leader company must rely to a set of residual demand curves in which the aggregated competitors' information is somehow embedded. In this regard, note that the residual demand curve for a given hour is not coupled with those of previous or future hours, whereas the detailed bids handled by the market operator are usually coupled along the 24-hour period.

4 CASE STUDY

The oligopolistic generation company simulated in this work is composed of eight thermal units plus two hydro plants, coupled by the same water stream, whose power-water characteristics, water dynamics and spill out behavior have been modeled as piece-wise linear functions.

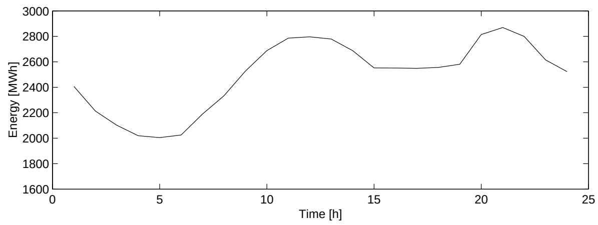

Figure 2: Hourly demand for the simulated market.

A 24-hour period for a day-ahead auction market has been considered. Figure 2 shows the total hourly demand corresponding to the scenario simulated. For simplicity, the demand is considered inelastic, which is equivalent to replacing the first term in (18) by a constant value D_h. In this study, the following residual demand curves have been modeled: 1) second-degree polynomial approximation; 2) piecewise linear approximation; 3) step-wise approximation. Such models are represented in Figure 3 for hour 21.

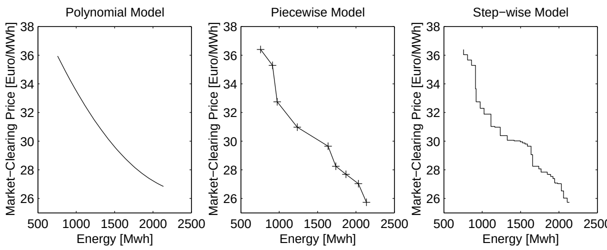

Figure 3: Residual demand approximations for the same hour.

4.1 Polynomial approximation

In this case, the residual demand curve is expressed as

$$\lambda^h(\phi) = \alpha_1^h + \alpha_2^h \phi^h + \alpha_3^h (\phi^h)^2$$

where the respective coefficients are obtained by second-order polynomial regression.

Figure 4 shows the optimal and market prices, along with the hourly gap between both prices. The average error is 1.13%, the largest errors taking place at hours 4, 5 and 6, when the energy cleared is smaller.

One of the sources which can justify in part the resulting error lies in the fact that the market-clearing algorithm takes into account the ramp limits of rival generating units, which are ignored by the leader company. In order to analyze the influence of this factor, the experiment is repeated after removing such constraints from the auction-based algorithm. Figure 5 shows that, as expected, the error decreases during hours 4, 5, and 6, the average error for the 24-hour period being just 0.84%. This result suggests that energy ramps, which do not explicitly appear in the aggregated bidding curves of rival generating units, constitute a crucial decision variable when formulating the optimal bidding problem of a leader company. Ramp limits will be considered in the sequel.

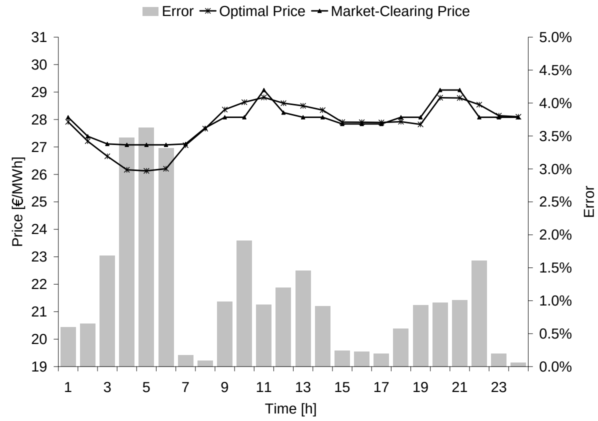

Figure 4: Prices and error for the polynomial model with ramp constraints.

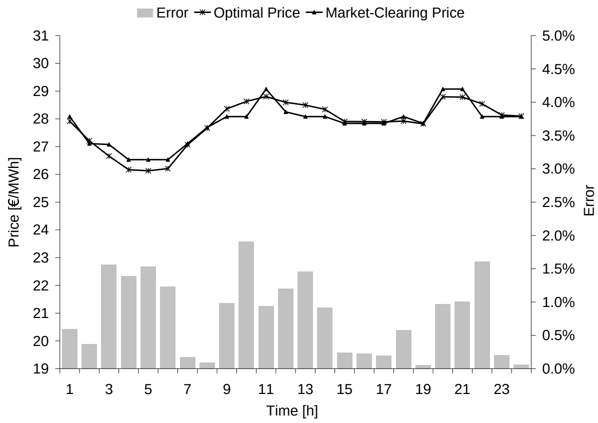

Figure 5: Prices and error for the polynomial model without ramp constraints.

4.2 Piecewise linear approximation

In this case, the residual demand curves have been approximated by a piecewise linear function comprising eight segments.

The comparison between the optimal and market-clearing prices is shown in Figure 6. Note that the market prices are smaller than the expected optimal prices, except for hours 4, 5, 15 and 19. The average error is this time 1.58%, the largest and smallest differences being 3.75% and 0.059% at hours 10 and 4 respectively.

4.3 Step-wise model

Figure 7 shows the difference between the optimal and market clearing prices when the residual demand curves are represented by a multi-step model. Compared to previous models, it is apparent that the error is much larger in this case at peak hours; in fact, it is null for the first 8 hours. The average error for the 24-hour period is 0.92%, compared to 1.13% for the polynomial regression and 1.93% for the piecewise linear approximation.

Therefore, it can be concluded that the multi-step model for residual demand curves provides a closer agreement between expected and resulting prices when the leader company is interested in exercising its potential market power. Accuracy of the piecewise linear approximation could be improved at the expense of increasing the number of linear intervals.

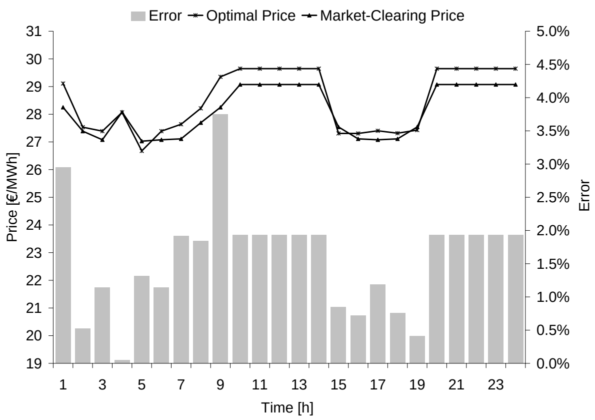

Figure 6: Prices and error for the piecewise linear model.

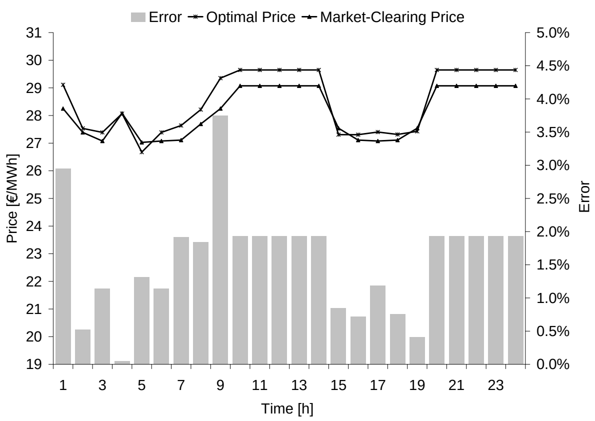

Figure 7: Prices and error for the step-wise model.

4.4 *Oligopolistic company profit and demand share*

Even more important than the gap between the market expected prices and the resulting prices are the noticeable differences among the hourly prices of Figures 4, 6 and 7, which may significantly affect the net income of all market agents, in particular that of the leader company. This is somewhat contrary to intuition, considering the minor quantitative differences among the three models adopted to represent the residual demand curves.

Table 1 compares the average market price, the total amount paid by consumers, the fraction of the total energy which is actually supplied by the leader company, as well as its profit, for the three residual demand models adopted. Note that the multi-step model yields both the largest daily profit and the largest percentage of market share for the leader company, whereas the piecewise linear model leads to opposite results.

Such differences can be partly attributed to the complexity and nonconvexity of the resulting optimization problems, comprising many integer variables and nonlinear constraints, which are hence prone to converge to different local optimum points. In this kind of applications, the optimization tools employed behave like black boxes, whose output is rather sensitive to minor model changes.

For comparison, the perfect market case, in which the leader company does not exercise market power, is included in the last row. It is worth noting that, in spite of the leader company serving a larger portion of the total load, its total profit is, as expected, much smaller in this case as a consequence of the lower clearing prices.

Model	Mean price [Euros/MWh]	Demand cost [Euros]	Market share [Euros]	Daily profit [Euros]
Polynomial	27.93	1,678,390	57.65	420,964
Piecewise	28.13	1,690,913	56.66	419,242
Multi-Step	27.89	1,677,260	59.46	427,054
Perf. Market	25.39	1,531,484	71.64	336,538

Table 1: Average market-clearing price, cost to consumers, leader company's market share and daily profit.

5 CONCLUSIONS

This paper analyzes the influence of the model adopted to represent the residual demand curves of an oligopolistic company, which is willing to exercise market power, on the clearing prices of a day-ahead auction-based energy pool. First, the nonlinear mixed-integer optimization problems respectively faced by the market operator and the leader company are briefly reviewed. Then, three residual demand curve models have been simulated to compare the gap between optimal prices predicted by the leader company and actual prices resulting from a complex auction process. It is shown that energy ramps of rival companies are partly responsible for these differences, which are smaller when the multi-step model is adopted. Finally, it is analyzed how such models affect the net profit and market share of the leader company.

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