

Volume fluctuations and compressibility of a vibrated granular gas

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The volume fluctuations in the steady state reached by a vibrated granular gas of hard particles confined by a movable piston on the top are investigated by means of event-driven simulations. Also, a compressibility factor, measuring the response in volume of the system to a change in the mass of the piston, is introduced and measured. From the second moment of the volume fluctuations and the compressibility factor, an effective temperature is defined by using the same relation as obeyed by equilibrium molecular systems. The interpretation of this effective temperature and its relationship with the granular temperature of the gas, and also with the velocity fluctuations of the movable piston, is discussed. It is found that the ratio of the temperature based on the volume fluctuations to the temperature based on the piston kinetic energy obeys simple dependencies on the inelasticity and on the piston-particle mass ratio.

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I. INTRODUCTION

Granular matter in general [1,2] and granular gases in particular [3–5] have been recently the object of intensive theoretical and experimental research. Not in the least this is because of the opportunity they offer to investigate fundamental questions of nonequilibrium physics. One of these issues is the nature and properties of nonequilibrium fluctuations and their relevance for the description of the macroscopic behavior of the system. The simplest fluctuations that can be considered are those of global properties of a system. The total-energy fluctuations of an isolated granular gas, modeled as an ensemble of smooth inelastic hard spheres, have already been investigated elsewhere [6,7]. They are specifics of granular systems and a consequence of the localized character of the energy dissipation in collisions.

In this paper, the volume fluctuations of a vibrated granular gas are studied by means of event-driven simulations [8,9]. The volume of the system changes because the wall on top of the grains is a piston that can move in the vertical direction. The equilibrium position of the piston is determined by equating its weight per unit of area with the pressure of the granular gas just below it. On the other hand, the properties of the fluctuations of the piston around that position, and therefore the volume fluctuations of the system, are not known. If the system were a molecular gas at equilibrium, therefore without being energized by vibration the total volume fluctuations are known to be Gaussian, and with a second moment that is proportional to the temperature and the isothermal compressibility of the gas [10]. There is no reason to expect the above properties to hold in granular gases that are inherently in nonequilibrium states. Even more, to give a meaning to them, a first point to be addressed is to specify the notion of temperature to be used. Actually, the equilibrium expression for the volume fluctuations can be used to define a temperaturelike quantity that can be experimentally measured. This is somehow an extension of the usual way of defining “effective temperatures” from the extension to nonequilibrium states of the fluctuation-dissipation theorem. A revision of these ideas in the context of granular media is given in [11]. See also [12] for a discussion of the

validity of the Einstein relation in externally driven granular gases.

The emerging natural question is whether the effective temperature defined from the relationship between volume fluctuations and compressibility has some other conceptual interpretations, and if it is related to other possible sensible definitions of temperature in the system. The first obvious candidate to be considered is the granular temperature of the gas, defined from the second moment of the velocity distribution of the grains. Actually, this parameter is known to play in the hydrodynamic description of granular gases a role similar to the usual temperature in molecular hydrodynamics [13]. But there is another temperature parameter that is relevant for the description of the system, the one defined from the second moment of the velocity distribution of the movable piston. Both temperatures, defined from the velocity of the gas and of the piston, respectively, are the same in equilibrium systems, but they can differ strongly in granular systems, as a manifestation of the violation of energy equipartition [14]. Clarifying the relationship among the effective temperature defined from the volume fluctuations, the granular temperature, and the temperature parameter of the piston is one of the aims of this work.

The structure of the remaining of this paper is as follows. In Sec. II, the system is described and the macroscopic steady state to be considered is characterized. Also, the parameter region to be investigated is specified. It is shown that when the mass of the piston is much larger than the mass of the grains, the volume fluctuations of the system exhibit a Gaussian distribution. The second moment of this distribution depends much stronger on the inelasticity of collisions between particles than on the elastic or inelastic character of the collisions of the particles with the piston. In Sec. III, a compressibility factor is defined for the granular gas. As usual, it is a measurement of the change in the volume of the system as a consequence of a change in the external pressure under well-defined constraints. From the values of the second moment of the volume fluctuations and of the compressibility factor, an effective temperature is defined, as indicated above. This temperature turns out to be quite simply related with the temperature parameter of the piston, but its relationship with the granular temperature of the gas ap-

pears to be very intricate. Finally, Sec. IV contains a short summary of the main results in the paper and some general comments.

II. STEADY FLUCTUATIONS OF THE POSITION OF THE PISTON

Consider a system composed of N smooth inelastic hard disks of mass m and diameter d , in the presence of gravity, and confined by a movable piston of mass M located on the top. By definition, the piston can only move in the direction of the gravity field. Moreover, there is no friction between the piston and the lateral walls of the vessel containing the gas. The system of particles is kept fluidized and at low density by injecting energy through the bottom wall, which is vibrating. Inelasticity in collisions between particles is modeled by means of a constant coefficient of normal restitution α , defined in the interval $0 < \alpha < 1$. Thus, when two particles i and j collide their velocities change instantaneously from the initial values $\mathbf{v}_i, \mathbf{v}_j$ to the postcollisional ones given by

$$\mathbf{v}'_i = \mathbf{v}_i - \frac{1 + \alpha}{2} (\hat{\boldsymbol{\sigma}} \cdot \mathbf{v}_{ij}) \hat{\boldsymbol{\sigma}}, \quad (1)$$

$$\mathbf{v}'_j = \mathbf{v}_j + \frac{1 + \alpha}{2} (\hat{\boldsymbol{\sigma}} \cdot \mathbf{v}_{ij}) \hat{\boldsymbol{\sigma}}, \quad (2)$$

where $\mathbf{v}_{ij} \equiv \mathbf{v}_i - \mathbf{v}_j$ is the relative velocity and $\hat{\boldsymbol{\sigma}}$ is the unit vector joining the centers of the two particles at contact. The z axis will be taken in the direction of the gravitational field, so that the particles are submitted to an external force of the form $\mathbf{f} = -mg_0 \hat{\mathbf{e}}_z$, with g_0 being a positive constant and $\hat{\mathbf{e}}_z$ being the positive unit vector along the z axis. Collisions of particles with the movable piston on the top are also considered as smooth and inelastic, with α_p being the coefficient of normal restitution for them. Therefore, in a collision between particle i and the piston, the component $v_{i,x}$ of the velocity of the particle perpendicular to the z axis remains unchanged,

$$v'_{i,x} = v_{i,x}, \quad (3)$$

while the component $v_{i,z}$ of the velocity of the particle and the velocity V_z of the piston are instantaneously modified accordingly with

$$v'_{i,z} = v_{i,z} - \frac{M}{m + M} (1 + \alpha_p) (v_{i,z} - V_z), \quad (4)$$

$$V'_z = V_z + \frac{m}{m + M} (1 + \alpha_p) (v_{i,z} - V_z), \quad (5)$$

respectively. When the transversal section of the system, i.e., the size of the piston W , is smaller than a critical value [15,16], the gas reaches, after a transient time interval, a stationary state with gradients only in the direction of the gravitational field. If, in addition, the inelasticity of the system is small, the inelastic hydrodynamic Navier-Stokes equations with the appropriate boundary conditions [17,18] provide an accurate description of the stationary state [19]. As the coefficient of normal restitution of the gas α decreases,

significant deviations from the predictions following from the Navier-Stokes equations show up. They are due to the coupling between inelasticity and gradients that exists in the stationary state, in such a way that strong inelasticity implies large gradients of the hydrodynamic fields. This coupling is a peculiarity of the steady states of inelastic fluids, following from the balance between the energy dissipated because of inelastic cooling and the hydrodynamic energy fluxes.

A previous analysis, carried out in [19], focused on the macroscopic description of the granular gas in terms of the density and granular temperature fields, with the velocity field being zero. At this level of description, the role of the movable piston on top of the gas is to partially determine the boundary conditions needed to solve the hydrodynamic equations for the steady state under consideration. Here, the interest will be on the fluctuations of the movable piston, namely, on its position fluctuations. Some results for the velocity fluctuations have been reported elsewhere [14]. There, it was shown that the steady-state velocity fluctuations of the piston are Gaussian with zero mean for $\alpha_p \geq 0.6$ and $\alpha \geq 0.8$. Nevertheless, no simple relationship between the second moments of the velocity distributions of the piston and the gas next to it was found. It is worth to remark that there is no reason to expect such a relation to exist at a macroscopic level of description, i.e., involving only the hydrodynamic fields and the parameters of the system. Actually, the simulation results reported in [14] indicate that the details of the velocity distribution of the gas, beyond its first few moments, are relevant to determine the second moment of the velocity distribution of the piston.

It is clear that the position fluctuations of the movable piston are related with the volume fluctuations of the inelastic gas. Actually, this relationship can be made direct and exact by properly choosing the nature of the vibrating wall located at the bottom of the system. The mission of the latter is to energize the system, keeping the particles fluidized. The expectation is that the behavior in the bulk of the system is independent of the details of the way in which this wall is being vibrated. Consequently, the simplest possible choice has been used in all the results to be reported in the following. The bottom wall is vibrated with a sawtooth velocity profile, having a velocity v_w . This means that all the particles colliding with the wall find it moving upward with that velocity [20,21]. Besides, the amplitude of the wall motion is considered much smaller than the mean free path of the particles in its vicinity, so that the position of the wall can be taken as fixed at $z=0$. Therefore, the dynamics of the vibrating wall at the bottom does not induce directly any change in the volume (or area) occupied by the granular gas. Also, and again for the sake of simplicity, collisions of the particles with this wall will be considered as elastic.

In the event-driven simulations carried out, periodic boundary conditions were used in the x direction. The width of the system and the number of particles in it were fixed to $W=70d$ and $N_z \equiv N/W=6d^{-1}$, respectively. Moreover, the velocity of the vibrating wall v_w was chosen in each case large enough, not only to fluidize the system, but also to guarantee that the density remains small throughout the granular gas and, consequently, the dilute limit can be expected to be accurate. In this case, the dependence of the hydrodynamic

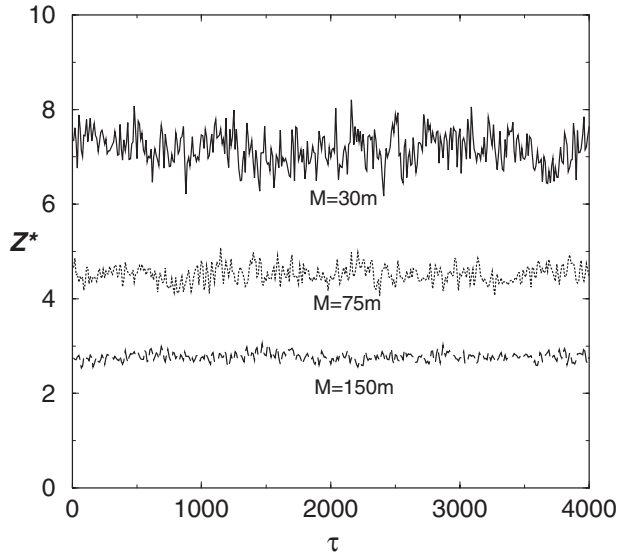


FIG. 1. Time evolution of the dimensionless scaled position $Z^* \equiv Zg_0/v_W^2$ of the movable piston located on top of the system, once the steady state has been reached. There are 420 disks in a box of width $W=70d$. In all cases the coefficient of normal restitution of the particle collisions is $\alpha=0.95$ and that for the particle-piston collisions is $\alpha_p=0.99$. Results for three values of the mass of the piston M are shown, as indicated. Time τ is measured in accumulated number of collisions per particle.

profiles on v_W is very simple and follows by dimensional analysis [19]. The dependence of the position fluctuations of the piston on this velocity will be discussed later on. The value of the coefficient of normal restitution for the collisions between particles has been varied within the interval $0.85 \leq \alpha < 1$. This includes a range of values for which the Navier-Stokes hydrodynamic description is not accurate due to the coupling between inelasticity and gradients already pointed out [19]. The coefficient of restitution for the collisions of the particles with the movable piston has been set to $\alpha_p=0.99$ in most of the simulations being reported, but it has been verified that the results depend very weakly on the value of this coefficient, remaining practically the same when α_p is decreased, at least up to $\alpha_p=0.8$. Some examples of this behavior will be given below.

In all the simulations being presented, it was observed that the height Z of the movable piston oscillates about an average value $\langle Z \rangle = L$, once the steady state is reached. As an example, in Fig. 1 the time evolution of the scaled position of the piston, $Z^* \equiv Zg_0/v_W^2$, is shown for $\alpha=0.95$, $\alpha_p=0.99$, and three choices of the mass of the piston: $M=30m$, $75m$, and $150m$. Time τ is measured in accumulated number of collisions per particle. It is observed that both the value of L and the amplitude of the fluctuations decrease as M increases. Of course, this is the expected behavior. Also notice that the trajectory of the piston does not exhibit systematic oscillations, but it is apparently random. This indicates that the motion of the piston does not have any hydrodynamic component induced, for instance, by the vibrating wall at the bottom.

From the steady trajectory of the piston, the probability distribution for its position can be built. To increase the sta-

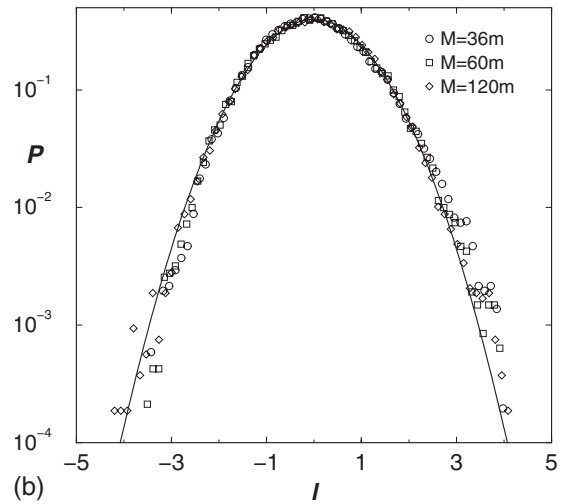
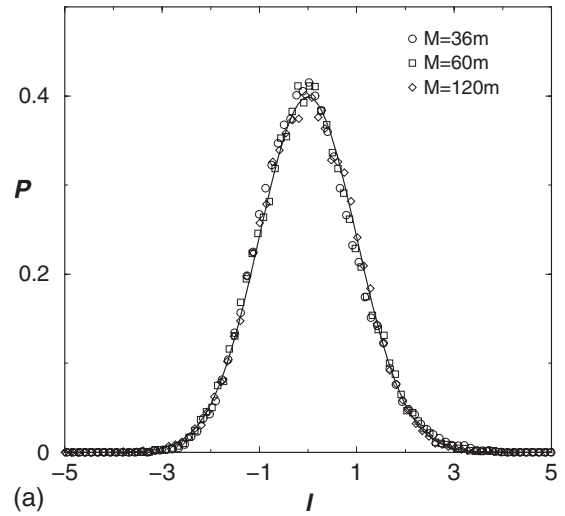


FIG. 2. Steady position distribution of the piston $P(l)$ in both normal and logarithmic scales. The symbols are from the simulations while the solid lines are Gaussian with unity dispersion. The data correspond to three systems differing in the mass of the piston, as indicated. The coefficient of normal restitution for the particle collisions is $\alpha=0.94$. The dimensionless length ℓ is defined in Eq. (6).

tistics, several trajectories have been generated for each set of values of the parameters. As already mentioned, the fluctuations of the piston depend on the parameters defining the system. To see whether this dependence occurs only through the first two moments of the probability distribution, a normalized length ℓ has been defined as

$$\ell \equiv \frac{Z-L}{\sigma_Z} = \frac{Z^*-L^*}{\sigma_Z^*}, \tag{6}$$

where σ_Z is the square root of the second central moment or standard deviation of Z , $\sigma_Z^2 \equiv \langle Z^2 \rangle - L^2$, and the star indicates that lengths are being measured in the dimensionless scale defined above. In Fig. 2, the obtained probability distribution of ℓ , $P(\ell)$, is plotted for a system with $\alpha=0.94$. Again, three values of the mass ratio have been considered, namely, $M=36m$, $60m$, and $120m$. It is seen that the probability distri-

TABLE I. Third and fourth moments of the position distribution of the piston, obtained from the simulations. The position ℓ is given in the dimensionless scale defined in Eq. (6).

α	α_p	M/m	$\langle \ell^3 \rangle$	$\langle \ell^4 \rangle$
0.98	0.99	24	0.250	3.083
		60	0.172	3.190
		120	0.0385	2.898
	0.8	24	0.254	3.230
		60	0.136	3.047
		120	0.057	2.922
0.94	0.99	24	0.242	3.138
		60	0.125	2.996
		120	0.047	3.006
	0.8	24	0.145	2.964
		60	0.213	3.088
		120	0.101	2.985

butions are accurately fitted by a Gaussian (solid lines), at least up to values of the probability density on the order of 10^{-4} . A similar behavior has been found in all the simulated systems with parameters within the ranges mentioned above, although it seems that a small but systematic deviation shows up as the mass of the piston M becomes smaller, approaching the mass of the particles. A possible explanation for this behavior is that, as the mass of the piston decreases, the amplitude of its position fluctuations increases, and the effect of the external gravitational field breaks the symmetry of the fluctuations around the average position. To check this idea and to quantify the deviations from the Gaussian of the position fluctuations, the third and fourth moments of ℓ have been computed from the simulation data. The results for some of the simulations are given in Table I. For a Gaussian distribution it is $\langle \ell^3 \rangle = 0$ and $\langle \ell^4 \rangle = 3$. The deviations of the third moment from the Gaussian value are much stronger than those of the fourth one, supporting the idea that the main cause of the deviation from the Gaussian is due to the symmetry breaking produced by the external field, when the mass of the piston is not much larger than the mass of the particles. In any case, the deviations are rather weak and it can be concluded that, in the explored parameter region, the position fluctuations of the piston can be considered as Gaussian with a very good accuracy.

In Ref. [19] it was shown that the values of the average position of the piston L scale with v_w^2 in the low-density limit and, consequently, L^* does not depend on the velocity of the vibrating wall in this limit. The extension of this result to volume fluctuations requires one to go beyond hydrodynamics. To investigate whether σ_Z^* also has the above scaling property, several series of simulations have been performed varying the value of v_w , while keeping constant all the other parameters. It is important to stress that v_w was always chosen large enough as to fluidize the system and to avoid the presence of regions with density above what is considered the low-density range. In all the cases investigated, there was no dependence of σ_Z^* on v_w , within the statistical uncertainties. Therefore, in the low-density limit, the standard deviation

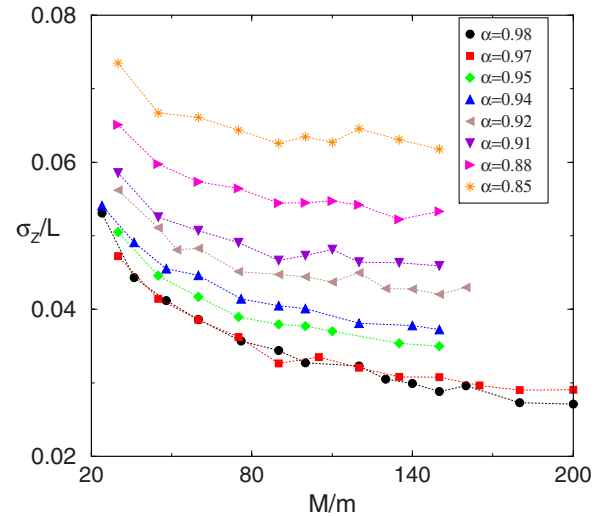


FIG. 3. (Color online) Dimensionless relative standard deviation σ_Z/L of the position of the movable piston as a function of the mass ratio M/m , for several values of the coefficient of normal restitution of the gas α , as indicated in the inset. The curves are guides for the eye. In all the cases, the coefficient of restitution for the gas-movable piston collisions is $\alpha_p = 0.99$.

of the piston position seems to scale with the square of the velocity of the vibrating wall, i.e., in the same way as the average position L .

In Fig. 3, the relative standard deviation $\sigma_Z/L = \sigma_Z^*/L^*$ is plotted as a function of the mass ratio M/m , for several values of the coefficient of normal restitution of the gas, α , in the interval $0.85 \leq \alpha \leq 0.98$. The coefficient of normal restitution for particle-movable piston collisions is in all the cases $\alpha_p = 0.99$. It is observed that, for a given α , there is a region in which σ_Z/L decreases as the mass ratio increases. This effect is less pronounced the smaller is the coefficient of restitution α , i.e., the more inelastic the collisions. For large values of M/m , the results in the figure indicate that σ_Z/L tends to a plateau with a constant value. The values of the mass ratio needed to reach the plateau monotonically decrease as α decreases. The data in Fig. 3 also indicate that, for a constant mass ratio, the relative fluctuations increase as α decreases.

To show that the influence of the inelasticity of the particle-movable piston collisions is much weaker than that of the particle-particle collisions, in Fig. 4 the relative standard deviation is plotted as a function of the mass ratio for two pairs of data. Each pair corresponds to the same value of α (namely, 0.98 and 0.94), but different values of α_p (namely, 0.8 and 0.99). Although the variation of α_p is almost five times the variation of α , it is seen that the data corresponding to the same α are much closer than those corresponding to the same α_p . On the other hand, there is a relevant qualitative feature to be stressed. While decreasing α produces an increase in the relative fluctuations, decreasing α_p has the opposite effect: the relative fluctuations also decrease.

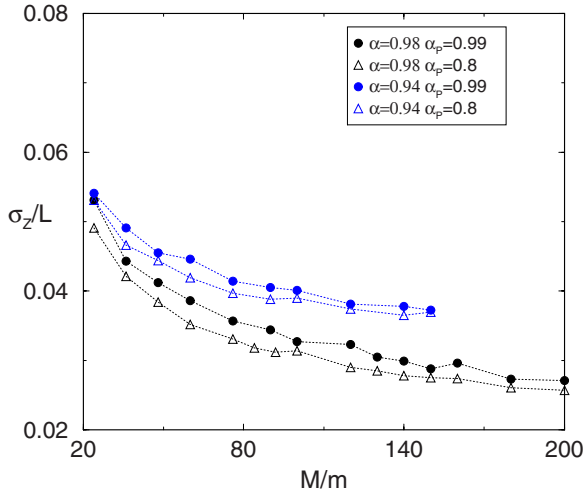


FIG. 4. (Color online) Dimensionless relative standard deviation σ_z/L of the position of the movable piston as a function of the mass ratio M/m . The symbols are simulation data and the lines are guides for the eye. The two upper curves correspond to $\alpha=0.94$, and the two lower ones correspond to $\alpha=0.98$. In each case, two values of α_p have been employed: 0.99 (circles) and 0.8 (triangles).

III. COMPRESSIBILITY AND EFFECTIVE TEMPERATURE

To measure the facility of the system to be compressed, define a coefficient of compressibility k by

$$k \equiv -\frac{1}{\langle V \rangle} \left(\frac{\partial \langle V \rangle}{\partial p_L} \right)_{v_W}, \quad (7)$$

where p_L is the pressure of the granular gas in the vicinity of the movable piston and V is the volume (area) of the system. The derivative in the above equation is computed at constant values of all the parameters defining the system, α , α_p , N_z , and v_W , except $p_L = Mg_0/W$, as it follows from the definition of the pressure. In the following, p_L will be modified by changing the mass of the piston M , keeping g_0 and W unchanged. The reason for this choice is twofold. First, changing g_0 is equivalent to modifying v_W and, second, increasing W can lead to the setup of transversal instabilities, as already mentioned. The notation stresses the constancy of the velocity of the vibrating wall. Although other compressibility coefficients could be defined, the one in Eq. (7) has the advantage of being easy to implement in experiments, at least at a conceptual level. On the other hand, it is worth to remark that by keeping v_W constant and changing M , both the hydrodynamic profiles inside the fluid and the power injected into it through the vibrating wall are modified. In particular, the latter is given by [19]

$$Q_0 = W \left(N_z + \frac{M}{mW} \right) mg_0 v_W. \quad (8)$$

Therefore, increasing M while keeping v_W constant produces an increase in the power Q_0 . For the two-dimensional systems being considered here, definition (7) is equivalent to

$$k = -\frac{W}{Lg_0} \left(\frac{\partial L}{\partial M} \right)_{v_W}. \quad (9)$$

The idea of introducing a compressibility for vibrated granular fluid was already used in Ref. [22] in the context of hydrodynamical stability analysis. In a molecular system at equilibrium, the isothermal compressibility,

$$k_T = -\frac{1}{\langle V \rangle} \left(\frac{\partial \langle V \rangle}{\partial p} \right)_T, \quad (10)$$

is related with the volume fluctuations by

$$\sigma_V^2 \equiv \langle (V - \langle V \rangle)^2 \rangle = k_B T \langle V \rangle k_T, \quad (11)$$

where k_B is the Boltzmann constant. In nonequilibrium states, there is no reason to expect for the above relationship to hold, but it is tempting to employ it to define an effective temperature parameter of the system, T_{eff} , expecting it to have some intrinsic physical meaning. Therefore, taking into account that the volume fluctuations in the case being considered are associated with fluctuations of the position Z of the movable piston, we define T_{eff} through

$$\sigma_Z^2 = -\frac{T_{eff}}{g_0} \left(\frac{\partial L}{\partial M} \right)_{v_W}. \quad (12)$$

The Boltzmann constant has been set equal to unity as it is usually done when defining the granular temperature from the average kinetic energy of the grains. From Eq. (12) it follows that T_{eff} relates the volume response to pressure perturbations with the steady volume fluctuations of the system. Of course, Eq. (12) by itself is just a mathematical definition and does not add anything to the physical understanding of the system. On the other hand, the definition would become relevant if this effective temperature were related to other temperaturelike parameters of the system. Two main candidates clearly stand out: the granular temperature of the gas in the vicinity of the piston, T_L , and the temperature parameter of the piston, T_p , defined through $T_p = M \langle V_z^2 \rangle$ [19]. Both parameters, T_L and T_p , are not at all the same, as it should be the case if the system under consideration were at equilibrium and energy equipartition would apply. The violation of equipartition is a general feature of granular systems known since long ago [23] and that has attracted a lot of attention in the last years. A more detailed discussion of this issue for the setup being considered here is given in [14]. In Fig. 5, event-driven simulation results for the ratio T_p/T_L are plotted as a function of M/m . Several values of the coefficient of normal restitution of the gas in the interval $0.85 \leq \alpha \leq 0.98$ have been considered, while again $\alpha_p = 0.99$ for all the data shown. It is observed that the behavior of this temperature ratio is quite intricate. For instance, T_p/T_L decreases as M/m increases for $\alpha > 0.95$, but it happens the other way round for $\alpha < 0.95$. Actually, as α decreases below this value, the increase in the temperature ratio is rather fast, and T_p reaches to be up to four times larger than T_L . If the parameters T_p and T_L were interpreted as real temperature parameters, this latter behavior would be fully counterintuitive. The temperature of the heated body (the piston) is larger than the temperature of the heating one (the gas next to the piston).

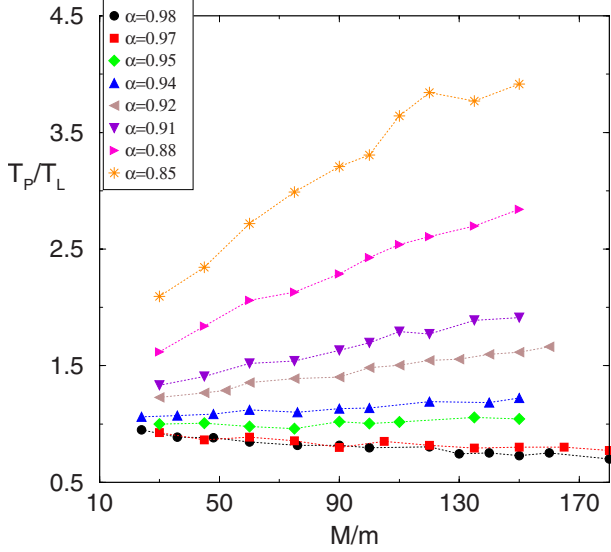


FIG. 5. (Color online) Ratio of the temperature parameter of the piston, T_P , to the granular temperature of the gas next to it, T_L , versus the mass ratio M/m , for several values of the coefficient of restitution of the gas α , as indicated in the inset. The curves are guides for the eye. In all the cases, the coefficient of restitution for the gas-movable piston collisions is $\alpha_p=0.99$.

From the values of σ_Z and $(\partial L/\partial M)_{v_w}$ obtained from the event-driven simulation data, the effective temperature T_{eff} has been computed by means of its definition [Eq. (12)]. In the Appendix some details are given of the way in which the above derivative was actually evaluated. Then, in Figs. 6 and 7 the temperature ratios T_{eff}/T_L and T_{eff}/T_P are plotted, respectively, as functions of the mass ratio for the same systems as in Fig. 5. A clear difference is observed in the behavior of the two temperature ratios for a given constant

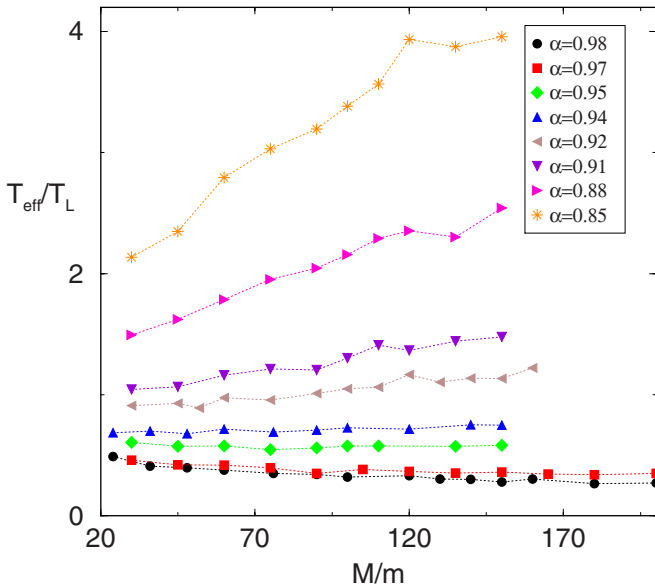


FIG. 6. (Color online) Ratio of the effective temperature, T_{eff} , defined in Eq. (12) to the temperature of the gas in the vicinity of the piston, T_L , as a function of the mass ratio for the same systems as in Fig. 5.

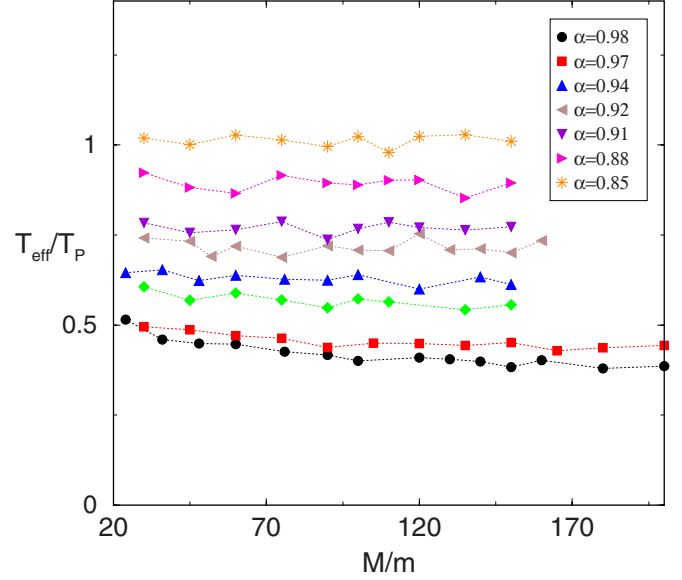


FIG. 7. (Color online) Ratio of the effective temperature, T_{eff} , defined in Eq. (12) to the temperature parameter of the piston, T_P , as a function of the mass ratio for the same systems as in Fig. 5.

coefficient of restitution α . While T_{eff}/T_L exhibits a strong dependence on M/m and does not seem to tend to a well-defined limit as it increases, the dependence of T_{eff}/T_P on M/m is very weak, being only appreciable for the least inelastic cases and when the mass ratio is small. Thus, it is concluded that for large enough mass ratio M/m , the effective temperature is proportional to the temperature parameter of the piston with a coefficient of proportionality that is independent of the mass of the piston, i.e.,

$$T_{eff} = b(\alpha, \alpha_p) T_P, \quad (13)$$

where a possible dependence of the coefficient on α_p has been included.

To identify the dependence of the coefficient b on α , in Fig. 8 the plateau values of T_{eff}/T_P , reached upon increasing

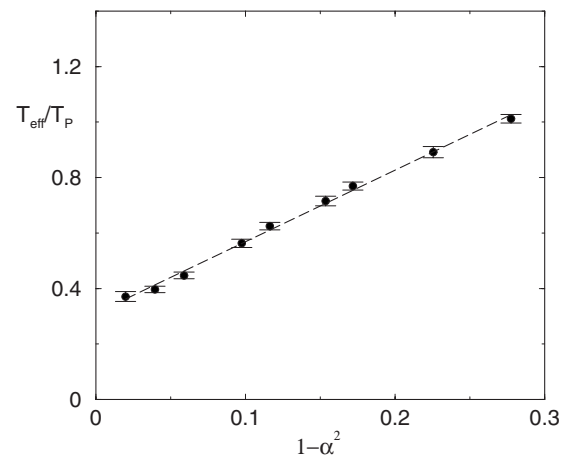


FIG. 8. Ratio T_{eff}/T_P for large values of M/m as a function of $1-\alpha^2$ for the same systems as in Fig. 5. The symbols are simulation results and the dashed line is a linear fit of them.

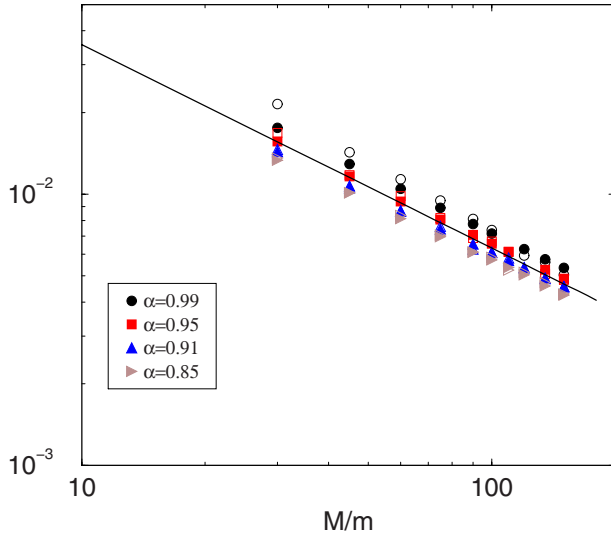


FIG. 9. (Color online) Dimensionless compressibility $-mL^{-1}(\partial L/\partial M)_{v_w}$ (filled symbols) and scaled position fluctuations of the piston $\sigma_z^2 mg_0/Lb(\alpha, \alpha_p)T_p$ (empty symbols) as functions of the mass ratio M/m . The different symbols correspond to different values of the coefficient of restitution for the collision between particles, α , as indicated in the inset. The straight line has a slope of $-3/4$, and it is a guide for the eye.

the value of the mass ratio M/m , are plotted versus $1-\alpha^2$. The points are very well fitted by a straight line, indicating that T_{eff}/T_p grows linearly with $1-\alpha^2$, at least in the considered interval, $0.85 \leq \alpha \leq 0.98$. It is worth to note that some care is needed when extrapolating to the elastic limit $\alpha \rightarrow 1$ these results. In this limit, a stationary state is only possible if, in addition, the vibrating wall is arrested, i.e., also the limit $v_w \rightarrow 0$ is taken. But all the previous discussion has been carried out at constant velocity of the vibrating wall. This explains why the extrapolation of the linear fitting in Fig. 8 does not lead to $b=1$, and this does not mean any kind of contradiction.

In Fig. 9, the dimensionless compressibility $-mL^{-1}(\partial L/\partial M)_{v_w}$ and the scaled second moment of the position fluctuations $\sigma_z^2 mg_0/Lb(\alpha, \alpha_p)T_p$ have been plotted as functions of M/m . A logarithmic representation is employed. Data for several values of the coefficient α have been included. For all the data $\alpha_p=0.99$. If T_{eff} had been used instead of $b(\alpha, \alpha_p)T_p$, the two plotted quantities would be the same by definition, i.e., the filled and empty symbols would agree in all the cases. It is seen that the dependence on M/m of the dimensionless compressibility is accurately described by a power law of the form $(M/m)^{-3/4}$, indicated in the figure by the solid straight line. Nevertheless, it is important to realize that the interval of values of M/m for which the above dependence is identified is rather narrow, just one order of magnitude, so that its range of validity can be limited.

IV. DISCUSSION AND SUMMARY

The aim here has been to investigate the volume fluctuations of a vibrated low-density gas of inelastic hard disks in the presence of gravity and confined by a movable piston on

the top. The study has been restricted to the parameter region in which the system reaches a steady state with gradients only in the direction of the external gravitational field, i.e., perpendicular to the movable piston. In practice, this has limited the values of the coefficient of normal restitution of the gas particles to the interval $0.85 \leq \alpha < 1$. Due to the coupling between inelasticity and hydrodynamic gradients, which is peculiar of steady states of granular systems, the above limitation also implies restriction to small gradients. Nevertheless, the analysis carried out in Ref. [19] indicates that the range of hydrodynamic gradients considered here exceeds the limit of validity of the Navier-Stokes approximation.

Some of the main results can be summarized as follows:

(i) for large mass of the movable piston compared with the mass of the gas particles, the volume fluctuations are Gaussian with very good accuracy; (ii) the square root of the second moment of their distribution scales with the square of the velocity of the vibrating wall at the bottom, i.e., in the same way as the amplitudes of the hydrodynamic fields in the gas; (iii) by requiring the same relation between volume fluctuations and compressibility as in equilibrium systems to be verified, an effective temperature can be defined; (iv) the effective temperature turns out to be proportional to the second moment of the velocity fluctuations of the piston, with a proportionality parameter that depends on the inelasticity of both the particle-particle and particle-piston collisions, but it seems to be independent of the mass of the piston; and (v) the effective temperature cannot be related in a simple way to the temperature of the granular gas; even more, the relationship between both parameters is not monotonic.

A relevant open question is the relationship between the granular temperature of the gas in the vicinity of the piston and the temperature parameter of the piston, with the latter being defined from the second moment of its velocity distribution. An explanation of the simulation results seems to require a detailed knowledge of the velocity distribution function of the gas next to the piston [14]. If this is the case, approximated solutions of the Boltzmann equation—as provided, for instance, by the Chapman-Enskog procedure in the first Sonine approximation—would not be of enough accuracy as to describe the deviation from equipartition between the gas and the movable piston.

The present study complements the one in Ref. [14], in which the velocity fluctuations of the piston were investigated in detail. A natural issue now is whether the velocity fluctuations and the position fluctuations of the piston are correlated. We have computed from the simulation data the joint probability distribution for the position and velocity of the piston and compared it with the product of the marginal distributions for the position and the velocity. Both results agree within the statistical uncertainties, indicating the absence of correlations.

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APPENDIX

In Ref. [19], an expression for the average position of the piston, L , was derived by using a hydrodynamic description of the granular with the appropriate boundary conditions. The theoretical prediction was shown to be in reasonable agreement with the simulation results. Although this expression could have been used to compute $(\partial L / \partial M)_{v_w}$, here this quantity has been obtained from the simulation data for the sake of consistency.

When trying to compute from the values of L as a function of M/m the derivative of the former with respect to the latter, the technical problem arises that the considered values of M/m are separated by intervals on the order of 10–15 units. Decreasing this interval would require one to strongly increase the simulation time. In addition, some smoothing

process should be used. Here, a different approach has been followed. The simulation data for L have been fitted by an analytical function of the mass ratio whose derivative has been afterward computed. The trivial choice of a polynomial in m/M does not work; so, motivated by the equilibrium elastic result, an expression of the form

$$L = A \frac{g_0}{mv_w^2} \ln \left(1 + \frac{Bm}{M} \right), \quad (\text{A1})$$

where A and B are two adjustable dimensionless parameters, was used. It turned out that it fits very well the results for all the values of α and α_p considered here. Moreover, the values of the two fitting parameters are quite stable, in the sense that their values seem to converge very fast when the number of fitted data is increased.

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- [1] H. M. Jaeger, S. R. Nagel, and R. P. Behringer, *Rev. Mod. Phys.* **68**, 1259 (1996).
- [2] J. Duran, *Sands, Powers and Grains: An Introduction to the Physics of Granular Materials* (Springer Verlag, New York, 2000).
- [3] I. Goldhirsch, *Annu. Rev. Fluid Mech.* **35**, 267 (2003).
- [4] N. V. Brilliantov and T. Pöschel, *Kinetic Theory of Granular Gases* (Oxford University Press, Oxford, 2004).
- [5] I. S. Aranson and L. S. Tsimring, *Rev. Mod. Phys.* **78**, 641 (2006).
- [6] J. J. Brey, M. I. García de Soria, P. Maynar, and M. J. Ruiz-Montero, *Phys. Rev. E* **70**, 011302 (2004).
- [7] P. Visco, A. Puglisi, A. Barrat, F. van Wijland, and E. Trizac, *Eur. Phys. J. B* **51**, 377 (2006).
- [8] M. P. Allen and D. J. Tildesley, *Computer Simulations of Liquids* (Clarendon Press, Oxford, 1987).
- [9] T. Pöschel and T. Schwager, *Computational Granular Dynamics* (Springer, Berlin, 2005).
- [10] L. D. Landau and E. M. Lifschitz, *Statistical Physics Part 1* (Pergamon Press, Oxford, 1978).
- [11] A. Baldassarri, A. Barrat, G. D'Anna, V. Loreto, P. Mayor, and A. Puglisi, *J. Phys.: Condens. Matter* **17**, S2405 (2005).
- [12] A. Puglisi, A. Baldassarri, and A. Vulpiani, *J. Stat. Mech.: Theory Exp.* **2007**, P08016.
- [13] C. S. Campbell, *Annu. Rev. Fluid Mech.* **22**, 57 (1990).
- [14] J. J. Brey and M. J. Ruiz-Montero, *J. Stat. Mech.: Theory Exp.* **2008**, L09002.
- [15] J. J. Brey, M. J. Ruiz-Montero, F. Moreno, and R. García-Rojo, *Phys. Rev. E* **65**, 061302 (2002).
- [16] E. Livne, B. Meerson, and P. V. Sasorov, *Phys. Rev. E* **66**, 050301(R) (2002).
- [17] J. J. Brey, J. W. Dufty, C. S. Kim, and A. Santos, *Phys. Rev. E* **58**, 4638 (1998).
- [18] J. J. Brey and D. Cubero, in *Granular Gases*, edited by T. Pöschel and S. Luding (Springer-Verlag, Berlin, 2001).
- [19] J. J. Brey and M. J. Ruiz-Montero, *Phys. Rev. E* **79**, 031305 (2009).
- [20] S. McNamara and J.-L. Barrat, *Phys. Rev. E* **55**, 7767 (1997).
- [21] S. McNamara and S. Luding, *Phys. Rev. E* **58**, 813 (1998).
- [22] M. Argentina, M. G. Clerc, and R. Soto, *Phys. Rev. Lett.* **89**, 044301 (2002).
- [23] J. T. Jenkins and F. Mancini, *J. Appl. Mech.* **54**, 27 (1987).