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Assessment of Generalized State Estimators including Ampere Measurements

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Abstract— **Conventional State Estimators (SE) can not take advantage of all available measurements, like currents flowing through circuit breakers and individual external injections. Current measurements are frequent at the lower levels and their loss implies a reduction of the global redundancy and hence a deterioration of the SE filtering capability. In this paper, a methodology is presented allowing all current measurements to be included in the model. A Generalized State Estimator (GSE) is used and the substations are represented in an efficient way by means of an Implicit Model. To illustrate the performance of the proposed formulation, several simulations are presented. Results are validated by checking the accuracy of the estimated states and by showing the improved ability to detect and identify bad data and topology errors.**

Keywords— **Substation Models, Generalized State Estimators, Topology Errors.**

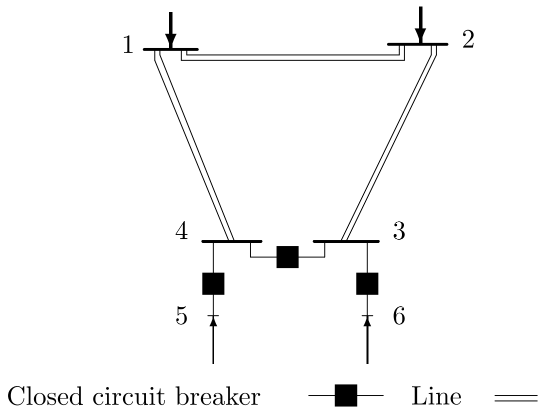

Fig. 1. Test system.

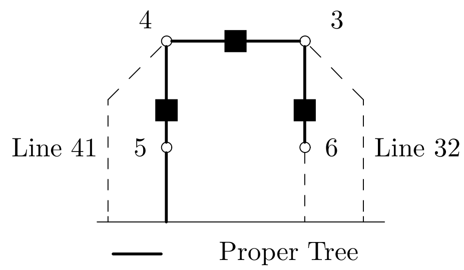

Fig. 2. Proper tree of the detailed substation

I. Introduction

EXISTING State Estimators resort to the bus-branch model of the system in which only nonzero impedances corresponding to lines and transformers remain. The state vector is composed of bus voltage magnitudes and phase angles. Based on the status of switching elements, electrical buses are determined beforehand by the Topology Processor (TP). One of the shortcomings of this conventional model lies in the fact that connectivity information is assumed correct, which complicates significantly the process of detecting and identifying topology errors. Also, certain measurements, like voltage magnitudes and power injections, must be aggregated into a single one, preventing in this way the possibility for individual bad data to be identified.

In this context, it is possible to include branch current measurements, as shown for the first time in [1] and extended later in [2]. However, ampere measurements corresponding to circuit breakers, as well as certain individual current injections, can not be added to this model, because these data are associated to branches of null or unknown impedance, which are not retained.

The Generalized State Estimator (GSE) paradigm is based on a detailed representation of substations at the physical level [3]. Two types of substation models exist: the complete or explicit model and the implicit one. The complete model augments the state vector with power flows through every circuit breaker (CB). The number of conventional state variables is also very large, as a consequence of the detailed model adopted. Such a large number of state variables is compensated by a proportional number of extra topological constraints, representing CB statuses. The size of this full model is huge and, for this reason, only a reduced number of suspect substations are modeled in detail [4]. On the other hand, the implicit substation model developed in [5], [6] reduces the size of the problem significantly, making it possible the representation of all substations of the system and, hence, the inclusion of all internal measurements available. Topology consistency can be subsequently checked by unfolding the information provided by the implicit model.

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II. Review of the implicit model

This model is aimed at minimizing the number of extra variables which must be included in the state vector to model topological connectivity. In order to elaborate the implicit model, a graph is used for each substation [6]. Nodes of this graph correspond with busbar sections (i.e. physical nodes) plus an extra node modeling the outer system, while branches represent lines, transformers, shunt elements, external injections and switching devices.

On this graph, the so-called *proper tree* is built, allowing the necessary state variables to be automatically selected so that the substation can be modeled at the physical level. Branches of the proper tree are selected keeping the following rules in mind:

- Include as many closed CBs as possible.
- For each electrical bus select one of the associated injections (a zero-injection branch should be retained for buses without external injections).
- If necessary, complete the tree with open CBs.
- Exclude regular non-zero impedance branches.

In Fig. 1, a 3-bus network is shown in which one of the substations is represented in detail, along with its graph and proper tree.

As explained in [5], the set of extra variables added to the state vector by the implicit model is composed only of power injections and power flows through closed CBs which are excluded from the tree. In terms of this slightly augmented state vector, any power flow (active or reactive), f_{ij}, within the substation can be expressed as

$$f_{ij} = \sum f_L(V_i, V_j, \theta_i, \theta_j) + \sum f_e \qquad (1)$$

where f_L is the power flow through regular branches with non-null impedance, which is a function exclusively of the classical state variables (V_i, V_j, θ_i, θ_j), and f_e refers to the flows through the proper tree links [6]. The above expression is easily obtained for each branch by means of the respective cut-set equation.

In the example of Fig. 1 only the power injection at bus 6 has to be added to the state vector. For instance, the power flow through CB 5-4 is expressed as:

$$f_{54} = f_{L41} + f_{L32} - f_6$$

where f_L refers indistinctly to active or reactive power and power flows through lines 4-1 and 3-2 are well-known nonlinear functions of state variables.

Topological information associated with CB statuses are not explicitly handled by the implicit model. Instead, topology error analysis proceeds from the results provided by the SE. It is proved in [5] that Lagrange multipliers associated with topological constraints and their covariance matrix can be computed from

$$\lambda = T \begin{bmatrix} r \\ \mu \end{bmatrix} \qquad (2)$$

where r is the measurement residual vector, μ is the vector of Lagrange multipliers corresponding to zero-injection constraints, and T is the so-called *Topology Sensitivity Matrix*. Normalized multipliers can then be obtained and used for topology error detection and identification.

III. Formulation of current measurements within the implicit model

Ampere measurements corresponding to regular non-zero impedance branches can be expressed in terms of conventional state variables and branch parameters [1]. It has long been known that this kind of measurements gives rise to several numerical and convergence problems when included in state estimators with low redundancy (see for instance [1], [2]). In some cases, however, they are essential to assure full network observability, particularly at the subtransmission levels.

On the other hand, when the GSE paradigm is adopted, there is a need for internal current measurements, i.e., those associated with switching devices, to be added to the model. This can be done by resorting to the basic equation:

$$I_{ij} = \frac{\sqrt{p_{ij}^2 + q_{ij}^2}}{V} \qquad (3)$$

where I_{ij} is the current through branch i-j within the substation, p_{ij} and q_{ij} are the active and reactive power flows and V is the respective bus section voltage. Note that the implicit model allows the power flows p_{ij} and q_{ij} to be expressed in terms of the state variables obtained from the proper tree by means of (1).

Jacobian entries corresponding to (4) and (5) can be obtained as linear combinations of existing Jacobian elements, as follows:

$$\frac{\partial I_{ij}}{\partial x} = \frac{1}{I_{ij}V^2}\left[p_{ij}\frac{\partial p_{ij}}{\partial x} + q_{ij}\frac{\partial q_{ij}}{\partial x}\right] \qquad (4)$$

$$\frac{\partial I_{ij}}{\partial V} = \frac{1}{I_{ij}V^2}[p_{ij}\frac{\partial p_{ij}}{\partial V} + q_{ij}\frac{\partial q_{ij}}{\partial V}] - \frac{I_{ij}}{V} \qquad (5)$$

where x denotes any state variable other than V. Note that these terms get undefined for flat start, which is solved by adding ampere measurements only after the second iteration.

IV. Assessment of Estimation Accuracy

Accuracy of the estimated state with and without internal ampere measurements is analyzed and compared in this section.

A. Gaussian errors

For the system of Fig. 1, 300 measurement sets are generated by adding Gaussian noise ($\sigma = 0.02$) to the results of a load flow. Table I and II show the system parameters and the exact measurement set.

For each set of measurements, two simulations are carried out, first using a conventional SE and then resorting to a GSE including current measurements. The measurement vectors are composed of the following elements:

$$z_{SE} = [V_1, p_{12}, q_{12}, p_{23}, q_{23}, p_1, q_1, p_{14}, q_{14}, p_{21}, q_{21}]$$

$$z_{GSE} = [V_1, p_{12}, q_{12}, p_{23}, q_{23}, p_1, q_1, p_{14}, q_{14}, p_{21}, q_{21}, I_{54}, I_{63}, I_{43}]$$

As the exact state is known in simulation environments the following performance index can be used to compare the results:

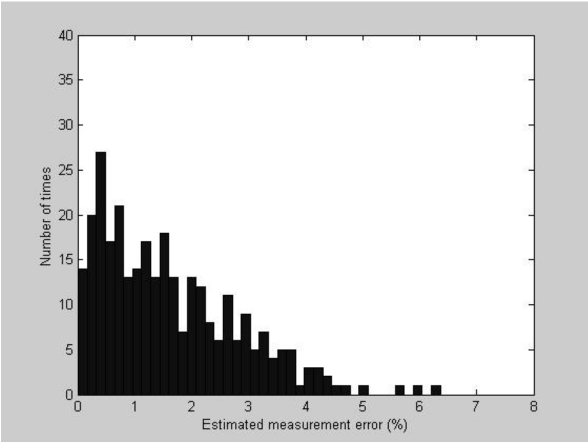

Fig. 3. Estimated measurements errors provided by the SE.

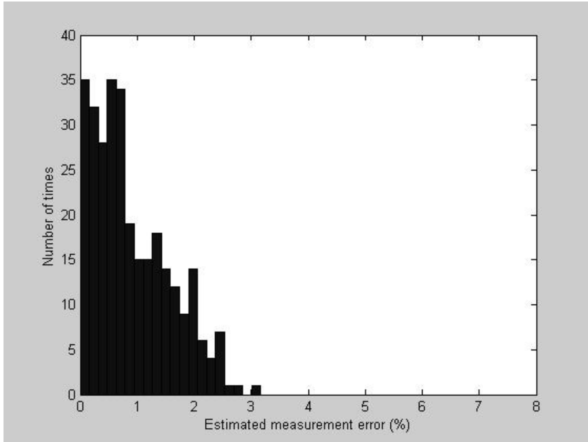

Fig. 4. Estimated measurements errors provided by the GSE.

TABLE I
SYSTEM DATA

Line	R	X	B_s
Line 12	0.019	0.059	0.050
Line 23	0.010	0.030	0.028
Line 14	0.015	0.040	0.040

TABLE II
EXACT MEASUREMENTS

Measurement	Value
V1	1.0100
p12	1.0000
q12	0.2000
p21	-0.9804
q21	-0.1887
p23	0.7500
q23	0.1000
p1	3.0355
q1	0.5350
p14	2.0355
q14	0.3350
I54	2.2895
I63	0.5771
I43	0.2912

$$\varepsilon = \frac{\sum \frac{|\widehat{z}_i - z_i^t|}{|z_i^t|}}{N} \tag{6}$$

where $|\cdot|$ denotes absolute value, $\widehat{z}_i$ are the estimated measurements, z_i^t are the exact measurements and N the number of considered measurements.

Figures 3 and 4 present the histograms corresponding to the 300 simulations performed. While the mean relative error of the unfiltered input measurements is 1.669%, that of the estimated measurements is 1.613% when the ordinary SE is adopted and reduces to 0.904% when the GSE, including current measurements, is employed. Note that the conventional estimator does not use any of the internal measurements available at the substation which is modeled in detail. Such a low redundancy explains its limited capability to filter measurement noise.

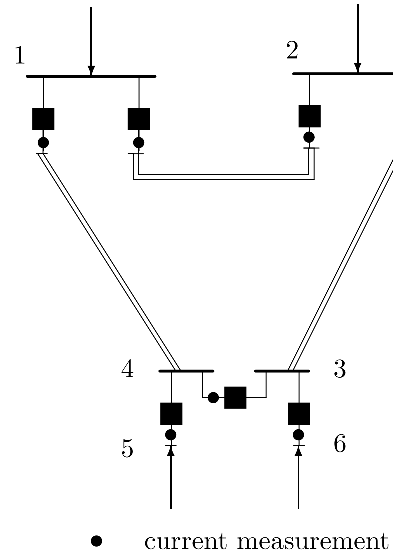

Fig. 5. Detailed substations and ampere measurements considered

Similar results are obtained when different error levels are simulated. In order to confirm the above conclusions, the experiments are repeated by successively modeling in detail the remaining two substations (Fig. 5 shows the one-line diagram for the whole system). In Fig. 6, a comparison is performed between both estimators for different error levels and a total of 500 simulations, when one, two or three substations are included in the GSE model. Note that the larger the number of substations added to the model, i.e. the higher the redundancy, the better the results.

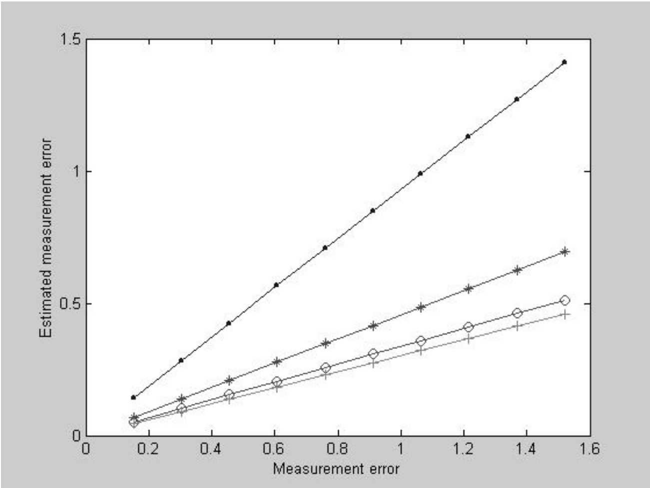

Fig. 6. Errors in: [.] SE, [*] GSE (Sub. 3), [o] GSE (Sub. 1 and 3), [+] GSE (Sub. 1,2 and 3)

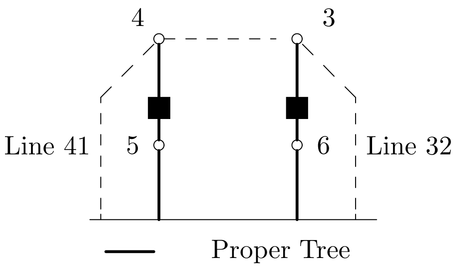

Fig. 7. Proper tree of substation with erroneus status in CB 4-3

B. Topology errors

Results obtained in the presence of topology errors are reported in this section. In the test system of Fig. 1 an erroneous status is adopted for the circuit breaker 4-3, which is assumed open when it is really closed. This is called a split error, by which the conventional estimator assumes two rather than one electrical nodes within the substation.

Figure 7 presents the proper tree associated to the graph of substation 3, when CB 4-3 is assumed open. Note that all links correspond to lines or open CB. For this reason, no new state variables are required for the implicit model to properly perform detailed representation of the substation and topology error identification.

Table III shows the exact measurements and constraints, as well as the residuals and normalized multipliers obtained by means of both a standard SE and a GSE including current measurements. With the adopted measurement set, the SE cannot detect the topology error (note the null residuals). However, the presence of abnormally high normalised multipliers associated to the respective topological constraints (21.0 and 21.7), allows the GSE to correctly identify the wrong status assumed for the circuit breaker 3-4. This is a consequence of ampere measurements through CBs 5-4 and 6-3 being considered by the GSE.

TABLE III
TOPOLOGY ERROR

Meas./Const.	Value	S.E.	G.S.E.
V1	1.0100	0	5.9
p12	1.0000	0	7.1
q12	0.2000	0	1.5
p23	0.7500	0	12.3
q23	0.1000	0	3.1
p1	3.0355	0	1.3
q1	0.5350	0	0.2
p2	-0.2304	0	6.6
q2	-0.0887	0	0.6
p14	2.0355	0	8.8
q14	0.3350	0	1.9
I54	2.2895	-	16.1
I63	0.5771	-	18.7
p43	0	-	21.0
q43	0	-	20.7

TABLE IV
BAD DATA

Meas./Const.	Value	S.E.	G.S.E.
V1	1.0100	3.3	10.0
p12	1.0000	13.7	13.1
q12	0.2000	1.3	0.7
p23	0.7500	24.8	17.3
q23	0.1000	2.0	2.3
p14	2.4426	24.8	27.2
q14	0.3350	1.3	0.7
p21	-0.9804	12.9	12.8
q21	-0.1887	0.7	1.3
I54	2.2895	-	11.0
I63	0.5771	-	11.0
I43	0.2912	-	11.0

C. Bad data

Assuming the measurement set is exact, the value of p_{14} is increased by 20% to simulate a bad data. Table IV shows the measurement set considered as well as the normalised residuals corresponding to both estimators. With a plain SE the bad data is detected but not identified (the low redundancy leads to a pair of residuals being 24.8). When the GSE and current measurements are employed, the error in p_{14} is detected and identified (the largest normalised residual, 27.2, corresponds to p_{14}).

V. Conclusions

In order to include all existing measurements within the state estimation process in an efficient way, particularly ampere measurements, a generalized state estimator based on the recently introduced implicit model is adopted in this paper to represent substations. Experimental results show a significant improvement in the accuracy of estimated states and a reinforced capability to detect and identify both topological and analogue errors when internal current measurements are not discarded.

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Biographies

Antonio de la Villa Jaén was born in Riotinto, Spain, in 1960. He received his electrical and doctor engineering degrees from the University of Seville. He is presently an Assistant Professor at University of Seville. His primary areas of interest are computer methods for power system state estimation problems.

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