

# On the Use of PMUs in Power System State Estimation

Antonio Gómez-Expósito  
University of Seville  
Seville, Spain  
age@us.es

Ali Abur  
Northeastern University  
Boston, USA  
abur@ece.neu.edu

Patricia Rousseaux  
University of Liège  
Liège, Belgium  
p.rousseaux@ulg.ac.be

Antonio de la Villa Jaén  
University of Seville  
Seville, Spain  
adelavilla@us.es

Catalina Gómez-Quiles  
University of Seville  
Seville, Spain  
catalinagq@us.es

**Abstract** - Synchronized phasor measurement units (PMUs) are becoming a reality in more and more power systems, mainly at the transmission level. This paper presents, in a tutorial manner, the benefits that existing and future State Estimators (SE) can achieve by incorporating these devices in the monitoring process. After a review of the relevant PMU technological aspects and the associated deployment issues (observability, optimal location, etc.), the alternative SE formulations in the presence of PMUs are revisited. Then, several application environments are separately addressed, regarding the enhancements potentially brought about by the use of PMUs.

**Keywords** - PMUs, state estimation, observability, measurement placement, dynamic estimation, multi-area estimation

## 1 Introduction

STATE estimation (SE) has been for decades one of the essential applications in Energy Management Systems (EMS), allowing secure operation of transmission grids. Measurements received and processed by the state estimators typically include power flows, net power injections and voltage and current magnitudes [1]. A basic assumption behind the SE is that the measurement set constitutes a single snapshot of the system being monitored, which is not fulfilled in practice because it takes a while to remotely capture and centrally gather all the information to be processed by the SE. In fact, it would be very difficult, if not impossible to assure that all measurements are synchronized (i.e., refer to the same instant). However, as long as the time elapsed between the first and the last component in the measurement set is small enough, compared to the time constant of the system load, this assumption will be acceptable in practice.

Nowadays, there is a clear trend to broaden the geographical scope of many SEs, in accordance to the needs of regional electricity markets, in which long-distance energy transactions have to be accurately and permanently monitored. In this context, the task of collecting wide area measurements and synchronizing the solutions provided by each control area will be a challenging one [2].

Recently, the introduction of more sophisticated protection and measurement components, such as the Intelli-

gent Electronic Device (IED), has provided the capability of certain phase angle differences between adjacent (voltage and current) phasors to be added at the local area or substation level [3, 4]. While this may benefit to a certain extent the accuracy of the SE at the TSO level, it can hardly be helpful in the multi-TSO case.

Occasionally, a seemingly simple technical innovation can become a major player in changing the entire industry. There are several historical examples of such innovations one can identify, including the light bulb, transistor, laser, etc. Global positioning satellite (GPS) system can be included in this vein, perhaps not unlike the internet (or the original DARPA-net), which was also created initially for a small group of users, but then rapidly became a universal tool employed by almost anyone worldwide. The GPS system provides two important benefits which were previously not readily or easily acquired. One is the ability to determine geographical coordinates and the other is to have global access to a very accurate clock allowing to time stamp measured quantities irrespective of the physical coordinates at which these measurements are taken. Development and installation of the GPS system was soon followed by numerous engineering applications, which mainly consisted of a receiver and a processor. The receiver's function is to capture the signals transmitted from a redundant set of satellites and then process them in various different ways based on the objective of the implemented application. One example of such an application is the phasor measurement unit (PMU). These devices are installed at substations in electric power systems, and their objective is to accurately determine the frequency of the alternating current and voltage, and also produce the phasor representation of these signals defined with respect to the global clock.

Deployment of PMUs started at a slow pace about a decade ago and accelerated after a number of successive blackouts experienced in power systems all around the globe.

Phasor measurements are synchronized with respect to the time reference provided by the GPS satellites. Thanks to this accurate global time reference, synchrophasors with identical time-stamp received from various substations allow to create a coherent picture of the system state at a given instant, eliminating in this way the need to artificially set a phase angle, arbitrarily taken as the reference

angle in conventional state estimators.

Devices that can measure synchronized phasors were developed and potential benefits of PMU measurements were recognized over twenty years ago by Phadke et al. [5, 6]. The first implementation of GPS-synchronized phase angle measurements in an industrial power system SE was presented in [7].

Utilization of PMU measurements will impact state estimation in different ways. On the one hand, since the number of PMUs installed in existing power systems is not yet sufficient to carry out SE exclusively based on PMU measurements, SE formulation and solution remains non-linear and iterative respectively. More imaginative solutions, sequentially handling conventional and PMU measurements in a two-step procedure have been also proposed [8].

On the other hand, SE related issues such as network observability and measurement placement [9, 14], solution accuracy and reliability (convergence rate), processing of bad data and other (parameter and topological) types of errors [15, 16] will have to be reconsidered. Moreover, in view of the higher sampling rates at which PMUs can work, the possibility of estimating the dynamic evolution of certain critical variables is being explored [17].

This paper covers in a succinct manner all of the above issues. Then, with the help of small tutorial examples, the potential benefits provided by the incorporation of PMUs in several SE environments are shown.

## 2 Phasor Measurement Units

**A** PHASOR Measurement Unit is a digital device providing synchronized voltage and current phasor measurements, referred to as synchrophasors [18].

PMU features were first implemented in stand alone units whose most relevant function is its capability to provide synchrophasors. Nowadays, many IEDs (RTUs, protective relays, ...) have been upgraded to produce synchrophasor measurements in addition to their own function.

### 2.1 General PMU architecture

Most generally, PMUs provide multi-channel input so that in addition to the voltage at the installation bus, currents in more than one line and possibly in all incident lines can be processed by a single unit. The three-phase voltages and currents are converted to appropriate analog inputs by instrument transformers and anti-aliasing filtering. Each analog signal is digitized by the A/D converter with sampling rate usually varying from 12 to 128 samples per cycle of the nominal power frequency. The sampling clock is phase-locked with the GPS clock pulse which provides the Universal Time Coordinated (UTC) time reference used to time-tag the outputs.

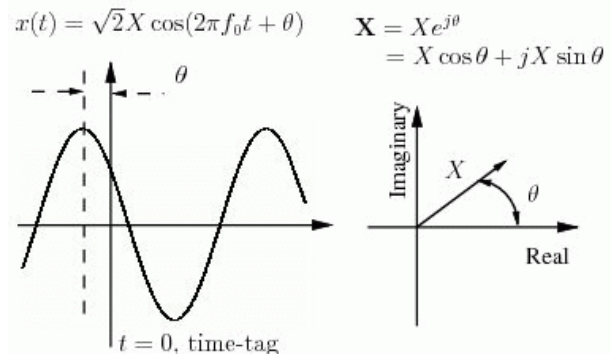
Phasors of phase voltages and currents are computed from sampled data by the PMU microprocessor using a signal processing technique as described in section 2.2 below. The calculated phasors are combined to form the

positive sequence phasor measurements (as well as negative and zero sequences in case of unbalanced conditions). Other estimates of interest are frequency and rate of change of frequency.

Computed phasor measurements are transmitted through a digital communication network to higher level applications at a rate of 10 up to 60 frames per second. Many PMUs offer storage capacity enabling local exploitation of synchrophasors. However, in many real-time applications, and in particular state estimation, the phasor measurements are not used locally but rather at remote locations. Phasor data from a number of PMUs is then collected by a special-purpose computer, called Phasor Data Concentrator (PDC), which correlates phasor data by time-stamp to create a system-wide measurement set. PDCs can provide a number of specialized outputs such as a direct interface to a SCADA, EMS system or an upper level PDC.

### 2.2 Synchrophasors and measurement techniques

The basic definition of the phasor representation of a sinusoidal waveform at nominal frequency  $f_0$ , both in its polar and rectangular form, is illustrated in Fig. 1. The phasor angle is given by the angular difference between the peak of the sinusoid and the reference time  $t = 0$ . When considering synchrophasor, this reference time corresponds to the time-tag. If the waveform is not a pure sine signal, the computed phasor represents its fundamental frequency component.



**Figure 1:** Sinusoidal waveform and its phasor representation

There have been several digital algorithms proposed to estimating the phasor data. The most commonly used technique relies on the Discrete Fourier Transform (DFT) [19]. The signal phasor is computed in a continuous process from successive samples in a moving data window of one or several fundamental cycles. Adding the contribution of the new sample while removing that of the oldest one provides the more computationally efficient recursive DFT algorithm. The sampling clocks are usually kept constant at a multiple of  $f_0$ . When frequency varies by a small amount around its nominal value, the leakage error introduced in phasor estimates can be compensated with high accuracy by a post-processing filtering. It can also be shown that the computed phasor rotates in the complex plane with an angular velocity equal to the difference between  $f_0$  and the actual frequency so that frequency and rate of change of frequency estimates are given by the first

and second derivatives of the phasor angle.

An alternative consists in replacing the DFT by a wavelet transform [20]. A number of algorithms based on nonlinear estimation techniques (nonlinear weighted least squares estimation, Kalman filtering, neural networks, etc.) have also been proposed in the literature [21, 22, 23]. According to these approaches, the phase signal is modeled as a nonlinear function of the phasor data (amplitude, phase angle, frequency, rate of change of frequency) considered as parameters to be estimated from the waveform samples.

### 2.3 PMU performances and standards

Performances of a PMU device, in terms of accuracy or processing time, are dictated by its components, mainly, the instrumentation channel, the A/D converter and the parameters of the phasor estimation algorithm.

Regarding state estimation, the accuracy of PMU data is a very important issue. It is recognized that synchrophasor measurements are usually more precise than conventional SCADA ones. Conceptually, PMU data are time tagged with precision better than 1 microsecond and magnitude accuracy that is better than 0.1%. However, this potential performance is not achieved due mainly to errors from instrumentation channels and system imbalances. Presently, evaluation of PMU data accuracy is still a challenging problem discussed in the scientific literature [24].

PMU are manufactured by a variety of companies defining specifications for each particular PMU device. The specifications concern: the window length, sampling rate and type of phasor estimation algorithm, the phasor estimate reporting rate, the communication protocol and the measurement accuracy.

In order to achieve interoperability among PMUs, it is essential that their behavior complies with a common standard. The most recent IEEE C37.118-2005 standard [25] defines the synchrophasor convention and the time-tagging process, provides the definition of an accuracy measure as well as requirements for measurement performances under steady-state conditions. It also defines data communication formats. Requirements for response to power system transients are not considered.

## 3 Background on State Estimation

Given the following measurement equation [26]:

$$z = h(x) + e \quad (1)$$

where:

$x$  is the state vector (size  $n = 2N - 1$ ),

$z$  is the measurement vector (size  $m > n$ ),

$h$  is the vector of functions, usually nonlinear, relating error free measurements to the state variables,

$e$  is the vector of measurement errors, customarily assumed to have a Normal distribution with zero mean and known covariance matrix  $R$ . When errors are

independent  $R$  is a diagonal matrix with values  $\sigma^2$ , where  $\sigma$  is the standard deviation of the measurement errors.

the maximum likelihood estimate  $\hat{x}$  is obtained by minimizing the Weighted Least Squares (WLS) function:

$$J = \sum_{i=1}^m [z_i - h_i(\hat{x})]^2 / \sigma_i^2$$

The minimum of the scalar  $J$  is reached by iteratively solving the so-called Normal equations:

$$G_k \Delta x_k = H_k^T W [z - h(x_k)] \quad (2)$$

where:

$H_k = \partial h / \partial x$  is the Jacobian evaluated at  $x = x_k$ ,

$G_k = H_k^T W H_k$  is the gain matrix,

$W = R^{-1}$  is the weighting matrix,

$\Delta x_k = x_{k+1} - x_k$ ,  $k$  being the iteration counter.

Iterations finish when  $\Delta x_k$  is within an appropriate tolerance. It can be shown that the covariance of the estimate is:

$$\text{cov}(\hat{x}) = \hat{G}^{-1} \quad (3)$$

where  $\hat{G}$  is the gain matrix computed in the last iteration.

Upon convergence, the bad data processing function is activated to detect, identify and eliminate bad analog measurements. Bad data detection is accomplished based on the largest normalized residual test [27]. If the detection test fails, then the measurement corresponding to the largest normalized residual will be declared bad and its value will be removed or corrected [1].

In conventional bus-branch SE models the state vector is composed of voltage magnitudes and phase angles, whereas the measurement vector typically comprises measurements of power injections, branch power flows and voltage magnitudes. At lower voltage levels, though, line current magnitudes and bus current injection magnitudes can play a key role to obtain a sufficiently redundant system. The inclusion of PMU measurements is the subject of this paper.

In the so-called generalized SE model the state vector is augmented with power flows through circuit breakers (CB) at certain substations where a topology error is suspected, and the measurement vector may likewise include existing current or power flow measurements through any CB.

Transformer taps and suspected network parameters can also be handled, if sufficient redundancy exists, both by conventional and generalized SE.

#### 4 Network observability and measurement placement

WHEN using conventional measurements network observability tests can be carried out based on the properties of the measurement Jacobian  $H$  which is the gradient of the nonlinear measurement equation evaluated at an arbitrary operating point such as the flat start. If the Jacobian has full column rank, then the system will be declared fully observable. In principle, the same approach will work when the Jacobian is modified in the presence of synchronized phasor measurements, which are either of voltage or current type. On the other hand, considering the long term outlook where sufficient number of PMUs are available to carry out state estimation exclusively using PMUs and disregarding all the conventional measurements, network observability analysis can be formulated as a simple graph covering problem.

Phasor measurement units may have different number of channels, i.e. they may support different number of inputs for voltage and current signals. It should also be noted that each input will be connected to one phase of a three phase voltage or current. While all three phases of voltage and current signals are monitored, most PMUs will output only the positive sequence values or the corresponding three phase quantity. Given the fact that power systems are sparsely connected, i.e. each bus has only a limited number of neighbors irrespective of the system size, channel limits on PMUs may be assumed to be sufficient to monitor as many signals as needed at a given bus. This assumption will be relaxed later and its impact will be investigated.

Assuming that a PMU is placed at bus  $k$ , the following quantities can then be assumed to be available:

- Voltage phasor at bus  $k$ ,
- Current phasors along all lines/branches incident to bus  $k$ .

Network model and parameters being known, the above information will allow computation of phasor voltages at all the neighboring buses as well. Hence, placing a PMU at a given bus implies observability of all branches incident to that bus. This simple observation will lead to the following integer programming formulation of the network observability problem using only phasor measurements:

$$\text{Minimize } C^T x_i \quad (4)$$

$$\text{Subject to } AX \geq 1 \quad (5)$$

where

$C$  is the cost vector for installation of PMUs,

$X$  is a binary vector indicating the presence (1) or absence (0) of PMUs at buses,

$A$  is a binary matrix mapping nonzero entries of the bus admittance matrix to ones,

$1$  is a vector of ones.

Buses corresponding to the nonzero values in the solution of (5) will yield the locations to place PMUs for full network observability [10]. While the conventional measurements are excluded in this formulation, equality constraints can be incorporated in order to reduce the number of required PMUs. The most common equality constraints in power systems are those provided by the net zero injections at passive buses with no generation or load. These can be readily incorporated into the problem of (5) as done in [11].

##### 4.1 Methods of placing PMUs for different objectives

While the ultimate goal is to populate power systems with enough PMUs to facilitate full observability based only on PMU measurements, this will still be a few years away. In the meantime, as investment decisions are to be made where and how many PMUs to place in a given system, different objectives may be considered. Furthermore, since PMUs may be considered for specific applications such as special protection schemes, secondary voltage control, voltage or angle stability monitoring, etc. there may be already a constrained set of buses where PMUs may have to be placed. In such cases, secondary considerations in order to make the best of these investments will be important. This section will briefly review two such cases.

##### 4.2 PMU placement to detect topology errors

Topology errors are caused by incomplete or wrong information about one or more circuit breakers at the substations. These errors can be very difficult to detect and identify due to the specific measurement configuration around the affected substation. A certain type of topology error that is referred to as *branch topology error* is defined as the error in the status of a given branch, i.e. whether or not the branch is in or out of service [12]. These types of errors are relatively easier to detect and identify compared to the more complex ones involving several breakers leading to bus splits or mergers.

Detectability of a branch topology error is closely linked to the measurement redundancy and configuration. Hence, it is possible to make a previously undetectable branch topology error detectable by strategic meter placement. If a limited number of PMUs are being considered to be placed in a given system, one consideration may be to improve branch topology detectability. The optimal case would be to have all branches topology error detectable, but even making a subset of branches topology error detectable would be a welcome improvement.

This can be accomplished in three steps:

- Identify all branches that are topology error undetectable,
- For each identified branch, determine all candidate PMU locations so that if a PMU is placed, this will make the branch topology error detectable,
- Set up an optimal selection problem so that a minimum number of the candidates identified in the pre-

vious step can be selected to make topology errors associated with all identified branches in step 1, detectable.

Details of the problem formulation along with illustrative examples can be found in [13]. It should be noted that, as shown in [13], phasor measurements have some unique advantages over the conventional measurements when it comes to topology error detection and identification.

#### 4.3 PMU placement for measurement error detection

Every measurement system may have vulnerability pockets where errors in one or more measurements can not be detected. Such measurements are referred to as *critical* and their measurement residuals will be null irrespective of their measured values [15]. A robust measurement design will address these vulnerabilities and strategically place meters in order to transform these critical measurements into redundant ones whose errors will be detectable. Techniques for identifying all critical measurements in a given power system exist and can be used to determine all such measurements. Once they are identified, a numerical factorization based approach (whose details are given in [15]) can be used to determine all possible locations where PMUs can be placed so that a given critical measurement will be transformed. These candidate PMU locations are then considered simultaneously and a minimum number that will transform all critical measurements, can be determined again using an integer programming formulation.

Simulation results shown in [15] clearly imply dependence of the optimal number of required PMUs on the network topology and existing measurement configuration. However, there are cases where with only a handful of PMUs, a very large number of critical measurements can be transformed, thus drastically improving the robustness of state estimation against bad data.

#### 4.4 PMU placement for parameter error detection and identification

Every power system data base requires constant maintenance due to changes in network parameters either due to environmental conditions such as temperature, humidity, wind, etc. or due to human error in entering data corresponding to equipment parameters such as transformer taps, shunt capacitor banks, etc. A typical power system model will have a huge number of parameters associated with its line, transformer, shunt capacitor/reactor models. Hence, use of state estimator as a tool to detect and identify parameter errors has been a topic of numerous investigations. These investigations mainly focused on parameter estimation based on the assumption that a suspect set of parameters have already been identified. However, selecting a suspect set which is guaranteed to contain erroneous parameters simply based on measurement residuals is not always possible. A method that overcomes this limitation is recently developed and then applied to the case of strategic placement of PMUs for parameter error detection and identification [16].

It is noted that, if strategically placed, PMUs will en-

able error identification of certain parameters which are not possible to identify using conventional measurements, no matter how high of a measurement redundancy is introduced. This result is validated with some simple examples in [16]. Along with the branch topology error detection problem, the parameter error identification problem constitutes one of the examples where PMUs will have a unique edge over conventional measurements.

## 5 State estimation formulation in the presence of PMUs

UP to the advent of the PMU technology the measurement vector  $z$  contained only power (flow and injection), voltage magnitude and, in certain particular cases, current magnitude measurements, all of them taken in a non-synchronized manner.

At the local level [4], phase angle differences among adjacent voltage and/or current waveforms can also be provided by the new generation of intelligent electronic devices (IEDs).

This section discusses several modeling issues arising in the formulation of the SE problem when PMUs are to be incorporated.

### 5.1 Measurement models

As stated previously, PMUs can provide both voltage and current phasors, collectively denoted as  $z_V$  and  $z_I$  respectively.

Depending on whether the state vector is represented in polar or rectangular coordinates, different expressions will result for the measurement model, as follows:

- State vector in polar coordinates:

$$x = \begin{bmatrix} V \\ \theta \end{bmatrix}$$

In this case, the error-free models corresponding to each type of measurements take the form:

$$z = h(x) \quad (6)$$

$$z_V = Kx \quad (7)$$

$$z_I = h_I(x) \quad (8)$$

where it is assumed that  $z_V$  is represented in polar coordinates so that  $K$  is a trivial matrix with a single 1 in each row. If phase angle differences,  $\theta_i - \theta_j$ , are also considered, then the respective rows in  $K$  will contain a 1 and a  $-1$ . Note that the measurement model associated with  $z_I$  is nonlinear, irrespective of  $z_I$  being expressed in polar or rectangular coordinates. The rectangular version is preferable, however, owing to the numerical problems (undefined Jacobian terms) that may arise for very small currents.

The expressions for  $h(\cdot)$  can be found elsewhere [1] while those corresponding to  $h_I(\cdot)$  are given in [28].

- State vector in rectangular coordinates:

$$x_r = \begin{bmatrix} V_{Re} \\ V_{Im} \end{bmatrix}$$

The error-free models corresponding to each type of measurements take the form:

$$z = h_r(x_r) \quad (9)$$

$$z_{V_r} = Kx_r \quad (10)$$

$$z_{I_r} = Y_r x_r \quad (11)$$

where the measurement vectors  $z_{V_r}$  and  $z_{I_r}$  are assumed in rectangular coordinates,

$$z_{V_r} = \begin{bmatrix} V_{Re} \\ V_{Im} \end{bmatrix} ; \quad z_{I_r} = \begin{bmatrix} I_{Re} \\ I_{Im} \end{bmatrix}$$

because this way the measurement models (10) and (11) become linear, which is one of the key advantages associated with PMUs. In this case,  $h_r(\cdot)$  reduces to a set of quadratic functions, provided  $V^2$  rather than  $V$  measurements are included in  $z$ .

Note that, unlike in the polar state vector case, null injection constraints can be linearly formulated if null currents rather than null powers are enforced.

The expressions for  $h_r(\cdot)$  can be found elsewhere [1] while those corresponding to  $Y_r$  are given in [29].

### 5.2 Simultaneous SE formulation

The information provided by PMUs can and should be handled in theory at the same time the conventional measurements are processed by the SE. This requires modifications to the existing software in order to accommodate the new Jacobian terms and components of the residual vector [30].

Depending on the proportion of conventional versus PMU measurements, and the number of PMU channels, the polar or rectangular model will be preferable, as follows:

- If  $z_I$  is empty, that is, PMUs provide only voltage phasors, then the conventional polar model will be preferable. This requires minor adaptations of existing SEs to accommodate phase angle measurements (see the subsection 5.4 below for a discussion on the reference angle issue).
- In future environments, however, where PMU measurements will eventually replace conventional RTU measurements, the rectangular model will be preferable. Eventually, in the absence of any power or isolated voltage magnitude measurement, the resulting SE model will become fully linear in rectangular coordinates, which is a nice feature to exploit.

In certain cases, conventional raw measurements could be previously manipulated so that they are converted to PMU-like pseudomeasurements. For instance, a pair of power (flow or injection) measurements associated with a bus whose voltage phasor is provided by a local PMU, can be transformed into an equivalent current phasor, increasing in this way the linearity of the resulting model. This requires of course that the covariance of the pseudomeasurements be computed from that of the raw measurements.

Note also that, in this approach, the PMU measurements are taken into account from the very beginning during the observability and bad data analyses.

### 5.3 Sequential SE formulation

In this scheme, conventional measurements are first processed in the usual manner, and then a new SE is designed aimed at improving the initial estimates by incorporating the information provided by PMUs. A nonlinear transformation is required in between to switch between polar and rectangular coordinates [31].

The three stages involved are as follows:

- 1) Disregarding PMU measurements, obtain a preliminary estimate  $\tilde{x}$  by solving the conventional nonlinear SE problem, given by (1)-(2). This requires that the entire network be observable in the presence of just RTU measurements. As a byproduct, the inverse of the gain matrix arising in the last iteration provides the estimate's covariance, according to (3):

$$\text{cov}(\tilde{x}) = \tilde{G}^{-1}$$

- 2) The estimate  $\tilde{x}$  is transformed to rectangular coordinates:

$$\tilde{x}_r = f(\tilde{x}) \quad (12)$$

where the nonlinear functions  $f(\cdot)$  represent the well-known relationships,

$$\begin{aligned} \tilde{V}_{Re} &= \tilde{V} \cos \tilde{\theta} \\ \tilde{V}_{Im} &= \tilde{V} \sin \tilde{\theta} \end{aligned}$$

In addition to  $\tilde{x}_r$  its covariance is required. This is obtained from:

$$\text{cov}(\tilde{x}_r) = \tilde{F} \cdot \text{cov}(\tilde{x}) \cdot \tilde{F}^T \quad (13)$$

where  $\tilde{F}$  is the Jacobian of  $f(\cdot)$  computed for  $\tilde{x}$ . Note that this Jacobian is a  $2 \times 2$ -block diagonal square matrix.

- 3) The phasor measurements provided by PMUs, in rectangular coordinates, along with the estimate  $\tilde{x}_r$ , lead to the following linear measurement model:

$$\tilde{x}_r = x_r + \varepsilon_x \quad (14)$$

$$z_{V_r} = Kx_r + \varepsilon_V \quad (15)$$

$$z_{I_r} = Y_r x_r + \varepsilon_I \quad (16)$$

where the covariance of  $\varepsilon_V$  and  $\varepsilon_I$  is a known diagonal matrix and that of  $\varepsilon_x$  is given by (13). Accordingly, the final estimate  $\hat{x}$  is the solution to the Normal equations arising at this linear stage:

$$\begin{aligned} \begin{bmatrix} I \\ K_x \\ Y_r \end{bmatrix}^T \begin{bmatrix} W_x & & \\ & W_V & \\ & & W_I \end{bmatrix} \begin{bmatrix} I \\ K_x \\ Y_r \end{bmatrix} \hat{x} &= \\ \begin{bmatrix} I \\ K_x \\ Y_r \end{bmatrix}^T \begin{bmatrix} W_x & & \\ & W_V & \\ & & W_I \end{bmatrix} \begin{bmatrix} \tilde{x}_r \\ z_{V_r} \\ z_{I_r} \end{bmatrix} & \quad (17) \end{aligned}$$

where the weighting matrices  $W_x$ ,  $W_V$  and  $W_I$  are the inverse of the respective covariance matrices. Note that  $W_V$  and  $W_I$  are customarily considered diagonal matrices, which ignores the fact that the measurement noises for the set of “raw” measurements gathered by a single PMU are correlated. On the other hand, the matrix  $W_x$  is given by:

$$W_x = \text{cov}^{-1}(\tilde{x}_r) = \tilde{F}^{-T} \cdot \tilde{G} \cdot \tilde{F}^{-1} \quad (18)$$

where the inverse of  $\tilde{F}$  is trivially obtained by computing the  $2 \times 2$  inverse of its constitutive diagonal blocks. This matrix is no longer diagonal but its sparsity should be exploited.

The main advantage of this alternative is that a conventional SE is resorted to at the beginning, allowing existing software to be adopted. Moreover, the third stage consists of a linear SE, which implies that the solution is obtained in a single iteration, preventing the risk of divergence in the presence of bad data. Although the solution reached through this three-stage approach is not the optimal one, it is sufficiently accurate for practical purposes. Its main drawback is that PMU measurements cannot be used during the first step to potentially enlarge the observable network and to enhance the bad data detection and identification process.

#### 5.4 Reference bus issues

State estimation problem is commonly formulated by choosing a reference bus (typically but not necessarily the same as the slack bus used for the power flow analysis) and setting its voltage phase angle equal to zero. This also implies that the reference phase angle will be excluded from the state vector and the corresponding column of  $H$  will be removed when building the measurement Jacobian. Alternatively, the reference phase angle can be retained in the state vector but then a phase angle pseudo-measurement of arbitrary value (zero for convenience) must be added for each observable island.

In the absence of any phase angle measurement, this practice presents no problems and provides a suitable framework to define the system state where the actual value of the reference bus voltage phase angle is irrelevant.

However, as the phasor measurements start populating the systems, the choice of a reference bus will no longer be an arbitrary decision. There are two possibilities:

- 1) Choose a bus where no PMU exists: This will create inconsistencies between the arbitrarily assigned reference angle at the chosen bus and actual phase angle measurements provided by PMUs at other buses.
- 2) Choose one of the buses with PMUs as the reference bus: This will work as long as the PMU at the chosen bus functions perfectly. If the measurements provided by this PMU contain errors, then these errors will not be detectable and will bias the estimated state.

This issue has been recognized early on and alternative approaches were considered. Among them is a document [32] which is produced by the Eastern Interconnection Phasor Project (EIPP) group. In this document a virtual bus angle reference, which is computed as the average of several phase angle measurements by PMUs located in the vicinity of a chosen bus is introduced. This approach still remains vulnerable to errors in individual PMU measurements despite the use of averaging.

In the presence of numerous PMUs it will be logical to use the absolute phase angle information provided by those devices. Hence, the measurement Jacobian will have to include columns corresponding to all bus voltage magnitudes as well as phase angles, the dimension of the system state vector being twice the number of buses [33].

In this case, the system will be declared observable if no zero pivots are encountered while factorizing  $G$ . When there is more than one observable island in the system excluding the phasor measurements, then there has to be at least one phase angle measurement in every observable island to make the overall system observable.

When there is only one phase angle measurement in the system, then this case can be reduced to the conventional formulation with an assigned reference bus. Since the value of its phase angle is irrelevant, errors in this measurement will not affect the estimation results (critical information).

A more realistic case is when there are two or more phasor measurements in the system. In this case, detection of phasor measurement errors requires higher redundancy as discussed below. Disregarding the phasor measurements, conventional network observability analysis [1] will yield the number of observable islands in a given system. Having at least one phasor measurements in every observable island will ensure observability for the entire network. In order to be able to detect and identify errors in the phasor measurements, their redundancy should be further increased in their respective observable islands. Definition of critical  $k$ -tuples can be found in [1]. Following this definition, it can be shown that two phasor measurements will ensure detectability and three will be necessary for identification of bad data associated with any phasor measurement in a given observable island.

## 6 Application environments

**I**N this section, the improvement in SE performance due to the incorporation of PMU measurements is illustrated. First, the effect of including PMU measurements in the accuracy of the TSO-level estimator is assessed. Then, the enhanced synchronization capability provided by PMUs in the multi-TSO SE case is addressed. Owing to space limitations, only small tutorial examples are considered, but the main conclusions remain valid for realistic networks.

Other issues, such as the improved monitoring of smart distribution systems by combining SE and PMUs, or the use of the high volume of historical data collected by PMUs to develop reliable load forecasting, still in their

infancy, are not addressed for the sake of brevity.

For the simulations below the state variables are expressed in polar form. The complex voltage phasors provided by PMUs are assumed in polar form while the current phasors are represented in rectangular form in order to avoid ill-conditioning problems [4]. Under these assumptions, the PMU measurements can be expressed as follows:

$$\begin{aligned} V_i^m &= V_i + \varepsilon_{V_i} \\ \theta_i^m &= \theta_i + \varepsilon_{\theta_i} \\ I_{Re,ij}^m &= V_i(A \sin \theta_i + B \cos \theta_i) + \\ &\quad V_j(C \sin \theta_j + D \cos \theta_j) + \varepsilon_{I_{Re,ij}} \\ I_{Im,ij}^m &= V_i(E \sin \theta_i + F \cos \theta_i) + \\ &\quad V_j(G \sin \theta_j + H \cos \theta_j) + \varepsilon_{I_{Im,ij}} \end{aligned}$$

where the constants  $A$  to  $H$  depend on the parameters of the  $\pi$  model associated with branch  $i$ - $j$  [28]. In all simulated scenarios the PMU and conventional measurements have been handled simultaneously during the estimation process.

### 6.1 Utilization of PMUs at the TSO level

The accuracy improvement arising by incorporating PMU measurements in a conventional SE at the TSO level has been assessed with the help of the 5-bus network sketched in Fig. 2. Different scenarios with increased redundancies have been considered:

- 1) Conventional measurements only: the set of measurements used for these simulations is shown in Fig. 2. The standard deviation for this type of measurements has been set to 0.01.
- 2) Inclusion of 1 PMU: in addition to the conventional set of measurements a single PMU ('PMU1' in Fig. 3) has been located at node 3. This device has several channels measuring the complex voltage at bus 3 as well as the current phasors through lines 3-4 and 3-5.
- 3) Inclusion of 3 PMUs: two more PMUs are added at nodes 4 and 5 ('PMU2' and 'PMU3' in Fig. 3). These PMUs measure only the voltage phasor at the corresponding node.

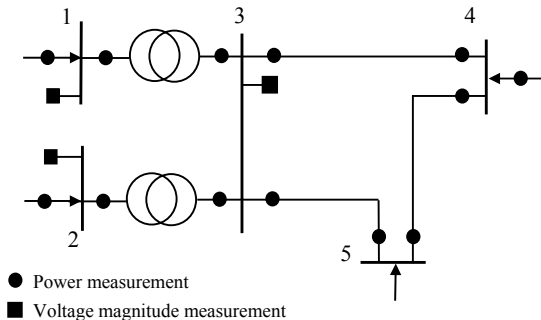


Figure 2: 5-bus illustrative network with conventional measurements

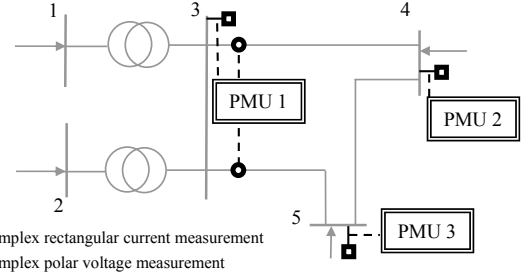


Figure 3: 5-bus illustrative network with PMU measurements

Different values for the standard deviation associated with PMU measurements have been tested, ranging from 0.0005 to 0.01 (same quality as conventional measurements). A parameter  $K$ , relating the standard deviation of conventional ( $\sigma_c$ ) and PMU ( $\sigma_{pmu}$ ) measurements, has been defined:

$$K = \sigma_c / \sigma_{pmu}$$

Hence,  $K = 10$ , for instance, means that the PMU measurements are 10 times more accurate in average than conventional measurements.

The measurements for the different scenarios have been created by adding gaussian noise to the 'exact' measurements corresponding to a given network state. The randomly generated noise has been scaled according to the standard deviation of the corresponding measurement, as follows:

$$z_i^m = z_i^{ex} + k_i \sigma_i \quad (19)$$

where  $z_i^m$  is the  $i$ -th measurement,  $z_i^{ex}$  the exact calculated value,  $k_i$  a randomly generated gaussian number  $N(0,1)$  and  $\sigma_i$  the standard deviation assumed.

For each value of  $K$  one hundred Monte Carlo simulations have been performed. In order to evaluate the improvement brought about by PMUs, the accuracy of voltage magnitude and power flow estimates are separately analyzed (these are the most interesting magnitudes for EMS operators). For this purpose, the following indices have been defined:

$$S_V = \sum_{i=1}^n |\tilde{V}_i - V_i^{ex}| / n$$

$$S_{PQ} = \sum_{i=1}^{n_{PQ}} |\tilde{P}Q_{ij} - PQ_{ij}^{ex}| / n_{PQ}$$

where  $n$  and  $n_{PQ}$  are the number of nodes and power flow (active and reactive) measurements, respectively and  $PQ_{ij}$  represents any power flow measurement.

Fig. 4 shows the resulting  $S_V$  index for the different scenarios. It can be observed how, as more PMUs are incorporated, lower values are obtained, which means more accurate estimates. Moreover, the estimates improve as the parameter  $K$  increases, which happens when the standard deviation associated with the PMU measurements decreases. Fig. 5 shows similar results for the index  $S_{PQ}$ .



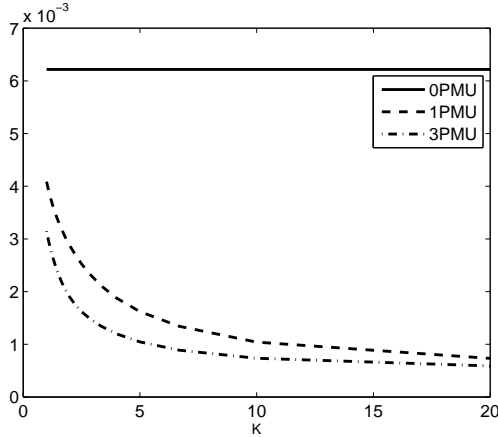


Figure 4: Simulation results for index  $S_V$

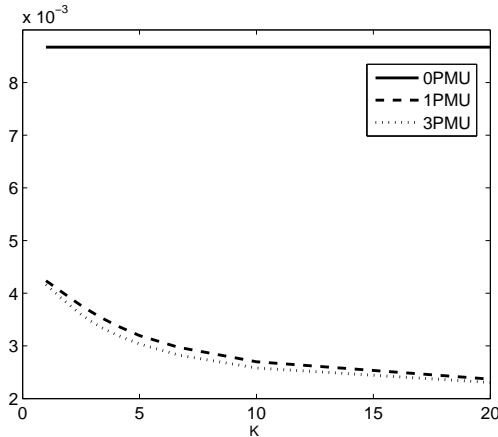


Figure 5: Simulation results for index  $S_{PQ}$

The following remarks are in order:

- The incremental benefit of adding voltage phasors ('PMU2' and 'PMU3') is less noticeable than that of current phasors ('PMU1'), particularly when power flows are considered (Fig. 5).
- For values of  $K$  larger than 10 the accuracy improvement brought about by PMUs somewhat saturates. In general, however, this will also depend on the redundancy of the conventional measurement set.
- From the point of view of estimate accuracy, it is probably better to invest in further improving the quality of PMU measurements rather than increasing the number of PMUs. However, robustness against failure or loss of PMU channels is higher when a larger number of PMUs are installed.

## 6.2 Utilization of PMUs at the regional multi-TSO level

In regional power systems, where real-time measurements are gathered within neighboring areas by the various control centers distributed over the grid, the Multi-Area State Estimation (MASE) has got renewed interest. In this environment, the need of properly monitoring energy transactions across TSO borders via large interconnections, while at the same time processing the real-time data at the most appropriate place, make the MASE a good

alternative. In general, MASE relies on some kind of decomposition-coordination scheme, taking advantage of the usually weaker geographical or measurement coupling among areas.

The MASE consists of a sequence of hierarchical SE processes comprising two main stages [2]: 1) each TSO independently solves the state estimation of its own area, including the tie-lines, border nodes and border measurements of adjacent areas; 2) the results of these decoupled estimators are then used by an *ad hoc* procedure which coordinates the estimate for the entire system. Under this scheme, when the different areas are not synchronized in time with PMU measurements, each of the areas sets a local phase angle reference for the TSO-level estimation process (first step). This requires the introduction of new state variables  $u$ , one for each area, relating the different phase angle references of the system. These variables, whose values are estimated at the second step, coordinate the results of the areas by referring the estimates to a global phase angle reference.

When synchronized PMU measurements are available at all areas, the local phase angle references are no longer needed. The PMU measurements will implicitly coordinate the independent estimates to the Universal Time Coordinated reference (UTC). In case some areas do not have PMU measurements available, only those areas will need to set a local phase angle reference, and for each one a  $u$  variable will have to be defined and estimated in order to coordinate the local estimates to the UTC.

Since the  $u$  variables coordinate the estimates of different areas, their role is crucial in the computation of power flows through tie-lines. The quality of the  $u$  estimates will affect the accuracy of the estimated flows through the tie-lines and, as a consequence, the estimates of the energy transactions among TSOs. If PMUs are available, the lack of  $u$  variables along with the enhanced accuracy usually provided by the PMU measurements, imply better estimates of power flows at tie-lines.

Some simulations have been carried out in order to evaluate the improvement of the multi-area state estimation in the presence of PMUs. The network used in these tests is made up of three IEEE 14-bus test networks, connected to each other as shown in Fig. 6, where only the tie-lines and border buses are represented. Table 1 shows the tie-line parameters adopted for these experiments.

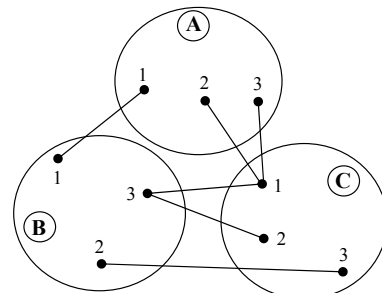


Figure 6: Multi-area system composed of 3 IEEE 14-bus networks

**Table 1:** Tie-line parameters

From	To	$R$ (p.u.)	$X$ (p.u.)	$B_{sh}$ (p.u.)
1A	1B	0.0194	0.0592	0.0528
2A	1C	0.0540	0.2230	0.0528
3A	1C	0.0470	0.1980	0.0528
2B	3C	0.0581	0.1763	0.0528
3B	1C	0.0570	0.1739	0.0528
3B	2C	0.0670	0.1710	0.0346

Different sets of ‘realistic’ measurements have been generated from a given state, following the procedure described in the previous section, in accordance to (19). The base case contains a complete set of conventional measurements comprising: voltage magnitudes at all nodes ( $\sigma = 0.01$ ), active and reactive power injections at all nodes ( $\sigma = 0.02$ ) and active and reactive power flows at both ends of all branches ( $\sigma = 0.015$ ). Four scenarios with additional PMUs have been considered:

- 1) A single PMU located at node 5 of TSO A. In this case, the local phase angle reference of TSO A can be taken as the global phase angle reference. The  $u$  variables for TSOs B and C will have to be computed in order to coordinate the estimates and calculate the power flows through the tie-lines.
- 2) PMU at node 5 of TSOs A and C. TSO B is the only area without PMU measurements and, therefore, it needs to set a local phase angle reference in order to perform its own SE. Since the estimate at TSO B is not synchronized to the rest of the system, its corresponding  $u$  variable will have to be estimated.
- 3) All TSOs with PMU at node 5. In this case,  $u$  variables are not necessary since the independent TSO estimates will be synchronized. Note that node 5 is an internal node, which is not directly connected to tie-lines.
- 4) A single PMU at node 1 of TSO C. Unlike in the previous scenarios, in this case the PMU is located at a border node, directly connected to TSOs A and B through tie-lines. Since the TSO areas of influence used for the first step include the tie-lines, the three TSOs can ‘see’ the PMU measurements in this case and, therefore, the estimates of the first step will be directly synchronized without the need of  $u$  variables.

Each PMU incorporates the voltage phasor measurement at the corresponding node and the current flow phasors measurements at the adjacent lines. The standard deviation of the PMU measurements has been set to 0.001.

Tables 2 and 3 show the exact and estimated values of the  $u$  variables for scenarios 1 and 2 respectively. For scenarios 3 and 4, no  $u$  variables are involved. In scenario 2 the estimate of  $u_B$  is more accurate (i.e., closer to the exact value) than in the first scenario, due to the higher redundancy level.

**Table 2:** Only one PMU at TSO A

Variable	Exact	Estimate
$u_B$	-0.0972	-0.0955
$u_C$	-0.1345	-0.1324

**Table 3:** PMU at TSOs A and C

Variable	Exact	Estimate
$u_B$	-0.0972	-0.0964

For each scenario, the tie-line power flows have been estimated and the index  $S_{PQ}$  defined in the previous section has been computed. In this case, an  $S_\theta$  index, similar to the  $S_V$  index employed before, has been defined:

$$S_\theta = \sum_{i=1}^n |\tilde{\theta}_i - \theta_i^{ex}|/n$$

Table 4 shows the indices obtained for the different scenarios. For scenarios 1 to 3 the quality of the estimates increases (lower errors) with the number of PMUs incorporated. Note that scenario 4, with a single PMU located at a strategic border node so that no  $u$  variables are needed, gives better estimates of power flows through tie-lines than scenario 3, with 3 PMUs installed and no  $u$  variables involved either. However, in terms of phase angles estimates, scenarios 2 and 3 provide better results.

**Table 4:** Average estimation errors for the scenarios considered in MASE

Indices	Scen. 1	Scen. 2	Scen. 3	Scen. 4
$S_{PQ}$	0.00373	0.00338	0.00324	0.00118
$S_\theta$	0.00234	0.00134	0.00119	0.00196

From these results, it can be concluded that PMU measurements improve the estimates of power flows through the tie-lines in a MASE process, due to the removal of  $u$  variables and the increased accuracy of the added measurements. Not only is the number of PMUs important, but also the location. A lower number of PMU measurements in carefully selected locations may give better estimates of the tie-line power flows. However, as expected, the number of phase angle measurements provided by PMUs is directly related to the accuracy of the phase angles estimates.

## 7 Use of PMUs in dynamic state estimation

**I**N the electric power engineering terminology, SE refers to static SE; this computes the state vector at one time instant from measurements captured at the same time instant. The process is repeated at successive times  $k$  but does not include any physical modeling of the time behavior of the system. The Dynamic State Estimation (DSE) method on the contrary relies on the following general dynamic model, written here in its discrete state transition form [34]:

$$x_{k+1} = f(x_k, w_k, k) \quad (20)$$

with noise  $w_k$  accounting for modeling errors.

Most DSE algorithms rely on the extended Kalman filter consisting of alternate sequences of filtering and prediction steps: at time  $k$

$$\hat{x}_k = \bar{x}_{k-1} + K_k (z_k - h(\bar{x}_{k-1})) \quad (21)$$

$$K_k = (H_k^T W H_k + M_k^{-1})^{-1} H_k^T W \quad (22)$$

$$\bar{x}_{k+1} = F_k \hat{x}_k + u_k \quad (23)$$

$$M_{k+1} = F_k (H_k^T W H_k + M_k^{-1})^{-1} F_k^T + Q \quad (24)$$

These equations are obtained after proper linearization of (20) and (1) where:

$\hat{x}_k(\bar{x}_k)$  is the estimated (predicted) state at time  $k$ ,

$F_k$  is the Jacobian of  $f$  evaluated at time  $k$ ,

$u_k$  acts as a command term coming from the linearization,

$M_k$  is the covariance matrix of predicted state  $\bar{x}_k$ ,

$Q$  is the covariance matrix of noise  $w$ , assumed to have a Normal distribution with zero mean.

Benefits which could be encountered from DSE are linked to its predictive ability which provides the necessary information to perform preventive analysis and control and can also help observability analysis, identification of bad data and detection of topology errors.

In practice, however, DSE is faced with the problem of the availability of a reliable modeling of the system state evolution. According to the power system state estimation paradigm, the system dynamics are modeled as a succession of steady states, the transitions between states being caused by the variations of the loads and by the corresponding adaptations of the generations. Up to the advent of the PMU technology, the available measurements were limited to slow varying quantities captured at relatively low rates thus limiting the application of DSE. The introduction of phasor measurements, precisely synchronized and available at higher rates, makes it possible to derive dynamic estimators capable of following faster system variations.

PMU measurements  $z_V$  and  $z_I$  (7), (8) are easily embedded in the EKF filtering step. The simultaneous formulation described in section 5.2 is used by handling at the same time the PMU and the conventional measurements in (21).

Regarding the dynamics modeling two main approaches can be distinguished. The first one [35] relies on the generic linear model :

$$x_{k+1} = F_k x_k + d_k + w_k \quad (25)$$

Here  $F_k$  is a diagonal matrix accounting for the state transition and  $d_k$  is associated with the trend component. These parameters are identified on-line from archived data of the system state using the Holt's linear exponential smoothing method.

The second approach [36] recognizes that, rather than the voltage state vector components, the variables which actually drive the system dynamics considered are the nodal power injections. The prediction is as follows : (1) the load flow data are predicted at the next time instant using a short-term nodal load forecasting technique; (2) the predicted state vector  $\bar{x}_{k+1}$  is obtained through a standard load flow calculation.

The concept of DSE can be extended to short-term dynamics such as the generator speed or acceleration using the additional information of frequency and rate of change of frequency provided by PMU devices [37]. The state of

the system is composed of the voltage phasor at each bus and also of the frequency of the voltage phasor at each bus of the system. Additional internal dynamical or algebraic states are also introduced for each device. The model of the system is described by a set of differential and algebraic equations as follows :

$$\frac{dx(t)}{dt} = f(x(t), y(t), t), \quad 0 = g(x(t), y(t), t) \quad (26)$$

where  $x$  and  $y$  are the dynamic and algebraic state vector respectively.

The estimation is distributed at the substation level using a three-phase breaker oriented, instrumentation channel inclusive model. The set of physical measurements  $z$  comprises all available data coming from PMUs, relays and other IEDs. These values are compared to the computed values  $h(\bar{x})$  deduced from the model forming a measurement error vector  $e$ . To ease the computations, the model is quadratized and digitized through a numerical integration technique. A standard WLS estimation is performed on the sum of the squared errors  $e$ .

This algorithm can be run at rates comparable to those recommended in the synchrophasors standard IEEE-C37.118, thus enabling to track the system real-time dynamic evolution. This estimator can be used in several applications dealing with power system wide area monitoring and control, such as transient stability monitoring [38].

## 8 Conclusions

THIS paper aims to provide a candid evaluation of the way large, multi-area power systems will be monitored as their operations become more interdependent. Multi-areas can be defined either geographically or based on voltage levels. This paper's contributions build on the numerous innovative works done so far by various researchers and the hierarchical perspective to monitor very large scale power systems which is arising in the upcoming smart grid context. An important technological driver in this development is the synchronized phasor measurements, which provide benefits in identification of topological and parameter errors, maintaining network observability, improving statistical as well as numerical robustness of the estimators. They also pave the way of developing estimators with very high scan rates, making it possible to capture system dynamics which are currently ignored by existing state estimators. Another important driver is the set of computational and communication technologies that are rapidly becoming available at all substations, facilitating the implementation of hierarchical solutions like those discussed in this paper. Such hierarchical decoupling appears inevitable in order to efficiently address the growing complexity of the system due to the penetration of renewable distributed generation and storage, primarily at the lower voltage levels. In a near future, as the operation becomes more heavily dependent on these technologies and their automation, issues of cyber and physical security will need to be addressed.

## 9 Acknowledgements

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