Proc. of the 5th WSEAS/IASME Int. Conf. on Electric Power Systems, High Voltages, Electric Machines, Tenerife, Spain, December 16-18, 2005 (pp574-578)

A Comparison Between Kalman Filters and STDFT for Harmonic Estimation in Power Systems

J. A. ROSENDO MACÍAS A. GÓMEZ EXPÓSITO
Department of Electrical Engineering
University of Sevilla
Camino de los Descubrimientos s/n, Sevilla
SPAIN

Abstract: - This paper presents a comparison between Kalman filter and the running DFT for the computation of harmonics in power systems applications. The performance of both filters is compared for events like voltage dips or those in which a decaying DC component is present. The comparison considers also the presence of higher order harmonics.

Key-Words: - Digital filters, Harmonic analysis, Kalman filtering, Power system harmonics, Power system protection, short time discrete Fourier transform

1 Introduction

Many power systems applications, like digital protection or power quality assessment, rely on the accurate computation of harmonics. Although the use of the short-time discrete Fourier transform (STDFT) has been extensively considered for this purpose, other techniques are also available [1], [2], [3].
Among them, Kalman filtering shows great promise because its estimate is unbiased and optimal in the sense that the covariance of the estimation error is minimized.

In this paper the basic theory of the STDFT and Kalman filter will be first reviewed, as well as the Kalman filter models available for harmonic estimation.

The comparison presented in this paper considers three main scenarios: A) Single harmonic model and sudden amplitude dips at different phase angles of the purely sinusoidal signal. B) Decaying DC component at different phase angles, and C) Presence of higher harmonics to be modeled in the Kalman filter.

2 Short-time Discrete Fourier Transform

The Short-time discrete Fourier transform (STDFT) or running discrete Fourier transform of a signal $z(n)$ and window length N can be seen as the discrete Fourier transform (DFT) of the sequence $z(n-N+1)$, $z(n-N+2)$, …, $z(n-1)$, $z(n)$:

$$Z_n(k) = \sum_{i=0}^{N-1} z(n-N+1+i)e^{-j\frac{2\pi i}{N}} \qquad (1)$$

where $Z(k)_n$ is the harmonic number k ($k = 0,1,\dots N-1$) at time instant n.

The computation of every harmonic can be seen as the output a digital filter, with a frequency response similar to that of figure 1, for the fundamental harmonic ($k = 1$), where it can be easily verified that each filter rejects perfectly the presence of other harmonics:

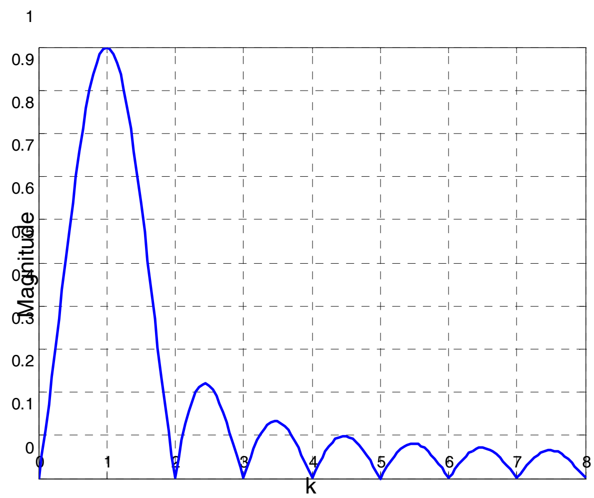

Fig. 1: Frequency response of the fundamental harmonic filter $Z(1)_n$.

Direct computation of a single harmonic using equation (1) involves O(N) floating operations but several algorithms have been developed to reduce this computational cost. Recursive algorithms like [4] or [5] have low cost, independent of N, but long-term accuracy deterioration has been reported [6]. Non-recursive algorithms, like [7], [8] are free of these problems and still have a low computational cost, O($\log N$).

3 Kalman Filters

Kalman filtering theory assumes a system model with the following equation in the state space

$$x(n+1) = \Phi(n)x(n) + w(n)$$

and a measurement model given by

$$z(n) = H(n)x(n) + v(n)$$

where

- $x(n)$ is the state vector
- $\Phi(n)$ is the state transition matrix
- $H(n)$ is the measurement matrix.
- $w(n)$ and $v(n)$ are the model and the measurement error vectors. Each of their components is considered to be a white noise. The covariance matrices, $Q(n) = E(ww^T)$ and $R(n) = E(vv^T)$ respectively, are assumed to be diagonal matrices of constant terms: $Q(n) = Q_0 I$ and $R(n) = R_0 I$.

Having *a priori* knowledge of error covariances Q and R, the filter can be implemented with the following equations

$$\hat{x}^-(n) = \Phi(n-1)\hat{x}^-(n-1)$$

$$P^-(n) = \Phi(n-1)P(n-1)\Phi^T(n-1) + Q(n)$$

$$K(n) = P^-(n)H^T(n)\left[H(n)P^-(n)H^T(n) + R(n)\right]^{-1}$$

$$\hat{x}(n) = \hat{x}^-(n) + K(n)\left[z(n) - H(n)\hat{x}^-(n)\right]$$

$$P(n) = \left[I - K(n)H(n)\right]P^-(n)$$

where $\hat{x}(n)$ is the estimation of $x(n)$, $K(n)$ is the Kalman gain, $P(n)$ is the covariance matrix of the estimation error, $P(n) = E\left[(x-\hat{x})(x-\hat{x})^T\right]$, $\hat{x}^-(n)$ and $P^-(n)$ are estimations at instant n using only information available at instant $n-1$.

The filter needs initial values for the state estimation vector $\hat{x}(0)$ and estimation error covariance $P(0)$, which is considered to be diagonal, $P(0) = P_0 I$. However, these values are not especially relevant since their effect only last for the first instants.

3.1 Model for a single harmonic

Estimating a single harmonic leads to the state vector

$$x(n) = \begin{bmatrix} C_1 \\ S_1 \end{bmatrix}$$

where C_1 and S_1 are the real and imaginary components.

For the fundamental harmonic, two approaches are possible for the transition and measurement matrices. Let N be the number of samples for a period of the signal, so, the first approach leads to a constant phasor in steady state:

$$\Phi(n) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H(n) = \left[\cos(2\pi n/N) \quad -\sin(2\pi n/N)\right]$$

while the second one leads to a rotating phasor:

$$\Phi(n) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$H(n) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where $\phi = 2\pi / N$.

Although the same results are obtained in both cases provided the rotation is taken into account, the second approach has the advantage of having matrices of constant coefficients.

3.2 Model for decaying DC component

This is a usual case in digital protection applications where an exponentially decaying component is present during the faulted period.

In this case, the state vector

$$x(n) = \begin{bmatrix} X_0 \\ C_1 \\ S_1 \end{bmatrix}$$

should be used, being X_0 the decaying DC component.

With this state vector and considering the rotating phasor approach, the transition and measurement matrices are:

$$\Phi(n) = \begin{bmatrix} e^{-1/\lambda} & & \\ & \cos(\phi) & -\sin(\phi) \\ & \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$H(n) = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

where λ is the time constant of decay, which can be estimated from the equivalent impedance of the network, $\lambda = L_{eq} / R_{eq}$.

3.3 Model for higher harmonics

When p harmonics have to be modeled:

$$x(n) = \begin{bmatrix} C_1 \\ S_1 \\ \vdots \\ C_p \\ S_p \end{bmatrix}$$

The transition and measurement matrices are

$$\Phi(n) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & & & \\ \sin(\phi) & \cos(\phi) & & & \\ & & \ddots & & \\ & & & \cos(p\phi) & -\sin(p\phi) \\ & & & \sin(p\phi) & \cos(p\phi) \end{bmatrix}$$

$$H(n) = [1 \quad 0 \quad \ldots \quad 1 \quad 0]$$

4 Comparison

Performance of STDFT and Kalman filters is compared in this section for several cases. The signal under study is sampled $N = 16$ times every period and the Kalman filter uses $Q_0 = 1$, $R_0 = 0.01$, $\hat{x}(0) = 0$ and $P_0 = 10000$, like in [9].

4.1 Single harmonic

To compare both techniques, a signal which is purely sinusoidal except for a two-cycle dip is supposed:

$$z(n) = \begin{cases} 0 & 32 < n < 64 \\ 100\cos(2\pi n / N + \varphi) & elsewhere \end{cases}$$

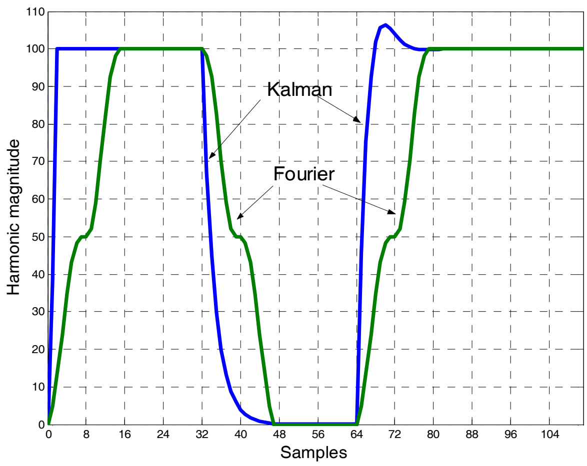

Fig. 2: Performance for sinusoidal signal, $\varphi = 0$

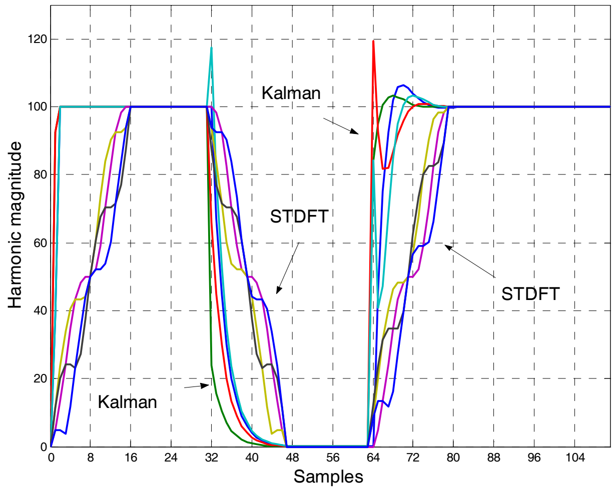

Fig. 3: Performance for different phase angles

In figure 2, the performance for Kalman and STDFT is shown for the signal with $\varphi = 0$. The Kalman filter converges within the first 3 samples due to the high value of P_0.

Convergence time at the subsequent dip transitions still remains faster than that of SFDFT, which is one period ($N = 16$).

Figure 3 shows the performance for different values of φ, including $\varphi = 0$ and $\varphi = \pi / 2$. It can be seen that the Kalman filter presents strong oscillations for some sinusoids, while SDTFT has a quite uniform behavior, but a little bit slower than the Kalman filter.

4.2 Decaying DC component

This comparison considers the case of a decaying DC component added to the sinusoidal signal from the instant $n = 32$ on:

$$z(n) = \begin{cases} s(n) + 100e^{-(n-32)/\lambda} & 32 < n \\ s(n) & elsewhere \end{cases}$$

where $s(n) = 100\cos(2\pi n / N + \varphi)$ and $\lambda = 16$.

Figure 4 shows that the Kalman filter is able to converge to the correct value of the harmonic in the presence of the decaying DC component, while the STDFT remains with oscillations even after one period. This good behavior of the Kalman filter is checked for different phase angles in the sinusoid, as shown in figure 5. In this figure, it can seen that the convergence time and the results of the DC component do not depend on the phase angle.

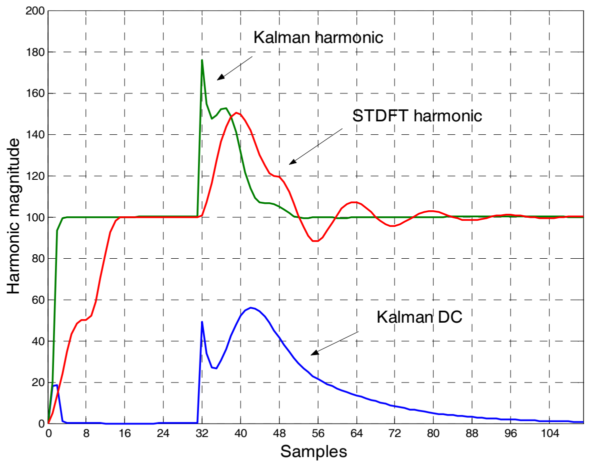

Fig. 4: Comparison for decaying DC component

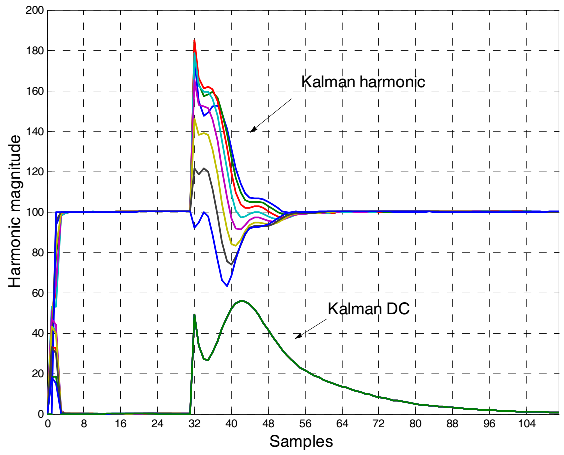

Fig. 5: Kalman performance for decaying DC component and several phase angles

4.3 Higher harmonics

This test is carried out improving the model of the Kalman filter to consider up to $p = 4$ harmonics. The result is shown in figure 7, where it is clear that even the model for $p = 2$ harmonics slows down the Kalman filter convergence time compared with that of the STDFT. Convergence is similar to that for larger values of p.

These results are not significantly affected by the presence of the modeled harmonics, as shown in Figure 7 for the case $p = 4$ and purely sinusoidal harmonics 2, 3 and 4 of amplitude 10.

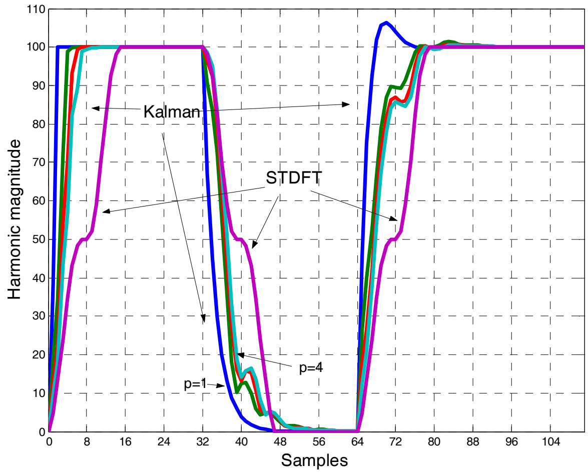

Fig. 6: Performance for large number of harmonics in the model

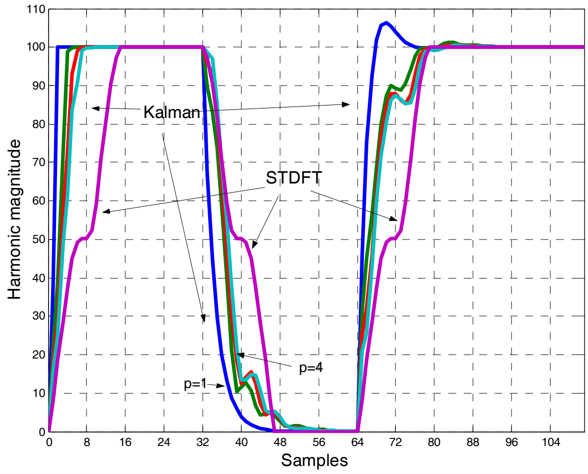

Fig. 7: Performance for large number of harmonics in signal and model.

5 Conclusion

The STDFT shows quite a uniform convergence within a period to the proper value of the harmonic, despite the presence of higher harmonics. However it can not deal properly with the presence of decaying DC component, which produces ripple around the correct harmonic magnitude.

On the other hand, the Kalman filter shows faster convergence than a period when a single harmonic is present, but sensitivity of this filter to non-modeled components makes it necessary to enlarge the Kalman model. The enlarged model comprising higher harmonics slows down the filter convergence, but in the case of decaying DC component provides the correct output as soon as the DC component is properly estimated.

References:

[1] A.G. Phadke, J.S. Thorp, *Computer Relaying for Power*, Ed. Somerset, England: Research Studies Press, 1988.

[2] K.K. Li, C. Cheung, Y.Q. Xia, Performance comparison and evaluation of digital distance protection algorithms, *International Conference on Advances in Power System Control Operation and Management, 1991. APSCOM-91*., Vol.1, Nov 1991, pp.455-460.

[3] E.O. Schweitzer, D. Hou, Filtering for Protective Relays, *WESCANEX 93, "Communications, Computers and Power in the Modern Environment" Conference Proceedings*, IEEE, May 1993, pp. 15-23.

[4] J.H. Halberstein, Recursive, complex Fourier analysis for real-time applications, *Proc. IEEE*, Vol. 54, no. 6, Jun, 1966, p. 903.

[5] A. Gómez, J. A. Rosendo, J. L. Ruiz, Discrete Fourier Transform Computation for Digital Relaying, *Int. Journal of Electrical Power & Energy Systems*, Vol. 16(4), Aug 1994, pp. 229-233.

[6] J.A. Rosendo, A. Gómez, Efficient Moving-Window DFT Algorithms, *IEEE Transac. on Circuits and Systems, part II Analog and Digital Signal Processing*, Vol. 45 (2), Feb. 1998, pp. 256-260.

[7] A. Gómez, J.A. Rosendo, Fast Non-Recursive Computation of Individual Running Harmonics, *IEEE Transac. on Circuits and Systems, part II Analog and Digital Signal Processing*, Vol. 47 (8), Aug. 2000, pp. 779-782.

[8] A. Gómez, J.A. Rosendo, Fast Harmonic Computation for Digital Relaying. *IEEE Transac. on Power Delivery*, Vol. 14 (4), Oct. 1999, pp. 1263-1268.

[9] J.A. Rosendo, A. Gómez, Kalman Filter Tuning for Digital Protection Applications, *IEEE PowerTech 2003. Bolonia, Italy*, Paper 240, Jun. 2003.