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Designing fixed income securities investment portfolios for SMEs in different scenarios

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Designing fixed income securities investment portfolios for SMEs in different scenarios

The management of fixed income securities investment portfolios in Small and Medium Enterprises is gaining increasing interest in capital markets. This paper analyses Robust Optimisation Models as efficient tools for risk management of fixed income securities. The study includes the analysis of scenario-based optimisation models applied to the portfolio selection and on the basis of indeterminate initial endowment. A detailed analysis is made for a case study involving the composition of fixed income investment portfolios, which is solved using robust scenario-based optimisation models. Finally, a sensitivity analysis is carried out for different scenarios occurring for each of the models.

Keywords: Investment portfolio; robust optimisation; scenario; expected return; coordinated error

1. Introduction

The management of fixed income securities investment portfolios in Small and Medium Enterprises (SMEs) is gaining an increasing interest in the capital markets (Cheung and Chan, 2002; Shirai, 2004; Jobst, 2005; Banerjee, 2006). The shortening life cycle of products and the usefulness of fixed investment combined with large fluctuations in share and property prices tend to reduce the certainty that collateral can offer in cases of repayment problems.

There is an extensive tradition of sector studies conducted in relation to investments in fixed income securities in the investment market, with numerous investment funds investing exclusively in fixed income securities. These are "fixed income funds" available from any bank, savings bank or securities firm. The main reason for choosing them is that, as well as offering greater liquidity, they are also a secure investment; in other words, they offer greater stability.

The problem of portfolio selection is highly complex, since there are many sources of uncertainty and multiple selection criteria. The risk/return pairing is the basic principle applied in models, with the subsequent conflict of objectives, since, in order to achieve higher yield, more risky strategies must be adopted.

The main historic reference in this area is the Markowitz Model (1952). The aim of the author was to propose a model of rational behaviour for the decision-maker for the selection of securities with immediate liquidity (in this model, the liquidity of the security is immediate at the end of the reference period). He proposed a mean-variance (MV) model for portfolio selection. In this model, the expected return and risk of the investment are measured using the mean and variance of past yield.

Subsequently, other authors proposed diverse models that introduced different modifications and improvements to the Markowitz Model. Sharpe (1965) proposed a simplification which assumed the existence of a linear relationship between the yield of the security and that of the market portfolio. This meant that portfolio risk could be defined without using covariances, thereby greatly simplifying calculations. Michaud (1989) examined a method for dealing with the instability of the solutions in light of changes in expected yield. It is grounded in the principle that the efficient frontier is not a simple line but rather a band that represents a confidence interval. Changes in the parameters can lead to portfolios falling within that same confidence interval or outside of it. When these new portfolios derived from new parameters lead to solutions within the same confidence level, the portfolio found initially should not be modified. It should only be modified when the new solution falls outside of the confidence interval found previously. And, finally, Chopra and Ziemba (1993) studied the consequences of estimation errors on expected yield, and variances and covariances in the Markowitz model. They reached the conclusion that errors in expected yield are important to the order of 11 times greater than errors in the estimation of variance, and 20 times greater than errors of covariance on the effects of deviation from the true optimum. Furthermore, for more risk-adverse investors, errors in expected yield are much more important than for those with a lower risk-aversion. In other words, the solution obtained is further from the true optimum.

Roy (1952), Kataoca (1963) and Telser (1955) carried out studies in relation to the Safety First Model, which places the safety of the investor above any possible increase in yield. One alternative to the models cited above are models that require measurements based on past data and the experience of expert managers (Tanaka and Guo, 1999), as well as the consideration of scenarios and their application in robust optimisation, based on target-based programming and the consideration of weak restrictions (Mulvey, Vanderbei and Zenios, 1995). Although dealing with different scenarios is traditional in scientific-business management, and the idea was already present in the states of nature of Decision Theory developed by Neumann and Morgenster, portfolio selection was not tackled until the Scenario and Factors Model developed by Markowitz and Perold (1981). The use of robust optimisation in decision-making enables risk aversion to be considered directly (Bai, Carpenter and Mulvey, 1997), which is one of the advantages it offers over stochastic programming. Dembo (1992), Escudero (1995), Golub (1994) and Vassiadou-Zeniou and Zenios (1996) applied their models to the planning and financial management of robust optimisation models. This aspect is the focal point of this paper.

Robust optimisation involves the integration of target-based programming on the basis of possible scenarios. Its application generates a series of solutions that make the possible manifestations of different scenarios progressively less sensitive. The optimum solution for the problem is robust in terms of optimality if it remains "closed" in the optimum. If it remains admissible for any manifestation, the model is said to be robust.

The application of Robust Optimisation Models greatly strengthens the decision-making process of companies with investments in fixed income securities (Fabozzi et al. 2007; Goel et al. 2009; Gülpinar et al. 2011).

The variety of scenarios and volatilities of the different markets in which a manager can operate make the in-depth study of investment risk essential. The purpose of said study is to equip the manager with tools that facilitate treatment, guaranteeing and securing maximum profitability for investments, and consequently improving the competitiveness of the company. The systematic management of risks ensures the success of investments and the foreseeability of their indicators. The management of Interest Rate Risk (Dembo, 1991), in which the random parameters of the problem (interest rates) are described through scenarios with their corresponding probabilities, allows the subjectivity of the decision-maker to be introduced when choosing scenarios and assigning probabilities. And also the *Total Resource Model* (Rockafellar and Wets, 1993 and Escudero 1995), in which all decisions are adjusted in time when new information is obtained about them, except those corresponding to the first period of time, so that all decisions are consistent with the information available in any given time period.

In this paper, Robust Optimisation Models are applied to a study into Risk Management in fixed income investment portfolios. For this purpose, it identifies analyses and quantitatively analyses risk.

Furthermore, it resolves a scenario-based optimisation model applied to the problem of portfolio selection with an indeterminate initial endowment.

The model proposes the investment capital and a different portfolio for each of the scenarios. Since it is unlikely that a solution might simultaneously fulfil criteria of robustness for admissibility and optimality, a coordinated model is constructed that adequately combines the two concepts.

The second section shows the mathematical programming models developed for decision-making based on robust scenario-based optimisation models, seeking to minimise coordinated error. The third section offers an analysis of the results obtained, as well as a sensitivity analysis for each of the different models. Finally, the fourth section provides a comparison between the results of the different models, and the main conclusions are presented in section five.

2. Modelling the investment portfolio decision-making

To tackle with the presented problem is necessary to introduce the required modelling process helping the decision support.

2.1. Modelling the problem

This study begins with the determination of a set of scenarios, $s = 1, \ldots, S$, and expected yield for each bond, $i = 1, \ldots, n$, known for each scenario (which will be noted as r_{is}). From here, we can obtain the efficient frontier of each scenario, by resolving the following model based on that of Markowitz, which determines the efficient portfolio for each of the scenarios independently. This has been termed the *IOP (independent optimum portfolio) model*

Minimize $\sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_{is}x_{js}$

Subject to: $\sum_{i=1}^{n} r_{is}x_{is} = r_s C_s \quad s = 1,...S$

$$\sum_{i=1}^{n} x_{is} = C_s \quad s = 1,...S$$

$$x_{is} \geq 0 \quad \forall i = 1,...,n \quad (1)$$

Where σ^2 determines the variance of the portfolio; x_{is} is the amount invested in asset i in scenario s; r_{is} is the expected yield on asset i in scenario s, which is considered a datum as indicated; C_s is the total capital invested in the portfolio in scenario s; and r_s is the expected yield of the portfolio in scenario s. Both are data of the model.

The resolution of the model for each scenario, varying the value of the parameter r_s, allows the efficient frontier to be obtained for that scenario.

The consideration of the fact that the scenario that will ultimately be produced at the end of the process is unknown but which, on the other hand, can be estimated with a certain probability associated with the occurrence of one scenario or another, leads to the consideration of a coordinated model that adequately combines the concepts of admissibility and optimality. In other words, a robust model that sufficiently controls the risk taken by the investor when making the investment and which sticks to the optimum solution as closely as possible, regardless of the scenario that occurs. The concept of robustness introduced here aims to mitigate the problems presented by the *Markowitz Model*. The first problem is excessive exposure to variations in future yield, through the consideration of different scenarios. The values of the variances with regard to expected yield in short intervals of times (for example, one year) are very large. In other words, the prediction error in these periods is also high. Furthermore, it is extremely sensitive to the values of the expected yield, so that small variations can yield portfolios that vary greatly in terms of their composition.

The aim of the model is to minimise the *coordinated error of the investment in the asset with regard to scenario s.* This error is the relative deviation in expected profits with regard to the optimum profits for each scenario. We define the *coordinated error of x with regard to scenario s* as the relative deviation in the expected profit of solution *x* pertaining to the profit of the optimum portfolio for each scenario, expressed in percentage terms instead of monetary units. Furthermore, different expected yields have been considered for each security in accordance with the expected scenario.

Hence, two coordinated models are proposed. In the first, the maximum error produced in the most unfavourable scenario, *s*, is minimised, and has been termed the *MM Model* (min-max error).

Mininimize ε

Subject to: $P_s \varepsilon_s(x) \leq \varepsilon$, $s = 1,....,S$

$$C_{\min}\left(1 - \frac{C_{\min} - C_s}{C_{\min}} y^s\right) \leq C(x),\ s = 1,...,S-1$$

$$C_{\max}\left(1 - \frac{C_{\max} - C_{s+1}}{C_{\max}} y_s\right) \geq C(x),\ s = 1,...,S-1$$

$$L(x) \leq L(x_{s+1})y_s + L(x_S)(1 - y_s),\ s = 1,...,S-1$$

$$1 \leq \sum_{s=1}^{S-1} y_s \leq 2$$

$$\varepsilon, x \geq o\ , y_s \in \{0,1\}$$

For the construction of these models, portfolio risk will be measured through the variance of its past yield, where σ_{ij} is the covariance of the yield earned on asset, *i*, with that of the yield earned on asset, *j* .

The variables of the model are the coordinated error (ε); the amount invested in asset *i* (x_i) which now corresponds to the investment eventually undertaken and is independent of the scenario which might or might not be produced aprioristically; and the total associated coordinated capital, $C(x)$, which remains, therefore, indeterminate. The model also calculates the investment risk in the assets, $L(x)$, and the binary variable that establishes the observed scenario, y_s.

The data taken into account are the probability of occurrence of each scenario, P_s, the minimum capital to be invested, $C_{\min}$ and the maximum capital to be invested, $C_{\max}$.

Finally, the set of different bond ratings, i, and the series of scenarios are the parameters to include.

So then, the solution of the model provides the real portfolio and the capital to be invested in it.

The second model presented coincides with the previous one and is only differentiated in terms of the target function, in which the aim is to minimise average weighted error through the probability of occurrence of scenario *s*. It has been termed the *MAD Model* (*minimum absolute deviation*), with the aforementioned target function $Min\ \varepsilon = \sum_{s=1}^{S} P_s \left| \varepsilon_s \right|$.

2.2. Preparing the data for the study

Firstly, the data pertaining to each bond are compiled; then the portfolio risk is calculated, the scenarios are constructed and, finally, the expected yields are calculated in each scenario.

The bonds used for the experimentation of this paper are US corporate bonds. The bonds are generally issued in multiples of $1,000 (also known as nominal value or par value of the bond). As investment possibilities, this study deals with a group of 24 bonds belonging to the US secondary corporate bonds market, with maturity periods of between 3 and 10 years. Specifically, eight bonds with maturity at 3 years (2012), eight with maturity at 5 years (2014), and eight with maturity at 10 years (2019). In all cases, 2009 is taken as the initial year.

On the basis of the data for all the transactions carried out in relation to each bond during the year 2008 (obtained from FINRA at www.finra.org), for each bond, a table is constructed showing the past prices for the year 2008. Table 1 depicts the case for bond WFC.JL.

The trading prices of each of the 24 bonds for all the transactions carried out during the year 2008, as well as the other general information for each bond, are the data used to construct the scenarios, calculate the expected yield for each bond and calculate the matrix of covariance, required to evaluate risk.

With regard to bond selection criteria, the choice of the 24 bonds was made in accordance with the following criteria:

- Very active bonds: The problem examined is the selection of portfolios with an investment time horizon of one year, after which, the securities mature, and the money earned is either consumed or reinvested. To avoid having to liquidity risk in the model, or the risk that the possibilities of selling the bond are limited, bonds were chosen with a high number and volume of transactions in 2008, thereby, presumably, diminishing the exposure to this risk.
- Non-Callable Bonds: The risk of early redemption has similarly not been taken into account in the models. For this purpose, the bonds used in this study do not incorporate this option for the investor (Non-callable bonds).
- Bonds with similar ratings. In other words, although it is possible to take the risk of insolvency into account when analysing the expected yield of bonds, in this case it has not been considered, selecting, as investment possibilities, bonds with similar ratings to ensure their compatibility in terms of insolvency risk. All the bonds considered belong to the group defined as "above average" or "above average grade" investment, with ratings between A- and A+ as rated by Standard & Poor's and Fitch and between A3 and A1 by Moody's.

Then the hypotheses of the bonds considered in the models are low liquidity risk, since they are non-callable and have similar ratings.

The market price for the bond in each month was taken to be the average trading price of all transactions carried out during that month. The prices are normally expressed as a percentage of the nominal ($1,000 for this type).

This table is used to construct the table showing past yield (table 2) using the definition of current yield ($ra = \frac{Q}{P} \times 100$, where Q is the rate of interest of the coupon).

MONTH	PRICE (P)
Jan-08	106.615
Feb-08	107.077
Mar-08	104.958
Abr-08	104.606
May-08	101.116
Jun-08	103.329
Jul-08	102.438
Aug-08	101.417
Sep-08	99.123
Oct-08	100.522
Nov-08	102.474
Dec-08	102.471

Table 1. Past Prices

MONTH	YIELD (ra)
Jan-08	5.76
Feb-08	5.76
Mar-08	5.72
Abr-08	5.82
May-08	5.89
Jun-08	5.95
Jul-08	5.97
Aug-08	5.99
Sep-08	6.08
Oct-08	6.13
Nov-08	6.08
Dec-08	5.98

Table 2. Past yield

Repeating the operation gives a 12 element vector for each bond, R_i, with the current yield for each month in the year 2008. These vectors are used to obtain the matrix of covariances, where the covariance of two bonds i and j is equal to $\sigma_{ij} = \frac{1}{12}\sum_{t=1}^{12}(R_{it} - \overline{R_i})(R_{jt} - \overline{R_j})$.

For past yields, current yields have been used instead of the yield to maturity, since the latter gives the internal rate of return (*IRR*) of the investment, and, given that bonds with different maturity terms have been considered, it would not be correct to use this measure when constructing the matrix of covariances. Hence, to obtain the expected yield, firstly, the variation in yield to maturity was calculated, followed by the market price at the end of the investment period, and finally, the expected yield.

The scenarios were constructed on the basis of the existing structure of interest rates on 01/01/2009 for A-rated corporate bonds. Through the webpage of *FINRA* the data were obtained for yield to maturity for the different maturity terms offered by the set of corporate bonds with this rating.

Five possible scenarios have been assumed, which could represent the temporal structure of interest rate (TSIR) at the end of the investment period (01/01/10). As for the movements of the curve, it has been assumed that there will be parallel movements on the yield curve. Of these 5 assumed scenarios, the central scenario 3 represents the maintenance of interest rates, for which the yield curve would be the same as at the start of the investment period. Furthermore, from this central scenario, there are two favourable scenarios, scenario 4 and scenario 5, with decreasing interest rates that entail a decrease in the yield to maturity (IRR) for all the bonds considered of 100 and 200 base points respectively (minimum percentage variation scale used to compare yield for fixed income securities, representing a one hundredth of a percentage point. 100 base points equals 1%.); and two unfavourable scenarios, scenario 2 and scenario 1, with rising interest rates which entail an increase in yield to maturity of 100 and 200 base points.

The yield curves representing the 5 different scenarios are shown in Figure 1.

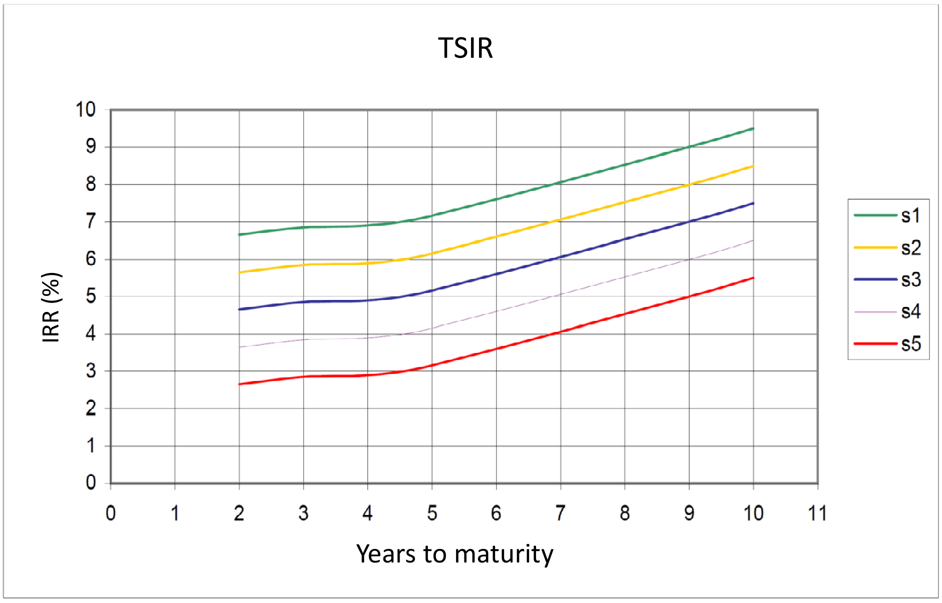

Figure 1. Yield curves of different scenarios

The central curve represents the TSIR from the start of the investment period to the end in scenario 3 representing the case of rates not varying at the end of the period.

Based on the yield curves (figure 1) the percentage variation of the yield to maturity (IRR) of the A-grade bonds market is calculated from the start to the end of the investment, depending on which scenario occurs.

The arguments required for these two functions are organised as shown in table 3.

Bond details		
Nominal value	$ 1,000	
Yield to maturity as % of nominal	100.00	
Annual coupon rate	7.630%	
Desired annual yield	7.070%	
Buying date	01/01/2009	
Maturity date	01/07/2019	
Frequency of payment	2	
Basis	0	
Bond value as % of nominal	104.04	← =Price(C7;C8;C5;C6;C4;C10;C11)
Bond value in dollars	$ 1,040.43	← =C13/100*C3
Calculation of yield		
Current Yield	7.33%	← =(C5*C4)/C13
Yield to Maturity	7.07%	← =Yield(C7;C8;C5;C13;C4;C10;C11)

Table 3. Calculation of the price and yield of the bond at the start of the investment using the Excel tool

Then, calculations were performed to obtain the market price that a bond would have if, at the end of the investment, it has to provide 20% higher yield than it had at the start to compete in the bonds market.

In table 4, the buying date is replaced by the selling date (01/01/2010) and the desired annual yield, 7.07%, by 8.49%, in other words, 20% more than provided at the start of the investment.

Bond details		
Nominal value	$ 1,000	
Yield to maturity as % of nominal	100.00	
Annual coupon rate	7.630%	
Desired annual yield	8.490%	
Buying date	01/01/2010	
Maturity date	01/07/2019	
Frequency of payments	2	
Basis	0	
Bond value as % of nominal	94.45	← =Price(C7;C8;C5;C6;C4;C10;C11)
Bond value in dollars	$ 944.51	← =C13/100*C3
Calcluation of yield		
Current Yield	8.07%	← =(C5*C4)/C13
Yield to Maturity	8.49%	← =Yield(C7;C8;C5;C13;C4;C10;C11)

Table 4. Calculations of the market price at the end of the investment period using Excel tool

On the basis of table 3, and adding the selling date (01/01/2010) and the estimated selling price for this scenario ($944.51), table 5 is then constructed.

Bond details		
Nominal value	$ 1,000	
Yield to maturity as % of nominal	100.00	
Annual coupon rate	7.630%	
Annual desired yield	7.070%	
Buying date	01/01/2009	
Maturity date	01/07/2019	
Selling date	01/01/2010	
Frequency of payments	2	
Day Count Basis	0	
Bond value as % of nominal	104.04	← =Price(C7;C8;C5;C6;C4;C10;C11)
Bond value in dollars	$ 1,040.43	← =C13/100*C3
Bond value at selling date	94.45	
Calculation of yield		
Current Yield	7.33%	← =(C5*C4)/C13
Yield to Maturity	7.07%	← =Yield(C7;C8;C5;C13;C4;C10;C11)
Yield up to selling date	-1.94%	← =Yield(C7;C9;C5;C13;C16;C10;C11)

Table 5. Calculation of expected bond yield using Excel tool

By carrying out the three steps for each of the 24 bonds in each of the five scenarios, the expected yield for each bond in each scenario are obtained (table 6).

		r_{is} (%)				
		r_{i1}	r_{i2}	r_{i3}	r_{i4}	r_{i5}
3 YEARS	BAC.NU	1.89	4.43	6.91	9.55	12.12
	BAC.OJ	1.76	4.43	7.04	9.84	12.55
	BAC.WG	1.34	4.45	7.48	10.74	13.93
	CAT.LO	2.08	4.50	6.86	9.40	11.85
	HBC.GBA	1.44	5.24	8.96	12.99	16.94
	SPG.KD	3.11	8.30	13.45	19.09	24.68
	TGT.GJ	2.74	5.61	8.43	11.44	14.39
	WFC.JL	1.46	3.52	5.71	8.04	10.31
5 YEARS	BAC.GEK	-1.55	4.26	10.02	16.02	22.62
	BAC.GEO	-1.48	3.90	9.22	14.73	20.78
	BAC.GER	-1.46	3.77	8.95	14.31	20.16
	BAC.GFM	-1.52	3.53	8.83	14.31	20.33
	CMA.HJ	-2.26	5.71	13.73	22.25	31.78
	COF.HE	-2.25	3.51	9.22	15.48	21.70
	HBC.GCB	-2.04	3.33	8.64	14.16	20.21
	SPG.JP	-1.62	5.55	12.72	20.27	28.66
10 YEARS	APA.GG	-1.94	3.84	10.39	16.79	23.51
	BAC.GWX	-2.30	3.56	10.19	16.65	23.41
	BAC.HIK	-1.98	4.63	12.20	19.68	27.62
	HBC.IKN	-2.80	4.76	13.45	22.05	31.20
	HBC.IKX	-2.62	4.62	12.91	21.10	29.80
	HBC.IMH	-2.82	5.85	15.95	26.08	37.02
	PRU.HT	-2.35	4.06	11.32	18.44	25.94
	UTX.GB	-1.81	3.33	9.13	14.77	20.67

Table 6. Expected yield for each bond in each scenario

3. Analysis of the results

In this section, we analyse the results provided by the robust optimisation model for the previously presented case study.

3.1. Results for the portfolio model of each independent scenario, IOP

In accordance with the data obtained in section 2, looking at, for example, the expected yield for the bonds in scenario s3, we see that the minimum value is 0.0571 (5.71%) for the bond *WFC.JL* and the maximum was 0.1595 (15.95%) for *HBC.IMH*. By resolving the IOP model with $r_3 = 0.0571$ and $C_3 = 238{,}956.06\$$ the optimum solution reached is that all capital must be invested in the bond *WFC.JL*. This is clearly the only capital distribution possibility that gives 5.71% as the expected yield of the portfolio. Any other distribution of this capital would give higher portfolio yield. Therefore, this would be the lower limit of the efficient frontier, in other words, the portfolio with the minimum expected yield within the efficient frontier. If the IOP model is resolved for $r_3 = 0.1595$ and $C_3 = 339{,}394.51\$$, the result is the all the capital should be invested in *HBC.IMH* and that is the only portfolio that would give those expected results. This portfolio would be at the upper limit of the efficient frontier. For any other expected portfolio yield r_3 between 0.0571 and 0.1595 there would be numerous possibilities to distribute capital among the 24 bonds to obtain such expected yield. Of all these capital distribution possibilities for each r_3, the model provides the solution with minimum risk defined as the variance of the portfolio. By resolving the model for various values of r_3 within this interval, we obtain the efficient portfolio frontier for scenario 3, in other words, the set of portfolios that for each level of yield have the minimum variance (minimum risk). The efficient frontier for this scenario is shown in figure 2.

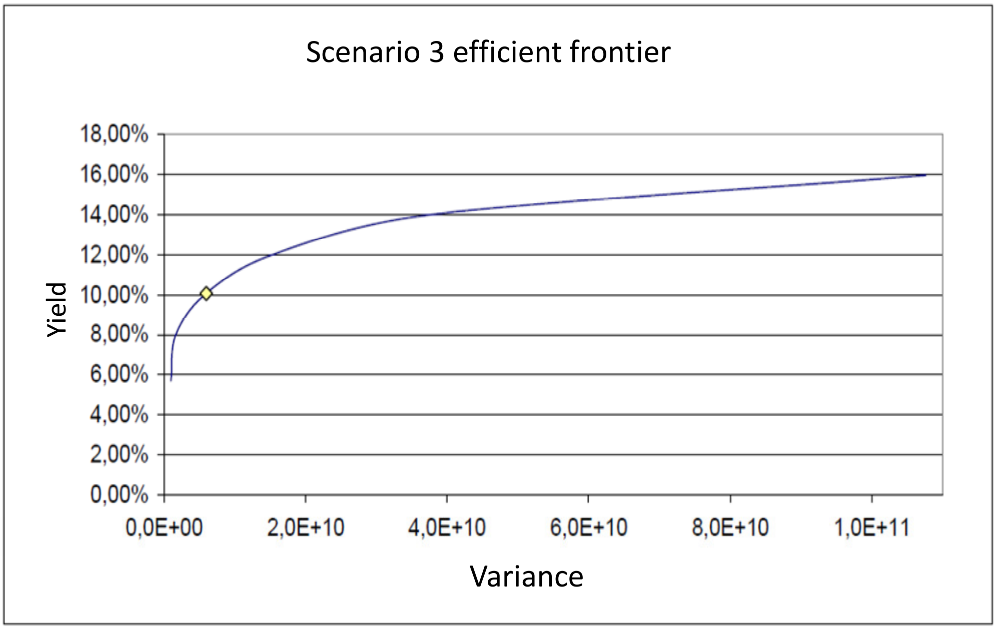

Figure 2. Efficient portfolio frontier for scenario s3

From the set of efficient portfolios for scenario s3, represented in figure 2 for their efficient frontier, we have highlighted the one defined as the optimum portfolio for the scenario, which would deliver an expected yield equal to the mean of the expected yield for all the bonds in this scenario, which will be used as a reference in the coordinated model ($r_3 = 10.07\%$).

The capital invested in this portfolio is $C_3 = 317{,}352.33\$$ and the risk associated with this portfolio is 5.99×10^{9}.

This must be carried out for each scenario, taking the optimum portfolio for each scenario to be the portfolio, within the set of portfolios that make up the efficient frontier, whose expected yield is the mean of the expected yield for all the bonds for this scenario and with minimum possible variance. Only one exception was made, for scenario *1*. In fact, the expected yield for the 5 and 10 year bonds in this scenario is negative (table 5), and by taking the mean of all the yields, a negative value is obtained. Since investments were deliberately not made in a bond that was estimated to have negative yield, the value of the expected yield for the optimum portfolio of scenario 1 is taken to be the mean of the positive yields.

The table below shows the composition of the optimum portfolios for each scenario, as well as the expected yield, investment capital and expected risk of each of them (table 7).

		s1	s2	s3	s4	s5
	r_s	1,98%	4,53%	10,07%	15,76%	21,76%
	C_s	$140.623,61	$212.808,60	$317.352,33	$396.957,18	$466.449,35
	$L(x_s)$	412.107.714,58	815.339.865,51	5.991.862.077,55	8.287.578.555,80	9.879.275.128,02
		x_s				
3 YEARS	BAC.NU	0,00	0,00	0,00	0,00	0,00
	BAC.OJ	$70.020,54	$207.378,41	0,00	0,00	0,00
	BAC.WG	0,00	0,00	0,00	0,00	0,00
	CAT.LO	$58.492,99	0,00	0,00	0,00	0,00
	HBC.GBA	0,00	0,00	0,00	0,00	0,00
	SPG.KD	0,00	$5.430,19	$85.262,68	0,00	0,00
	TGT.GJ	$12.110,08	0,00	0,00	0,00	0,00
	WFC.JL	0,00	0,00	0,00	0,00	0,00
5 YEARS	BAC.GEK	0,00	0,00	0,00	0,00	0,00
	BAC.GEO	0,00	0,00	0,00	0,00	0,00
	BAC.GER	0,00	0,00	0,00	0,00	0,00
	BAC.GFM	0,00	0,00	$232.089,65	$300.548,53	$386.514,61
	CMA.HJ	0,00	0,00	0,00	0,00	0,00
	COF.HE	0,00	0,00	0,00	0,00	0,00
	HBC.GCB	0,00	0,00	0,00	0,00	0,00
	SPG.JP	0,00	0,00	0,00	$96.408,64	$79.934,75
10 YEARS	APA.GG	0,00	0,00	0,00	0,00	0,00
	BAC.GWX	0,00	0,00	0,00	0,00	0,00
	BAC.HIK	0,00	0,00	0,00	0,00	0,00
	HBC.IKN	0,00	0,00	0,00	0,00	0,00
	HBC.IKX	0,00	0,00	0,00	0,00	0,00
	HBC.IMH	0,00	0,00	0,00	0,00	0,00
	PRU.HT	0,00	0,00	0,00	0,00	0,00
	UTX.GB	0,00	0,00	0,00	0,00	0,00

Table 7. Optimum portfolios for each scenario

3.2. Result for the coordinated error model of investment in the asset with regard to scenario s according to the MM model

Having obtained the optimum portfolios for each scenario and the capital to be invested in each of them, all this information is incorporated into the coordinated model (*MM Model*) which will provide the composition of the real portfolio and the investment capital required so as to minimise the coordinated error with regard to each of the scenarios.

For this analysis, the first assumption made is the equiprobability of scenarios. In other words, $P_s = 0.2$ (20%) for the five scenarios, C_{min} = C_1 = 140,623.61 \$ and C_{max} = C_5 = 466,449.35 \$.

Then, the composition of the solution portfolio for the *MM Model* will be as shown in table 8.

		x_i
3 years	BAC.NU	73,64
	BAC.OJ	246.700,49
	BAC.WG	4.289,94
	CAT.LO	3.378,30
	HBC.GBA	5.670,43
	SPG.KD	9.533,27
	TGT.GJ	78,42
	WFC.JL	110,70
5 years	BAC.GEK	6.526,57
	BAC.GEO	6.962,83
	BAC.GER	6.634,08
	BAC.GFM	47.545,01
	CMA.HJ	6,68
	COF.HE	6.758,55
	HBC.GCB	3.731,88
	SPG.JP	10.602,79
10 years	APA.GG	7.564,83
	BAC.GWX	5.740,67
	BAC.HIK	8.270,86
	HBC.IKN	7.709,59
	HBC.IKX	6.617,19
	HBC.IMH	11.006,78
	PRU.HT	8.651,41
	UTX.GB	6.281,20
	$C(x)$	\$420.446,12
	$L(x)$	8.257.308.969,00

Table 8. Coordinated portfolio of the MM Model

Figure 3 shows the coordinated errors and weighted coordinated errors with regard to each of the scenarios for this solution.

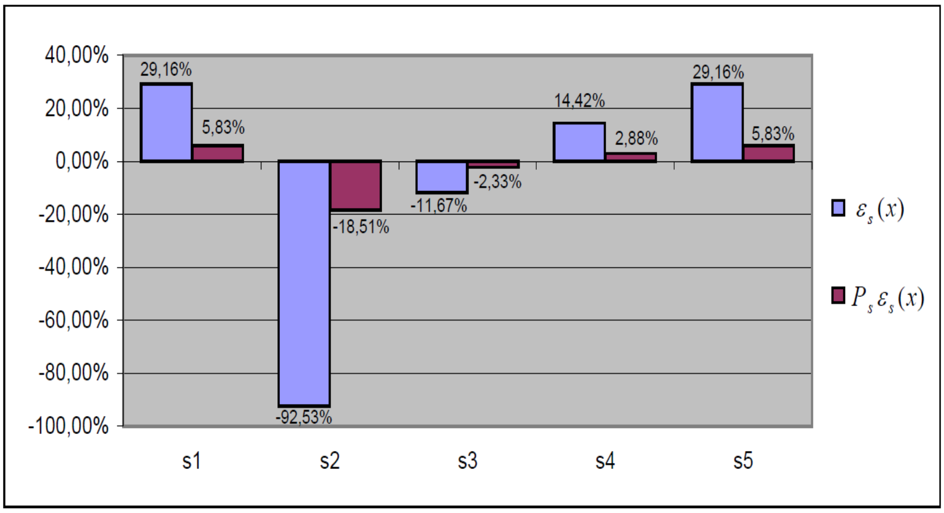

Figure 3. Coordinated errors for the coordinated portfolio of the MM Model

Table 9 below compares the optimum portfolio for each scenario with the coordinated portfolio, in the case that such a scenario should occur.

SCENARIO 1

		Capital	Variance	Yield	Profits
Optimum Portfolio		C_s	σ^2	r_s	$r_s C_s$
		$140,623.61	412,107,714.58	1.98%	$2,780.83
Coordinated Portfolio		$C(x)$	σ^2	$\sum_{i=1}^{24} r_{is} x_{is} / C(x)$	$\sum_{i=1}^{24} r_{is} x_{is}$
		$420,446.12	8,257,308,969.0	0.47%	$1,969.93
Coordinated Error	$\varepsilon_s(x)$	29.16%			

SCENARIO 2

		Capital	Variance	Yield	Profits
Optimum Portfolio		C_s	σ^2	r_s	$r_s C_s$
		$212,808.60	815,339,86551	4.53%	$9,637.57
Coordinated Portfolio		$C(x)$	σ^2	$\sum_{i=1}^{24} r_{is} x_{is} / C(x)$	$\sum_{i=1}^{24} r_{is} x_{is}$
		$420,446.12	8,257,308,969.0	4.41%	$18,554.8
Coordinated Error	$\varepsilon_s(x)$	-92.53%			

SCENARIO 3

		Capital	Variance	Yield	Profits
Optimum Portfolio		C_s	σ^2	r_s	$r_s C_s$
		$317,352.33	5,991,862,077.6	10.07%	$31,961.35
Coordinated Portfolio		$C(x)$	σ^2	$\sum_{i=1}^{24} r_{is} x_{is} / C(x)$	$\sum_{i=1}^{24} r_{is} x_{is}$
		$420,446.12	8,257,308,969.0	8.49%	$35,691.36
Coordinated Error	$\varepsilon_s(x)$	-11.67%			

SCENARIO 4

		Capital	Variance	Yield	Profits
Optimum Portfolio		C_s	σ^2	r_s	$r_s C_s$
		$396,957.18	8,287,578555.8	15.76%	$62,550.53
Coordinated Portfolio		$C(x)$	σ^2	$\sum_{i=1}^{24} r_{is} x_{is} / C(x)$	$\sum_{i=1}^{24} r_{is} x_{is}$
		$420,446.12	8,257,308,969.0	12.73%	$53,528.04
Coordinated Error	$\varepsilon_s(x)$	14.42%			

SCENARIO 5

		Capital	Variance	Yield	Profits
Optimum Portfolio		C_s	σ^2	r_s	$r_s C_s$
		$466,449.35	9,879,275,128.1	21.76%	$101,487.72
Coordinated Portfolio		$C(x)$	σ^2	$\sum_{i=1}^{24} r_{is} x_{is} / C(x)$	$\sum_{i=1}^{24} r_{is} x_{is}$
		$420,446.12	8,257,308,969.0	17.10%	$71,893.37
Coordinated Error	$\varepsilon_s(x)$	29.16%			

Table 9. Comparison of the optimum portfolios for each scenario with the coordinated portfolios

So, for an equiprobable distribution of scenarios, a coordinated portfolio is obtained in which expected yields are higher than those of the optimum portfolio of the central scenarios and lower than those of the extreme scenarios. The coordinated error for the extreme scenarios is the same. The coordinated portfolio is more diversified than the optimum portfolios where investment capital was concentrated in 2 or 3 securities. The investment capital in the coordinated portfolio (420,446.12 \$) is closer to $C_{\max}$ than to $C_{\min}$. Finally, the risk of the coordinated portfolio (8,257,308,969.00) is closer to the maximum risk than to the minimum risk.

3.3 Sensitivity analysis: MM Model

In order to conduct a sensitivity analysis for different scenario occurrence probabilities, the influence of probabilities assigned to each scenario has been analysed in the final solution provided by the model. To do this, another distribution of probabilities is considered in which the most probable scenario is s1. An analysis is made of whether this improves the results obtained by the optimum portfolio in this scenario and the influence it has on the other scenarios. The distribution of probabilities was taken as follows**:** 40% for scenario 1, 30% for scenario 2, 15% for scenario 3, 10 % for scenario 4 and 5% for scenario 5. The composition of the portfolio provided by the MM Model is shown below in Table 10.

		x_i
3 years	BAC.NU	1170,45
	BAC.OJ	249005,62
	BAC.WG	5163,27
	CAT.LO	4278,83
	HBC.GBA	5987,33
	SPG.KD	14329,72
	TGT.GJ	5341,47
	WFC.JL	121,78
5 years	BAC.GEK	7221,28
	BAC.GEO	6695,34
	BAC.GER	6469,05
	BAC.GFM	6955,14
	CMA.HJ	25,23
	COF.HE	6840,87
	HBC.GCB	5789,80
	SPG.JP	9503,88
10 years	APA.GG	7705,36
	BAC.GWX	7387,30
	BAC.HIK	8609,83
	HBC.IKN	8899,63
	HBC.IKX	8098,49
	HBC.IMH	10815,80
	PRU.HT	7906,61
	UTX.GB	6240,98

$C(x)$	$400.563,06
$L(x)$	8.354.996.763,34

Table 10. Coordinated portfolio of the MM Model with asymmetric distribution of probabilities

When the probability of occurrence increases for scenario 1, the investment capital decreases, towards the capital of the optimum portfolio for this scenario, although remaining between the capitals for scenarios 4 and 5. Risk remains at a similar level, below the risk of the last scenario as imposed by the fourth restriction.

Figure 4 shows the coordinated errors and weighted coordinated errors with regard to each of the scenarios for the optimum portfolio.

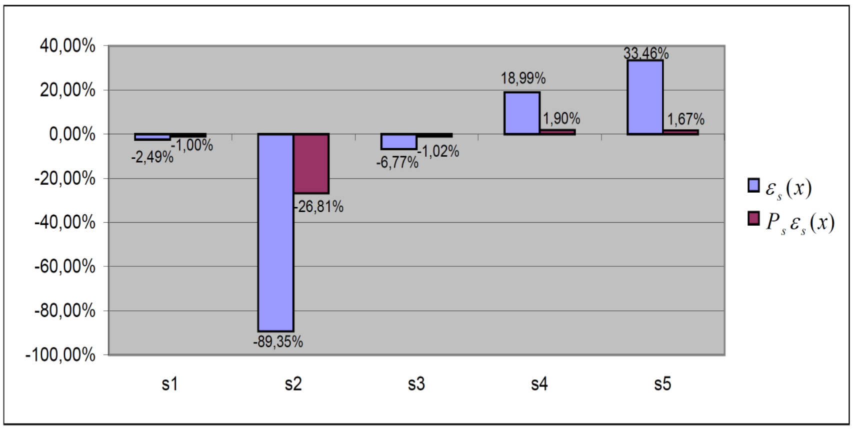

Figure 4. Coordinated errors of the coordinated portfolio of the MM Model with asymmetric distribution of probabilities

On the basis of these data, several important observations can be made. On the one hand, the coordinated error with regard to scenario 1 is now negative. This means that the coordinated portfolio has a higher expected yield than the optimum portfolio for this scenario, which is now the most probable. The coordinated errors with regard to scenarios 2 and 3 are still negative although not to such an extent as in the previous case, especially for scenario 3. For scenarios 4 and 5, coordinated errors are still positive and also higher than in the previous case, now that the probabilities of occurrence for both scenarios are lower. The maximum coordinated error is produced for the last scenario, the least probable one, with 33.46%. However, since the probability of this scenario is half that of scenario 4, it is this latter scenario in which minimise average weighted error takes the maximum possible value (1.90%) which is, therefore, the value of the target function in the optimum portfolio.

Therefore, it could be said that the MM Model responds favourably to this new distribution of probabilities for the scenarios approaching in terms of expected yield the optimum portfolio for the most probable scenario and moving away from the least probable scenarios.

3.4. Result for the coordinated error model of investment in the asset with regard to scenario s according to the MAD model

A study was then initiated using the target function associated with the *MAD Model*, and initially considering conditions of equiprobability for the scenarios. Hence, the composition of the portfolio provided by the *MAD Model* is shown in table 11.

		x_i
3 YEARS	BAC.NU	6604,92
	BAC.OJ	245347,92
	BAC.WG	2680,99
	CAT.LO	2120,29
	HBC.GBA	3770,86
	SPG.KD	7141,82
	TGT.GJ	5241,95
	WFC.JL	1319,89
5 YEARS	BAC.GEK	3615,31
	BAC.GEO	4234,40
	BAC.GER	3975,30
	BAC.GFM	45016,70
	CMA.HJ	1350,58
	COF.HE	3870,73
	HBC.GCB	8163,60
	SPG.JP	7092,50
10 YEARS	APA.GG	8133,64
	BAC.GWX	6053,06
	BAC.HIK	5090,13
	HBC.IKN	2009,68
	HBC.IKX	3153,01
	HBC.IMH	7015,33
	PRU.HT	5533,57
	UTX.GB	7200,62
	$C(x)$	$395.736,77

$L(x)$	6.090.337.539,99

Table 11. Coordinated portfolio of the MAD Model

Figure 5 shows the coordinated errors and weighted coordinated errors with regard to each of the scenarios.

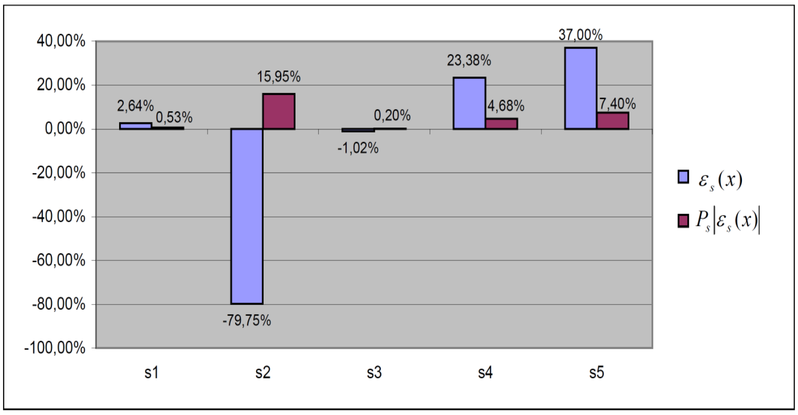

Figure 5. Coordinated errors for the coordinated portfolio of the MAD Model

3.5 Sensitivity analysis: MAD Model

To observe the robustness or sensitivity of the proposal to possible modifications in the input data regarding the probability of scenario occurrence, the problem has been considered with the following distribution of probabilities: 40% for scenario 1, 30% for scenario 2, 15% for scenario 3, 10 % for scenario 4 and 5% for scenario 5, giving the following coordinated portfolio (table 12):

		x_i
3 YEARS	BAC.NU	2627,04
	BAC.OJ	200950,33
	BAC.WG	2638,22
	CAT.LO	2604,50
	HBC.GBA	2648,95
	SPG.KD	4050,67
	TGT.GJ	9078,04
	WFC.JL	10727,22
5 YEARS	BAC.GEK	4947,46
	BAC.GEO	4834,47
	BAC.GER	4384,03
	BAC.GFM	6604,74
	CMA.HJ	5348,88
	COF.HE	5075,34
	HBC.GCB	5231,27
	SPG.JP	3808,91
10 YEARS	APA.GG	3739,24
	BAC.GWX	3503,44
	BAC.HIK	3556,09
	HBC.IKN	4408,31
	HBC.IKX	4108,26
	HBC.IMH	1674,02
	PRU.HT	1308,70
	UTX.GB	4894,29
	$C(x)$	$302.752,42
	$L(x)$	3.987.875.973,40

Table 12. Coordinated portfolio of the MAD Model with asymmetric distribution of probabilities

Figure 6 shows the coordinated errors and weighted coordinated errors with regard to each of the scenarios for this solution.

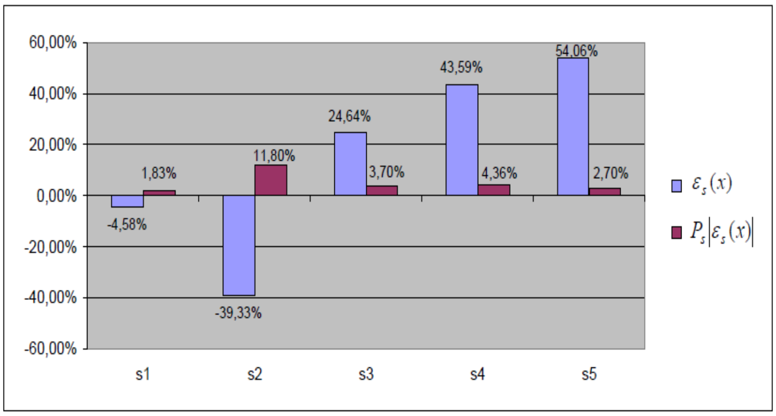

Figure 6. Coordinated errors of the coordinated portfolio for the MAD Model with asymmetric distribution of probabilities

4. Comparative analysis of the results of the models.

Having studied and compared the solutions provided by each model for the case of equiprobable scenarios, the information reflected in table 13 is obtained.

Analysing the results obtained for each scenario, as well as the solution of the coordinated models (for the case of asymmetric scenario distribution), it is argued that coordinated capital and risk are greater in the *MM Model* than in the *MAD Model*. Expected yield (the sum total of the expected yields in each scenario) is also higher for this model. Therefore, the yield is slightly higher for the solution portfolio of the *MM Model*. However, the difference between the maximum and minimum coordinated errors is slightly lower for the *MAD Model*, and the sum total of the coordinated errors with regard to each of the scenarios is more negative for the *MM Model*. This latter finding gives an idea of the deviation of the expected yield for the coordinated portfolio from the expected yield of the optimum portfolio. Having obtained the coordinated error (in terms of absolute value), the robustness index (12) was defined, based on the variation in the expected yield of the investment project from the optimum portfolio of each scenario. Having analysed the comparative tables for both models, we see that the robustness index is higher for the *MAD Model*. The robustness index, therefore, expresses how close each of the two models is to the ideal model. An ideal coordinated model, with a coordinated profile that gives the same expected yield in each scenario as the expected yield for the optimum portfolio of each of them, would have a robustness index equal to 1 (100%), since all its coordinated errors would be zero.

Table 14 shows the case of asymmetrical distribution of scenario probabilities. If the distribution of scenario probabilities is asymmetric, for the *MAD Model*, the investment capital of the coordinated portfolio more rapidly approaches the capital amount to be invested in the optimum portfolio of the most probable scenario. In fact, the *MAD Model* responds more favourably to this new distribution of probabilities, providing a variance with a lower level of risk. The average scenario-based expected yield for the coordinated portfolio of the *MM Model* is still higher for this distribution of probabilities. As with the equiprobable distribution, the average of the weighted coordinated errors, in terms of absolute value, is lower for the *MAD Model*. For the same reason as in the previous case, the robustness index for the *MAD Model* is still higher than for the *MM Model.*

		MM MODEL	MAD MODEL
		Min ε s.t. $P_s\varepsilon_s(x) \le \varepsilon$	Min $\varepsilon = \sum_{s=1}^{S} P_s\left\|\varepsilon_s\right\|$
(1)	$C(x)$	$420,446.12	$395,736.77
(2)	$L(x)$	8,257,308,969.00	6,090,337,539.99
(3)	$\sum_{s=1}^{5}\sum_{i=1}^{24} r_{is}x_{is}$	$181,637.56	$164,174.45
(4)	$\sum_{s=1}^{5}\sum_{i=1}^{24} r_{is}x_{is} \Big/ 5$	$36,327.51	$32,834.89
(5)	$\dfrac{\sum_{s=1}^{5}\sum_{i=1}^{24} r_{is}x_{is} \Big/ 5}{C(x)}$	8.64%	8.30%
(6)	$P_s\varepsilon_s(x)\max$	5.83%	7.40%
(7)	$P_s\varepsilon_s(x)\min$	-18.51%	-15.95%
(8)	$P_s\varepsilon_s(x)\max - P_s\varepsilon_s(x)\min$	24.34%	23.35%
(9)	$\sum_{s=1}^{5} P_s\varepsilon_s(x)$	-6.29%	-3.55%
(10)	$\sum_{s=1}^{5} P_s\left\|\varepsilon_s(x)\right\|$	35.39%	28.76%
(11)	$\sum_{s=1}^{5} P_s\left\|\varepsilon_s(x)\right\| \Big/ 5$	7.08%	5.75%
(12)	$\Re = 1 - \sum_{s=1}^{5} P_s\left\|\varepsilon_s(x)\right\| \Big/ 5$	92.92%	94.25%

Table 13. Comparative analysis of the results of the two models

		MM MODEL	MAD MODEL
		Min ε s.t. $P_s\varepsilon_s(x) \le \varepsilon$	Min $\varepsilon = \sum_{s=1}^{5} P_s\left\|\varepsilon_s(x)\right\|$
(1)	$C(x)$	$400,563.06	$302,752.42
(2)	$L(x)$	8,354,996,763.34	3,987,875,973.40
(3)	$\sum_{s=1}^{5}\sum_{i=1}^{24} r_{is}x_{is}$	$173,427.43	$122,335.75
(4)	$\sum_{s=1}^{5}\sum_{i=1}^{24} r_{is}x_{is} \Big/ 5$	$34,685.49	$24,467.15
(5)	$\dfrac{\sum_{s=1}^{5}\sum_{i=1}^{24} r_{is}x_{is} \Big/ 5}{C(x)}$	8.66%	8.08%
(6)	$P_s\varepsilon_s(x)\max$	1.67%	4.36%
(7)	$P_s\varepsilon_s(x)\min$	-26.81%	-11.80%
(8)	$P\varepsilon(x)\max - P_s\varepsilon_s(x)\min$	28.48%	16.16%
(9)	$\sum_{s=1}^{5} P_s\varepsilon_s(x)$	-25.24%	-2.87%
(10)	$\sum_{s=1}^{5} P_s\left\|\varepsilon_s(x)\right\|$	32.39%	24.39%
(11)	$\sum_{s=1}^{5} P_s\left\|\varepsilon_s(x)\right\| \Big/ 5$	6.48%	4.88%
(12)	$\Re = 1 - \sum_{s=1}^{5} P_s\left\|\varepsilon_s(x)\right\| \Big/ 5$	93.52%	95.12%
(13)	$P_1\varepsilon_1(x)$	-1.00%	-1.83%

Table 16. Comparative analysis of the results of the two models with asymmetric distribution of probabilities

5. Conclusions

This paper has studied and analysed robust optimisation models as efficient tools for the risk management of fixed income securities, which can represent a relevant financial service for SMEs and larger companies. It presented a problem entailing the composition of fixed income investment portfolios starting with an indeterminate initial endowment.

The theoretical part of the work is complemented by analysis and resolution of the *scenario-based optimisation model* (applied to the problem of portfolio selection and on the basis of an indeterminate initial endowment). To do this, *MM and MAD models* were used, having carried out a sensitivity analysis in relation to the various possibilities of different scenarios occurring for each of the models.

In view of the results presented in sections 3 and 4, we see that, regardless of whether the distribution of scenario probabilities is symmetric or asymmetric, the *MAD model* offers the most robust behaviour.

It is important to highlight that the weighted coordinated risk (13), for the most probable scenario (*s1*), is negative for both models, although better (more negative) for the *MAD model*. In general terms, we can say that the models developed to minimise the impact of risk on investments in the fixed income markets are solutions to prevent possible bankruptcies derived from said risk, greatly strengthening the decision-making processes of investment firms.

Finally, the results obtained are better than would have been achieved using immunising strategies since these kinds of strategies are used to combine the interest rate risk of a portfolio of assets to a flow of assets, with the idea that exposure to the risk of said portfolio is zero. Therefore, to follow an immunising strategy, periodic readjustments of the portfolio are required in order to ensure its duration remains equal to the Risk of Reinvestment (IPH) except when zero coupon bonds are being used. The problem presented is that maintaining the immunised portfolio throughout the IPH implies a continual restructuring of the portfolio, owing to the temporal evolution of the duration, which entails high transaction costs, thereby reducing the profitability of the portfolio.

An additional problem of immunising strategies is the risk of reinvestment to which they are subjected.

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