ROBUST PID TUNING. APPLICATION TO A MOBILE ROBOT PATH TRACKING PROBLEM

Julio E. Normey-Rico * Ismael Alcalá ** Juan Gómez-Ortega ** Eduardo F. Camacho **

* *Dpto. de Automação e Sistemas, Federal Univ. of Santa Catarina, Brazil, e-mail:julio@lcmi.ufsc.br*
** *Dpto. Ingeniería de Sistemas y Automática, Univ. of Seville, Spain, e-mail:[ismael,juango,eduardo]@cartuja.us.es*

Abstract: This paper presents a methodology for tuning PIDs considering the nominal performance and the robustness as control specifications. The synthesis procedure is similar to the Ziegler-Nichols method for PID controllers and can be easily used for industrial processes. As a workbench for testing the PID controller a mobile robot has been used. The path tracking problem of a mobile robot has been used as a workbench for testing the PID controller. *Copyright © 2000 IFAC*

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1. INTRODUCTION

In spite of the fact that the most frequently controllers used in industry are PIDs, studies presented by many authors have shown that a great amount of control problems are caused by inappropriate tuning of the PID parameters. Thus, study of PID tuning methods is of great importance.

Many methods have been proposed in literature to tune PIDs. One of the most known methods for tuning PID controllers when the process can be modeled as a first order transfer function with a delay, either stable (P_e) or integrative (P_i) is the tuning method proposed by Ziegler and Nichols (1942). This method generates acceptable responses in stable processes with low ratio between the dead time (L) and time constant (T) of the process, but the responses are very oscillating or very slow with high ratio L/T or integrative processes. In this paper, a PID is tuned approximating the dead time by a transfer function (Léonard, 1998) and taking into account robustness considerations.

As a workbench for testing the proposed PID controller a mobile robot has been used. The problem treated in this work is known as Path Tracking (PT) which is that of driving a mobile robot to follow a previously calculated reference path, defined as a set of consecutive points. A synchro drive mobile robot is used and a robust tuned PID controller is proposed to solve the PT problem. This controller has been tuned for integrative processes and allows the use of simple models.

The paper is organized as follows: In section 2 the description of the proposed tuning for the PID controller is presented for stable plants and integrative plants are described in section 3. The robustness of the proposed PID is discussed in section 4. The model considered for the mobile robot kinematics is presented in section 5 and the way in which the PID is applied to the PT problem is shown in section 6. In section 7, experimental results on a Nomad 200 mobile robot are shown. The paper ends with the conclusions.

In the following two sections, the PID tuning is discussed for stable and integrative plants modeled with $P_e(s)$ and $P_i(s)$ defined as:

$$P_e(s) = \frac{K_p}{1+Ts}e^{-Ls}, \quad P_i(s) = \frac{K_v}{s}e^{-Ls} \qquad (1)$$

2. PID TUNING FOR STABLE PROCESSES

A simple and frequently used (Léonard, 1998) way for tuning a PID controller for P_e plants is using a rational first order function instead of the dead time, which provides a good approximation in the frequency domain:

$$e^{-sL} = \frac{e^{-sL/2}}{e^{sL/2}} \doteq \frac{1 - sL/2}{1 + sL/2} \qquad (2)$$

This approximation allows a low frequencies second order transfer function model given by:

$$P_1(s) = \frac{K_p(1 - sL/2)}{(1+Ts)(1+sL/2)} \qquad (3)$$

A PID controller with a low pass filter (which reduces high frequency derivative gain) has a transfer function:

$$C(s) = K_c(1 + \frac{1}{T_i s} + T_d s)\frac{1}{1+T_f s} \qquad (4)$$

The parameters of the controller are tuned using the *root locus* method, looking for quick response without oscillations (see figure 1)

- $T_d = \frac{2LT}{L+2T}$
- $T_i = T + L/2$
- T_f is chosen to reduce high frequency gain. A simple way is to chose $T_f = \alpha L$, with $0 < \alpha < T/L$, that allocates filter poles to the left of $-1/T$ (see figure 1).
- K_c is chosen in such a way that exist two equal closed loop real poles in $s = -1/T_0$:

$$K_c = \frac{L+2T}{LK_p}(4\alpha + 1 - 4(\alpha^2 + \alpha/2)^{1/2}) \qquad (5)$$

with:

$$T_0 = \frac{\alpha}{(\alpha^2 + \alpha/2)^{1/2} - \alpha}\frac{L}{2} \qquad (6)$$

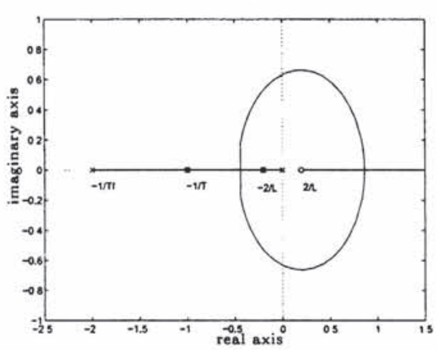

Fig. 1. Root locus diagram for PID tuning

From these relations and from root locus diagram it can be seen that low α values generate quick responses (low T_0 values). If the model is equal to the process, this tuning guarantees quick responses without oscillations and governed by $s = -1/T_0$ real poles. But in real processes, low T_0 means allocate poles in complex plane where cannot be used the approximation given by equation 2. In these cases, errors between the model and the process are important and the time response has high frequency components that generate oscillations. If $\alpha > T/L$ a slow response is obtained due to low pass PID filter that cancels derivative effects. Also the system performance is limited by model errors for small α values. This problem is similar to that of high order plants modeled with low order models, where closed loop poles must be allocated considering unmodeled dynamic.

In real practice, dynamic model errors and parameters estimation uncertainties must be considered. As will be proved in this work, parameter α is related to the controller robustness. This create a compromise between performance and robustness.

Once a PID is well tuned for perturbation rejection (with an adequate α) the responses show a higher overshoot than the one specified. This corresponds to the zeros allocated in the closed loop and can be corrected introducing a reference filter (as shown in figure 2) that reduces the effect of these zeros. The filter can be tuned in a simple way because zeros are introduced by the controller. This problem is known as two degrees of freedom control and is analyzed in some works (Morari and Zafiriou, 1989) (Åström and Witenmark, 1984). Also, a study of the zeros influence in system response based on basic tools (time response and frequency domain) is presented in (Normey-Rico, 1995).

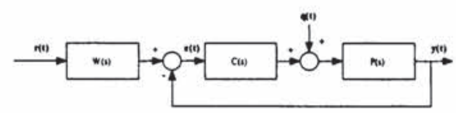

Fig. 2. System controlled with PID and reference filter

In particular, the reference filter can be chosen as follows:

$$W(s) = \frac{1 + 0.4T_r s}{1 + sT_r} \qquad (7)$$

and the zeros effect can be canceled with T_r. The slow PID zero is allocated in $s = -2/L$, and then T_r can be chosen equal to $L/2$. Thus, the number of parameters remains the same and the controller simplicity is also maintained. A better performance is shown in figure 3 where the simulation conditions and PID parameters are held.

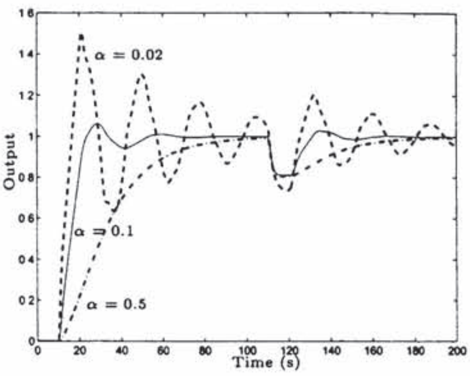

Fig. 3. Closed loop system responses with PID and reference filter for stable plant

3. PID TUNING FOR INTEGRATIVE PROCESSES

Dead time and integrative behavior makes PID tuning more difficult. As was mentioned, Ziegler-Nichols method is not good enough in these cases. Alternative methods have been proposed (Morari and Zafiriou, 1989) (Chien and Fruehauf, 1990) (Luyben, 1996). In this work, simplicity is maintained with two steps based method. In the first step a proportional control K_o (see figure 4) is tuned. This transforms integrative system dynamic into a stable process. Due to the system

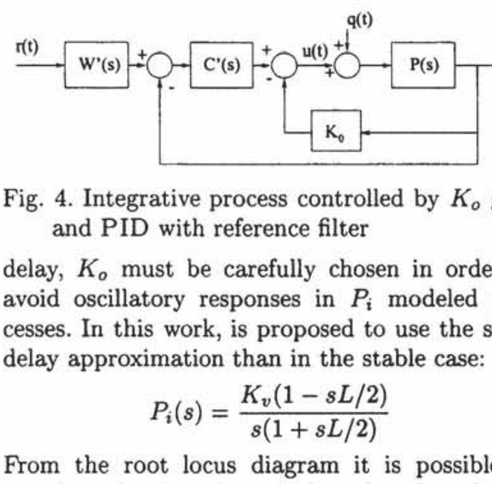

Fig. 4. Integrative process controlled by K_o gain and PID with reference filter

delay, K_o must be carefully chosen in order to avoid oscillatory responses in P_i modeled processes. In this work, is proposed to use the same delay approximation than in the stable case:

$$P_i(s) = \frac{K_v(1 - sL/2)}{s(1 + sL/2)} \qquad (8)$$

From the root locus diagram it is possible to calculate the K_o value which makes that the inner closed loop poles generate responses with an overshoot less than 5%. Thus, the K_o final value is $K_o = \frac{0.5}{K_v L}$ and the new transfer function between control and output (with the delay approximation) is given by:

$$P_2(s) = \frac{2K_v/L(1 - sL/2)}{1 + 1.5Ls + L^2s^2} \qquad (9)$$

In the second step, the PID controller $C'(s)$ is tuned using the same ideas presented for the stable case:

$$C'(s) = K_c(1 + \frac{1}{T_i s} + T_d s)(\frac{1}{1 + T_f s}) \qquad (10)$$

and the following values are selected:

- $T_i = 1.5L$ e $T_d = 2L/3$,
- T_f proportional to L: $T_f = \gamma L$, with $0 < \gamma < 0.75$ (for the allocation of the filter poles to the left to the $P_2(s)$ poles)
- K_c for real poles is a function of γ and is given by:

$$K_c = \frac{1.5}{LK_v}(4\gamma + 1 - 4(\gamma^2 + \gamma/2)^{1/2}) \qquad (11)$$

that allocates the poles in $s = -1/T_1$ with:

$$T_1 = \frac{\gamma}{(\gamma^2 + \gamma/2)^{1/2} - \gamma}\frac{L}{2} \qquad (12)$$

The obtained relations are similar to that of the stable case expressions. Again, a new parameter (γ) defines robustness characteristics of the system and also, it is used a reference filter $W'(s) = \frac{1+0.4T_r s}{1+T_r s}$ for better transient responses in reference changes, and it is tuned as in the stable case. In this case, $C'(s)$ has complex conjugate zeros whose real part is used for tuning T_r; then:

$$T_r = 0.75L \qquad (13)$$

It is clear that the scheme presented in figure 4 can be transformed into the scheme of figure 2. The relation between $W'(s)$, K_o and $C'(s)$ and $C(s)$ and $W(s)$ is given by: $C(s) = K_o + C'(s)$, $W(s) = \frac{W'(s)C'(s)}{C(s)}$ where can be observed that although $C(s)$ is a PID controller, $W(s)$ is no more a first order filter and the simple procedure used with $C'(s)$ cannot be repeated. In figure 5 it is shown responses to different γ values for the proposed $C'(s)$ e $W'(s)$.

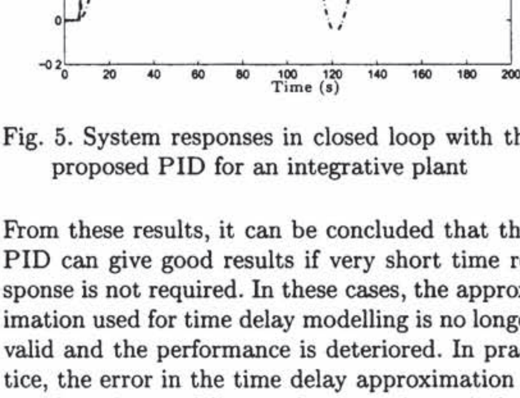

Fig. 5. System responses in closed loop with the proposed PID for an integrative plant

From these results, it can be concluded that this PID can give good results if very short time response is not required. In these cases, the approximation used for time delay modelling is no longer valid and the performance is deteriored. In practice, the error in the time delay approximation is not the only one difference between the real plant and its model and all of them must be considered in the controller design in order to guarantee the robustness. This will be analyzed in the following section.

4. ROBUST TUNING OF PID CONTROLLER

The robust stability in closed loop can be analyzed in a simple way taking into account that P can be written as $P = P_n + \delta P$, where δP are the modeling errors and P_n represents the nominal plant. This type of uncertainties model is not structured because they are not linked to specific parameters (gain, delay, etc). In this case, $P_n(s)$ is the transfer function used to tune the PID: $P_n(s) = P_e(s)$ in the stable case, and $P_n(s) = P_i(s)$ in the integrative case. The closed loop characteristic equation is: $1 + C(s)P(s) = 1 + C(s)(P_n + \delta P) = 0$, where $C(s)$ is the total control action in cascade with the process. A polar diagram of the system is shown in figure 6.

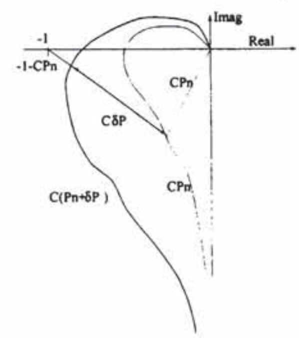

Fig. 6. Polar diagram for robust stability study of PID

First, it is calculated the polar diagram in open loop and nominal case $CP_n(j\omega)$. With the tuning chosen in the nominal case, the -1 point is not included. Then, the distance between diagram points $CP_n(j\omega)$and -1 point are computed by $\|-1 - CP_n(j\omega)\|$. This is a measure of the closed loop robustness. From the characteristic equation of the system it is obtained:

$$|-1 - CP_n| = |1 + CP_n| = |C\delta P| \qquad (14)$$

Now it is computed the polar diagram for $C(P_n + \delta P)$. If the -1 point is not included in the new diagram, then the real system is stable and the distance between real and nominal curves are always greater than $|1 + CP_n|$ for all frequencies. The maximum distance between nominal and real plants (ΔP) that holds robust stability in closed loop is calculated by:

$$|\delta P(j\omega)| \leq \Delta P(\omega) = \frac{|1 + C(j\omega)P_n(j\omega)|}{|C(j\omega)|} \quad \forall\omega. \qquad (15)$$

or relative to P_n:

$$\left|\frac{\delta P(j\omega)}{P_n(j\omega)}\right| \leq dP(\omega) = \frac{|1 + C(j\omega)P_n(j\omega)|}{|C(j\omega)P_n(j\omega)|} \quad \forall\omega. \qquad (16)$$

$dP(\omega)$ coincides with the inverse of the system closed loop transfer function for $W(s) = 1$. This shows the relation between performance and robustness. Using expressions of $P_n(s)$ and $C(s)$ with the proposed tuning for the stable case:

$$\Delta P_e(\omega) = \frac{|(1 + T_0 j\omega)^2|}{|1 - j\omega L/2|} \quad \forall\omega \qquad (17)$$

in integrative case is:

$$\Delta P_i(\omega) = \frac{|(1 + T_1 j\omega)^2|}{|1 - j\omega L/2|} \quad \forall\omega \qquad (18)$$

As T_0 is a function of α and T_1 is a function of γ, these parameters can be used for tuning the system robustness in closed loop. Greater α (γ) values, mean greater T_0 (T_1) values, and therefore, greater $dP(\omega)$ in high frequencies. On the other hand, a robustness improvement implies slower responses and performance losses. For instance, in figure 7 $dP_i(\omega)$ is shown for different values of γ.

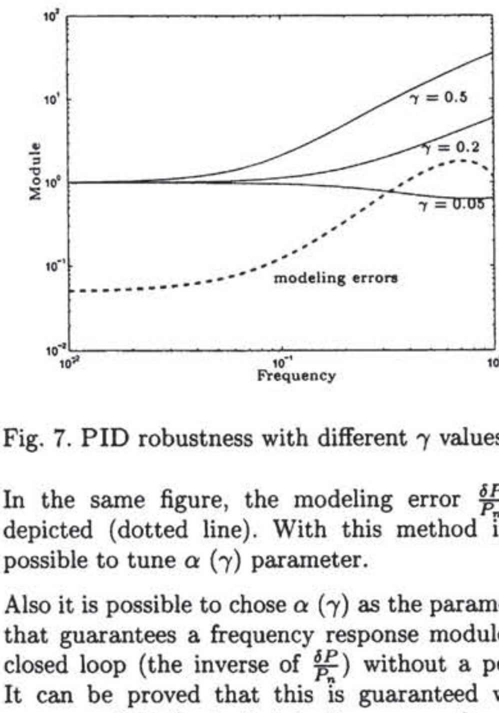

Fig. 7. PID robustness with different γ values.

In the same figure, the modeling error $\frac{\delta P}{P_n}$ is depicted (dotted line). With this method it is possible to tune α (γ) parameter.

Also it is possible to chose α (γ) as the parameter that guarantees a frequency response module in closed loop (the inverse of $\frac{\delta P}{P_n}$) without a peak. It can be proved that this is guaranteed with $\alpha = \gamma = 0.13$ (note that in the proposed cases, the function is the same for stable and integrative cases). Chosing $\alpha = \gamma = 0.13$, the tuning parameters can be written in a tabular form like Ziegler-Nichols known rules. Table 1 shows the tuning for $PID(s) = K_c(1 + \frac{1}{T_i s} + T_d s)(\frac{1}{1+T_f s})$, reference filter $Filter(s) = \frac{1+0.4T_r s}{1+T_r s}$ and K_o (in the integrative case).

5. THE MOBILE ROBOT PATH TRACKING PROBLEM

As a application for testing the proposed robust tuned PID controller a Nomad 200 mobile robot is

Model	K_c	T_i	T_d	T_f	K_o	T_r
$\frac{K_p}{1+sT}e^{-Ls}$	$\frac{0.375(L+2T)}{K_pL}$	$T+L/2$	$\frac{LT}{L+2T}$	$0.13L$	0	$L/2$
$\frac{K_v}{s}e^{-Ls}$	$\frac{0.563}{LK_v}$	$1.5L$	$2L/3$	$0.13L$	$\frac{1}{2K_vL}$	$0.75L$

Table 1. PID parameters value for the proposed tuning

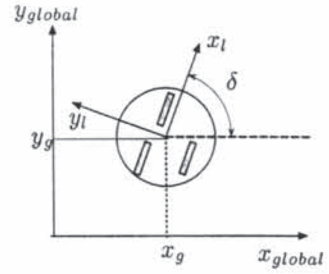

Fig. 8. Global reference frame for the mobile robot Nomad 200

used. This robot has a *synchro-drive* type locomotion system which consists of three drive wheels which turning speed and orientations vary simultaneously. In figure 8 it is shown the synchro-drive configuration and the steering angle δ referenced to the global axis.

The robot provides a position estimation system based on odometry. It is well known that odometry is a technique which has an accumulative error which implies the need for update the estimation from environmental data provided from sensor systems with a predetermined frequency. This problem is considered to be decoupled with the PT problem and has not been accomplished in this work.

5.1 *Synchro-drive steering angle model*

The mobile robot steering angle δ must follow a steering angle reference δ_r defined by an approximation point located at a fixed distance, λ, in the path (see figure 9). As the control variable the

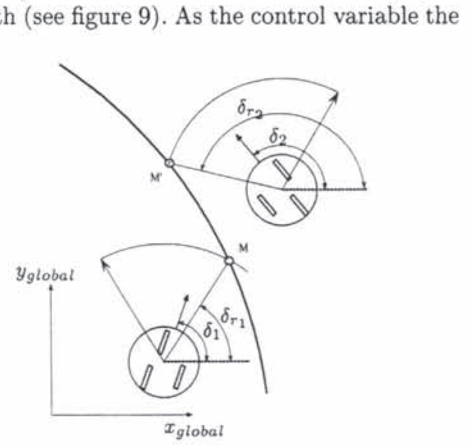

Fig. 9. Approximation points and their respective reference steering angles in two different positions

steering velocity reference $\dot{\delta}_r$ has been chosen, and a constant linear velocity V of the mobile robot has been used. The model of the robot for the steering angle is given by a kinematic equation and a non linear dynamics related to a rate limiter in the steering velocity. As was mentioned, the control variable used has been the steering velocity reference $\dot{\delta}_r$. The steering velocity $\dot{\delta}(t)$ follows its reference with a non linear dynamics. When a change in $\dot{\delta}_r$ occurs, the steering velocity has a small dead time $L = 0.2$ *seconds* and is limited by the maximum acceleration ($\ddot{\delta}_{max}$) given by the steering motors.8

6. PATH TRACKING WITH THE PROPOSED PID

For testing the behaviour of the robust tuned PID, the block diagram that appears in figure 10 was implemented in a Nomad 200 mobile robot. The reference steering angle δ_r is computed in the *approximation point* block from the robot position (x_g, y_g) and the points that define the path. In order to tune the proposed PID, a linear model of the process is needed. The linear model has been obtained from the hypothesis that the increments in the reference steering velocity are small and the dynamic of the rate limiter block is not present. In this case the model can be simplified by an integrative process with dead-time given by the following equation:

$$P_i(s) = \frac{e^{-0.2s}}{s} \tag{19}$$

Then, for the mobile robot, the reduced model was defined with $Kv = 1$, $L = 0.2\ s$ and γ was chosen equal to 0.5 for compensate uncertainties in the simplified model given by equation 19.

Using $\gamma = 0.5$ the parameters for the reference filter $W(s)$, $C(s)$ and K_o are computed. These transfer functions were calculated for discrete time and the final control law programmed in the robot mobile main processor.

7. EXPERIMENTAL RESULTS

For the experimental tests the controller parameters have been chosen as follows: $\lambda = 0.6\ meters$, $V = 0.4\ m/s$ (90% of the maximum robot velocity), $\ddot{\delta}_{max} = 30\ degrees/s^2$ and the sampling time T was chosen equal to 0.2 seconds

Figure 11 shows the performance of the proposed controller for two different real dead times. The

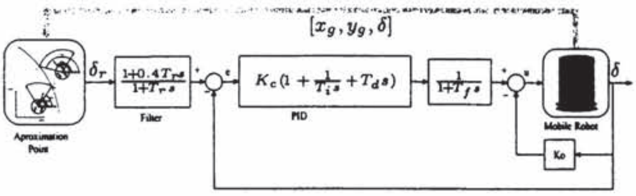

Fig. 10. Block diagram of the proposed PID controller implemented in the mobile robot for path tracking

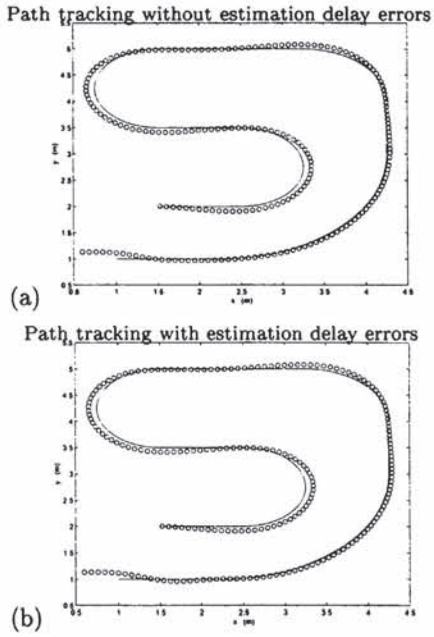

Fig. 11. Path tracking for the mobile robot: (a) with real delay L, (b) with real delay $1.4L$

initial position for the mobile robot was $x_0 = 0.6$, $y_0 = 1.13$ and orientation $\delta_0 = 0$ (parallel to x_g axis). Note that the reference path chosen has small curvatures, which makes more difficult to follow the reference. The path tracking in the two cases of figure 11 are very similar without delay errors (figure 11(a)) and with a 40% in the delay error (figure 11(b)). The difference appears in the very first sample times, where the path following with estimation delay errors (b) is more oscillating.

In figure 12 also there are differences when the control demands more control effort in path tracking with estimation error (figure 11(b)).

Note the satisfactory performance obtained in the reference following (see figure 11) although the linear velocity V is 90% of the Nomad's top velocity and the reference steering angle δ_r changes continuously.

8. SUMMARY AND CONCLUSIONS

A method for robust tuning of PID controllers has been proposed based on classical control concepts

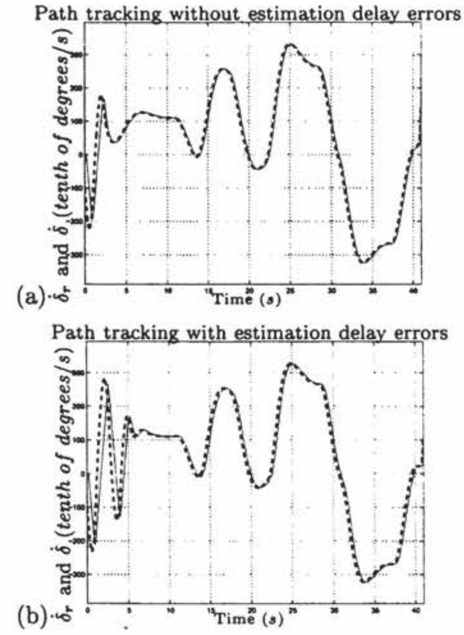

Fig. 12. Steering velocities $\dot{\delta}_r$ (continuous line) and $\dot{\delta}$ (dashed line): (a) with real delay L (b) with real delay $1.4L$

and can be applied to stable and integrative processes that are modeled like first order transfer functions with a delay. The synthesis procedure is similar to the Ziegler-Nichols method for PID controllers. The robustness of the controller can be changed with parameters (α and γ for stable and integrative plants) and allows a good performance with uncertainties in the dead time. Finally the proposed PID has been implemented for a mobile robot path tracking problem and some experimental results have shown the good performance obtained in spite of delay estimation uncertainties.

9. REFERENCES

Åström, K.J. and Witenmark (1984). *Computed Controlled Systems.* Prentice Hall, New York.

Chien, I.L. and Fruehauf (1990). Consider imc tuning to improve controller performance. *Ind. Eng. Chem. Res.* **86**(10), 33–41.

Léonard, F. (1998). Delay approximation comparison in a cacsd context. *IFAC LTDS'98* pp. 99–104.

Luyben (1996). Tuning proportional-integral-derivative controllers for integrator dead-time processes. *Ind. Eng. Chem. Res.* **35**(10), 3480–3483.

Morari, M. and E. Zafiriou (1989). *Robust Process Control.* Prentice-Hall.

Normey-Rico, J. (1995). Teaching classical controller design in a basic control course. Technical report. Proc. III LASAC, Chile.

Ziegler, J.G. and N.B. Nichols (1942). Optimum settings for automatic controllers.. *Trans. ASME 64* pp. 759–768.