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HYBRID SYSTEMS FOR SOLVING MODEL PREDICTIVE CONTROL OF PIECEWISE AFFINE SYSTEM

Miguel Peña ** **E. F. Camacho** * **Sandra Piñón** *,[1]

* *Escuela Superior de Ingenieros. Universidad de Sevilla, Camino de los Descubrimientos s/n. 41092 Sevilla, Spain*
** *INAUT, Fac. de Ingeniería, U. N. San Juan, Argentina*

Abstract: This paper presents a hybrid procedure to solve Model Predictive Control (MPC) of Piecewice Affine (PWA) system. The procedure uses the concepts of reacheble set, controllable set and State Transition Graph (STG) in order to reduce the number of Quadratic Problems (QP) needed to obtain an global minimum. The proposed algorithm reduces considerably the number of explorations needed during the search of a global minimum and thus the time required by the MPC can be reduced to a small fraction of the time required to the original problem. *Copyright, 2003, IFAC.*

Keywords: Model predictive control, hybrid systems, piecewise affine systems

1. INTRODUCTION

Different methods for the analysis and design of controllers for Hybrid Systems (HS) have been reported (Tomlin *et al.*, 2000), (Bemporad and Morari, 1999). Among them, the class of optimal controllers is one of the most studied. Most of the literature deals with optimal control of continuous-time HS and on the computation of optimal or sub-optimal solutions (Hedlund and Rantzer, 1999), (Riedinger *et al.*, 1999). Although some techniques for determining feedback control laws seem to be very promising and many of them suffer from the curse of dimensionality. MPC has become an accepted standard for complex constrained control problems in the process industry but some limitations to which processes MPC could be used on, due to the computationally expensive on-line optimization required. Explicit solutions to the MPC problem from linear constrained systems, which could increase the area of use for this kind of controller ((Bemporad *et al.*, 2000), (Johanssen *et al.*, 2000) and (Seron *et al.*, 2000)) have been derived. Unfortunately these approaches are so complex for HS. The application of MPC to HS requires to solve an optimization program with mixed, integer and real, decision variables. Mixed Integer Quadratic Program (MIQP) algorithm can be used to solve this problem (Bemporad and Morari, 1999). In spite of this combinatorial nature, several algorithmic approaches have been proposed and applied successfully to medium and large size application problems (Fletcher and Leyffer, 1995). However MIQPs are very time consuming and prohibitive for real time in most cases. To reduce the computed time, (Bemporad *et al.*, 1999) presents a Branch and Bound tree exploring strategy for solving MIQP involving time evolutions of MLD model.

This paper presents a hybrid procedure to solve MPC of PWA in order to reduce the comput-

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ing time needed to solve the problem. The approach presented here do not belong to the class of Branch and Bound (B&B) methods, instead it uses an enumeration method and an standard QP algorithm to solve the mixed integer-real optimization. The procedure uses the concepts of reachable set and one step controllable sets (Kerrigan, 2000) combined to the State Transition Graph (STG) concepts, in order to reduce the number of QP problems need to solves the optimization algorithm. The proposed algorithm reduces considerably the number of explorations needed during the search of a global minimum and thus the time required by the MPC can be reduced to a small fraction of the time required if the optimization algorithm is applied to the original problem.

The paper is organized as follows: in Section 2 the PWA systems are described and the used MPC strategy is developed. A simulation example is shown in Section 3 and concluding remarks are given in Section 4.

2. PROBLEM FORMULATION

Several modeling frameworks have been introduced for the discrete-time HS, one of them is the PWA. A PWA systems is defined as

$$\begin{aligned} x_{k+1} &= A^i x_k + B^i u_k + f^i \\ y_k &= C^i x_k + g^i \end{aligned} \quad \text{for}: \begin{bmatrix} x_k \\ u_k \end{bmatrix} \in \mathcal{X}_i \quad (1)$$

where $\{\mathcal{X}_i\}_{i=1}^{s}$ is a polyhedral partition of the states and input space. Each $\mathcal{X}_i$ is given by

$$\mathcal{X}_i \triangleq \left\{ \begin{bmatrix} x_k \\ u_k \end{bmatrix} | Q^i \begin{bmatrix} x_k \\ u_k \end{bmatrix} \preceq q^i \right\}$$

where x_k, u_k, y_k denote the state, input and output vector, respectively. Each subsystem $\mathbb{S}^i$ defined by the 7-uple $(A^i, B^i, C^i, f^i, g^i, Q^i, q^i)$, $i \in \{1, 2, ..., s\}$ is termed a component of the PWA system (1). $A^i \in \mathbb{R}^{n\times n}$, $B^i \in \mathbb{R}^{n\times m}$, and (A^i, B^i) is a controllable pair. $C^i \in \mathbb{R}^{r\times n}$ and $Q^i \in \mathbb{R}^{p_i\times(n+m)}$ and f^i, g^i, q^i are suitable constant vectors. Note that n is number of states, m is the number of inputs, r is the number of output and p_i is the number of hyperplanes that define the i-polyhedral.

Assume that a full measurement of the state is available at the current time k. Most formulation of MPC require that the problem

$$U = \arg(\min_{U} J) \quad (2)$$

$$\text{to}: J = \sum_{i=1}^{N} q_{ii}(y_{k+i|k} - w_k)^2 + \sum_{i=1}^{N-1} r_{ii} u_k^2 \quad (3)$$

$$\text{s.t.}: u_{\min} \leq u_{k+i} \leq u_{\max} \quad k = 1, ..., M \quad (4)$$

is solved at each time k, where $y_{k+i|k}$ denoted the predicted output vector at the $k+i$ time, obtained by applying the input sequence $U \triangleq \{u_k, ..., u_{k+N-1}\}$ to model (1) starting from the state x_k subjected to constraints. It be noted that w_k is the output reference. The first control input is then applied to the process. At the next sample, measurements are used to update the optimization problem, and the optimization is repeated. In this way, this becomes a closed-loop approach.

Let us consider the prediction problems associated to the MPC in the case of PWA system. The subsystem describing the process is known if x_k is known, but the following subsystems depends on the applied control sequence. It can be considered that a change (transition) of model is produced between a sampling instant and the next. In general a sequence of subsystems $I = \{I_k, I_{k+1}, ..., I_{k+N}\}$ is activated. Only the initial value $I_k = I_k(x_k)$ of this sequence is known. If no constraints are considered, the number of possible sequences for a prediction horizon N is s^{N-1}. In order to solve the MPC problem (3) the optimization sequence is added to the decision variables. The resulting optimization problem can be stated as

$$U = \arg(\min_{U,I} J) \quad (5)$$

where constraints relating the dependences of the possible sequences U and I have to be added, i.e.:

$$Q^{I_{k+j}} x_{k+1} \leq q^{I_{k+j}}, \; j = \{1, .., N\} \quad (6)$$

Due to the integer nature of sequence I, the problem of finding the optimum can be solved by finding the optimum of the solutions for all possible sequence of I, i.e.

$$U = \arg\left(\min_{I}\left(\min_{U}\left(J_{Q^{IU}u \preceq q^{IU}}\right)\right)\right) \quad (7)$$

where $Q^{IU}u \preceq q^{IU}$ indicate the constraints due to dependences between I and U.

2.1 Resolution of the Computational Problem

Equation (3) can be written as

$$J = (\bar{y} - \bar{w})^T \bar{Q}(\bar{y} - \bar{w}) + \bar{u}^T \bar{R}\bar{u} \quad (8)$$

where $\bar{R} = diag[r_{ii}]$, $(\bar{R} = \bar{R}^T \succ 0)$, $\bar{Q} = diag[q_{ii}]$, $(\bar{Q} = \bar{Q}^T \succ 0)$. Note that $\bar{Q}$ (weight matrix of the error) is different to Q^i (matrix that defines the polyhedral) and $\bar{y} = \left[y_{k+1}^T \cdots y_{k+N}^T\right]^T$, $\bar{w} = \left[w_{k+1}^T \cdots w_{k+N}^T\right]^T$, $\bar{u} = \left[u_k^T \cdots u_{k+N-1}^T\right]^T$ the predicted output vector can be written as

$$\begin{aligned} \bar{y} &= F_y x_k + H_y \bar{u} + f_{o_y} \quad (9) \\ F_y &= \mathbf{C}_y F_x, \; H_y = \mathbf{C}_y H_x, \; f_{o_y} = \mathbf{C}_y f_{o_x} \\ \text{where}: \; \mathbf{C}_y &= diag(C^{I_{k+1}}, C^{I_{k+2}}, \cdots, C^{I_{k+N}}) \\ H_x &= \left[h_1 \; h_2 \; \cdots \; h_N \right] \end{aligned}$$

$$F_x = \begin{bmatrix} A^{I_k} \\ A^{I_{k+1}} A^{I_k} \\ \vdots \\ A^{I_{k+N-1}} A^{I_{k+1}} \dots A^{I_k} \end{bmatrix}$$

$$h_1 = \begin{bmatrix} B^{I_k} \\ A^{I_{k+1}} B^{I_k} \\ \vdots \\ A^{I_{k+N-1}} A^{I_{k+N-2}} \dots A^{I_{k+1}} B^{I_k} \end{bmatrix}$$

$$h_2 = \begin{bmatrix} 0 \\ B^{I_{k+1}} \\ A^{I_{k+2}} B^{I_{k+1}} \\ \vdots \\ A^{I_{k+N-1}} A^{I_{k+N-2}} \dots A^{I_{k+1}} B^{I_{k+1}} \end{bmatrix}$$

$$h_N = \left[0\ 0\ 0 \vdots (B^{I_{k+N-1}})^T \right]^T$$

$$f_{o_x} = \left[f_1\ f_2\ \cdots\ f_N \right]$$

$$f_1 = \begin{bmatrix} I \\ A^{I_{k+1}} \\ \vdots \\ A^{I_{k+N-1}} A^{I_{k+N-2}} \dots A^{I_{k+1}} \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 0 \\ I \\ A^{I_{k+2}} \\ \vdots \\ A^{I_{k+N-1}} A^{I_{k+N-2}} \dots A^{I_{k+2}} \end{bmatrix}$$

$$f_N = \left[0\ 0\ 0\ \cdots\ I \right]^T$$

Note that the following equalities are fulfilled

$$\begin{aligned} \bar{x} &= F_x x_k + H_x \bar{u} + f_{o_x} \qquad (10) \\ \bar{y} &= \mathbf{C}_y \bar{x} \\ \bar{x} &= \left[x_{k+1}^T\ x_{k+2}^T\ \cdots\ x_{k+N}^T \right]^T \end{aligned}$$

Replacing (9) in (8) the index is

$$J(I, U) = \bar{u} H_{QP} \bar{u} + f_{QP}^T \bar{u} + g_{QP} \qquad (11)$$

where

$$\begin{aligned} H_{QP} &= \left[H_y^T \bar{Q} H_y + \bar{Q} \right] \\ f_{QP}^T &= \left[2x_k F_y^T \bar{Q} H_y + 2 f_o^T \bar{Q} H_y + 2 \bar{w}^T \bar{Q} H_y \right] \\ g_{QP} &= x_k^T F_y^T \bar{Q} F_y x_k + 2 f_o^T \bar{Q} F_y x_k + 2 f_o^T \bar{Q} \bar{w} \\ &\quad + f_o^T \bar{Q} f_o - 2 \bar{w}^T \bar{Q} F_y x_k + \bar{w}^T \bar{Q} \bar{w} \end{aligned}$$

The constrain over the control (4) can be written as $Q^u \bar{u} \preceq q^u$, $Q^u = \left[-I_{N \times N}\ I_{N \times N} \right]^T$, $q^u = \left[-(u_{\min})^T\ \ (u_{\max})^T \right]^T$ and the constraints due to I and U dependence (6) can be written as

$$Q^{I_x} \bar{x}_I \preceq q^{I_x} \qquad (12)$$

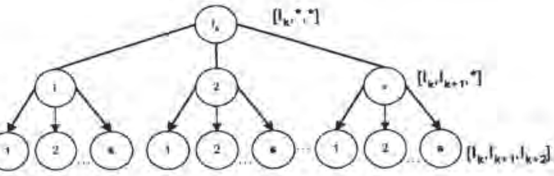

Fig. 1. Reacheble regions

where $\bar{x}_I = \left[x_{k+1}^T, \ldots, x_{k+N-1}^T \right]^T$, $Q^{I_x} = diag(Q^{I_{k+1}}, \ldots, Q^{I_{k+N-1}})$, $\bar{x}_I = \left[\left(q^{I_{k+1}}\right)^T, \ldots, \left(q^{I_{k+N-1}}\right)^T \right]^T$, $\bar{x}_I$ can be written by

$$\bar{x}_I = C_x \bar{x} \qquad (13)$$

where $C_x = \left[I_{n*N \times n*N}\ 0 \right]$. Replacing (13) and (10) in (12), the constraints due to the dependency between U and I results in $Q^{IU} \bar{u} \preceq q^{IU}$, $Q^{IU} = Q^{I_x} C_x H_x$, $q^{IU} = q^{I_x} - Q^{I_x} C_x F_x x_k - Q^{I_x} C_x f_{o_x}$. If constraints on the control actions are also considered then

$$Q_{QP} \bar{u} \preceq q_{QP} \qquad (14)$$

$Q_{QP} = \left[(Q^u)^T\ (Q^{IU})^T \right]^T$, $q_{QP} = \left[(q^u)^T\ (q^{IU})^T \right]^T$.

Therefore, once the sequence I is fixed, the problem can be solved by minimizing (11) subject to the constraints (14).

2.2 Reach set, controllable set and STG Algorithm

The key idea of the proposed algorithm is to determine the set of possible regions that can be reach from the actual region in next few sampling times. The reach set concept (Kerrigan, 2000) is used for this purpose. The set of regions that can be reached from a particular one can be organize as a STG as can be seen in figure 1. A search directed by this graph can then be organized. That is, all sequence that cannot be obtained following the transition graph are not considered.

Definition 1: (Kerrigan, 2000) **Reach set (RS)** $\tilde{\mathcal{R}}(\Omega)$. Given a discrete-time dynamic system $x_{k+1} = f(x_k, u_k, w_k)$ where $w_k \in \mathbb{W} \subset \mathbb{R}^d$ is an unknown disturbance ($\mathbb{W}$ are closed set in $\mathbb{R}^d$) and $u_k \in \mathbb{U} \subset \mathbb{R}^m$ ($\mathbb{U}$ is compact set in $\mathbb{R}^m$). The set $\tilde{\mathcal{R}}(\Omega)$ is the set of states in $\mathbb{R}^n$ to which the system will evolve at the next time step given any $x_k \subset \Omega$, admissible control input and allowable disturbance, i.e.

$$\tilde{\mathcal{R}}(\Omega) \triangleq \left\{ \begin{array}{l} x_{k+1} \in \mathbb{R}^n | \exists x_k \in \Omega, u_k \in \mathbb{U}, \\ w_k \in \mathbb{W}: x_{k+1} = f(x_k, u_k, w_k) \end{array} \right\} \qquad (15)$$

The RS for discrete-time linear system $x_{k+1} = Ax_k + Bu_k$ of a polytope set Ω of the state space, if it is consider that the control action $u_k \in \mathbb{U} \subset \mathbb{R}^m$ ($\mathbb{U}$ is a polytope (compact set) in $\mathbb{R}^m$) can be computed as Minkowski sum of the orthogonal projection of the set Ω over A and the $\mathbb{U}$ over B. See (Kerrigan, 2000) for more details.

One-Step Reachable Neighbors (1-SRN) can be defined based in the reach sets idea. The $\mathbb{S}^j$ subsystem is *one-SRN* of the $\mathbb{S}^i$ subsystem, denoted by $\mathbb{S}^i \underset{1}{\rightarrow} \mathbb{S}^j$, if the set $\mathcal{X}_j$ can be reached from the set $\mathcal{X}_i$ in one step. It is possible to determine if $\mathbb{S}^i \underset{1}{\rightarrow} \mathbb{S}^j$ due to $\mathbb{S}^i \underset{1}{\rightarrow} \mathbb{S}^j \iff \mathcal{R}(\mathcal{X}_i) \cap \mathcal{X}_j \neq \emptyset$. In the same way it can be determine the two-SRN if it is considered the two steps reach set $\mathcal{R}_2(\mathcal{X}_i) = \mathcal{R}(\mathcal{R}(\mathcal{X}_i))$ as $\mathbb{S}^i \underset{2}{\rightarrow} \mathbb{S}^j \iff \mathcal{R}_2(\mathcal{X}_i) \cap \mathcal{X}_j \neq \emptyset$. An so on for the n-SRN.

Definition 2: n-Step Reachable Neighbour $\mathbb{S}^i \underset{n}{\rightarrow} \mathbb{S}^j$.

$$\mathbb{S}^i \underset{n}{\rightarrow} \mathbb{S}^j \iff \mathcal{R}_n(\mathcal{X}_i) \cap \mathcal{X}_j \neq \emptyset$$

The index of all subsystems that are n-SRN of the subsystem i form the index set of n-SRN.

Definition 3: Index set of n-SRN $\mathbb{N}_n^i$

$$\mathbb{N}_n^i \triangleq \left\{ k \in [1, ..., s] / \mathbb{S}^i \underset{n}{\rightarrow} \mathbb{S}^k \right\}$$

Transition from state I_j to I_{j+1}remains bounded by the one-SRNs of the corresponding subsystem to I_j. This allow to prune the transition graph: a transition from I_j to I_{j+1} are not considered if $I_{j+1} \notin N_1^{I_j}$. If this concept is extended, the search tree can be pruned further considering that I_{j+k} should belong to $N_k^{I_j}$. Therefore, if only the one-SRN for a prediction horizon N are considered, the number of possible sequences to test by the QP algorithm is $s^{(I_k)} s^{(I_{k+1})} ... s^{(I_{k+N-1})}$ (instead of s^N, $s^{(I_k)} \leq s$), where s^{I_k} is the number of one-SRNs corresponding to subsystem I_k.

It should be noted that the determination of the 1 to n-SRNs for each subsystem can be done off-line. The neighbor list for each of the subsystems is included in the model description. Each subsystem is, therefore, defined by the $(7+N)$-uple $(A^i, B^i, C^i, f^i, g^i, Q^i, q^i, \mathbb{N}_1^i, ..., \mathbb{N}_N^i)$, $i \in \{1, 2, ..., s\}$ where, $\mathbb{N}^i$ is list containing the reachable neighbors of subsystem i. STG can be further pruned by considering the robust one step controllable set (SCS).

Definition 4: (Kerrigan, 2000) The robust one-Step Controllable Set $\tilde{\mathcal{Q}}(\Omega)$. The set $\tilde{\mathcal{Q}}(\Omega)$ is the set of states in $\mathbb{R}^n$ for which an admissible control input exists which will guarantee that the system will be driven to Ω in one step, for all allowable disturbances, i.e.

$$\tilde{\mathcal{Q}}(\Omega) \triangleq \left\{ \begin{array}{c} x_k \in \mathbb{R}^n \mid \exists u_k \in \mathbb{U} : \\ f(x_k, u_k, w_k) \in \Omega, \forall w_k \in \mathbb{W} \end{array} \right\} \quad (16)$$

In (Kerrigan, 2000) is show a method to compute the robust one-SCS. It is important to recognise that the one-SCS and the RS operate in different directions. The one-SCS is the set of states from which the system can be driven to a given set. The RS is the set of states to which the system can be driven from a given set. No explicit relation exists between the two sets.

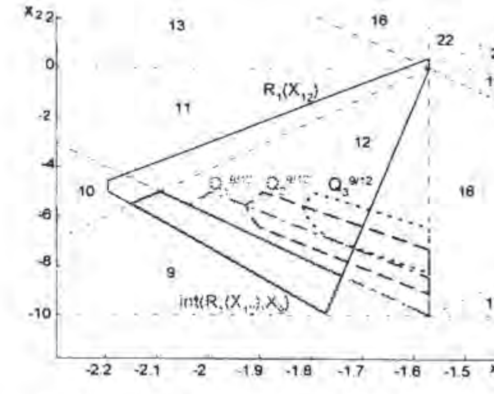

Fig. 2. Controllables set at one, two and three steps of de sector 9 over the sector 12

The 1-SCS of the subsystems j over the subsystem i $\mathcal{Q}_1^{j/i}$, is the subset of $\mathcal{X}_i$ from there subsystem j $(\mathcal{X}_j)$ can be reached in one step. That is,

$$\mathcal{Q}_1^{j/i} \triangleq \left\{ \begin{array}{c} x_k \in \mathcal{X}_i \mid \exists u_k \in \mathbb{U} : \\ f(x_k, u_k, w_k) \in \mathcal{X}_j, \forall w_k \in \mathbb{W} \end{array} \right\}$$

This set can be computed as

$$\mathcal{Q}_1^{j/i} = \mathcal{Q}_1(\mathcal{R}_1(\mathcal{X}_i) \cap \mathcal{X}_j) \cap \mathcal{X}_i$$

This concept can be extended from 1 to n steps as $\mathcal{Q}_n^{j/i}$. Note that the next state function corresponding to the system i is involved in the RS calculation as well as in the CS calculation. Figure 2 shows the controlable set to 1, 2 and 3 steps to the sector 9 over the sector 12.

To reduce the number of QP problems evaluations, the state of $x_k \notin \mathcal{Q}_1^{I_{k+1}/I_k}$ has been considered. If this statement is satisfied then the x_{k+1} state does not belong to I_{k+1} transition; therefore, the transition from I_k to I_{k+1} is not allowed. In order to apply these concepts, the definition of the $\mathcal{Q}_1^{j/i}$, ..., $\mathcal{Q}_N^{j/i}$ sets should be added to the definition of each subsystem.

3. ILLUSTRATIVE EXAMPLE

The proposed tree exploring strategy has been applied to the MPC of a simple pendulum system. A linearized equation of the discrete dynamic of the simple pendulum system is used as model. Consider the following linear system

$$ml\ddot{\theta} + kl\dot{\theta} + mg\sin(\theta) = \frac{\tau}{l}$$

where θ is pendulum angle, l is length of pendulum, m is mass of pendulum, g is the gravitational force, k is a friction coefficient and τ is a torque applied. Then, the state space model discretized to a sample time T_0 is

$$x_{k+1_1} = x_{k_1} + T_0 x_{k_2}$$

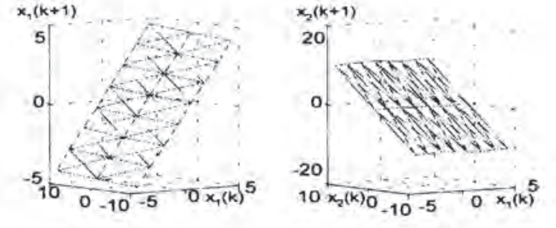

Fig. 3. PWA model representation of the pendulum

Table 1. Matlab Benchmark: PC performance, Window 98, Matlab 5.3

ODE	LU	Sparce	3-D	2-D
0.33	0.38	0.39	1.21	1.21

$$x_{k_1} = \left[1 - \frac{T_0 k}{m}\right] x_{k_2} - \frac{T_0 g}{l} \sin\left(x_{k_1}\right) + \frac{T_0}{l^2 m} u_k$$

where $u_k = \tau(k)$, $x_{k_1} = \theta(k)$ and $x_{2_k} = \dot{\theta}(k)$. Using $m = 1$, $l = 1$, $k = 0.5$, $g = 9.8$, $T_0 = 0.02$ as modelling parameters. Starting from a discretized model, a continuous PWA system as (1) has been obtained using a sectors linearization over the state space uniform grid. A partition $\mathcal{X}_i \triangleq \{x_k | Q^i x_k \preceq q^i\}$ is defined after obtaining 56 sectors over the state space. Matrixes C^i and B^i, are considered invariants, i.e.: $B^i = B = \left[0\ \frac{T_0}{l^2 m}\right]^T$ and $C^i = C = I_{(m,m)}$. Figure 3 shows the state linearization functions. Once the linearized model has been obtained, the neighbors corresponding to each sector are found using a reach set algorithm. In this example, the bounds of the torque are $10.78 \leq \tau \leq 10.78$ ($10.78 = 1.1\ mgl$), where the maximum torque in stable state to reach any position is $\tau = mgl$. All simulations test have been carried out in Matlab. Table 1 shows the performance of the computer used. Table 2 shows the prune method acronyms and their explanation. A prediction horizon N=4 is considered. The weights of the error and control action are $\bar{Q}_{x_1}$=1000, $\bar{Q}_{x_2}$=100, R=1, respectively.

Figure 4 shows the results with a reference $w = [0\ 0]$ and with different initial conditions. Dotted line and solid line show the behavior of the system to $\tau = 0$ (open loop) and the controlled system, respectively. It is possible to observe that the open loop state diverges for initials conditions $x_0 = [3\ \ 3]^T$.

Figure 5 shows a time parametric state curve and the number of QP evaluations for the N123 prune method. In this case the number of QP evaluations is an exclusive function of the belonged subsystem. Figure 6 compares the responses of (N1) with (N123). Note that in general the number of QP problems decreases when N123 is used.

Table 2. Acronyms of the method. SRN = steps reachable neighbors; SCOS = step controllables over sets

Acronym	Explanation
N1	Consider only 1-SRN
N123	Consider 1, 2, 3-SRN
N1-C1	Consider 1-SRN and 1-SCOS
N1-C12	Consider 1-SRN and 1, 2-SCOS
N1-C123	Consider 1-SRN and 1, 2, 3-SCOS
N123-C1	Consider 1, 2, 3-SRN and 1-SCOS
N123-C12	Consider 1, 2, 3-SRN and 1, 2-SCOS
N123-C123	Consider 1, 2, 3-SRN and 1, 2, 3-SCOS

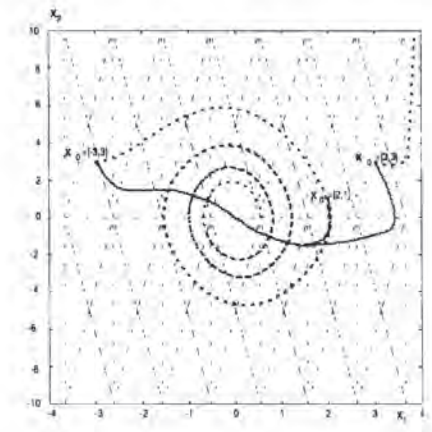

Fig. 4. Phase postrait with $w = [0\quad 0]$ and $x_0 = [2\ \ 1]^T$, $x_0 = [3\ \ 3]^T$, $x_0 = [-3\ \ 3]^T$

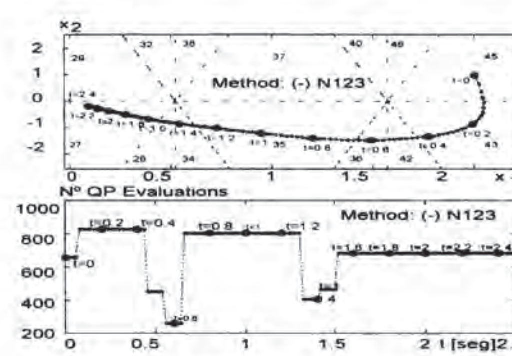

Fig. 5. Number of QP evaluations (N123)

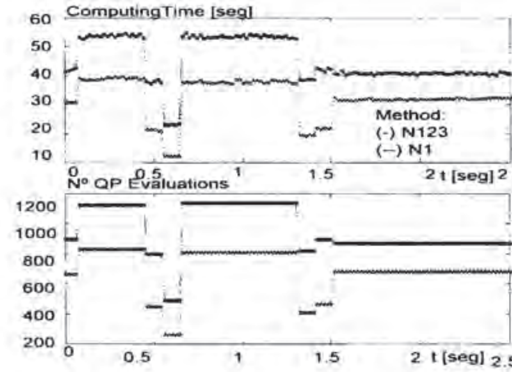

Fig. 6. Computing time and number of QP Evaluations [(-) N123 (:) N1]

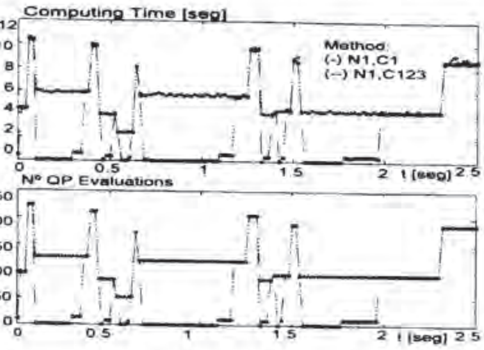

Fig. 7. Computing time and number of QP Evaluations [(-) N1-C1 (- -) N1-C123]

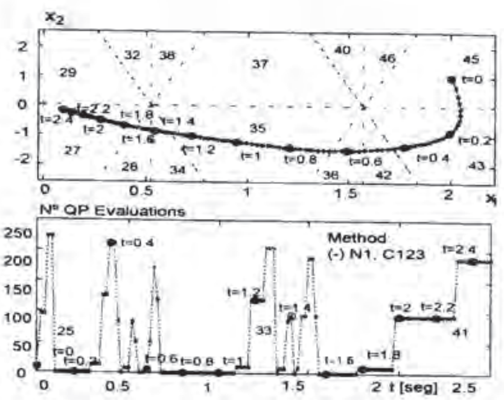

Fig. 8. Number of QP evaluations (N1- C123)

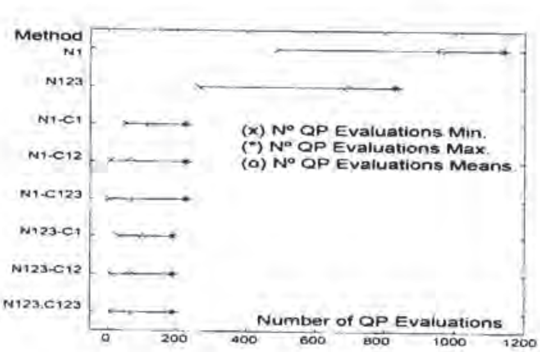

Fig. 9. Comparison table

Figure 8 shows that if N1-C123 is used, the number of QP decreases when the initial state is far away from the regions frontier, that is, when x_0 does not belong to any controllable sets at its neighbour. In this sectors the number of QP evaluations decreases when N1-C1 is compared with N1-C123 (Fig. 7). Figure 9 compares the number of (minimum, maximum and means values) QP evaluations for different pruning methods. Note that when a high number of controllable sets is considered although the quantity of evaluations minimum and means decrease, the maximum quantity of evaluations remains constant.

4. CONCLUSIONS

In this paper we have proposed a new approach for solving MPC control of PWA processes. The algorithm is based on the exploitation of state transition graphs and the concept of reach and controllable sets. The proposed tree exploring strategy chooses the reach set to diminish the number of possible realizations of the integer variables thus reducing considerably the number of QP problems needed to be solved.

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