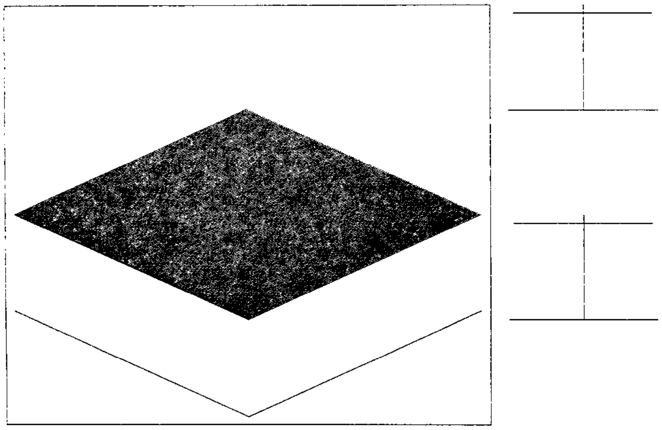

Fig. 6. Total frequency magnitude response of four 2-D QMF filters summation. The values are nearly constant, i.e., (0.999821 ~ 1.000179035) or (−0.000777 dB ~ 0.000777 dB).

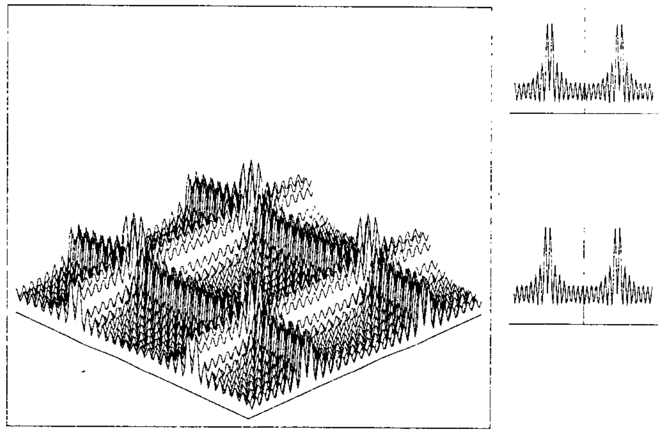

Fig. 7. The reconstruction error of Fig. 6 in enlarged scale. The largest error is located at the four corners of the 2-D QMF filters (0.999821 ~ 1.000179035) or (−0.000777 dB ~ 0.000777 dB).

nal signal can be proved to cancel each other, as long as the $F_0(Z_i)$'s are symmetric half-band filters. We can then perfectly reconstruct the original signal without any distortion. The factorization of the $F_0(Z_i)$'s into their maximum phase components $H_{\max}(Z_i)$ and minimum phase components $H_{\min}(Z_i)$ is the same as in 1-D cases [10]–[12].

IV. Experimental Results

Fig. 4 shows the four-band partition of the frequency spectrum in our 2-D subband coding scheme. First, we use the Parks–McClellan FIR filter program to design a 63-point symmetric half-band filter $F_0(Z_i)$, and the procedure in [12] is adopted to factor out $F_0(Z_i)$ into its 32-point maximum-phase $H_{\max}(Z_i)$ and minimum-phase part $H_{\min}(Z_i)$, respectively (see Tables I–III). The four 2-D QMF filters can be efficiently designed by the separable products of four 1-D QMF filters as in Fig. 3. The frequency magnitude responses of four 2-D 32×32 QMF filters are shown in Fig. 5(a)–(d). The reconstruction error is within ± 0.000777 dB (see Figs. 6 and 7), and the largest error is located at the four corners of 2-D QMF filters, mainly due to the factorization by homomorphic cepstrum.

V. Conclusions

In this paper, a new 2-D subband coding scheme is proposed, which adopts the 1-D QMF filter technique and Vetterli's separable scheme. Two-dimensional QMF filters can be efficiently designed by a separable product of identical 1-D QMF filters; this new 2-D subband coder can exactly reconstruct the original, free of any distortion. This will be very useful for image coding and transmission applications.

References

[1] N. S. Jayant and P. Noll, *Digital Coding of Waveforms*. Englewood Cliffs, NJ: Prentice-Hall, 1984, ch. 11, pp. 486–509.

[2] D. Esteban and C. Galand, "Application of quadrature mirror filters to split band voice coding schemes," in *Proc. 1977 IEEE Int. Conf. Acoust., Speech, Signal Process.*, (Hartford, CT), May 1977, pp. 191–195.

[3] M. J. T. Smith and T. P. Barnwell, III, "A procedure for designing exact reconstruction filter banks for tree-structured subband coders," in *Proc. 1984 IEEE Int. Conf. Acoust., Speech, Signal Process.* (San Diego, CA), Apr. 1984, pp. 27.1.1–27.1.4.

[4] M. J. T. Smith and T. P. Barnwell, III, "Exact reconstruction techniques for tree-structured subband coders," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-34, no. 3, pp. 434–441, June 1986.

[5] F. Mintzer, "Filters for distortion-free two-band multirate filter banks," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-33, no. 3, pp. 626–630, June 1985.

[6] M. Vetterli, "Multi-dimensional sub-band coding: some theory and algorithms," *Signal Process.*, vol. 6, pp. 97–112, Apr. 1984.

[7] J. W. Woods and S. D. O'Neil, "Sub-band coding of images," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-34, no. 5, pp. 1278–1288, Oct. 1986.

[8] R. E. Crochiere and L. R. Rabiner, *Multirate digital signal processing*. Englewood Cliffs, NJ: Prentice-Hall, 1983, pp. 155–157.

[9] F. Mintzer, "On half-band, third-band, and *N*th-band FIR filters and their design," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-30, no. 5, pp. 734–738, Oct. 1982.

[10] R. Boite and H. Leich, "A new procedure for the design of high order minimum phase FIR digital or CCD filters," *Signal Process.*, vol. 3, pp. 101–108, Apr. 1981.

[11] G. A. Mian and A. P. Nainer, "A fast procedure to design equiripple minimum-phase FIR filters," *IEEE Trans. Circuits Syst.*, vol. CAS-29, pp. 327–331, May 1982.

[12] S. C. Pei and S. T. Lu, "Design of minimum phase FIR digital filter by differential cepstrum," *IEEE Trans. Circuits Syst.*, vol. CAS-33, pp. 570–576, May 1986.

High-Frequency Design of the Wien-Bridge Oscillator Using Composite Amplifiers

A. RODRIGUEZ-VAZQUEZ, J. L. HUERTAS, AND B. PEREZ-VERDU

I. Introduction

The finite gain–bandwidth (GB) product of the operational amplifier (OA) severely degrades the high-frequency performance of the conventional one-OA realization of the Wien-bridge oscillator [1]. In [1], Budak and Nay have shown that the GB influence can be reduced by the expedient of using two-OA composite amplifiers. In particular, these authors focused on a realization of the Wien bridge using Reddy's amplifier [2] and developed an innovative design procedure that makes it possible to extend the range of operation of the oscillator. However, in spite of the advantages of using the amplifier in [1], there still

Manuscript received December 15, 1986.
The authors are with the Departamento de Electricidad y Electronica, Facultad de Fisica, Universidad de Sevilla, 41012 Sevilla, Spain.
IEEE Log Number 8613333.

0098-4094/87/0400-0441$01.00 ©1987 IEEE

remains a strong dependence between the frequency of oscillation and the GB value (see [1, eq. (8)]). Furthermore, the analysis in [1] relies on linear models, thus disregarding any influence due to either the amplitude-controlling mechanism or the OA nonlinearities. In particular, the influence of the slew rate is very strong at high frequencies, and as a result, important disparities appear between theoretical and actual behaviors [3].

The circuit in this letter is intended to overcome the above drawbacks. We consider a composite-amplifier realization of the Wien-bridge oscillator and use a nonlinear analysis technique for modeling the influence of the slew rate on the circuit performance. We then show that, by properly designing the amplifier, it is possible to obtain high-frequency, low-distortion sinusoidal signals whose frequency of oscillation is virtually independent of GB. Experimental data confirming these predictions are included.

II. Slew-Rate Oscillator Model

The Wien-bridge oscillator circuit is shown in Fig. 1. Two different blocks can be distinguished, namely an amplifier and a passive RC network. The transfer function for this latter network is

$$T_{RC}(S) = \frac{SWa}{S^2 + S3Wa + Wa^2} \tag{1}$$

where $Wa = 1/RC$.

If an ideal amplifier were available, its gain would have to be fixed to a value of 3, thus making the circuit oscillate at a frequency $Wo = Wa$. However, in practice this idealized picture is only valid at very low frequencies. When the frequency is increased, both the required gain value and the oscillation frequency deviate from those ideal values. Moreover, the influence of the amplifier nonlinearities gives rise to the onset of distortion. Clearly, all previous effects would have to be considered for a proper design, this requiring a careful choice of the amplifier.

Let us consider that the amplifier in Fig. 1 is realized using the two-OA composite structure of Fig. 2 [4]. The high-frequency nonlinear behavior of this amplifier can be accurately analyzed by resorting to the use of the OA model shown in Fig. 3 [6]. Assume that each OA in Fig. 2 is replaced by such a model. In the frequency domain, we may assume the signal V_1 to be sinusoidal and then analyze the circuit by using the describing-function technique. As a preliminary step, we have to identify which of the two OA nonlinearities is the dominant one. Obviously, it will depend on the actual values of both the K and the P parameter in Fig. 2. However, for the values to be considered herein, it can be shown that the nonlinearity of OA2 is the dominant one [3]. Taking this into account, we get the following transfer function for the amplifier in Fig. 2:

$$G(S,A) = K\frac{1+\frac{S}{GB}P}{1+\frac{S}{GB}\frac{K}{N(A)}+\frac{S^2}{GB^2}\frac{KP}{N(A)}} \tag{2}$$

where GB denotes the gain–bandwidth product of the OA (we have assumed $GB_1 = GB_2$) and A is the amplitude of the fundamental harmonic at the input of OA2. The describing function is given by

$$N(A) = \begin{cases} (2/\pi)\left[\sin^{-1}(\delta/A) + (\delta/A)\sqrt{1-(\delta/A)^2}\right] \\ \quad \text{for } (\delta/A) < 1 \\ 1 \quad \text{elsewhere.} \end{cases} \tag{3}$$

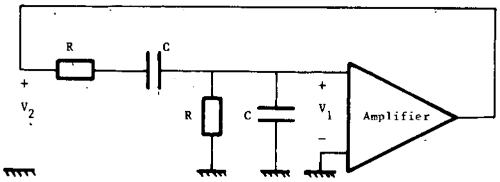

Fig. 1. Block diagram of the Wien-bridge oscillator.

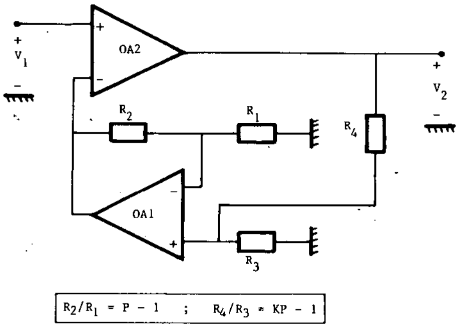

Fig. 2. Two-OA composite amplifier realization.

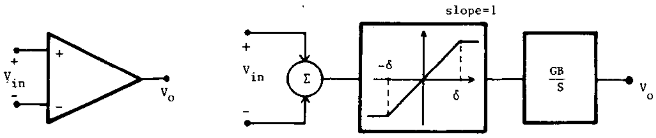

Fig. 3. Nonlinear model for the OA.

Consider now that we are using the previous amplifier model in Fig. 1. From the theory of nonlinear systems [7], it is known that there is a limit cycle oscillation for every solution of

$$G(S,A)T_{RC}(S)|_{S=jWo} = 1. \tag{4}$$

Using (2) in (4), we get the following characteristic equation:

$$\begin{aligned} S^4\frac{KP}{GB^2N(A)} &+ S^3\left[\frac{K}{GBN(A)}+\frac{3WaKP}{GB^2N(A)}\right] \\ &+ S^2\left[1+\frac{3WaK}{GBN(A)}+\frac{Wa^2KP}{GB^2N(A)}-\frac{WaKP}{GB}\right] \\ &+ S\left[3Wa+\frac{Wa^2K}{GBN(A)}-KWa\right]+Wa^2 = 0 \end{aligned} \tag{5}$$

which, after some algebraic manipulation, can be shown to exhibit the following solution:

$$\begin{aligned} Wo &= Wa \\ K &= \frac{3}{1+9[Wa/GBN(A)]^2} \\ P &= \frac{3}{N(A)}. \end{aligned} \tag{6}$$

This is a very interesting solution in that the actual frequency of oscillation is equal to the ideal one. As a matter of fact, given a value of the Wa of the RC network, previous equations give us the required values of K and P for achieving the goal of making $Wo = Wa$. However, in order to solve these equations, prior knowledge of $N(A)$ is required. In practice, we may resort to the use of some distortion measure related to $N(A)$. In particular, we propose to use the ratio (expressed in decibels) of the amplitudes

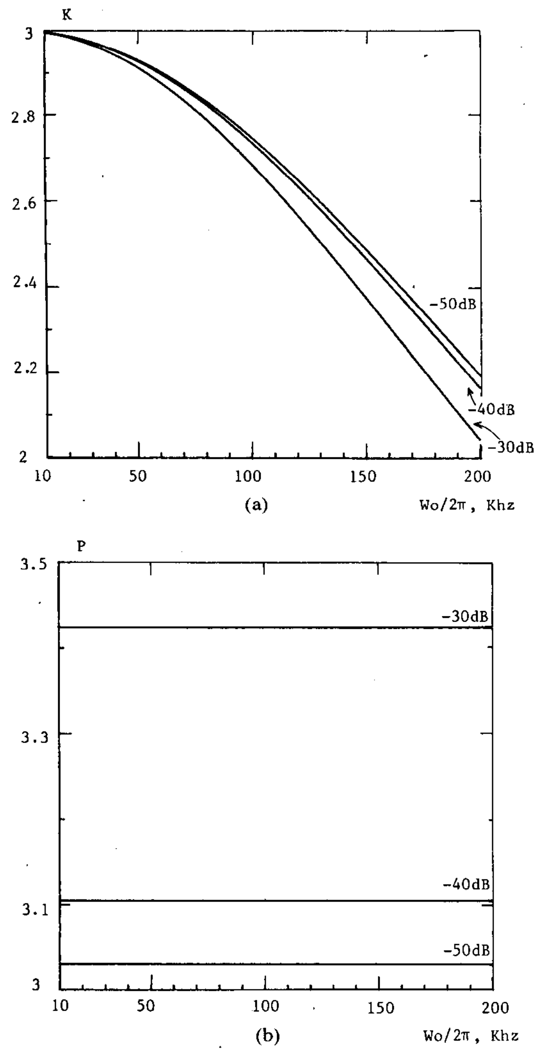

Fig. 4. (a) Plot of K versus oscillation frequencies for different distortion levels. (b) Plot of P versus oscillation frequencies for different distortion levels.

of the third and the fundamental harmonic at the output of OA2 (this is meaningful because the nonlinearity we are dealing with is odd). Such a ratio can be calculated from the corresponding value of $N(A)$, as in [7]. Using the results of these calculations in (6), we can obtain the plots in Fig. 4. There we show K versus Wo (Fig. 4(a)) and P versus Wo (Fig. 4(b)) for different values of the distortion parameter. Given Wo and the maximum distortion level, we can then obtain the required values of K and P.

At this point, the only question which remains to be answered is whether or not the limit cycle oscillations are stable. Applying Loeb's criterion [7], we obtain the following condition for stability [3]:

$$-5\frac{Wa^2}{GB^2}KP^2+9-3K\leqslant 0 \tag{7}$$

which can be shown to be fulfilled for the values given in (6).

III. Discussion of Results

The design procedure introduced in the paper has been tested via experimental measurements. Table I is a summary of the results we have obtained using LM747 dual operational amplifiers and passive components with a 1-percent tolerance. The first part of the table shows the theoretical values obtained during the design phase. These values have been calculated from (6) taking $GB = 800$ kHz and assuming the distortion to be -40 dB. The second part shows the measured parameters. As can be seen, there is very close agreement between each value of Wa and the corresponding frequency of oscillation. In fact, the deviation is less than 2 percent for frequencies up to 100 kHz. It compares favorably with the model proposed in [1]; there, the deviation is 2 percent for a frequency less than 6 kHz (see [1, eq. (18)]).

TABLE I
Experimental Results for Different Oscillation Frequencies

Theoretical $Wa/2\pi$, kHz	Theoretical K	Experimental $Wo/2\pi$, kHz	Experimental Dist., dB
22.7	2.98	22.5	−42
49.1	2.89	49.2	−41.5
88.4	2.68	88.3	−41
107.8	2.55	105.5	−37
154.8	2.2	147.7	−36

As a final point, it is important to mention that the proposed modeling technique is not directly applicable to a composite-amplifier structure different from the one in Fig. 2. This is because instability problems can appear. In particular, such problems arise when trying to use Reddy's amplifier [3]. In fact, the composite amplifier in Fig. 2 has been selected from the whole catalog of amplifier structures [4], [5] as the one best tailored for getting $Wo = Wa$ [3].

References

[1] A. Budak and K. Nay, "Operational amplifier circuits for the Wien-bridge oscillator," *IEEE Trans. Circuits Syst.*, vol. CAS-28, pp. 930–934, Sept. 1981.
[2] M. A. Reddy, "Operational-amplifier circuits with variable phase shift and their application to high-Q active RC-filters and RC-oscillators," *IEEE Trans. Circuits Syst.*, vol. CAS-23, pp. 384–389, June 1976.
[3] B. Perez-Verdu, "Nonlinear modeling of operational amplifiers based RC-oscillators," (in Spanish), Ph.D. dissertation, University of Sevilla, 1985.
[4] A. Soliman, "Classification and generation of active compensated non-inverting VCVS building blocks," *Int. J. Circuit Theory Appl.*, vol. 8, pp. 395–405, Oct. 1980.
[5] J. L. Huertas and A. Rodriguez-Vazquez, "On the active compensation of operational amplifiers based VCVS," *IEEE Trans. Circuits Syst.*, vol. CAS-29, pp. 497–506, July 1982.
[6] P. Allen, "A model of slew-induced distortion in single-amplifier active filters," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 565–572, Aug. 1978.
[7] A. Gelb and W. VanderVelde, *Multiple Input Describing Functions and Nonlinear System Design*. New York: McGraw-Hill, 1968.

Delay-Time Sensitivity in Linear *RC* Tree

NAVNEET K. JAIN, V. C. PRASAD, AND
A. B. BHATTACHARYYA

***Abstract*—The study of *RC* networks is important to understand digital MOS integrated circuits. Several authors studied these networks from the point of view of bounds on voltage waveform [1], [2], signal delay [3], etc.**

Manuscript received June 11, 1986.
N. K. Jain and V. C. Prasad are with the Electrical Engineering Department, Indian Institute of Technology, Hauz Khas, New Delhi-110016, India.
A. B. Bhattacharyya is with the Centre for Applied Research in Electronics, Indian Institute of Technology, Hauz Khas, New Delhi-110016, India.
IEEE Log Number 8611587.

0098-4094/87/0400-0443$01.00 ©1987 IEEE