where

$$P = \text{block diagonal}\left(I_1, I_2, \cdots, I_{i-1}, P_i, I_{i+1}, \cdots, I_p\right) \tag{12}$$

where I_k is $k \times k$ identity matrix, and k is the dimension of component Σ_k.

Using the component connection approach, a model for the interconnected system Σ is described with

$$\dot{z} = (F + GLH) z + GMu \tag{13a}$$

$$y = Hz \tag{13b}$$

where

$$F = \text{block diagonal}\left(A_1, A_2, \cdots, A_{i-1}, F_i, A_{i+1}, \cdots, A_p\right) \tag{13c}$$

$$G = \text{block diagonal}\left(B_1, B_2, \cdots, B_{i-1}, G_i, B_{i+1}, \cdots, B_p\right) \tag{13d}$$

$$H = \text{block diagonal}\left(C_1, C_2, \cdots, C_{i-1}, H_i, C_{i+1}, \cdots, C_p\right). \tag{13e}$$

Equations (3c), (10c), (12), and (13c) imply

$$FP = PA. \tag{14a}$$

Equations (3d), (3e), (10d), (10e), (12), and (13d) imply

$$(GLH) P = PBLC. \tag{14b}$$

Therefore, (14a) and (14b) imply

$$(F + GLH) P = P(A + BLC). \tag{15a}$$

Similarly

$$G = PB \tag{15b}$$

$$HP = C. \tag{15c}$$

b) *Necessity*: The condition stated in the theorem results from the definition of the component connection model. In this framework the state vector of the composite system is obtained by stacking the state vectors of the individual components.

Remark: If (1b) is modified to

$$y_i = C_i x_i + D_i v_i$$

then, (8b) needs to be modified to

$$y = Hz + Ju$$

and conditions (9) will include

$$J = D. \tag{9d}$$

Consequently, the theorem still holds and the composite system model is described by [6]

$$\dot{x} = (A + BQLC) x + BQMu$$

$$y = (I + DQL) Cx + DQMu$$

where

$$D = \text{block diagonal}\left(D_1, D_2, \cdots, D_p\right)$$

$$Q = (I - LD)^{-1}.$$

The aggregate model for the composite system is described by

$$\dot{z} = (F + GQLH) z + GQMu$$

$$y = (I + DQL) Hz + DQMu$$

where the aggregate model state variables z are as defined above.

V. Conclusion

Whenever dealing with interconnected large-scale systems the size of the system becomes an issue. Model reduction techniques are used to alleviate these difficulties. Most of the techniques are applied to the linearized model of the composite system model. In this paper, it is shown that in the component connection model framework the composite system is aggregable if and only if at least one of its components is aggregable. Because of the relatively small size of the individual components, it is easier to aggregate the component model rather than the composite system model. The important consequence of this result is that when dealing with interconnected large scale systems in the component connection model framework one does not need to aggregate the large-scale composite system but rather can consider the aggregation of each, relatively smaller scale component, independently.

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Chaos in a Switched-Capacitor Circuit

ANGEL B. RODRIGUEZ-VAZQUEZ, JOSE L. HUERTAS, AND LEON O. CHUA

***Abstract*—We report chaotic phenomena observed from a simple nonlinear switched-capacitor circuit. The experimentally measured *bifurcation tree* diagram reveals a *period-doubling route* to chaos. This circuit is described by a first-order discrete equation which can be transformed into the *logistic map* whose chaotic dynamics is well known.**

Several nonlinear circuits which exhibit various types of chaotic phenomena have been reported recently [1]–[5]. Our objective in this letter is to report an experimental result showing the ubiquitous *chaotic* phenomena can also occur in a switched-capacitor circuit. Since switched-capacitor circuits are important in VLSI technology, any potential anomaly or failure mechanisms

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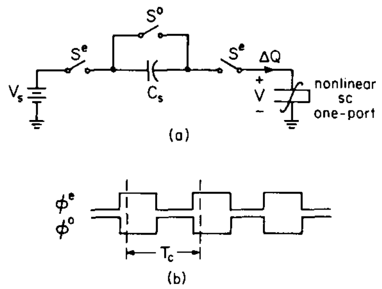

Fig. 1. A nonlinear switched-capacitor circuit and its associated timing diagram.

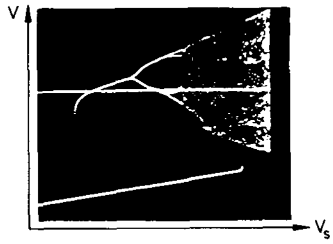

Fig. 2. Bifurcation tree.

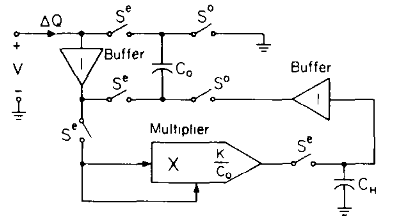

Fig. 3. Off-the-shelf realization of the FESC resistor in Fig. 1(a).

due to the onset of chaos should be fully analyzed. This chaotic circuit is also of circuit-theoretic interest because its dynamic equation is equivalent to the well-known *logistic* map [6] whose chaotic dynamics have been extensively studied and is now well understood. Since the logistic map is the simplest *chaotic* polynomial discrete map, the chaotic circuit to be described below is the *simplest* chaotic circuit described by a *first-order discrete* map.

Consider the switched-capacitor circuit in Fig. 1(a): it is made of a battery V_s, a linear capacitor C_s, a *nonlinear* switched-capacitor component [7], [8], and three analog switches. The state (on or off) of the switches is controlled by a standard two-phase clock defined by the timing diagram shown in Fig. 1(b). The switches labeled S^e (resp., S^o) turn *on* in synchronization with the rising edge of the clock signal ϕ^e (resp., ϕ^o).

The nonlinear switched-capacitor component—henceforth called an FESC (forward Euler switched capacitor) resistor—is defined by

$$Q_n - Q_{n-1} = kV_{n-1}^2 \triangleq \Delta Q_n \tag{1}$$

where ΔQ_n is the net charge flowing into the FESC resistor during the nth clock period, V_{n-1} is the voltage sampled across the FESC resistor during the $(n-1)$th period, and K is an arbitrary positive constant.

We have built the circuit in Fig. 1(a) with $C_s = 1$ nF and $k = 0.5$ nF/V using off-the-shelf components and observed the steady-state voltage waveform samples V for different values of the battery voltage V_s. Contrary to our intuitive expectation for a single-valued relationship between V and V_s, we found the relationship to be *multiple-valued* over some ranges of the "parameter" V_s, and undefined i.e., *chaotic*, for other ranges. This observation is summarized by the bifurcation tree measured experimentally from this circuit. The familiar cascades preceding the chaotic region implies a *period-doubling route* to chaos [6].

To derive a recursive relationship for V_n, we note that the net charge ΔQ_{n+1} flowing into the FESC resistor during the $(n+1)$th clock period must be equal to the net charge flowing out of the *linear* capacitor C_s (charge conservation principle), and hence:

$$V_{n+1} = V_s - \frac{k}{C_s} V_n^2. \tag{2}$$

We can transform (2) into several more familiar equivalent forms by defining

$$X_{n+1} = aV_{n+1} + b. \tag{3}$$

If we choose $a = 1/V_s$ and $b = 0$, we would obtain

$$X_{n+1} = 1 - \lambda X_n^2 \tag{4}$$

where

$$\lambda = kV_s / C_s.$$

If we choose

$$a = \frac{1}{4V_s}\left(-1 \pm \sqrt{1 + 4k\frac{V_s}{C_s}}\right)$$

and $b = \frac{1}{2}$, we would obtain the well-known *logistic map*

$$X_{n+1} = 4\lambda X_n(1 - X_n) \tag{5}$$

where

$$\lambda \triangleq \frac{k}{4aC_s}.$$

Both equations, (3) and (5) have been intensively studied [6] and their global qualitative behaviors are now well classified and understood. Consequently, Fig. 1(a) represents the *first* real physical circuit whose chaotic dynamics can be *completely* analyzed.

For readers interested in repeating our experiments, the FESC resistor in Fig. 1(a) can be realized by the circuit shown in Fig. 3.

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Nodal Voltage Simulation of Active *RC* Networks

P. V. ANANDA MOHAN, V. RAMACHANDRAN, AND M. N. S. SWAMY

***Abstract*— It is shown that the floating nodes present in active *RC* networks can be eliminated by nodal voltage simulation, leading to new active *RC* networks. The new active *RC* networks thus generated can be easily used to realize stray-insensitive *SC* networks. Equivalence of certain active *RC* topologies, resulting from application of this method, is also demonstrated.**

I. Introduction

The theory and design techniques of active RC filters are considerably mature at present. Integrated circuit implementation necessitated circuits using grounded capacitors, to eliminate the effect of bottom-plate parasitic capacitances [1]. With the advent of switched-capacitor technique, it has become necessary to eliminate the effect of parasitic capacitances altogether, since predistortion as well as trimming are unattractive [2]. It is natural, therefore, to attempt to derive stray-insensitive SC topologies from active RC filters using grounded capacitors. Three techniques viz., parasitic compensation [3], [4], nodal voltage simulation [5], [6], and stray-capacitance eliminating transformations [7] have been reported in the literature. In this letter, we examine the second approach in detail. It may be noted that this approach has been used in deriving bilinear SC ladder filters from doubly terminated LC filters [8]. Other variations such as nodal transfer function simulation [9], [10] are also available in the literature.

II. Nodal Voltage Simulation Method

We study the nodal voltage simulation technique with reference to specific active RC networks, in what follows, in order to demonstrate its utility.

A. Application to Multiple Feedback Type Single-Amplifier Low-Pass Filters

Consider the multiple-feedback active RC low-pass filter of Fig. 1(a). The internal node x in this network is described by KCL as

$$\frac{V_i}{R_1}+\frac{V_0}{R_3}=V_x\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+sC_1\right]. \qquad (1)$$

It is easy to realize V_x using OA's with grounded noninverting input as shown in Fig. 1(b), by the circuit within dotted lines. Augmenting this network by the remaining circuit consisting of

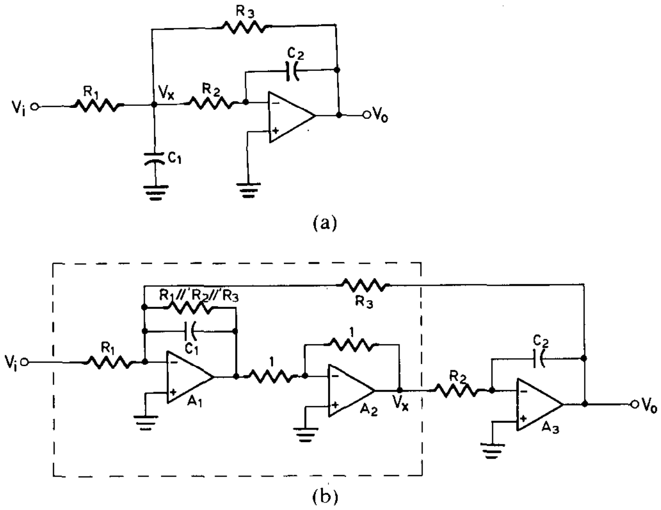

Fig. 1. (a) Multiple feedback low-pass filter. (b) Active RC network equivalent to Fig. 1(a).

R_2, C_2 and OA A_3 to realize V_0 from V_x, we obtain the network of Fig. 1(b), which interestingly, is the well-known Tow-Thomas biquad [11]. The availability of band-pass output at node x and low-pass output at V_0 in the circuit of Fig. 1(a) is transferred to the network in Fig. 1(b). Note that, both these circuits are based on inductance simulation [12], [13]. Further, we observe that the circuit of Fig. 1(a) is having damping due to resistors R_1, R_2, and R_3. This damping can be reduced by multiplying the effect of R_2 and R_3 through a negative resistance to ground at node x [14]. The resulting active RC filter is shown in Fig. 2(a). Proceeding in the same manner as in the case of Fig. 1(a), we obtain the equivalent RC active filter of Fig. 2(b). The transfer function realized is given by

$$\frac{V_0}{V_i}=\frac{-1/R_1}{s^2C_1C_2R_4+sC_2R_4\left(\dfrac{1}{R_2}-\dfrac{1}{R_5}\right)+\dfrac{1}{R_3}}. \qquad (2)$$

Thus both low-pass and band-pass transfer functions are still available, while large Q's can be obtained through the use of nearly equal R_2 and R_5 values. The Q-sensitivities are thus large:

$$S_{R_5}^{Q}=1+\frac{QR_3}{R_5} \quad \text{and} \quad S_{R_4}^{Q}=1-\frac{QR_3}{R_5}.$$

Note that Fischer and Moschytz [15] used such a modification for Fleischer–Laker's biquad [16] for reducing the capacitance spread.

In applications where equal capacitor values are preferred in the circuit of Fig. 1(a), positive feedback can be applied [17]. This leads to the modification of the Tow–Thomas biquad, as shown in Fig. 3(a). The transfer function realized is

$$\frac{V_0}{V_i}=\frac{1/R_1}{s^2C_1C_2R_4+s\left(\dfrac{C_2R_4}{R_2}-\dfrac{C_1R_4}{R_5}\right)+\left(\dfrac{1}{R_3}-\dfrac{R_4}{R_2R_5}\right)}. \qquad (3)$$

Note, however, that the band-pass transfer function is not realizable at the outputs of the OA's A_1 and A_2. The design equations

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