Trabajo Fin de Grado
Ingeniería Aeroespacial

Trajectory prediction in Standard Instrument Departure Routes considering weather effects.

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El tribunal nombrado para juzgar el Proyecto arriba indicado, compuesto por los siguientes miembros:

Presidente:

Vocales:

Secretario:

Acuerdan otorgarle la calificación de:

Sevilla, 2019

El Secretario del Tribunal

A mis padres
A mis abuelos

Agradecimientos

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Ignacio Bértiz Gutiérrez

Sevilla, 2019

Abstract

Automatization in the analysis of Standard Instrument Departure Routes (SIDs) is essential when comparing different ways of performing the same flight, and also to see how the aircraft trajectory varies with the influence of external factors. The main goal of this project is to develop an aircraft trajectory predictor in order to study different parameters and flight conditions considering, in an approach to the reality, the influence of the weather effects. It aims to contribute to the problem of the lack of automatization with a complete and versatile solution permitting that a user could easily programme a SID and obtain relevant kinematic and dynamic information. A deeper implementation includes the influence of the weather. Moreover, different studies of sensitivity with parameters such as the initial aircraft mass or the geographical location are carried out.

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FIGURE INDEX

Notation

a_0	Speed of sound.
APF	Airline Procedure File.
BADA	Base of Aircraft Data.
CAS	Calibrated Airspeed.
C_{d0}	Parasite drag coefficient.
c_E	Thrust specific fuel consumption.
D	Drag.
ECMWF	European Center for Medium-Range Weather Forecast.
g_0	Gravitational acceleration at mean sea level.
h	Geometric altitude.
H	Geopotential altitude.
$H_{p,trans}$	Mach transition geopotential pressure altitude.
H_p	Geopotential pressure altitude.
ISA	International Standard Atmosphere.
K	Induced drag coefficient.
L	Lift.
DME	Distance Measurement Equipment.
M	Mach number.
m	Aircraft mass.
OPF	Operations Performance File.
p_0	Standard atmospheric pressure at mean sea level.
PTD	Performance Table Data.
PTF	Performance Table File.
R_g	Gas constant for air.
R_E	Earth radius.
R	Turning radius
ROC	Rate of Climb.
ROCD	Rate of Climb and Descent.
SID	Standard Instrument Departure Route.
T	Thrust.
T_0	Standard atmospheric temperature at mean sea level.
T_{max}	Maximum Thrust.
V	True Airspeed.
V_{CAS}	True Airspeed associated with a CAS.
V_M	True Airspeed associated with a Mach number.
V_g	Ground speed.
w_{along}	Along-track wind.

w_{cross}	Crosswind.
w_m	Meridional wind.
w_z	Zonal wind.
W	Aircraft weight.
x	Distance measured from the departure airport in the North direction.
y	Distance measured from the departure airport in the East direction.
$\beta_{T,<}$	Temperature gradient with altitude below the tropopause.
γ	Aerodynamic flight path angle.
γ_g	Inertial flight path angle.
δ_{trans}	Pressure ratio at the transition altitude.
θ_{trans}	Temperature ratio at the transition altitude.
κ	Adiabatic index of air.
λ	Longitude.
λ_{AP}	Longitude of the departure airport.
λ_{ref}	Longitude of a reference location.
μ	Bank angle.
ρ_0	Standard atmospheric density at mean sea level.
ϕ	Latitude.
ϕ_{AP}	Latitude of the departure airport.
ϕ_{ref}	Latitude of a reference location.
χ	Heading.
χ_g	Course.
$\vec{e}_{track}$	Unit vector along the route in the direction of the ground speed.

1 INTRODUCTION

1.1 Motivation.

In an increasingly globalized world, the need for an efficient communication network is essential and even more and more requested by the consumers. They are not few people who still believe that air transport is a luxury unattainable to most of the pockets, but in fact, the increase in new routes and new lowcost airlines have made this sector one of the most versatile, as for the same flight you can find a wide range of ticket prices according to your preferences.

That versatility has supposed a remarkable increase in the number of people who travel by plane nowadays and so the number of airplanes flying the skies is growing. All those airplanes need to be perfectly controlled and organised in order to comply with the maximum of the aviation world: airworthiness, which means, ensuring the safety of the airplane and its surroundings at any time, including, for example, the ground personnel.

In this sense, a strong organization and standardization of the aircraft routes is needed focused on the most critical flight segments that are take-off and landing. The problem of establishing standardized routes has already been solved by the International Civil Aviation Organization (ICAO) in the Aeronautical Information Publication (AIP). AIP is designed to be a manual containing thorough details of regulations, procedures and other information pertinent to flying aircraft in the particular country to which it relates. It is usually issued by or on behalf of the respective civil aviation administration [1].

The Aeronautical Information Publication (AIP) is a huge source of information that provides general knowledge about route procedures and airport information. Each country develops its own AIP with the objective of organising and controlling air transport. With around 100,000 airplanes per day flying the skies, the standardization of routes is essential to ensure that airplanes are located in each moment were they are supposed to be. The segments of take-off and initial climb are especially important and critical as the traffic is higher and the response time for eventual risks is lower. In this sense, AIP includes standard departure routes of all the network to which it applies. The problem of analysing them lies in the lack of automatization of the SIDs (Standard Instrument Departure Routes) as they are found in a format in which the comparison between aircraft models or weather conditions, among others, is not permitted. A software tool to perform these tasks is needed as these routes aim to provide a network where Air Traffic Management (ATM) can organise and control the air traffic. It is particulary interesting that they are especially established for each airport, as the orography or the military restrictions put limits on the free trajectory of the aircraft. In addition, the influence of the weather, especially wind, might vary the conditions and variable values under which the airplane flies, and this must be included in the software formulation of the problem.

1.2 Aim of the project.

The aim of this project is to study the performance of an aircraft in its way along a SID and to contribute to the automatization of the SID descriptions, as they are not available in a software description yet, as well as study them in a wide variety of scenarios. In this project, a considerable level of automatization is achieved and relevant conclusions are obtained. Although the full state of automatization has not been reached yet, the presented tool contributes to its search. It is a tool that can be used by airlines to minimize the operational costs by reducing the fuel consumption or by ATM to predict where will the airplanes be after a certain time once they have departed. Moreover, the effect of the weather forecast is introduced at a particular location and time, making this prediction more realistic and obtaining the corrections a pilot should apply to follow the SID.

To achieve this objective, a detailed study of Flight Mechanics and aircraft performance is done and software tools are used. What is more, geometrical problems are solved and reasonable hypotheses and simplifications are taken into account. To validate the developed tool, named as *Trajectory Predictor,* multiple examples are presented along the project including different ways of defining the Standard Instrument Departure Routes.

1.3   Structure

The project has the following structure:

- **Chapter 1**: a brief introduction of the project is carried out. Its motivation, aim and structure is presented.

- **Chapter 2**: the Flight Mechanics equations are presented as they are the base of this project. Some hyphoteses are done and the consequences of including the weather effects are discussed.

- **Chapter 3**: main modules of BADA model are explained in order to establish a framework for the work done. The aerodynamic model, the operation performance and the airline procedures are included, among others.

- **Chapter 4**: the structure of the *Trajectory Predictor* is analysed. The inputs, outputs and information treatment are widely developed.

- **Chapter 5**: the interface with the user is presented by solving a particular example.

- **Chapter 6**: multiple examples are solved to validate the use of this tool. Four sensitivity studies are carried out in order to study different operation performance.

- **Chapter 7**: the conclusions of the project are presented.

- **Chapter 8**: future implementations and improvements are proposed.

2 EQUATIONS OF MOTION

The main branch of knowledge studied in this work has been Flight Mechanics. It is the science that studies the performance of air vehicles from a kinematic and dynamic point of view. The consideration of the aircraft as a mass point is adequate for generating aircraft trajectories over a few-minute time span. This movement is defined in terms of position, velocity and aircraft mass. At any moment, the aircraft is under the effects of a resultant force composed of the gravitational, aerodynamic and propulsive forces [2].

The reference system that is going to be used in the development of this project is the *topocentric inertial system*. It is defined as follow, being *xy* the horizontal plane:

- O – origin; any point of the Earth surface
- x – north direction
- y – east direction
- z – complete a right-hand trihedral

As it was previously told, the aircraft performance are going to be studied as the movement of the airplane mass center, considering it as a mass-variable point with three degrees of freedom. In this formulation, the hyphoteses that have been considered are the next ones:

- Aircraft as a solid rigid.

- Aircraft with a simetric plane.

- Thrust angle of attack equal to zero.

- Flat Earth.

- Constant gravity force, but using the geopotential altitude instead of the geometric one.

- Lateral force negligible as the movement of the aircraft is considered symmetric.

- Thrust sideslip angle equal to zero.

The reason why the sideslip angle is considered equal to zero is obvious in the case of a windless atmosphere, and in the presence of wind it is explained like that the airplane makes instant corrections to follow the wind direction making this angle equal to zero.

Regarding the Flight Mechanics equations and prior to their presentation, the simplification carried out in the trajectory angle must be highlighted. Its time derivative has been considered as negligible based on real commercial aircraft flights and, as a consequence, a direct relation between the lift and the weight is obtained as it is shown in equation (2.1). Also the lateral force is assumed as negligible comparing its contribution to the thrust.

$$L = \frac{W \cos \gamma}{\cos \mu} \tag{2.1}$$

The equations of motion without considering the wind effects are defined as follows. Note that equation (2.1) is also included as part of the problem equations.

$$\frac{dx}{dt} = V \cos\gamma \cos\chi \tag{2.2}$$

$$\frac{dy}{dt} = V \cos\gamma \sin\chi \tag{2.3}$$

$$\frac{dH}{dt} = V \sin\gamma \tag{2.4}$$

$$m\frac{dV}{dt} = T - D - mg_0 \sin\gamma \tag{2.5}$$

$$mV \cos\gamma \frac{d\chi}{dt} = L \sin\mu \tag{2.6}$$

$$\frac{dm}{dt} = -c_E T \tag{2.7}$$

The meanings of the variables not mentioned previously are as follow:

- H: Geopotential flight altitude. It has the opposite sign of the z coordinate of the topocentric inertial system.
- V: True airspeed.
- γ: Aerodynamic flight path angle.
- χ: Heading.
- L: Lift.
- T: Thrust.
- D: Drag.
- g_0: Gravitational acceleration at mean sea level.
- μ: Bank angle.
- m: Aircraft mass.
- c_E: Thrust specific fuel consumption.

In addition, equation (2.8) is included to consider the turning flight segments in the problem formulation. In this project, the turns are supposed to be circular. R is the turning radius. Furthermore, the relation between the aerodynamic flight path angle and the ROC is shown in equation (2.9).

$$R = \frac{V}{g_0 \tan\mu} \tag{2.8}$$

$$\sin\gamma = \frac{ROC}{V} \tag{2.9}$$

The introduction of the weather information affects to the aircraft trajectory and, as a consequence, the Flight Mechanics equations that govern the problem vary. A simple scheme of the new problem to study is shown below:
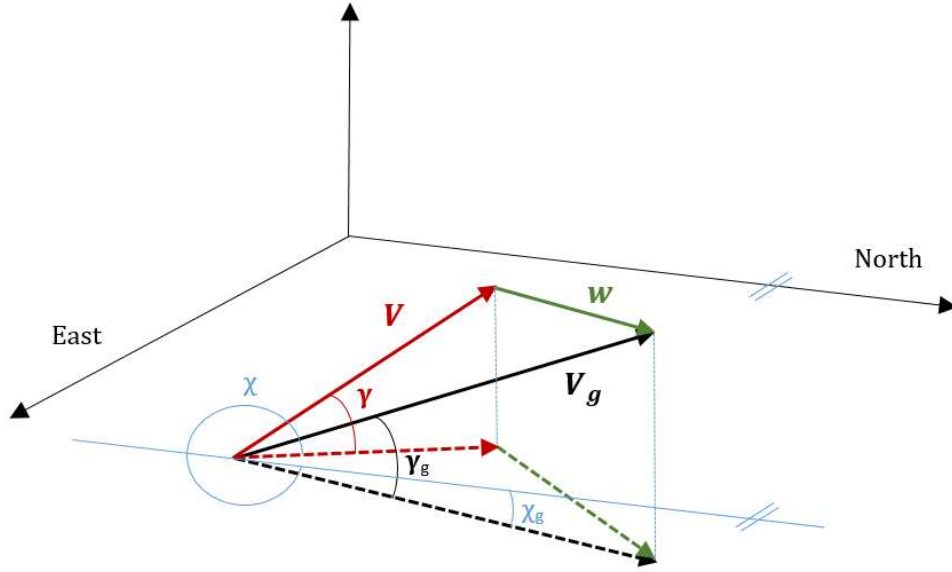
Figure 2-1. Perturbation in the aerodynamic velocity as a consequence of the wind effect.

V is the aerodynamic velocity considered in the previous sections, w is the 3-component wind vector for each position and V_g is the groundspeed. χ_g is the course and γ_g the inertial flight path angle. Projections in the horizontal plane of the three vectors are also provided to show the trajectory and courses that will take part in the problem formulation. To analytically solve this problem, it must be noted that it is still possible to apply the triangle of speeds since V_g and w define a plane. For this reason, a unit vector along the direction of the groundspeed is created and defined as it is shown below. Note that vectors are in bold.

$$\frac{V_g}{|V_g|} = e_{track} = \begin{bmatrix} \cos\gamma_g \cos\chi_g \\ \cos\gamma_g \, \text{sen}\,\chi_g \\ \text{sen}\,\gamma_g \end{bmatrix} \tag{2.10}$$

Taking into account Figure 2-1, the magnitud of the aerodynamic velocity considering the wind influence is expressed as:

$$V_g = \sqrt{V^2 - w_{cross}^2} + w_{along} \tag{2.11}$$

Where w_{cross} is the module of wind vector projection in the cross direction to V_g and w_{along} is the module of the wind vector projection in the same direction as V_g. To obtain these magnitudes and noticing that in general the problem is in 3D, the unit vector showed in equation (2.10) is used obtaining the following equations:

$$w_{cross} = w^T \left(I - e_{track} e_{track}^T \right)^2 w \tag{2.12}$$

$$w_{along} = e_{track}^T w \tag{2.13}$$

An important comment must be made prior to the presentation of the Flight Mechanics equations considering the wind influence: as it is widely explained in Section 4, one of the inputs of the trajectory predictor is the atmospheric model. There are two possible sources of information from which the information of the atmosphere can be obtained at every location. The first one is the International Standard Atmosphere (ISA) and the second

one is the European Center for Medium-Range Weather Forecast (ECMWF). The ISA model is the one that BADA uses and it is explained in Section 3, whereas the ECMWF defines the pressure, temperature and wind, and its methodology is going to be studied in Section 4. Both models are used in the development of this project and the fact of using one or another lies in the two main types of problems that have been solved:

- Firstly, the performance of the airplane have been calculated from a kinematic and dynamic point of view considering the ISA model. As a consequence of it, the real weather information has not been taken into account, prioritizing a more complex implementation of the Flight Mechanics equations that are equations (2.2)-(2.7).

- Secondly, the effects of a realistic model have been implemented by the introduction of the weather forecast provided by ECMWF. As it is a more complex way to define the atmospheric model, the decision of solving just the kinematic problem without losing accuracy in the calculation of the position, velocity, course or trajectory angle has been made This is justified by comparing two of the three types of climbs solved with the kinematic and dynamic formulation and seeing the similar results obtained. This is widely analysed in Section 6.

All these considerations lead to obtaining the system of equations that defines the problem under the wind influence:

$$\frac{dx}{dt} = V_g \cos\gamma_g \cos\chi_g \tag{2.14}$$

$$\frac{dy}{dt} = V_g \cos\gamma_g \sin\chi_g \tag{2.15}$$

$$\frac{dH}{dt} = V_g \sin\gamma_g \tag{2.16}$$

$$\frac{d\chi_g}{dt} = \frac{V_g}{R} \tag{2.17}$$

Note that equation (2.17) is the result of combining equations (2.6) and (2.18) in the case of considering the wind effects. Also, in equations (2.14)-(2.17), no aerodynamic or propulsive forces appear obtaining the problem formulation from the kinematic point of view mentioned above. To these four equations it is necessary to add two more ones: equations (2.18) and (2.19). They are equations (2.8) and (2.9) modified by the wind influence. R is the turning radius when the wind effects are considered.

$$R = \frac{V_g}{g_0 \tan\mu} \tag{2.18}$$

$$\sin\gamma_g = \frac{ROC}{V_g} \tag{2.19}$$

Equations (2.11) and (2.19) define, for each position, a non-linear equations system whose unkown variables are V_g and γ_g. It is solved by using the Newton-Rapshon method and implementing adequate initial values and a tight tolerance ensuring and controlling its convergence.

The aerodynamic velocity V is obtained from its direct relation with the CAS or Mach number as it is going to be seen in the Section 3. Note that the equations of motion when considering the weather effects are exclusively kinematic whereas the ones without that information include both the kinematic and dynamic formulation. That simplification is due to the complex problem that appears when both weather effects and dynamic formulation are included and, as a consequence, the decision of solving just the kinematic problem when the weather is taken into account has been made. The numerical justification of this simplification is issued in section 6.

A general 3D wind vector is depicted in Figure 2-1. However, the lack of information leads to consider only its components in the horizontal plane and the vertical component is assumed to be zero. Furthermore, the wind is considered to be stationary in time and the wind at the aircraft location is interpolated from a set of values at different horizontal locations, although the partial derivatives with respect to x and y are assumed to be zero [3]. More information regarding this issue is in the *Atmosphere model* section of Chapter 4.

3 BADA MODEL

BADA (Base of Aircraft Data) is a collection of ASCII files which specifies operation performance parameters, airline procedure parameters and performance summary tables for 438 aircraft types. This information is designed for use in trajectory simulation and prediction algorithms within the domain of Air Traffic Management (ATM). The coefficients include those used to calculate thrust, drag and fuel flow and those used to specify nominal cruise, climb and descent speeds. All the information below is extracted from [4]. The most relevant information that have been used from the BADA documentation is the following:

- International Standard Atmosphere model (ISA).
- CAS/TAS conversion.
- Mach/CAS transition altitude.
- Maximum climb and Take-Off Thrust.
- Fuel consumption.
- Aerodynamic model.
- Airline procedure models.
- Aircraft mass.

3.1 International Standard Atmosphere (ISA).

At every moment, it is needed to know the pressure, temperature and density in order to calculate relevant parameters and variables associated with the aircraft performance. The BADA model uses the International Standard Atmosphere (ISA), which defines the standard values in the Mean Sea Level:

$$T_o = 288.15 \ [K]$$

$$p_0 = 101325 \ [Pa]$$

$$\rho_o = 1.225 \ [kg/m^3]$$

$$a_o = 340.294 \ [m/s]$$

The physical constants that are used in the atmosphere model are the following:

Adiabatic index of air:	κ	$= 1.4$
Gas constant for air:	R_g	$= 287.05287 \ [m^2/ (K*s^2)]$
Gravitational acceleration at mean sea level:	g_o	$= 9.80665 \ [m/s^2]$
Temperature gradient with altitude below the tropopause:	$\beta_{T,<}$	$= -0.0065 \ [K/m]$

It is necessary to remark the fact that the altitude that is considered in the calculation of atmospheric parameters is not the altitude measured from the mean sea level, but the geopotential one, so it is necessary to obtain it. This is simply done by following the next equation:

$$H = \left(\frac{R_E}{R_E + h}\right) h \tag{3.1}$$

This equation is obtained from [5]. H refers to the geopotential altitude, R_E is the Earth radius and the mean value considered is 6371 kilometers; h is the geometric altitude measured from the mean sea level. H is the variable needed by the atmosphere model equations.

Note that neither in the Flight Mechanics equations nor the ISA definitions the geopotential pressure altitude (H_p) appears. This is due to the following consideration made by BADA: "Geopotential pressure altitude H_p is the geopotential altitude H that occurs in the ISA atmospheric conditions" [4]. So the geopotential pressure altitude that appears in the next sections is, according to the BADA model, approximately equal to the geopotential altitude.

3.2 CAS/TAS conversión.

Another important relation that defines the atmosphere model is the conversion between the TAS (True Air Speed) and the CAS (Calibrated Air Speed), being the first one the solution of the Fligth mechanics equations and the last one the format in which BADA and aircraft restrictions are provided as it is explained later. The relation between both parameters is the next one:

$$V_{CAS} = \left[\frac{2}{\nu}\frac{p}{\rho}\left\{\left(1 + \frac{p_o}{p}\left[\left(1 + \frac{\nu}{2}\frac{\rho_0}{p_o}CAS^2\right)^{1/\nu} - 1\right]\right)^{\nu} - 1\right\}\right]^{1/2} \tag{3.2}$$

Where the pressure and density equations in the troposphere and the equation of ν are below.

$$p = p_o \left(1 + \frac{\beta_{T,<}\text{h}}{T_0}\right)^{-\frac{g_o}{R_g \beta_{T,<}\text{h}}} \tag{3.3}$$

$$\rho = \rho_0 \left(1 + \frac{\beta_{T,<}\text{h}}{T_0}\right)^{-\frac{g_o}{R_g \beta_{T,<}\text{h}} - 1} \tag{3.4}$$

$$\nu = \frac{\kappa - 1}{\kappa} \tag{3.5}$$

3.3 Mach/CAS transition altitude.

As it could be seen in the previous section, it exists a direct relation between the CAS and the V_{CAS}. However, there is a flight altitude at which the CAS and the Mach number represent the same V_{CAS} value and it is called the geopotential Mach transition altitude. It is measured in feets and estimated as follow:

$$H_{p,trans} = \frac{3.28084}{\beta_{T,<}}[To(1 - \theta_{trans})] \tag{3.6}$$

Where θ_{trans} is defined as:

$$\theta_{trans} = (\delta_{trans})^{-\frac{\beta_{T,<}R_g}{g0}} \tag{3.7}$$

Where δ_{trans} is defined as:

$$\delta_{trans} = \frac{\left[1 + \left(\frac{K-1}{2}\right)\left(\frac{CAS}{a_0}\right)^2\right]^{\frac{K}{K-1}} - 1}{\left[1 + \frac{K-1}{2}M^2\right]^{\frac{K}{K-1}} - 1} \tag{3.8}$$

θ_{trans} represents the temperature ratio whereas δ_{trans} is the pressure ratio at the transition altitude.

3.4   Maximum climb and Take-Off Thrust.

The BADA model also includes the definition of the thrust in the take-off and initial climb. It is calculated as a function of the engine type, the geopotential pressure altitude, the true airspeed and the temperature deviation from standard atmosphere. In addition, there exist thrust models for three types of engines: jet, turboprop and piston. The jet engine type is the only that has been implemented, although the consideration of the other engines could be easily introduced with minor changes. The thrust for maximum climb for jet engines is defined as follow. It depends on the geopotential pressure altitude.

$$(Thr_{\max climb})_{ISA} = C_{Tc,1} * \left(1 - \frac{Hp}{C_{Tc,2}} + C_{Tc,3} * Hp^2\right) \tag{3.9}$$

Where $C_{Tc,1}$, $C_{Tc,2}$ and $C_{Tc,3}$ are coefficients specified for each aircraft model and provided by BADA.

3.5   Fuel consumption.

The BADA model specifies the thrust specific fuel consumption as a function of the true airspeed. In the case of aircraft with jet engines:

$$c_E = C_{f1} * \left(1 + \frac{V}{C_{f2}}\right) \tag{3.10}$$

Where C_{f1} and C_{f2} are coefficients specified for each aircraft model and provided by BADA. Examples of these and other relevant coefficients are shown in the Annex.

3.6   Aerodynamic model.

Equation (3.11) allows to obtain the lift coefficient. Lift coefficient permits to calculate the drag coefficient using the parabolic polar model defined by BADA and, at the same time, drag can be obtained.

$$C_L = \frac{2L}{\rho S V^2} \tag{3.11}$$

$$C_D = C_{D0} + K C_L^2 \tag{3.12}$$

$$D = \frac{1}{2}\rho V^2 S C_D \tag{3.13}$$

Note that C_{D0} and K coefficients vary along a SID as the influence of the extended landing gear just after the take off is contemplated in the BADA aerodynamic model.

3.7 Airline procedure model.

In the flight segment of climb that is going to be studied, the BADA model determines the Calibrated Air Speed for each altitude segment. In concrete, and for jet aircraft, the following CAS schedule is considered:

From 0 to 1,499 ft	CVmin · (Vstall)$_{TO}$ + VdCL, 1	(3.14)
From 1,500 to 2,999 ft	CVmin · (Vstall)$_{TO}$ + VdCL, 2	(3.15)
From 3,000 to 3,999 ft	CVmin · (Vstall)$_{TO}$ + VdCL, 3	(3.16)
From 4,000 to 4,999 ft	CVmin · (Vstall)$_{TO}$ + VdCL, 4	(3.17)
From 5,000 to 5,999 ft	CVmin · (Vstall)$_{TO}$ + VdCL, 5	(3.18)
From 6,000 to 9,999 ft	min (Vcl_1, 250 kt)	
From 10,000 ft to Mach transition altitude	Vcl_2	
Above Mach transition altitude	Mcl	

Where CVmin is the Minimum speed coefficient whose value according to the BADA model is fixed and equal to 1.3. The stall speed for take-off is a parameter that depends on the aircraft model and that is provided in a confidencial status by BADA.

The second part of the sum makes reference to the climb speed increment as a function of the flight altitude. They are fixed, measured in knots and do not depend on the aircraft model.

		[KCAS]
VdCL, 1	Climb speed increment below 1500 ft	5
VdCL, 2	Climb speed increment below 3000 ft	10
VdCL, 3	Climb speed increment below 4000 ft	30
VdCL, 4	Climb speed increment below 5000 ft	60
VdCL, 5	Climb speed increment below 6000 ft	80

Finally, Vcl_1 is the standard climb CAS between 6000 and 10000 feets, Vcl_2 is the standard climb CAS between 10000 feets and the Mach transition altitude and Mcl is standard climb Mach number above the Mach transition altitude. All these three parameters are given by BADA for each aircraft model.

3.8 Aircraft mass.

BADA specifies four mass values for each aircraft model. The interface with the user presented in Section 5 includes the possibility of choosen any aircraft mass within the limits defined by the minimum and maximum mass values. The four values provided by BADA for each aircraft model are the following:

- m_{min}: minimum aircraft mass.

- m_{max}: maximum aircraft mass.

- m_{ref}: reference aircraft mass.

- m_{pyld}: maximum payload mass.

Aircraft operating speeds vary with the aircraft mass. This variation is calculated according to the formula below. In this formula, the aircraft reference speed V_{ref} is given for the reference mass m_{ref}. The speed at another aircraft mass, m, is then calculated as V.

$$V = V_{ref} \sqrt{\frac{m}{m_{ref}}} \tag{3.19}$$

Note that the velocities that appear in equation (3.19) are true airspeeds whereas the schedule defined in Section 3.7 is expressed in terms of CAS or Mach number. In the first case, a conversion using equation (3.2) must be done in order to use equation (3.19). If the velocity is defined by a Mach number, then equation (3.20) is used:

$$V = M \sqrt{\kappa \frac{p}{\rho}} \tag{3.20}$$

Sensitivity studies with mass are presented in Chapter 6 to analyse the effect of the aircraft mass in the performance.

4 TRAJECTORY PREDICTOR

4.1 Concept and block diagram.

The main goal of this Project is to develop an aircraft trajectory predictor in order to study different parameters and flight conditions considering, in an approach to the reality, the influence of the weather. For this purpose, the predictor takes the aircraft model, the SID that is going to be analysed and the atmosphere model as inputs. As a result, the aircraft position at every time as well as the fuel consumption and the total time to complete the SID will be obtained as outputs. In a schematic way, the following conceptual diagram represents the ideas mentioned above:
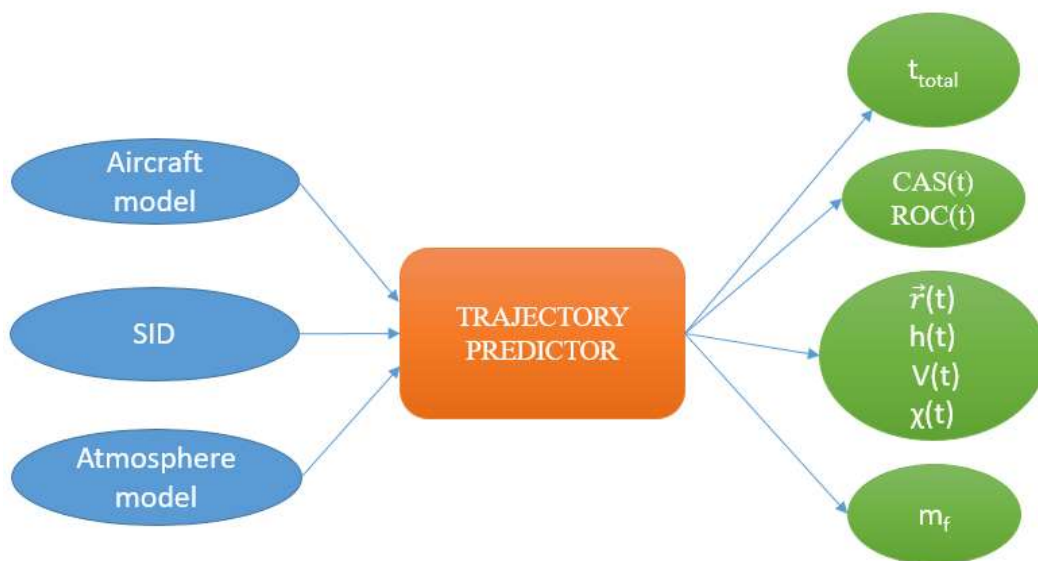
Figure 4-1. Block diagram of the trajectory predictor.

From now on, the different parts that constitute the trajectory predictor are going to be explained. The process carried out to solve several problems that have occurred during the development of the tool are also going to be discussed, mainly related with the software implementation. Therefore, the explanation of the inputs, the trajectory predictor and the outputs is going to be developed in the next sections.

4.2 Atmosphere model.

The atmosphere model is essential when considering the automatization of the SIDs as it defines the air conditions in which the aircraft is flying and, as a consequence, its performance may vary depending on such conditions. There have been considered two atmosphere models along the development of this project. Firstly, the International Standard Atmosphere (ISA) assumed by BADA, which has been already explained in Section 3.1. Then, the European Center for Medium-Range Weather Forecast (ECMWF) that defines itself as follows:

"The European Centre for Medium-Range Weather Forecasts (ECMWF) is an independent intergovernmental organisation supported by 34 states. ECMWF is both a research institute and a 24/7 operational service, producing and disseminating numerical weather predictions to its Member States. This data is fully available to the national meteorological services in the Member States. The Centre also offers a catalogue of forecast data that can be purchased by businesses worldwide and other commercial customers. The

supercomputer facility (and associated data archive) at ECMWF is one of the largest of its type in Europe and Member States can use 25% of its capacity for their own purposes. ECMWF's core mission is to:

- Produce numerical weather forecasts and monitor the Earth system;

- Carry out scientific and technical research to improve forecast skill;

- Maintain an archive of meteorological data".

This last topic is the one that is relevant for the project as ECMWF "produces operational ensemble-based analyses and predictions that describe the range of possible scenarios and their likelihood of occurrence. ECMWF's forecasts cover time frames ranging from medium-range, to monthly and seasonal, and up to a year ahead" [6].

The way in which data are provided is a key in the implementation of the problem in Matlab and this is what is discussed below. First of all, it is compulsory to define a range of altitudes, longitudes and latitudes in which the experiment is going to be done as the data of pressure, temperature and wind (2-axis) that is going to be used depends on the aircraft location and, as it is not possible to charge the data of the entire world, a region must be delimited. In the case of this project it will be a zone that covers a big part of Europe, including Seville, Madrid, Barcelona and London. This is the way to ensure that all the possible SIDs of these airports are covered and can be analysed.

Secondly, ECMWF provides the data of pressure, temperature, meridional wind, zonal wind, orography of the terrain and geopotential altitude for given values of the pressure levels in the region of study. These pressure levels tend to be more flat as the altitude is higher. The problem is then reduced to solve two different interpolations: firstly, a 2-dimension interpolation to localize the aircraft position in the range of ϕ and λ in which data is available and, secondly, a 1-dimension interpolation to associate the corresponding aircraft altitude to the values of pressure, temperature, meridional wind and zonal wind from the pressure levels provided by ECMWF. Note that the 2D interpolation will be done five times (pressure, temperature, meridional wind, zonal wind and geopotential altitude) whereas the 1D interpolation will be done four times (pressure, temperature, meridional wind and zonal wind). Both types of interpolations are done for a given aircraft location. Also, a 2D interpolation for the orography matrix is performed with the airport coordinates in order to obtain the initial altitude of the aircraft above the mean sea level.

Information from the ECMWF is obtained at a certain time and is related to particular date. They are obtained from a predictive method that uses previous data to estimate the weather forecast in a short time span. Particularly, the information that is going to be shown in Section 6 was available on January 7, 2019 at 12:00 hours, and it was referred to what would happen 36 hours later. ECMWF provides all the information needed to implement those effects, although the data format is not the adequate one for its software implementation, so it has been changed to create the data matrix that the predictor is able to read.

Data are provided by the ECMWF in a 3D-matrix format. The trajectory predictor receives six 3D matrices corresponding to pressure, temperature, meridional wind, zonal wind, orography of the terrain and geopotential altitude. The indeces of each matrix are: latitude, longitude and pressure levels. This is the way in which ECMWF provides the weather data. The region (ϕ, λ) selected to study is defined by the user who asks for the weather information, and the step size between the components of each index is also requested by the user as it is necessary to create the grid in which data is known and available. Matlab has a command called *meshgrid* to do this task.

A 2D interpolation is needed to approximate the values of the six matrices mentioned in the previous paragraph at the aircraft location. This is done because data is available in the four positions that surround the aircraft location as Figure 4-2 shows. With this first interpolation, the six 3D matrices have leaded in 6 vectors that go through the aircraft location.
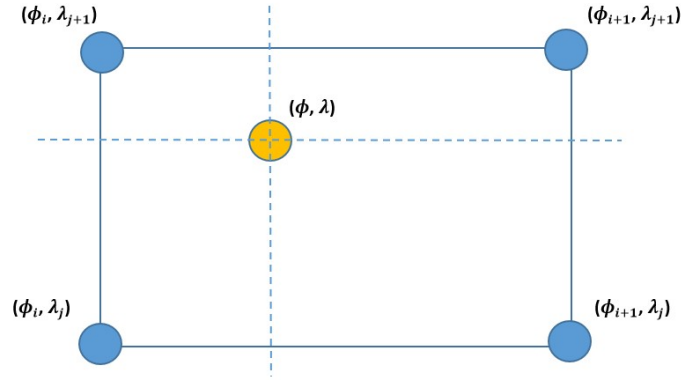
Figure 4-2. Sketch of 2D interpolation.

The interpolation of a generic variable *s* is:

$$s^{[k]} = s_{i,j,k}\, w_1 + s_{i+1,j,k}\, w_2 + s_{i,j+1,k}\, w_3 + s_{i+1,j+1,k}\, w_4 \tag{4.1}$$

The meaning of each parameter is the following:

- *k*: index that represents each one of the pressure levels provided by ECMWF.
- *i:* index that goes through each component of the latitude vector.
- *j:* index that goes through each component of the longitude vector.
- *w:* ponderation weight of each one of the four positions around the aircraft location (see Figure 4-2).

Once upon the six vectors at the aircraft position are obtained, a 1-dimension interpolation is carried out to associate the aircraft geopotential altitude calculated by the Flight Mechanics equations with the vector of geopotential altitudes obtained from the 2D interpolation. In this case, the process is done four times (pressure, temperature, meridional wind and zonal wind) by using the Matlab command *interp1*. Thus, everything is known at the exact position of the aircraft and the equations defined in Chapter 2 can be adequately integrated.

Figure 4-3. Problem for 1D interpolation.

Note that H (P) in Figure 4-3 denotes the reference that the ECMWF uses to provide the weather information, that is, the pressure levels. It should be also noted that the geopotential altitudes information that provides ECMWF takes into account the altitude above the mean sea 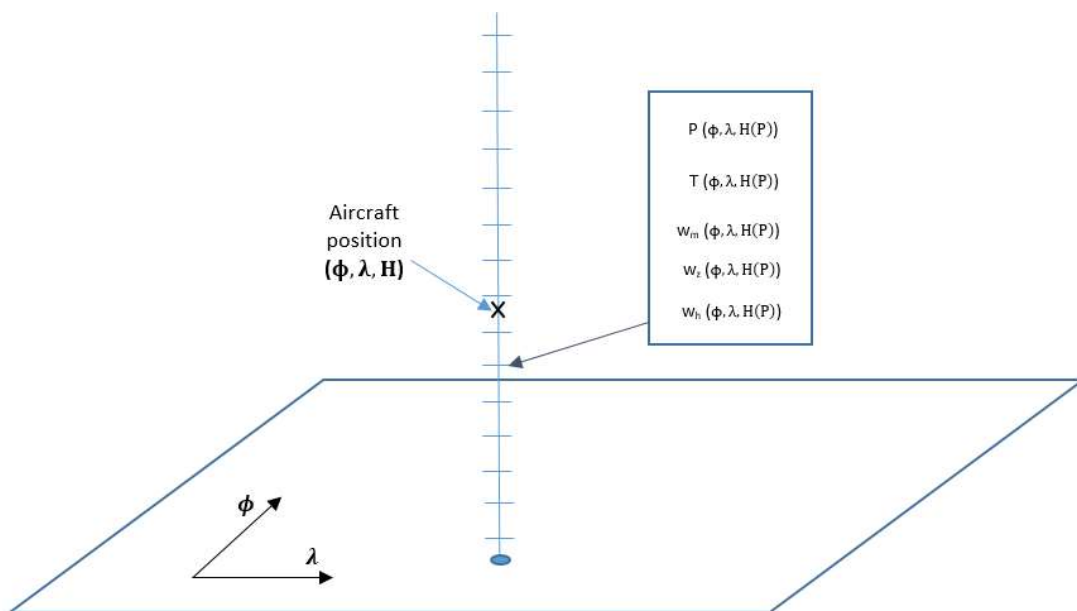level, but the results of the Flight Mechanics equations do not consider this fact unless some information of the terrain is supplied. That is why the orography matrix is also provided by ECMWF. The aircraft initial altitude is obtained by the 2D interpolation of that matrix at the aircraft location.

4.3   Aircraft model.

BADA provides information of more than 400 airplane models whose details are given in four different types of formats. First of all, the **Operations Performance File (OPF)** provides parameters for mass, drag, engine thrust and fuel consumption; the **Airline Procedure File (APF)** describes nominal manoeuvre speeds; the **Performance Table File (PTF)** is a summary of speeds, fuel consumption and climb/descent rates for various flight levels and the **Performance Table Data (PTD)** contains computed performance values at various flight levels. Examples of all of them are presented in the Annex, Section 10.1. Although the BADA files uses in this project are confidential, the examples shown on the Annex can be found in a BADA file whose access is public.

In the trajectory predictor some of these data have been needed as inputs. The present version of the trajectory predictor implements two types of aircraft: the Airbus A320 and the Boeing 767-300. They are similar models with the objective of comparing them. Anyway, the solution presented in this project is applicable to any type of aircraft, just needing the BADA files to be in the same file directory than the rest of the Matlab files.

4.4   SID treatment.

The third important input that the trajectory predictor requires is the SID description. This information is extracted from the Aeronautical Information Publication (AIP) that each country provides. In general, these documents are opened to the public and represent real trajectories that airplanes have to follow during their exit routes from the airport. The main source of information during the development of the project has been focused in the Spanish network, ENAIRE. Anyway, the versatility of the predictor obtained lies in the possibility of analysing any SID corresponding to any airport.

Each SID is given by its graphic representation and a text document that describes it by specifiying particular characteristics. The AIP information does not consider stop conditions in terms of altitude, though they do exist restrictions associated with the minimum altitude the aircraft has to reach at a particular location due to the presence of restricted areas or to air traffic management reasons. Other clarifications like this are widely developed in successive sections.
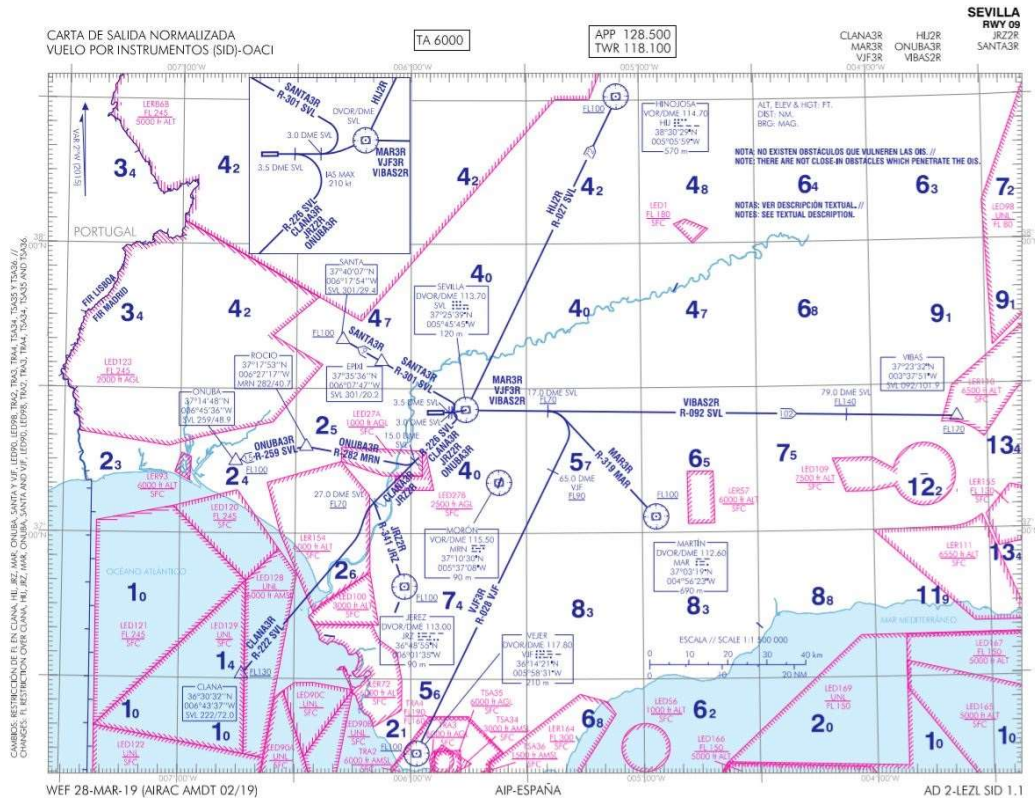
Figure 4-4. View of the available SID in SVQ airport at runway 09.

SEVILLA AD

→ SALIDAS NORMALIZADAS POR INSTRUMENTOS (SID)

STANDARD INSTRUMENT DEPARTURES (SID)

NOTA APLICABLE A TODAS LAS SID:
- AJUSTE DE VELOCIDAD:
  IAS MAX 250 kt hasta abandonar FL120.

NOTE APPLICABLE TO ALL SID:
- SPEED ADJUSTMENT:
  MAX IAS 250 kt to leave FL120.

PISTA 09

SALIDA CLANA TRES ROMEO (CLANA3R)
Subir en rumbo de pista hasta 3.5 DME SVL. Virar a la derecha (IAS MAX en viraje 210 kt), para interceptar y seguir R-226 SVL directo a cruzar 27.0 DME SVL a FL70 o superior. Virar a la izquierda para interceptar y seguir R-222 SVL directo a CLANA a FL130 o superior.
Pendiente mínima de ascenso 5.5% hasta FL70.
Pendiente mínima de ascenso debida a motivos operacionales.

SALIDA HINOJOSA DOS ROMEO (HIJ2R)
Subir en rumbo de pista hasta 3.0 DME SVL. Virar a la izquierda directo a DVOR/DME SVL. Proceder por R-027 SVL directo a VOR/DME HIJ a FL100 o superior.

SALIDA JEREZ DOS ROMEO (JRZ2R)
Subir en rumbo de pista hasta 3.5 DME SVL. Virar a la derecha (IAS MAX en viraje 210 kt), para interceptar y seguir R-226 SVL hasta 27.0 DME SVL. Virar a la izquierda para interceptar y seguir R-341 JRZ, directo a DVOR/DME JRZ a FL100 o superior.
Pendiente mínima de ascenso 5.5% hasta FL70.
Pendiente mínima de ascenso debida a motivos operacionales.

RUNWAY 09

CLANA THREE ROMEO DEPARTURE (CLANA3R)
Climb on runway heading up to 3.5 DME SVL. Turn right (turning MAX IAS 210 kt) to intercept and follow R-226 SVL, direct to cross 27.0 DME SVL at FL70 or above. Turn left to intercept and follow R-222 SVL direct to CLANA at FL130 or above.
Minimum climb gradient of 5.5% up to FL70.
Minimum climb gradient due to operational reasons.

HINOJOSA TWO ROMEO DEPARTURE (HIJ2R)
Climb on runway heading up to 3.0 DME SVL. Turn left direct to DVOR/DME SVL. Proceed on R-027 SVL, direct to VOR/DME HIJ at FL100 or above.

JEREZ TWO ROMEO DEPARTURE (JRZ2R)
Climb on runway heading up to 3.5 DME SVL. Turn right (turning MAX IAS 210 kt) to intercept and follow R-226 SVL up to 27.0 DME SVL. Turn left to intercept and follow R-341 JRZ, direct to DVOR/DME VJF at FL100 or above.
Minimum climb gradient of 5.5% up to FL70.
Minimum climb gradient due to operational reasons.

Figure 4-5. Example of the text that defines a SID.

The content and meaning of both the map and the text document allow to obtain the information enough to define the whole problem. Note that in both cases the descriptions are refered to the horizontal plane. This fact exemplifies the separation that exists between both the vertical and the horizontal motion in terms of studying and implementing them in the trajectory predictor. On the one hand, this is something that makes the problem easier to be developed, but, on the other hand, two different integrations have to be done simultaneously. In Section 5, an example of a SID definition is shown to understand how the information is extracted and, in Section 4.5, the Matlab implementation is explained.

4.4.1   Segment definition.

There are three types of variables that can be held constant by a Flight Management System (FMS): engine control (idle thrust, maximum thrust), speed (Mach or V_{CAS}) and vertical rate (ROC or γ_g). A vertical flight segment is defined by holding two of these variable types constant. The values of the constant variables must be within the performance limits of the aircraft.

There are three possible combinations of the two variables. These combinations hold either constant engine control and speed, or vertical rate and engine control, or vertical rate and speed. Furthermore, segments are divided into separate categories according to the choice of the speed variable (Mach or V_{CAS}). Exactly which variables are held constant in a segment depends on the type of aircraft, the type of segment and the standard pilot procedure [3].

Without considering the use of devices such as spoilers, leading-edge slats or trailing-edge flaps, there are two independent control inputs available for affecting the aircraft trajectory in the vertical plane. These are the throttle and the elevator. These inputs allow any pair among the three variables thrust, speed, or rate of climb or descent (ROCD) to be controlled. The other variable is then determined by the Flight Mechanics equations. The three resulting control possibilities are the next ones [4].

- Speed and Throttle Controlled   →   Calculation of Rate of Climb or Descent
- ROCD and Throttle Controlled   →   Calculation of speed
- Speed and ROCD Controlled   →   Calculation of Thrust

Regarding the equations (2.2) - (2.7) that allow to obtain the aircraft trajectory, a relation between the work done per unit of time by forces acting on the aircraft and the rate of increase in potential and kinetic energy can be written [4]:

$$(T - D)V \;=\; mg_0 \frac{dH}{dt} + mV \frac{dV}{dt} \tag{4.2}$$

Note that equation (4.1) applies to the case in which the ISA model is used, including the turning segments, as it is used in dynamic problems in which the forces appear. Also, the equations system when the weather effects are considered only takes into account the kinematic formulation.

Each one of the climbing configurations is going to be explained below.

4.4.1.1   Speed and Throttle controlled.

Assuming that velocity and thrust are independently controlled, then equation (4.1) is used to calculate the resulting rate of climb or descent (ROCD). This is a fairly common case for climbs and descents in which the throttle is set to some fixed position (maximum climb thrust or idle for descent) and the speed is maintained at some constant value of calibrated airspeed (CAS) or Mach number [4].

In this project, maximum thrust is assumed as well as the CAS schedule that varies with the flight altitude defined by BADA. Also, when the transition altitude is reached, the flight configuration with constant Mach number is stablished.

4.4.1.2   ROCD and Throttle controlled.

Assuming that the ROCD and thrust are independently controlled, then equation (4.1) is used to calculate the resulting speed. As in the previous control configuration, thrust is considered as the maximum and the ROC defines the trajectory angle of the airplane, as indicated in equation (2.8).

4.4.1.3   Speed and ROCD controlled.

Assuming that both ROCD and speed are controlled, then equation (4.5.1) can be used to calculate the necessary thrust. This thrust must be within the available limits for the desired ROCD and speed to be maintained.

Case of constant speed and maximum thrust is the most common such the equation (4.1) is most often used to calculate the rate of climb or descent. As BADA model specifies, "the ROCD is defined as the variation with time of the aircraft geopotential pressure altitude Hp". This is coherent with the fact of considering the geopotential altitude in the Flight Mechanics equations that is also considered by BADA to be approximately equal to the geopotential pressure altitude, as it was mentioned in Chapter 2.

4.4.2   Types of segments: state variables, control variables and restrictions.

Following the objective of calculating and implementing the software that analyses any given SID, the three different types of climb segments that can be found in whatever SID proposed to be studied are going to be analysed. A detailed explanation of each one is found below.

- Climb at a constant course.
- Climbing turn to a known radial.
- Climbing turn to a known location.

4.4.2.1   Climb at a constant course.

In this type of segment, the state variables are the position vector (x, y, h), the velocity and the weight. The thrust and the trajectory angle are the control variables and a fixed course and a turning radius equal to infinite are the restrictions imposed.

If any of the three types of climbs that can be performed is added, the problem is perfectly closed and a reasonable solution is calculated. Depending on the SID analysed, it can occur that the course of the segment under study was different to the final course of the previous one. It can be found in the same particular SIDs that do not model situations such as the route towards and from a DME position.

It should be also mentioned that a DME (Distance Measure Equipment) is a radio navigation equipment that measures the distance from a station to an aircraft and this is the main way in which distances are presented in the SID descriptions.
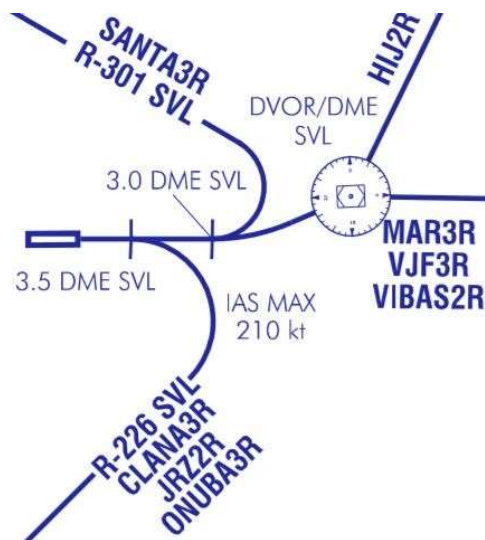
Figure 4-6. Example of distance marks measured from the DME SVL.

4.4.2.2   Climbing turn to a known radial.

Another type of segment that can easily be found in a SID description is the climbing turn to a know radial. In this case the state variables are the position vector (x, y, h), the velocity, the course and the weight. Again, the thrust and the trajectory angle are the control variables and they are defined for each of the three types of climb as it was explained previously. To close the problem, the turning radius or, what is equivalent as equations (2.8) or (2.18) show, the bank angle (μ) must be provided.

The calculation of the turning radius is not trivial at all. An analytical problem has to be solved considering the position at which the turn starts, the initial course and the final radial. Graphically, the problem is the next one:

Figure 4-7. Turning radius to a known course calculation problem.

The variables include in the Figure 4-5 are the following:

- **(x_0, y_0):** initial position where the turn starts.
- **(x_f, y_f):** reference position on the final course (provided by the SID).
- $\chi_{g,i}$: initial course.
- $\chi_{g,f}$: final course (provided by the SID).

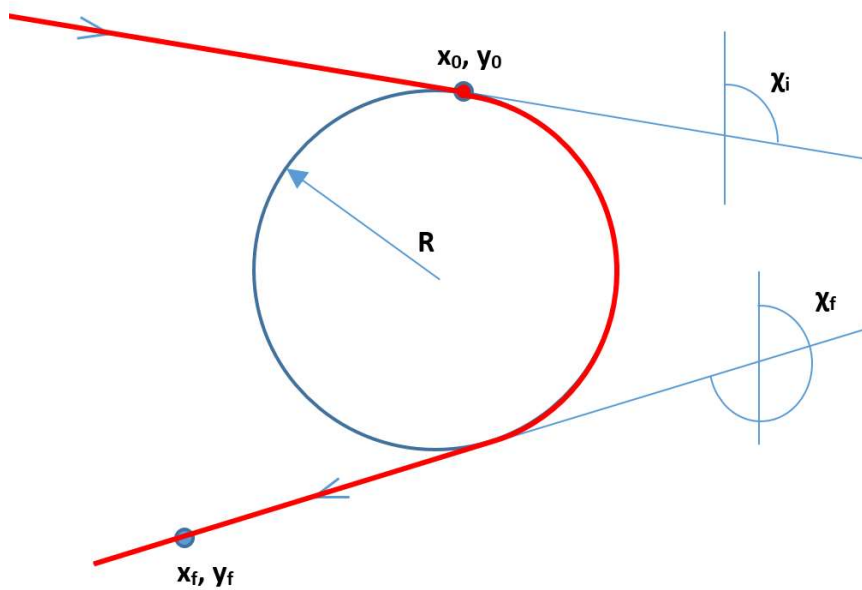In Figure 4-5, the flat Earth hypothesis consideration can be appreciated. It is a reasonable simplification as long as the distance that the aircraft covers during the turn is negligible compared to the Earth radius. Furthermore, there is a geometric problem to obtain the radius that can be summarized in this way: first of all, it is needed to obtain the angle bisector between the two lines defined by the initial and the final courses; secondly, a perpendicular line to the initial path containing the initial position must be computed. Then, the point intersection between this line and the angle bisector is obtained and, finally, the radius is obtained as the distance between that point and the initial position.

In this process, the fact that there are two possible angle bisectors is especially relevant as it is necessary to determine which one is the appropriate for the particular problem of each SID. To do so, *direction vectors* come up, but, previuously, the equations of the lines defined by the courses are obtained. These are their equations:

$$\frac{y - y_0}{\tan \chi_i} + x_0 - x = 0 \tag{4.3}$$

$$\frac{y - y_f}{\tan \chi_f} + x_f - x = 0 \tag{4.4}$$

Line defined by equation (4.2) represents the one that contains the initial position and equation (4.3) the one formed by the final radial. The two lines obtained from the angle bisector are the followings:

$$yZ + xT = M \tag{4.5}$$

$$yW - xS = N \tag{4.6}$$

Where the coefficients Z, T, M, W, S and N are given by:

$$Z = \frac{1}{\tan\chi_i \sin\chi_f} - \frac{1}{\tan\chi_f \sin\chi_i} \tag{4.7}$$

$$T = \frac{1}{\sin\chi_i} - \frac{1}{\sin\chi_f} \tag{4.8}$$

$$M = \frac{x_f}{\sin\chi_i} - \frac{y_f}{\tan\chi_f \sin\chi_i} - \frac{x_0}{\sin\chi_f} + \frac{y_0}{\tan\chi_i \sin\chi_f} \tag{4.9}$$

$$W = \frac{1}{\tan\chi_i \sin\chi_f} + \frac{1}{\tan\chi_f \sin\chi_i} \tag{4.10}$$

$$S = \frac{1}{\sin\chi_i} + \frac{1}{\sin\chi_f} \tag{4.11}$$

$$N = \frac{y_f}{\tan\chi_f \sin\chi_i} - \frac{x_f}{\sin\chi_i} - \frac{x_0}{\sin\chi_f} + \frac{y_0}{\tan\chi_i \sin\chi_f} \tag{4.12}$$

And the points obtained as a consequence of the intersection between the perpendicular line and the two angle bisector lines are the following:

$$y_A = \frac{M - x_0 T - y_0 T \tan\chi_i}{Z - T \tan\chi_i} \tag{4.13}$$

$$x_A = x_0 + \tan\chi_i (y_0 - y_A) \tag{4.14}$$

$$y_B = \frac{N + x_0 S + y_0 S \tan\chi_i}{W + S \tan\chi_i} \tag{4.15}$$

$$x_B = x_0 + \tan\chi_i (y_0 - y_B) \tag{4.16}$$

Finally, both possible turning radius are calculated as:

$$R_1 = \sqrt{(x_A - x_0)^2 + (y_A - x_0)^2} \tag{4.17}$$

$$R_2 = \sqrt{(x_B - x_0)^2 + (y_B - x_0)^2} \tag{4.18}$$

Determining which radius is the correct one is as simple as obtaining the direction vector of the bisector as:

$$\vec{u} = \frac{\vec{v_2} - \vec{v_1}}{2} \tag{4.19}$$

Where $\vec{v_1}$ is the direction vector of the line defined by the initial course and $\vec{v_2}$ is the direction vector of the one defined by the final course. It is important to correctly consider the sense of both vectors before the calculation of the bisector direction vector.

To conclude this section and focused on the software implementation, the predictor must have the ability to determine if it is a left or a right turn just by knowing the initial and the final courses. At this point, it is important to detect possible problems like the case in which the course goes, for example, from 350º to 30º, which is, going through 0º. If this occurred, detected by checking if the difference between the final and the initial courses is lower than $-\pi$, then 2π rad is summed to this angle difference. After that, if the angle difference is between 0 and π rad the turn is to the right. In any other case, it is a left turn.
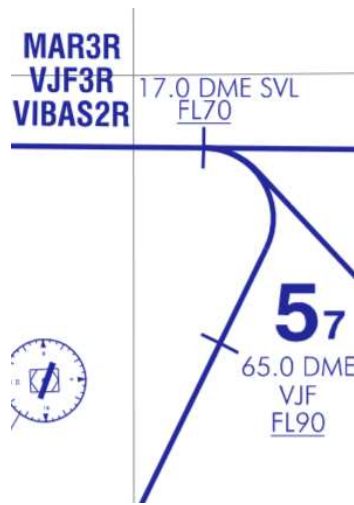
Figure 4-8. Example of a climbing turn to a known radial in a SID.

4.4.2.3   Climbing turn to a known location.

The third type of segment that can be found in a SID description is the climbing turn to a known location. It is important to emphasize that this final position is given by its spherical coordinates, but the software created works with Cartesian coordinates being necessary to convert them. To do so, the flat Earth hypothesis is considered and it should be remembered that Cartesian coordinates (x, y) are measured from the end of the runway with the x axis in the North direction and the y axis in the East direction. An example of this type of segment is available in Figure 4-6.

The estimation of the latitude is simple taking into account that the x coordinate moves along meridians so the radius is always the same, that is, the Earth radius. In the calculation of the longitude, the estimation of the radius of the circular sector in which the y variation is taking place has to be previously calculated. It can be done by multiplying the Earth radius by the cosine of the latitude in which the aircraft is. Taking into account these details the conversion from Cartesian coordinates to the spherical ones is as follows:

$$\phi = \phi_{AP} + \frac{180x}{\pi R_\oplus} \tag{4.20}$$

$$\lambda = \lambda_{AP} + \frac{180y}{\pi R_{\oplus} \cos \phi} \qquad (4.21)$$

Where ϕ_{AP} and λ_{AP} are the spherical coordinates of the departure airport. Note that according to both equations (4.20) and (4.21), ϕ and λ are expressed in degrees.

In this type of segment the state variables are the position vector (x, y, h), the aerodynamic velocity, the course and the weight. The control variables are the thrust and the trajectory angle obtained in a particular way depending on which one of the three types of climb is assumed and, like in the previous section, the turning radius must be obtained to close the problem. Like in the case of climbing turn to a known radial, it is done by solving an analytical geometric problem whose graphic representation is shown below. The variables include in the Figure 4-7 are the following:

- **(x_0, y_0):** initial position where the turn starts.
- **(x_f, y_f):** final position (provided by the SID).

Again, the hypothesis of flat Earth has been assumed as the distance that the aircraft covers during the turn is negligible compared to the Earth radius.
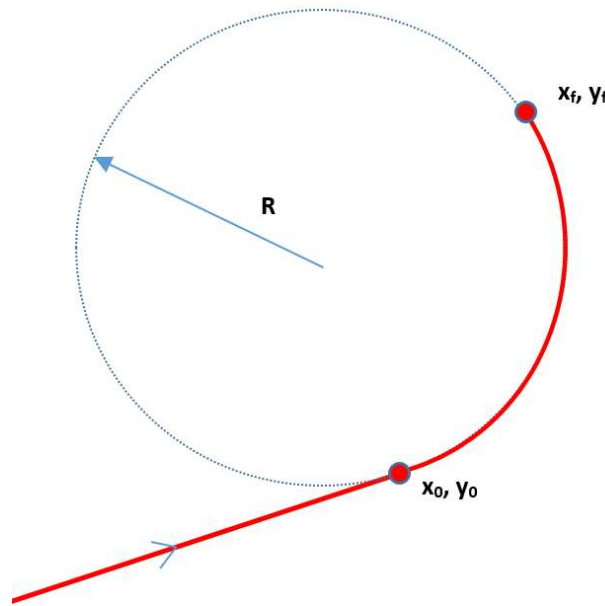
Figure 4-9. Turning radius to a known point calculation problem.

The problem of obtaining the turning radius can be resumed in this way: first of all, the equation of the line in which the aircraft is previously to the turn must be calculated. Then, a perpendicular line to it is obtained at the aircraft location. On the other hand, the mediatrix of the segment between the initial and final positions is computed. Finally, the intersection of both the mediatrix and the perpendicular line determines the turning center and the distance between this point and the initial position is the turning radius.

The equation of the line that is determined by the initial course and the initial position is below:

$$x - x_0 + \frac{y_0 - y}{\tan \chi_i} = 0 \qquad (4.22)$$

The perpendicular line to the one defined in (4.21) that goes through the initial position is:

$$x - x_0 + \tan\chi_i\,(y - y_0) = 0 \tag{4.23}$$

Then, the mediatrix of the segment between the initial and final positions is the following:

$$2x(x_0 - x_f) = x_0^2 + y_0^2 - x_f^2 - y_f^2 + 2y(y_f - y_0) \tag{4.24}$$

Finally, the turning center (x_R, y_R) is obtained as the intersection between the mediatrix (4.24) and the perpendicular line (4.23).

$$y_R = \frac{2x_0(x_0 - x_f) - x_0^2 - y_0^2 + x_f^2 + y_f^2 + 2(x_0 - x_f)y_0 \tan\chi_i}{2(x_0 - x_f)\tan\chi_i + 2(y_f - y_0)} \tag{4.25}$$

$$x_R = x_0 - \tan\chi_i\,(y_R - y_0) \tag{4.26}$$

To conclude this section, and as it was done in the previous one, the predictor must have the ability to determine the turning direction using the same information that is used when estimating the turning radius. In this case, it is a bit more complex than in the turn to a known radial problem as the final course is unknown. The way in which it has been solved is by using the *vector product* between the groundspeed velocity vector and the relative position vector. Ground speeds velocity vector is defined by the trajectory of the aircraft. Its components are the following:

$$V_{g,x} = V_g \cos\chi_{g,i} \tag{4.27}$$

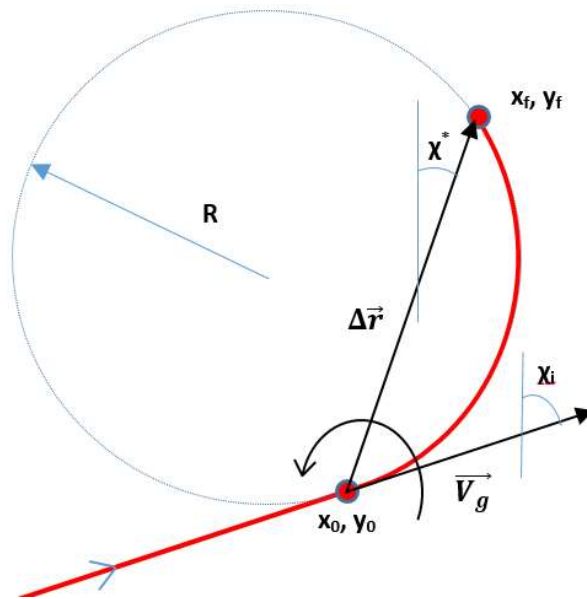$$V_{g,y} = V_g \sin\chi_{g,i} \tag{4.28}$$

Figure 4-10. Vector product to determine the turn direction

The relative position vector has its origin in the initial position (x_0, y_0) of the turn and its end in the final position (x_f, y_f). The vector product between these two vectors is presented below:

$$\vec{V_g} \wedge \Delta \vec{r} = \left(V_{g,x} \Delta r \sin \chi^* - V_{g,y} \Delta r \cos \chi^* \right) \vec{k} \tag{4.29}$$

Δr represents the norm of the position vector and χ^* is the angle between the north direction and the relative position vector. Note that this angle is not the same as the course of the aircraft (see Figure 4-8). If the norm of the vector product defined in equation (4.28) is greater than zero, then it is a left turn. If it is lower than zero, the turn is to the right.

The implementation in the trajectory predictor is not trivial: first of all, it is necessary to obtain χ^*. In this sense, the quadrant in which the angle is assigned has to be analysed. To see this, an angle called B is defined:

$$B = \tan^{-1} \left(\frac{x_f - x_0}{y_f - y_0} \right) \tag{4.30}$$

Depending on the relative position between the initial and the final points, B will be transformed to correctly define χ^* angle. Considering that $\Delta x = x_f - x_0$ and $\Delta y = y_f - y_0$, then the following table is assumed:

$\Delta x > 0$		$\Delta x < 0$	
$\Delta y > 0$	$\Delta y < 0$	$\Delta y > 0$	$\Delta y < 0$
$\chi^* = \dfrac{\pi}{2} - B$	$\chi^* = \dfrac{3\pi}{2} + B$	$\chi^* = \dfrac{\pi}{2} + B$	$\chi^* = \dfrac{3\pi}{2} - B$

Table 1. Criteria for χ^* angle.

4.4.3 Stop conditions.

The key of the trajectory predictor lies in its autonomy to detect when a flight segment ends and the next one starts. The end of a segment is defined by a capture condition. A capture condition occurs when a specific nonconstant variable reaches some desired value. The precision that defines a successful capture depends on the integrator and, in general, computer variables are more precise than the ones measured by current flight instruments [3].

- **Horizontal plane stop conditions:** these are the ones associated with distance, courses or reaching a particular location. They are obtained directly from the SID description and can be found in any of the three types of segments mentioned previously.

- **Vertical plane stop conditions:** these ones are related with the CAS variation with the altitude, the ROC as a function of the flight altitude or the variation of the coefficients that define the parabolic drag polar model.

4.4.3.1 Horizontal plane.

Roughly, there are two types of stop conditions regarding the horizontal plane which are adaptable to the type of segment analysed: distance and final course. The particular application in each type of segment is explained below.

- **Climb at a constant course.**

As its own name indicates, the course is the same in these segments, therefore, the stop conditions will always be in distance. The way in which distances are given in the SID descriptions is by referencing them to a DME. As it was told previously, it is a radio navigation system that measured the distance from itself to the aircraft in a radial way and, in general, it is shown in nautical miles (NM).

The way in which the predictor proceeds is simple: it asks the user to introduce a reference location that are the spherical coordinates of the DME indicated by the SID. Then, it asks for the distance from the DME at which the segment must stop. The idea is that the program integrates the Flight Mechanics equations and stops when the distance to the reference DME achieves the value provided by the user. This is detected by the trajectory predictor when the distance between two points reaches a particular value or, in other words, when equation (4.30) is verified.

$$D = \sqrt{\left(x - x_{ref}\right)^2 + \left(y - y_{ref}\right)^2} \qquad (4.31)$$

The meaning of each variable is the following:

- (x,y): Cartesian coordinates calculated by the trajectory predictor at each time step.
- (x_{ref}, y_{ref}): DME position given by its Cartesian coordinates. As they are originally provided in spherical coordinates by the user, equations (4.20) and (4.21) are used to convert them.
- D: Distance at each time span between the aircraft location and the reference DME.

An important detail must be commented: if the distance that the user introduces is zero, that means, the user wants the aircraft to reach the reference position, then the stop condition in distance will not be exactly zero, but a small value. This is done to avoid problems with the convergence of the integration.

- **Climbing turn to a known radial.**

In this case, the capture condition in distance is not applicable. As the name of segment indicates, the integration ends when the desire radial is reached. Note that a radial is defined for the course and for, at least, one location in that course. This data is provided by the user when asked to do so. This segment is referred to radials as just a final course does not define a segment because there are infinite solution that leaves the aircraft in the desire course, but only one that also accomplish with the condition of going through this reference position.

Just with these data (reference postion and final course) the segment is perfectly defined as long as the turning radius is obtained as shown in Section 4.4.2.2.

- **Climbing turn to a known location.**

The stop condition in distance is the one that applies now as the course is not relevant in terms of ending the segment. As long as the program detects that this is the type of segment after the information provided by the user, it calculates the distance between the initial and the final positions, being the last one a parameter that the user also has to provide.

4.4.3.2 Vertical plane.

As it was told in previous sections, stop conditions in the vertical plane are associated with:

- CAS variation with the altitude.
- ROC variation with the altitude.
- Stall speed, C_{D0} and K variation with the altitude.

Note that C_{D0} and K are the coeffcents of the parabolic drag polar model defined in equation (3.12). CAS schedule is defined by the BADA model as it was shown in Section 3.7. ROC schedule is also found in BADA, but in the files related to each aircraft model, specifically, in the *PTD* file. An example of it is in the Annex. The schedule for the stall speed, $C_{D0,}$ and K depends on the flight segment as defined in the *OPF* file, also provided by BADA.

Regarding the last mentioned schedule, it must be noted that as in a SID procedure the segments involved are take-off, initial climb and cruise, the effect of the landing gear drag has been taken into account for both the take-off and the initial climb segments. This is to model the flight phase in which the landing gear is not retracted, just after the take-off.

Regarding the Matlab implementation, a stop condition in the vertical plane must be treated in the same way as the ones in the horizontal plane, that is, the integration has to stop, the new stop condition has to be updated and a new integration will start. It can occur that two or more stop conditions are satisfied at the same time, whichever the plane is. In these cases, all the dynamics (horizontal and vertical) that have reached the stop conditions have to be updated. Also, the changes in the vertical dynamics are considered to be instantaneous, according to the CAS and the ROC schedules with the altitude. That is why it is very important to stop the integration once a change in the CAS or the ROC is reached because if the integration continues and an instant change occurs, then the software will try to join the aircraft trajectory before the stop condition and after it in some way and, as a consequence, the results might be incorrect.

4.5   Implementation with the ODE function in Matlab.

As the main goal of this project is to develop a trajectory predictor with the possibility of including the weather influence, a computational software is a necessary tool to complete this objective. A versatile tool able to solve differential equations and detect stop conditions for the different segments that complete a SID is needed, whichever the airplane, departure airport or weather conditions are. In this case, the computational software Matlab R2015a has been used, as it has been a tool frequently used along the Bachelor in Aerospace Engineering.

4.5.1   Stop conditions.

The type of stop condition influences in the way they are provided to the programme. Vertical plane stop conditions are directy implemented in the trajectory predictor taking into account the schedules stablished by the BADA model that have been explained in Section 3. The case of the stop conditions in the horizontal plane is different as they directly depend on the SID description and, in the lack of automatization, they have to be manually provided by the user. This process has been thought conscientiously to minimize the user's interaction so he is asked just for the essential information to perfectly define each flight segment.

There are three types of segments: climb at a constant course, climbing turn to a known radial and climbing turn to a known location. For all of them, and in the search of standardization, they are defined by a 6-component vector composed of: ϕ_{ref}, λ_{ref}, D, χ_i, χ_f and Rg. This is the meaning of each parameter:

- ϕ_{ref}, λ_{ref}

Reference latitude and longitude provided by the user in degrees and used by the predictor in radians. The values associated to these variables depends on the type of flight segment as indicated in the following table.

	$\phi_{ref}, \lambda_{ref}$
Climb at a constant course.	Position of the DME from which the distance is measured.
Climbing turn to a known radial.	N/A
Climbing turn to a known location.	Initial position of the flight segment.

Table 2. Parameters that defined each segment: reference position.

In the two types of segments in which the coordinates are given equation (4.31) applies. What is more, equations (4.20) and (4.21) are used to convert radians to meters measured from the departure airport. These parameters do not apply to the "Climbing turn to a known radial" as the stop condition in this case is the final course and the position provided by the user is only used to calculate the turning radius as it was explained in Section 4.4.2.2.

- **D**

D stands for the distance between the reference position and the real position of the aircraft at each time measured in meters. When it reaches a certain value, the integration stops and next segment starts.

	D
Climb at a constant course.	Distance between the DME and the final aircraft position desired.
Climbing turn to a known radial.	N/A
Climbing turn to a known location.	Distance between the initial and the final position.

Table 3. Parameters that defined each segment: distance.

As it was told when analysing the previous parameters, distance is not the stop condition that interest when studying the "Climbing turn to a known radial" segment so that is why it is not defined. Instead, the final course and one location of the final course must be provided as explained in Section 4.4.2.2.

- χ_i, χ_f

These variables represents the initial (i) and the final (f) courses measured in radians. In general, χ_i is equal to the final course of the previous flight segment. However, there are some SID descriptions in which an abrupt change of the course appears and that is the reason why it has been included the option of manually changing it, whichever the type of segment is. The reality is that the pilot would do a turning segment, but, in the scope of this project, an instant change is considered.

Regarding the final course as a stop condition, it only appears in the "Climbing turn to a known radial" type of segment. It is necessary to make some comments about critical situations than can occur in the study of this condition, for example, if during the development of the flight segment the course goes through zero, whichever the turning direction is. That can easily occurs as Figure 4-11 shows.
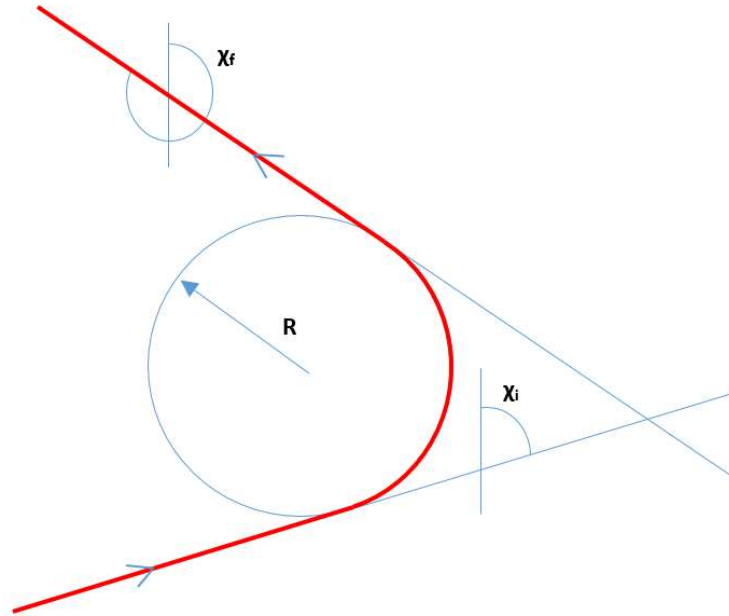

Figure 4-11. Critical case of course as the stop condition.

This is a phenomenon that can happen either in the left or the right turns as the variation of the course is linearly dependent with the time variation. This is a problem easily solvable. First of all, the trajectory predictor needs to determine if a critical case is had, then, it has to determine either if it is a left or a right turn. This point can be quickly accomplish attending to the sign of the turning radius. After that, there are two ways to proceed:

- If it is a left turn, then 2π radians are subtracted to the stop condition solving the problem of negative courses obtained by the Flight Mechanics equations, but not correctly defined considering its definition.

- If it is a right turn, 2π radians are added to the stop condition solving the problem of the course being linearly dependant with the time and reaching values over 2π radians.

Finally, it must be mentioned the fact that courses are measured from the DME at which they are referenced, but not towards it. Therefore, the course that has to be introduced in the trajectory calculator user interface may not be the one that directly appears in the SID, if the aircraft is heading towards the DME.

- **R**

This parameters stands for the turning radius and its calculus for both the "Climbing turn to a known radial" and the "Climbing turn to a known location" type of segments has been widely developed in previous sections. In the case of "Climb at a constant course", the radius is equal to infinite.

All these considerations lead to obtain a 6-component vector for each flight segment that contains the information enough to solve any segment. There are as many vectors as segments form the SID. In the end, a matrix with a number of raws equal to the number of segments and 6 colunms is obtained and saved in the trajectory predictor file so, in case the user wants to try a new combination of aircraft or atmosphere model in

the same SID, it is not necessary to entered the SID parameters again and it just have to be called it by the name with which it was saved.

4.6   Outputs.

The trajectory predictor generates a lot of results that include speeds, total time to complete the SID or fuel consumption, among others. In the block diagram of Figure 4-1, the most relevant parameters have been shown. They are the variation of the state variables with the time, the total time invested in completing the SID, the fuel consumption and the variation with the time of the CAS and the ROC. They offer important information about the aircraft performance and permit to compare the results obtained with other combinations of aircraft and atmosphere models in the search of optimization of the flight routes. The practical examples in Section 6 aims to demonstrate that utility.

5 TRAJECTORY PREDICTOR USER INTERFACE

The idea of this section is to show the trajectory predictor user interface as everything explained in previous sections is related to its software implementation. One of the advantages of the predictor that can be highlighted is the simplicity that the introduction of any SID supposes to the user, as the minimum necessary information is required to solve the problem. What is more, the information requested has been optimized according to the Aeronautical Information Publications so that the type of information needed is the one that directly appears in the SID description. There are different ways of solving the same SID by changing the type of climb, the aircraft or even the atmosphere model. Regarding this last point, the example presented in this section has been obtained considering the ISA atmosphere model. The introduction of the data provided by ECMWF might vary the results; however, the interface with the user is the same because, as it was detailed in Section 4.2, weather data are limited to a particular region and all this information is given in a matrix format that would be introduced by the programmer in the predictor software.

The different scripts between the trajectory predictor and the user in the solution of a SID are going to be presented. Specifically, the SID called *VJF3R* of Seville airport is the one choosen to be shown. The map view of it and its text description are below.

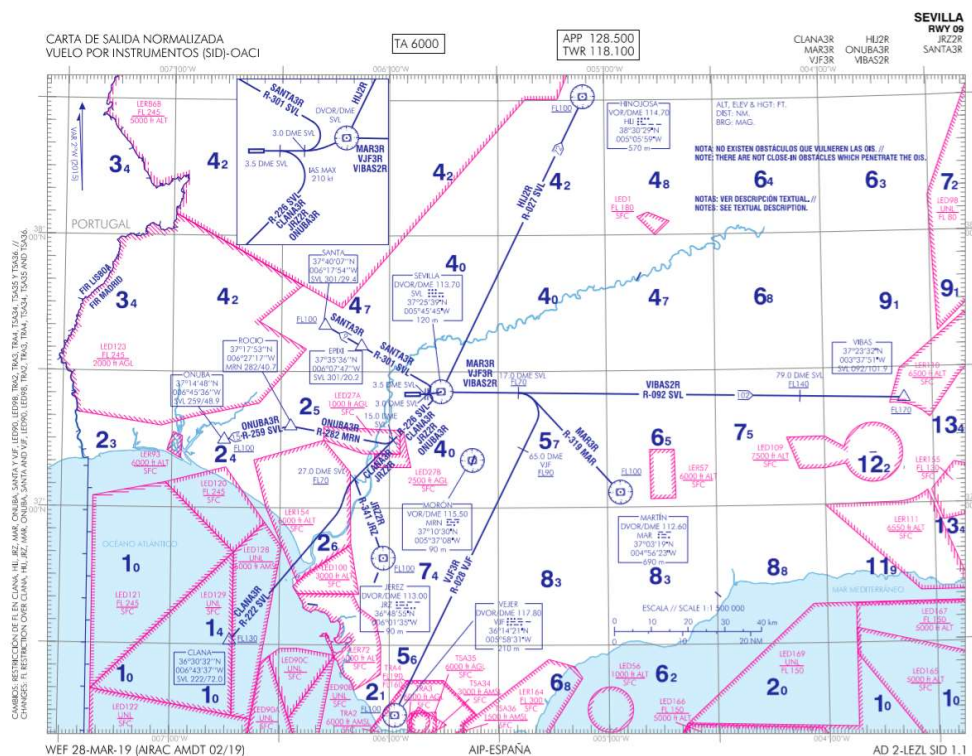Figure 5-1. General view of the SID available from RWY 09 in Seville airport [8].

VEJER THREE ROMEO DEPARTURE (VJF3R)
Climb on runway heading up to 3.0 DME SVL. Turn left direct to DVOR/DME SVL. Proceed on R-092 SVL up to 17.0 DME SVL at FL70 or above. Turn right to intercept and follow R-028 VJF, direct to cross 65.0 DME VJF at FL 90 or above. Direct to DVOR/DME VJF at FL100 or above.

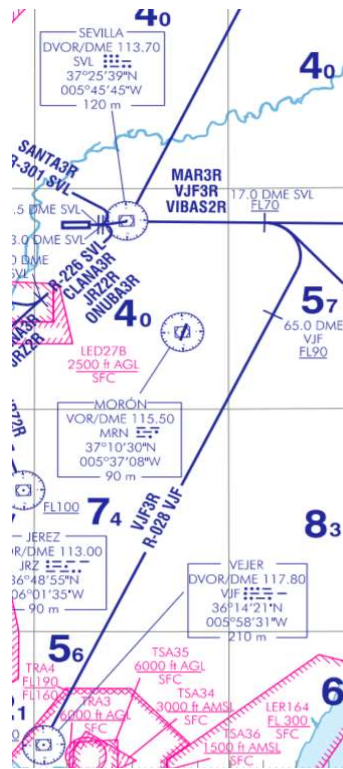Figure 5-2. Text description of VJF3R.

Figure 5-3. Detailed view of VJF3R.

This departure has been choosen as it contains all the types of segments that can be found in a SID. The steps a user has to follow to complete it are the following:

1) **Selection of the atmposphere model, airport, departure course, aircraft model, aircraft mass and type of climb**: by default, Seville, Madrid, Barcelona and London Heathrorw airports are already included. There is also another option to start in any airport. In relation with the aircraft model, there are available data for the Airbus A320 and the Boeing B763. In this example, Seville airport has been choosen, along with an initial course of 90º, as this is the runway heading, the A320 as the aircraft model, a initial mass of 60 tonnes, and the constant ROC and constant CAS type of climb. The atmosphere model is the International Standard Atmosphere.

```
Atmosphere model:  1 if ISA
                   2 if ECMWF
  1
Select airport:  1 if SVQ
                 2 if MAD
                 3 if BCN
                 4 if LHR
                 5 if other
  1
Departure course [°]:
 90
Aircraft model: 1 if A320
                2 if B763
  1
Aircraft mass
Min mass [t]: 39
Max mass [t]: 77
Ref mass [t]: 64

Insert aircraft mass [t]: 60

Type of climb: 1 if ROC=const, CAS=const
               2 if CAS=const, T=Tmax
               3 if ROC=const, T=Tmax
  1
```

2) **SID record, SID name and number of segments:** the programme contains an option regarding the possibility of saving the parameters that define a SID in order to be able to access them without starting the process again. That is why is requested a SID name. To obtain the number of segments it is necessary to look at the SID map and text. In this case there are five segments: climb at the runway heading, turn to DME SVL, climb from the DME SVL at R-092 SVL, turn to follow R-028 VJF and climb at constant course towards DME VJF.

```
0 if SID already implemented;  1 if new
 1
SID name:
 VJF3R_A320
Number of segments:
 5
```

3) **Segment number 1 (climb at a constant course):** as it was explained in sections 4.4.2.1 and 4.4.3.1, the only information required to define this type of segment is a reference position (that in this case is the DME SVL) given by its latitude and longitude and the distance from it at which the segment ends. An additional option of changing the initial course is included as there are some special cases in which the SID does not clarify if it is the same as the final course of the previous segment or not, being that issue something that pilots solve. In this case, the course is the same as the runway.

```
SEGMENT NUMBER 1

Type of segment:  1 si Climb at a constant course
                  2 si Climbing turn to a known location
                  3 si Climbing turn to a known radial
 1
Insert distance to the reference point [nm]:
 3
Insert latitude of the reference point [°]:
 37.428
Insert longitude of the reference point [°]:
 -5.7625
Initial course: 0 if the final of the previous segment
                1 if different
 0
```

4) **Segment number 2 (climbing turn to a known location):** as it was mentioned in sections 4.4.2.3 and 4.4.3.1, the user only has to provide the position at which the turn is going to finish. The predictor itself do all the necessary operations to solve the problem, incluing the estimation of the turning radius.

```
SEGMENT NUMBER 2

Type of segment:  1 si Climb at a constant course
                  2 si Climbing turn to a known location
                  3 si Climbing turn to a known radial
 2
Insert latitude of the final position [°]
 37.428
Insert longitude of the final position [°]
 -5.7625
Initial course: 0 if the final of the previous segment
                1 if different
 0
```

5) **Segment number 3 (climb at a constant course):** as in segment numer 1, distance and a reference position, in this case the DME SVL, are provided according to the SID description. In this segment, the initial course is different than the final course of the previous one as can be seen in Figure 5-1 and Figure 5-3. Then, the initial course is introduced by the user.

```
SEGMENT NUMBER 3

Type of segment:  1 si Climb at a constant course
                  2 si Climbing turn to a known location
                  3 si Climbing turn to a known radial
 1
Insert distance to the reference point [nm]:
 17
Insert latitude of the reference point [°]:
 37.428
Insert longitude of the reference point [°]:
 -5.7625
Initial course: 0 if the final of the previous segment
                1 if different
 1
Insert initial course [°]
 92
```

6) **Segment number 4 (climbing turn to a known radial):** as explained in sections 4.4.2.2 and 4.4.3.1, the information requested for this type of segment is the final course and the spherical coordinates of one position at this course that, in this case, it is the DME VJF. It is important to highlight that the final course is 208º and not 28º (as directly appears in the SID) because the aircraft is heading towards the DME.

```
SEGMENT NUMBER 4

Type of segment:  1 si Climb at a constant course
                  2 si Climbing turn to a known location
                  3 si Climbing turn to a known radial
 3
Insert latitude of a position in the final heading angle [°]
 36.239
Insert longitude of a position in the final heading angle  [°]
 -5.9753
Insert final course [°]
 208
Initial course: 0 if the final of the previous segment
                1 if different
 0
```

7) **Segment number 5 (climb at a constant course):** in the special case of this SID the final position coincides with the coordinates of the DME VJF and, as it has been considered as the reference point, the distance that has to be introduced is zero.

```
SEGMENT NUMBER 5

Type of segment:  1 si Climb at a constant course
                  2 si Climbing turn to a known location
                  3 si Climbing turn to a known radial
 1
Insert distance to the reference point [nm]:
 0
Insert latitude of the reference point [°]:
 36.239
Insert longitude of the reference point [°]:
 -5.9753
Initial course: 0 if the final of the previous segment
                1 if different
 0
```

After completing all the segments of the SID, the following information regarding the final position of the aircraft is shown:

```
xf =

   -1.3124e+05


yf =

   -2.9071e+03


hf =

    8.2294e+03


Vf =

    1.3848e+02


Xi =

    3.6303e+00


Wf =

    5.7637e+05


Total_time_s =

    1.0684e+03


Total_fuel_kg =

    1.2267e+03
```

These are the results of the Flight Mechanics equations at the final time measured in SI units. In addition, the total time to complete the SID and the mass of fuel consumption are also provided. Just as a reminder, local coordinates (x,y) are measured from the departure airport and represent the North and the East direction, respectively. Furthermore, some graphics are generated to visually analyse the results of the trajectory predictor. In this particular case, the type of climb choosen has been the one with constant ROC and constant CAS, so the following figures are obtained:
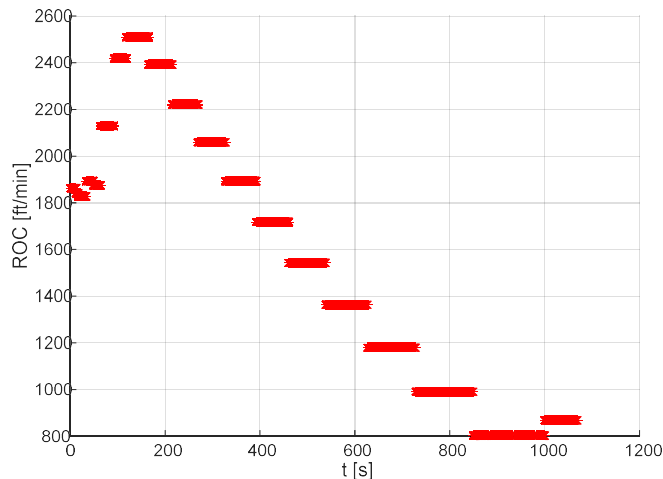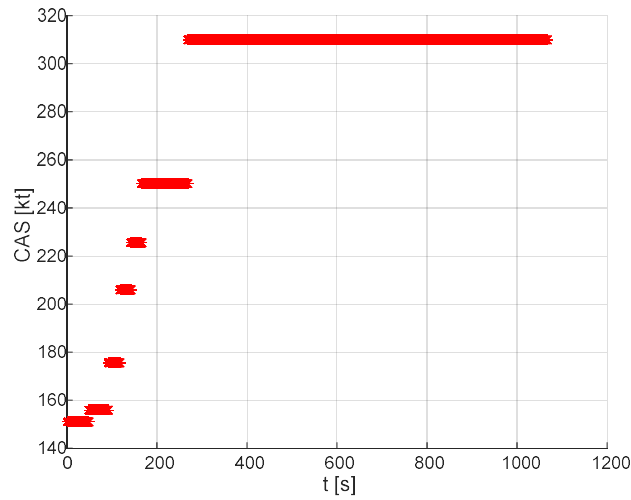
The schedule followed by the ROC is stablished by BADA for each aircraft model as well as the CAS schedule with the altitude. Note that the CAS is represented until the final time of the SID, meaning that the Mach transition altitude is not reached. Also, both representations are discontinuous as a consequence of the instant change of the values when reaching a particular altitude.

The trajectory predictor also provides figures of the variation of the latitude and the longitude with the time and a 3D graphic with the aircraft trajectory. All of them are shown below:
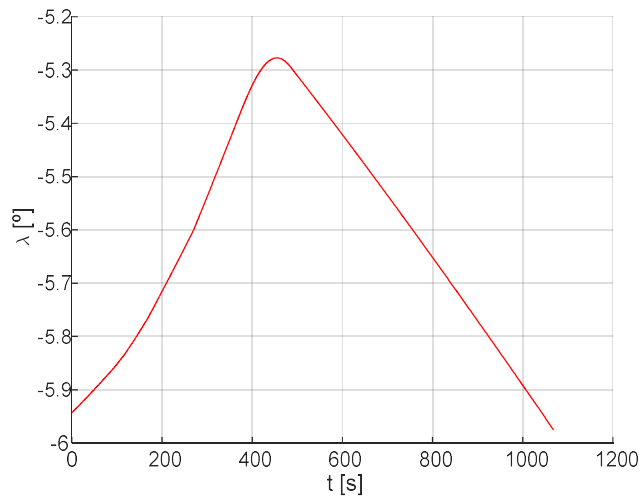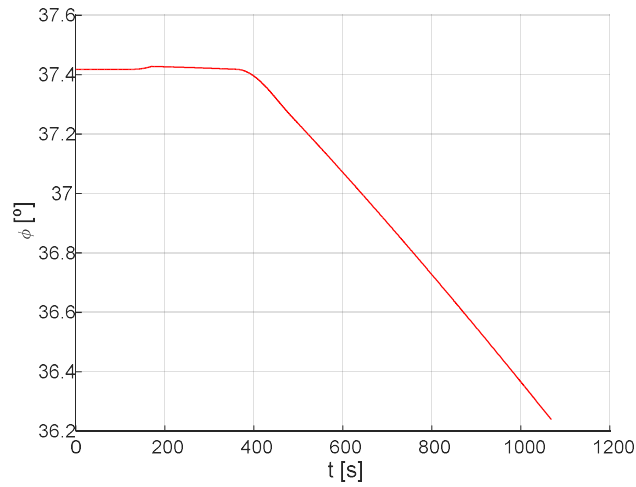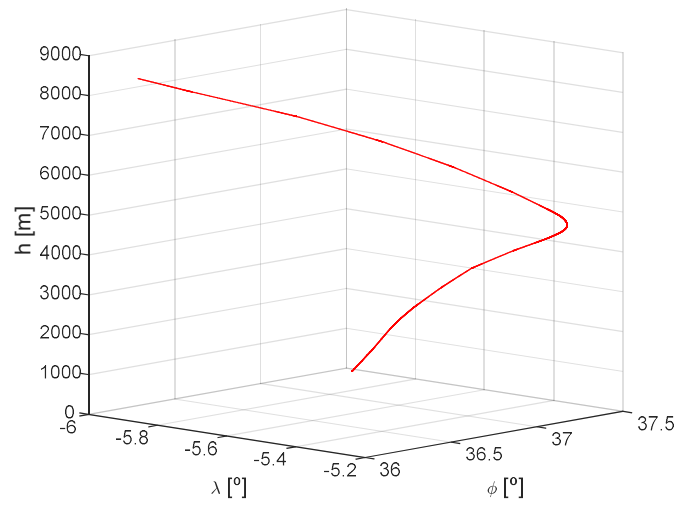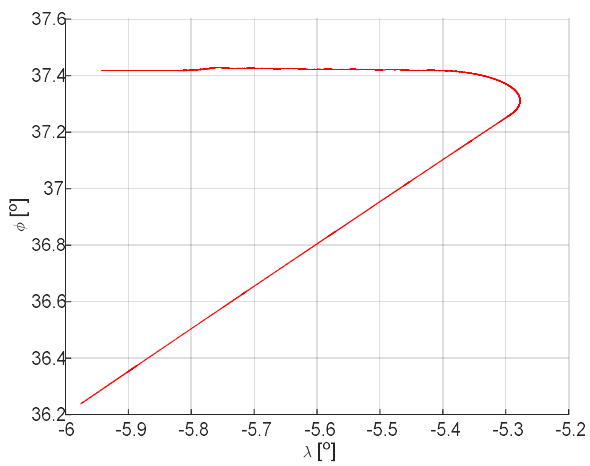
In the next section, this process is repeated with multiple combinations in the same and other SIDs to show all the information and comparisons that can be obtained. The influence of the weather is also included. The procedure to implement each combination follows the same steps as those shown in this section.

6 RESULTS

Up to this point all the theory and the base of the trajectory predictor has been widely explained. However, the best way to compare and analyse all the available options is by offering some examples of use from a graphic and a numerical point of view. The main advantage of the predictor created is that it is valid for any given SID, whichever the airplane or the atmosphere model are. For this reason, and in order to get the maximum information in the scope of this project, the results that are going to be shown below are related to Seville airport. The three different types of climbs are compared as well as the consequences of including the weather effects. Also the sensitivity in the performance with the aircraft mass is analysed. In Section 6.2, several examples of SIDs from other airports are analysed.

Firstly, focusing on Seville airport, the case without the wind influence is considered and the SID that is going to be analysed in multiple combinations will be the VJF3R of Seville. It is the same as the one implemented in the previous section in order to present the full process of data insert and results obtaining. The main variables of interest from an airline point of view are the total time spent and the fuel consumption. These variables depends on the aircraft model, the type of climb and also the weather influence.

So the three types of climb are shown below in the same figure for each aircraft model. The color code used to represent them is the following:

- Red: constant ROC and constant CAS.
- Blue: constant ROC and maximum thrust.
- Green: constant CAS and maximum thrust.
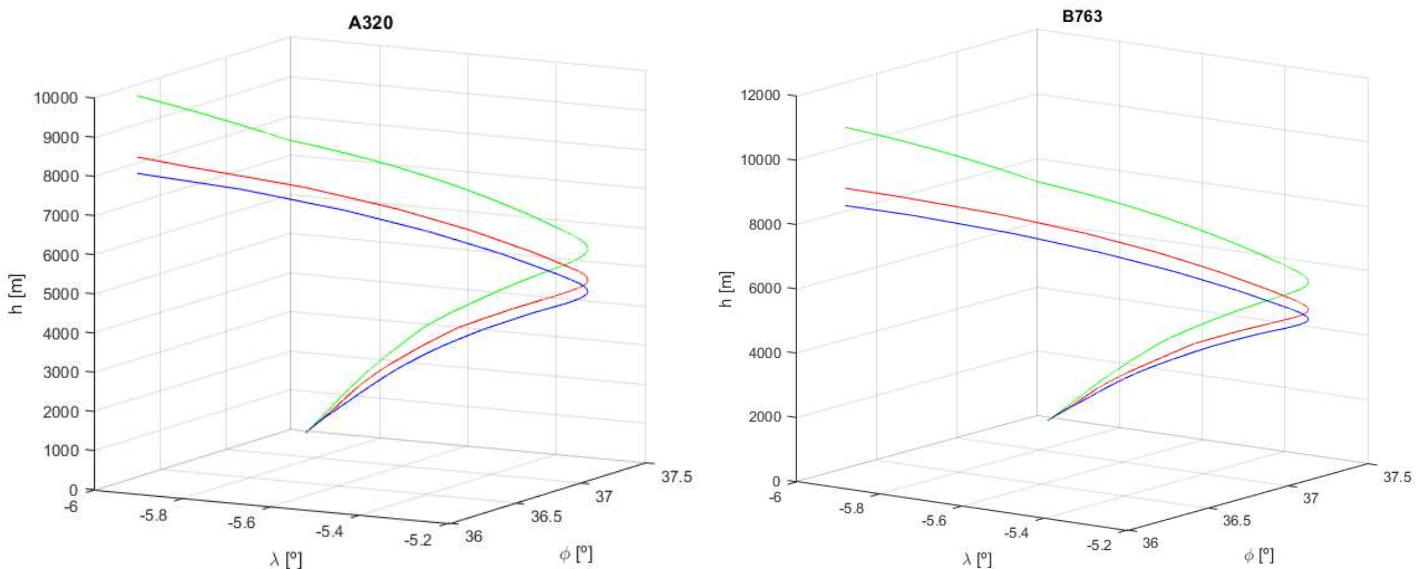
The comparisons of fuel consumption and total time spent for each combination are presented in the following tables:

Fuel consumption (kg)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	1278.8	1420.5	1297.6
B763	2811.9	3201.9	2890.8

Table 4. Fuel consumption: VJF3R.

Total time (s)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	1068.4	969.58	1022.1
B763	1046.5	924.40	1002.6

Table 5. Total time to complete the SID: VJF3R.

At a glance, the fuel consumption data could be excessive as they are in the order of the tonnes. However, the fuel quantity that an Airbus A320 is able to load is 20 tonnes and in the case of the Boeing B763 is approximately 60 tonnes, so taking into account the time spent in each SID combination that are shown in the table above, they turn to be logical results.

Note that in both aircraft models the constant ROC and maximum thrust type of climb is the most expensive in terms of the fuel consumption and the one with the least total time, whereas the constant ROC and constant CAS configuration implies the minimum fuel consumption and the maximum flight time to complete the SID. It should also be remarked the similarity between the results in the constant ROC and constant CAS type of climb and the constant CAS and maximum thrust configuration. It is not by chance as the maximum thrust model defined by BADA is the source of information from which the ROC schedule stablished for each aircraft model is computed. And in order to obtain a better view of the results mentioned above, Figure 6-1 is attached:
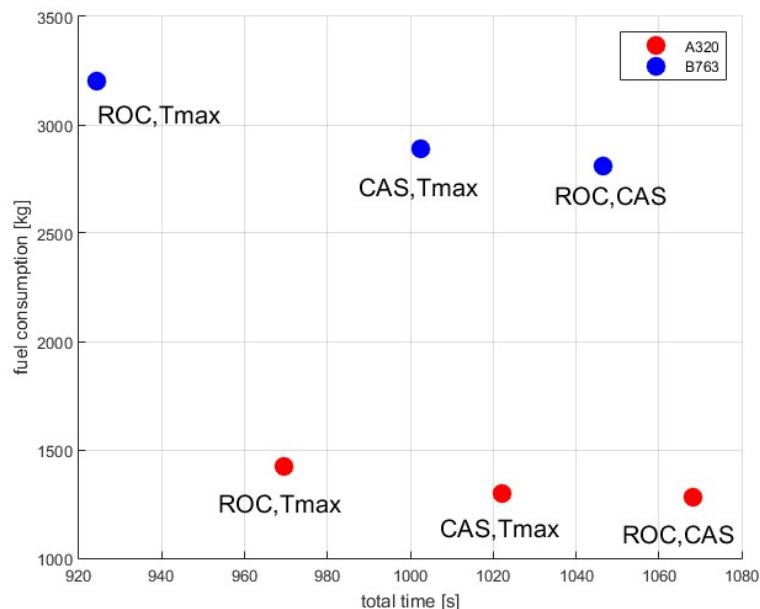
Figure 6-1. Fuel consumption vs. Total time combinations.

Going deeper in the study of this SID, weather effects are considered and, in order to make comparisons, the same SID, VJF3R, is the choosen to be solved. ECMWF provides all the information needed to implement those effects, although the data format is not the adequate one for its software implementation, so it has been changed to create the data matrix that the predictor is able to read.

Previous to the results presentation, a comment must be made about the consequences of the results shown above: in Section 2, it was explained that when the weather information is considered, the problem is solved only from a kinematic point of view favouring a less complex implementation of the Flight Mechanics equations. This idea comes from the study of the different types of climb when the ISA model is considered. Indeed, the results shown above reveal the similarity in the aircraft performance in two of the three types of climb: the constant CAS and maximum thrust climb and the constant ROC and constant CAS climb. The first one is a dynamic problem whereas the second one can be seen exclusively from a kinematic point of view, been this the proof that allows to make this simplification. From now on, when weather information is included, the constant ROC and constant CAS climb is the one that is been analysed.

The comparison between the trajectory without the weather effects and the one that considers them is going to be obtained. A fragment of the wind information for Seville region airport is also shown below. The latitude range is [36ºN, 37.5ºN] and the longitude one is [6ºW, 4ºW]. The red line represents the trajectory of the aircraft from a top view.

VJF3R - B763 - ECMWF (blue) - ISA (red)
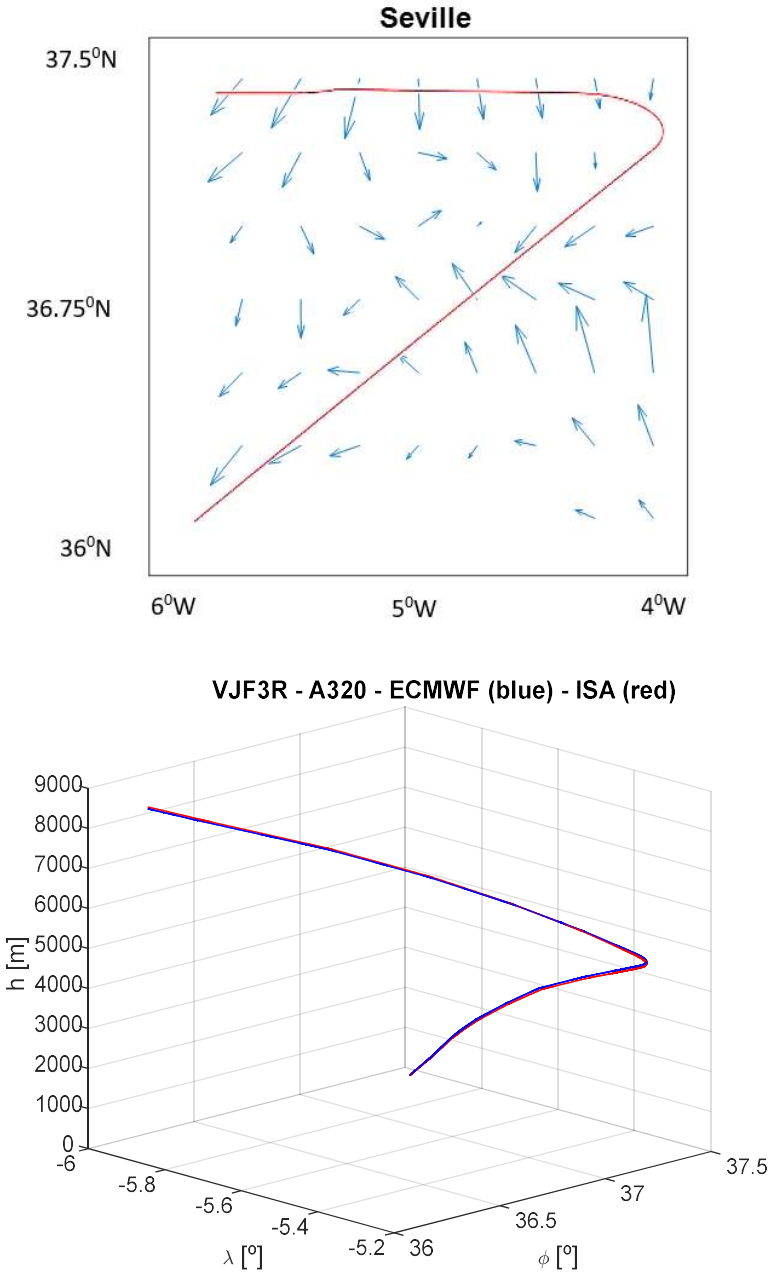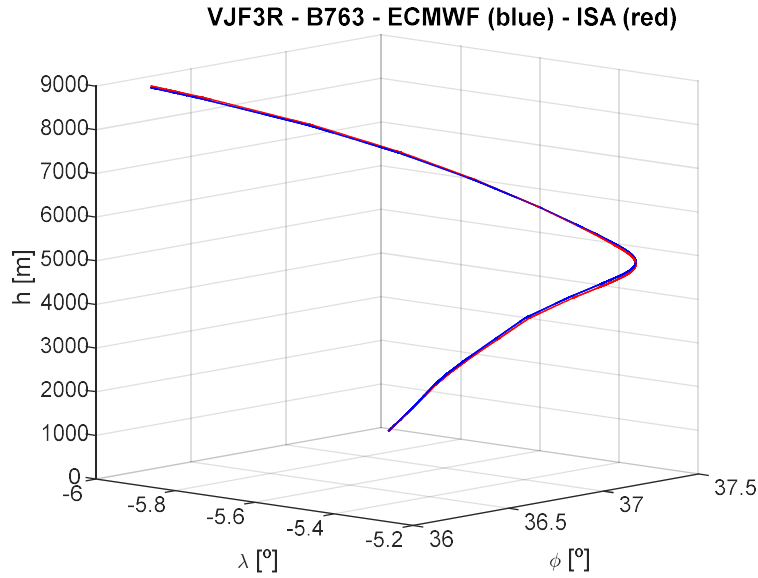
The mean value of the meridional wind is 2.17 m/s and, for the zonal wind, it is 1.52 m/s. It must be highlighted that both the integration with the weather information and the integration with ISA model include the orography of the terrain, so the initial altitude is different to zero if the airport is not at mean sea level, as this case. There are small differences in the trajectories followed by the aircraft for both models that are slightly appreciable in the figures. In the case of the A320, the altitude difference at the end of the SID between both atmosphere models is 45 m and for the B763 it is 40 m. Regarding the total time spent in completing the SID, the difference in the case of the A320 is 10.3 seconds, being higher the time spent in the case of considering the ISA model. For the B763, the variation is 9.4 seconds so there are not appreciable differences in terms of time spent to complete the SID for each aircraft model.

In addition, the influence of the initial aircraft mass in the performance is going to be analysed from a fuel consumption and total time to complete the SID points of view. To do so, the constant ROC and constant CAS climb is solved in three different scenarios: with the minimum aircraft mass, the maximum one and the reference one. These data are obtained from the BADA files. An example of them is in the Annex. The atmosphere model for the three cases is the ISA and the aircraft model is the A320. The results obtained for each aircraft model are shown below.

VJF3R - A320
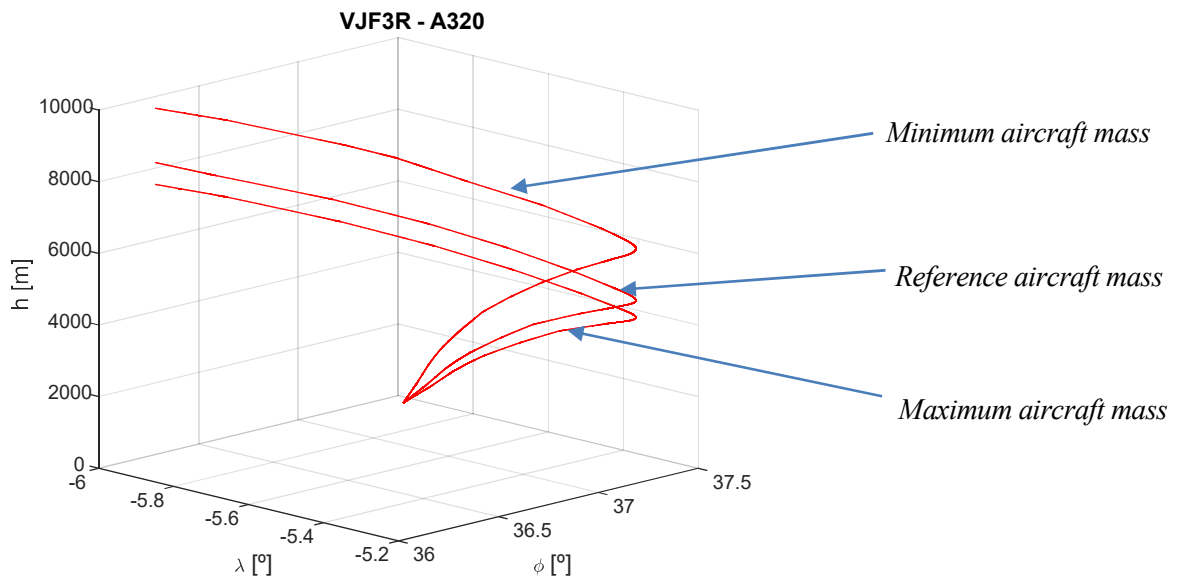
Fuel mass (kg)	Minimum mass	Reference mass	Maximum mass
A320	1081.5	1273.8	1380.9
B763	2440.8	2802.0	2989.3

Table 6. Fuel consumption as a function of the aircraft mass.

Total time (s)	Minimum mass	Reference mass	Maximum mass
A320	1503.1	1066.2	922.1
B763	1505.6	1044.3	901.1

Table 7. Total time spent to complete the SID as a function of the aircraft mass.

As it can be seen in Tables 6 and 7, the fuel consumption along the SID increases with higher values of the initial aircraft mass. On the other hand, the total time spent to complete it is lower. Furthermore, in the 3D figure shown above, the aircraft trajectory with the *minimum aircraft mass* presents the higher differences compared to the other two trajectories. This is due to the values that BADA considers as minimum, reference and maximum aircraft masses. For example, the data for the A306, which are public and can be obtained from [4], are 87 tonnes for the minimum aircraft mass, 140 tonnes for the reference one and 171.7 tonnes for the maximum one. Indeed, the difference between the minimum and the reference aircraft masses is almost the double than the difference between the reference and the maximum aircraft masses leading in the mentioned differences.

To conclude the sensitivity study with the initial aircraft mass, the differences in the CAS schedules with the altitude are shown in the figure below. As it can be seen, the higher the initial aircraft mass is, the higher CAS values are assumed.
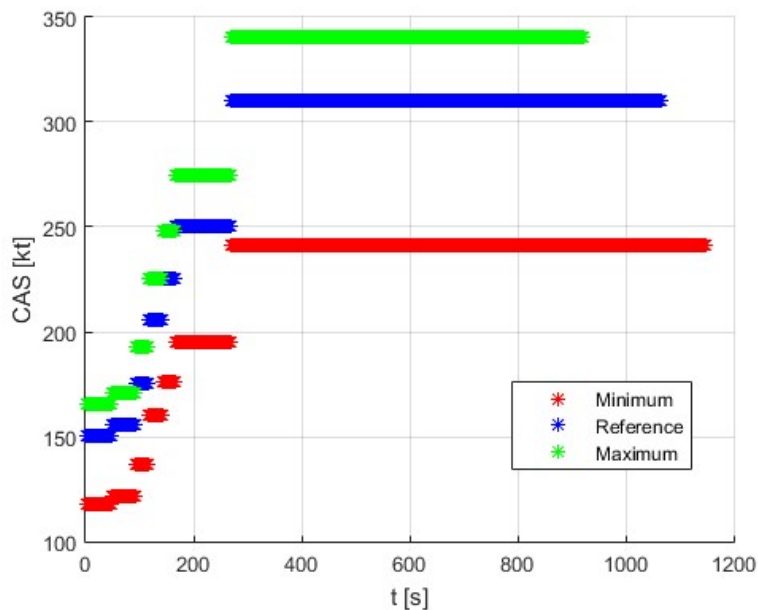
6.1 Other geographical locations.

The aim of this section is to demonstrate the versatility and flexibility of the developed tool. To do this, three SIDs have been analysed. They are from Madrid, Barcelona and London Heathrow airports. For each one, a comparison of the situations with the weather information and without it is presented. Note that the implementation as it was done in Section 5 is only done once for each airport because the generated matrix is valid in all of the combinations and the option of saving it included in the Matlab code has allowed to optimize the process. The initial aircraft mass for the A320 and the B763 considered along this section is the reference mass corresponding to each case .The weather information was available on January 7, 2019 at 12:00 hours, and it was referred to what would happen 36 hours later.

- **MADRID**

Madrid airport is the first Spanish airport in passenger traffic and flight operations, the fifth of Europe and the twenty fourth of the world. The choosen SID has been the PINAR2B from runway 14R. Pictures of its map and text descriptions are shown below. It has eight segments and includes two of the three types of them that define a SID.
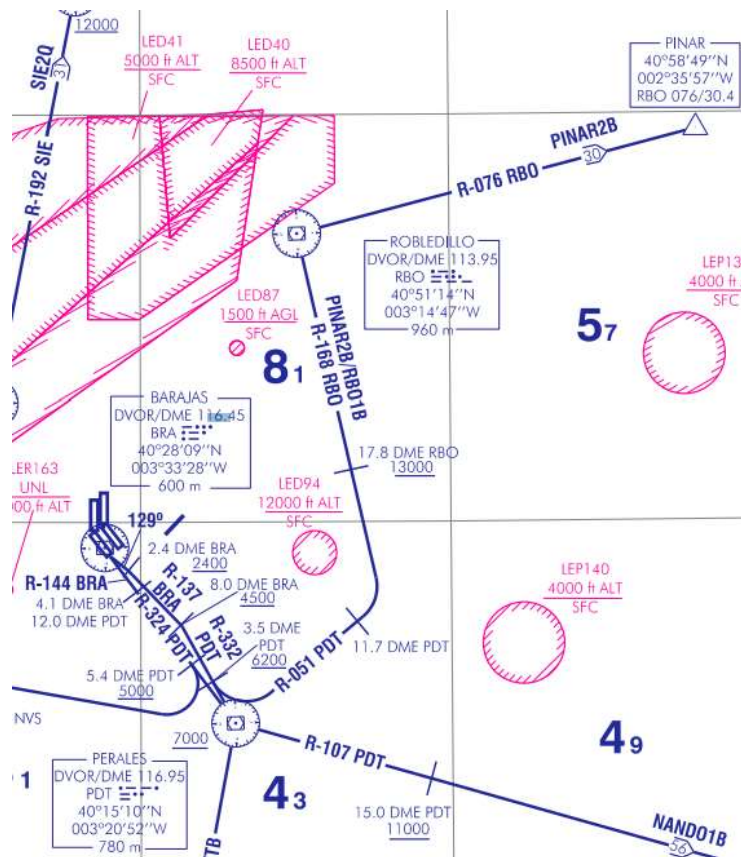
Figure 6-2. Visual description of PINAR2B.

PINAR TWO BRAVO DEPARTURE (PINAR2B)
Climb on magnetic heading 129° direct to cross 2.4 DME BRA at
2400 ft or above. Turn right to follow R-137 BRA direct to cross
8.0 DME BRA at 4500 ft or above. Turn right to follow R-332 PDT
direct to cross 3.5 DME PDT at 6200 ft or above. Turn left to follow
R-051 PDT until reaching 11.7 DME PDT. Turn left to follow R-168
RBO direct to cross 17.8 DME RBO at 13000 ft or above. Direct to
DVOR/DME RBO. Proceed on R-076 RBO direct to PINAR.

Minimum climb gradient of 6.1% up to 13000 ft.
Initial ATC clearance: Maintain 13000 ft and request flight level
change en-route.

Figure 6-3. Text description of PINAR2B.

The graphical results obtained from the trajectory predictor assuming a constant ROC and constant CAS climb, no weather influence and the Boeing 767-300 as the aircraft model are the following:
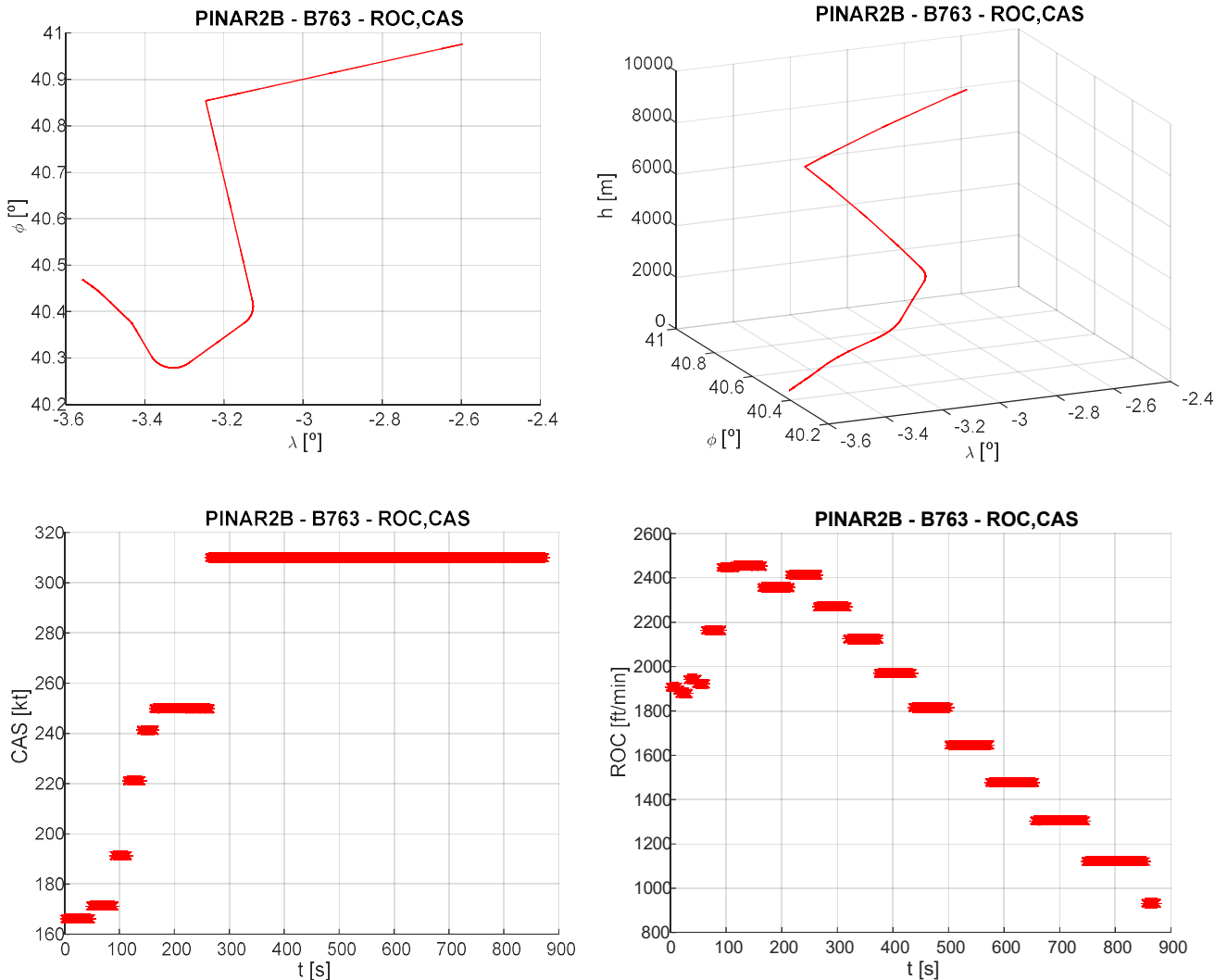
In the horizontal plane (ϕ, λ), the similarity to the visual description of the SID presented in Figure 6-2 is appreciated. Also, the CAS schedule follows the one defined by BADA confirming its correct implementation and the same occurs with the ROC schedule. Furthermore, the comparisons between aircraft models and types of climb are shown in the tables below:

Fuel consumption (kg)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	1111.5	1264.2	1141.4
B763	2465.7	2852.6	2561.9

Table 8. Fuel consumption: PINAR2B.

Total time (s)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	891.21	775.56	848.98
B763	874.23	733.64	831.71

Table 9. Total time to complete the SID: PINAR2B.

The results reveal how the B763 double the fuel consumption of the A320. It is something a priori logical taking into account that the B763 is an aircraft with more capacity and more available filght range. For the same reason, the A320 takes more time to complete the SID in all types of climb. It should me also mentioned the similarity in the results obtained in the constant ROC and constant CAS climb and the constant CAS and maximum thrust climb reinforcing the hypothesis when considering weather information of solving just from a kinematic point of view.

The introduction of the weather information and its comparison with the case shown above leads to obtain the following results. In this case, the latitude range for the wind information shown below is [40ºN, 41ºN] and the longitude one is [3.5ºW, 2.5ºW]. The red line represents the top view of PINAR2B.
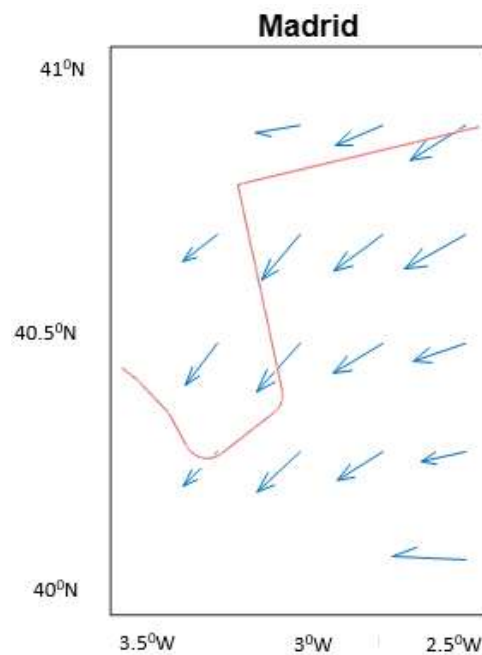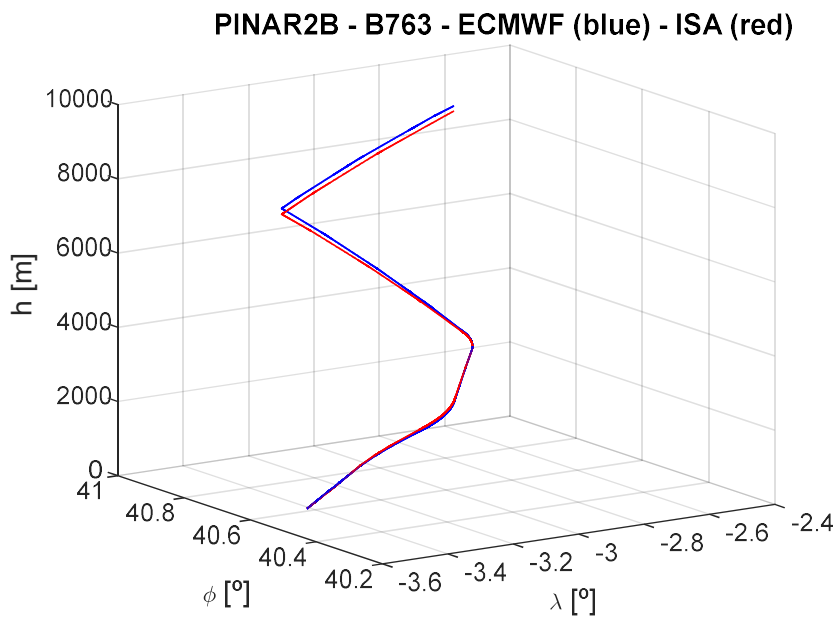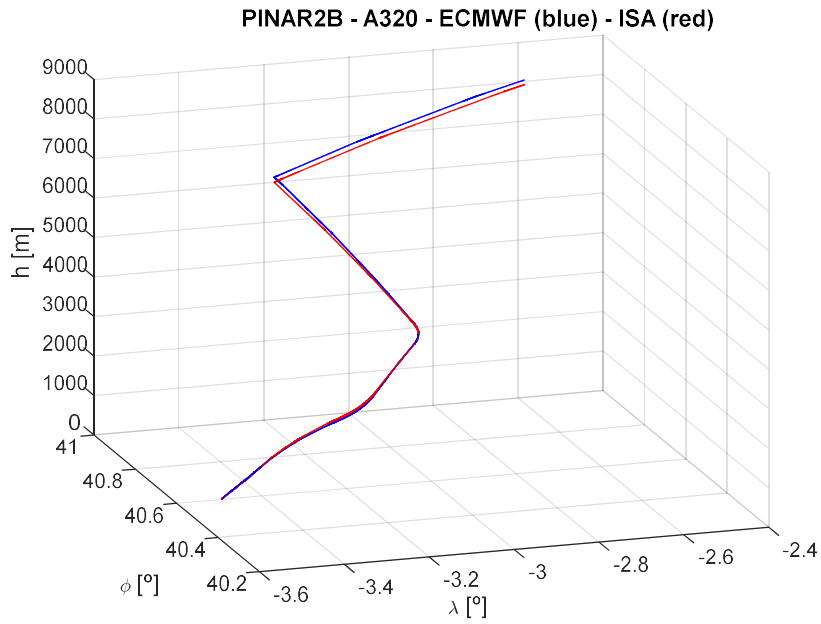
PINAR2B - A320 - ECMWF (blue) - ISA (red)

PINAR2B - B763 - ECMWF (blue) - ISA (red)

In the case of the A320, the difference in the total time spent to complete the SID between both atmosohere models is 29 seconds. The biggest differences between both trajectories are appreciated in the last segment of the SID. For the B763, this difference is 28 seconds. Regarding the final altitudes reached, the variation in the case of the A320 is 120 meters and for the B763 it is 138 meters.

- **BARCELONA**

Barcelona airport is the second busiest Spanish airport and it became the third in Europe in 2012. From this airport, SID VLA1B from runway 07L is randomly choosen to be calculated. Information of this departure is in the figures below.
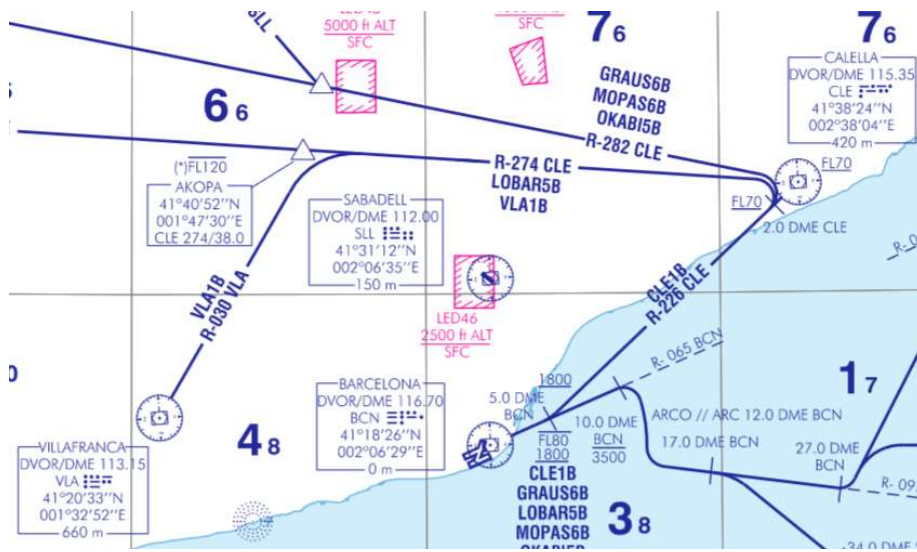
Figure 6-4. Visual description of VLA1B.

VILLAFRANCA ONE BRAVO DEPARTURE (VLA1B)
Climb on runway heading direct to DVOR/DME BCN. Proceed on
R-065 BCN until reaching 5.0 DME BCN between 1800 ft and FL80.
Turn left to intercept and follow R-226 CLE until reaching 2.0 DME
CLE at FL70 or above. Turn left to follow R-274 CLE to intercept and
follow R-030 VLA direct to DVOR/DME VLA.

Figure 6-5. Text description of VLA1B.

The graphical results obtained from the trajectory predictor assuming a calm atmosphere, constant ROC and constant CAS type of climb and the Airbus A320 as the aircraft model are the following:
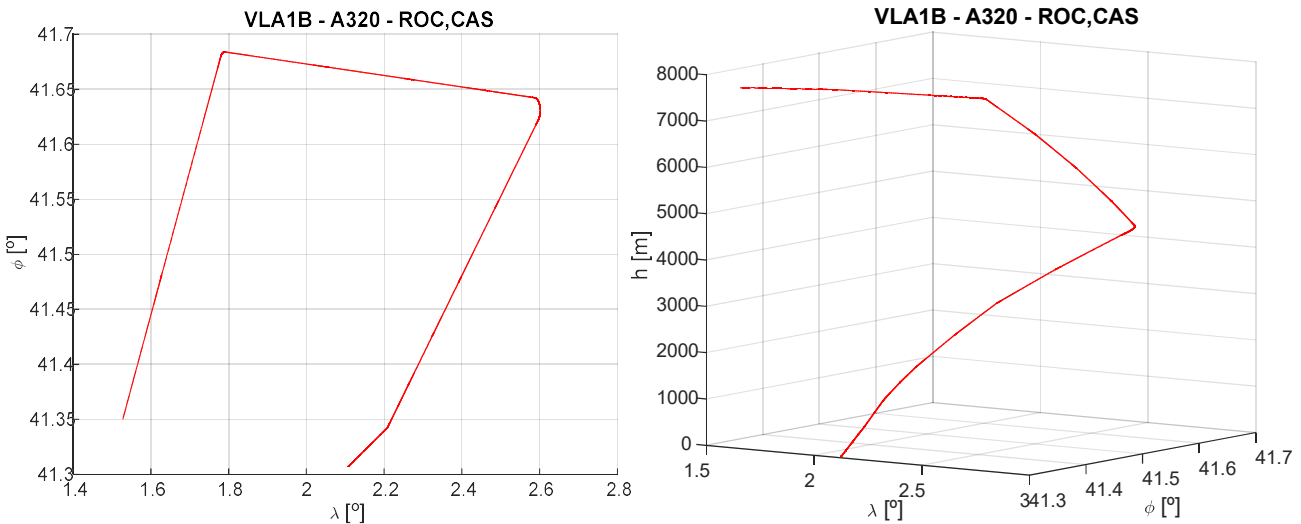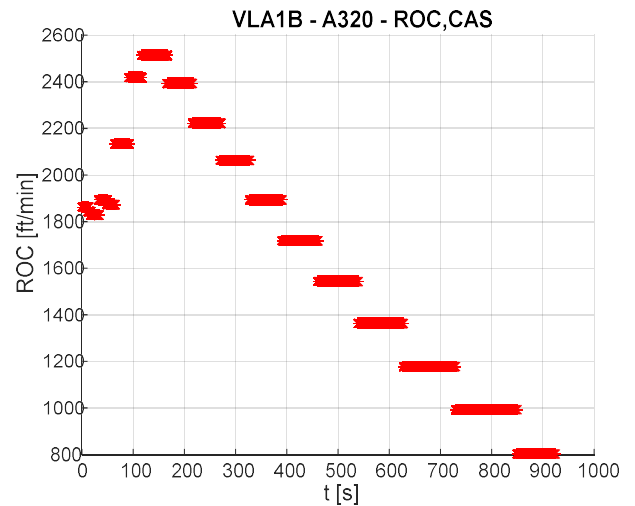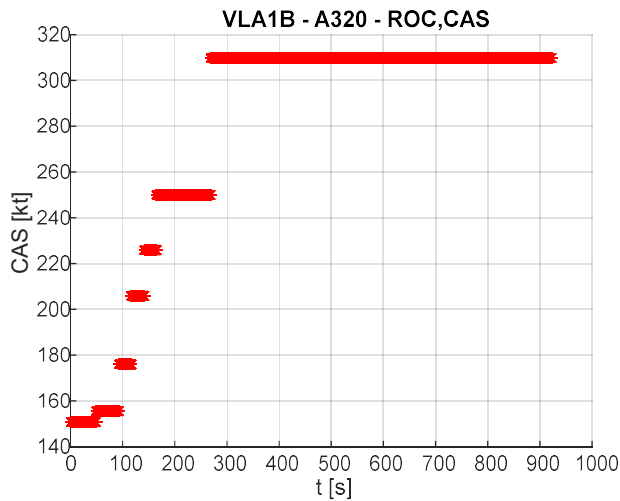
And the comparisons between aircraft models and types of climb are shown in the tables below:

Fuel consumption (kg)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	1168.3	1316.2	1189.6
B763	2606.9	2975.3	2674.7

Table 10. Fuel consumption: VLA1B.

Total time (s)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	923.55	816.54	885.49
B763	905.72	775.15	867.54

Table 11. Total time to complete the SID: VLA1B.

As in the previous example, the tendency is the same: the highest fuel consumption is reached at the constant ROC and maximum thrust type of climb and also the minimum total time to complete the SID. On the other hand, in the constant ROC and constant CAS type of climb, the fuel consumption is minimum and the time spent is maximum. The introduction of the weather information and its comparison with the case shown above leads to obtain the following results. In this case, the latitude range for the wind information shown below is [41ºN, 42ºN] and the longitude one is [1ºE, 3ºE]. The red line represents the top view of VLA1B.
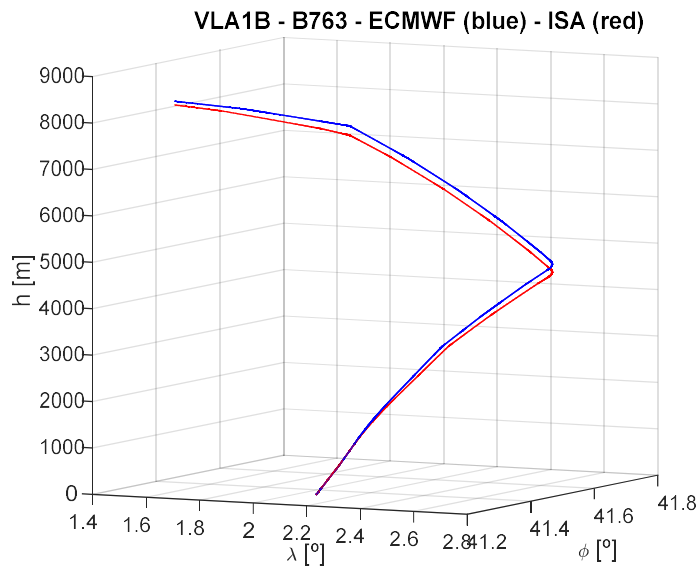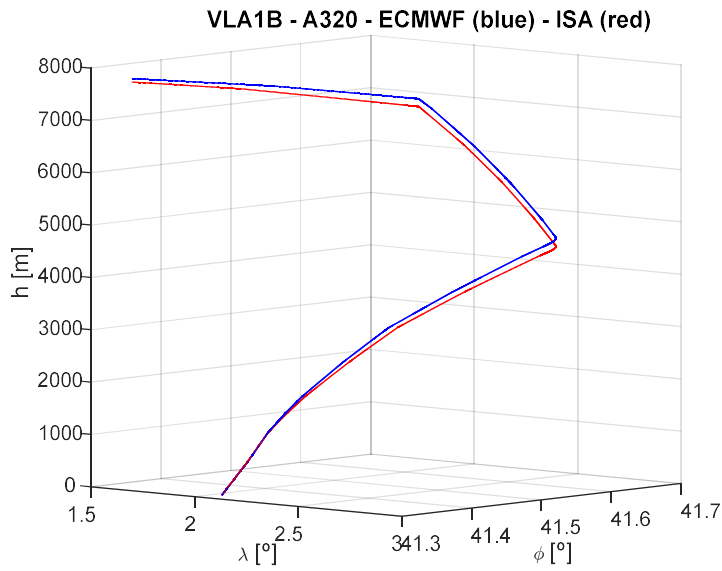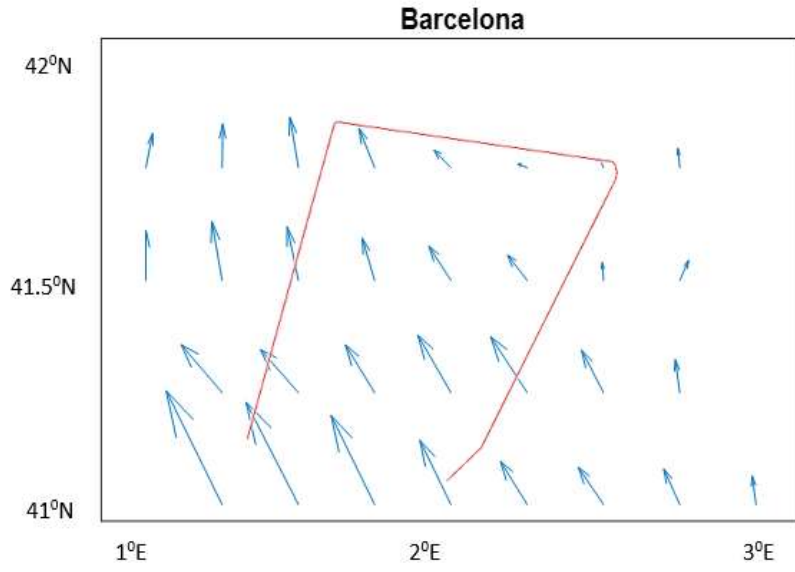
73

Barcelona

VLA1B - A320 - ECMWF (blue) - ISA (red)

VLA1B - B763 - ECMWF (blue) - ISA (red)

In the case of the A320, the difference in the total time spent to complete the SID between both atmosohere models is 16.2 seconds. The biggest differences between both trajectories are appreciated in the last segment of the SID. For the B763 this difference is 17 seconds. Regarding the final altitudes reached, the variation in the case of the A320 is 66 meters and for the B763 it is 80 meters.

- **LONDON HEATHROW**

London Heathrow is one of the busiest airports in the world and the first in the United Kingdom. It has two runways and a total of 28 Standard Instrumental Departures (SIDs). The SID that has been choosen to be solved is BPK 7F from runway 27R whose visual and text descriptions have been obtained from [7] and are shown in the figures below:

Figure 6-6. Text description of four possible SID including BPK 7F.
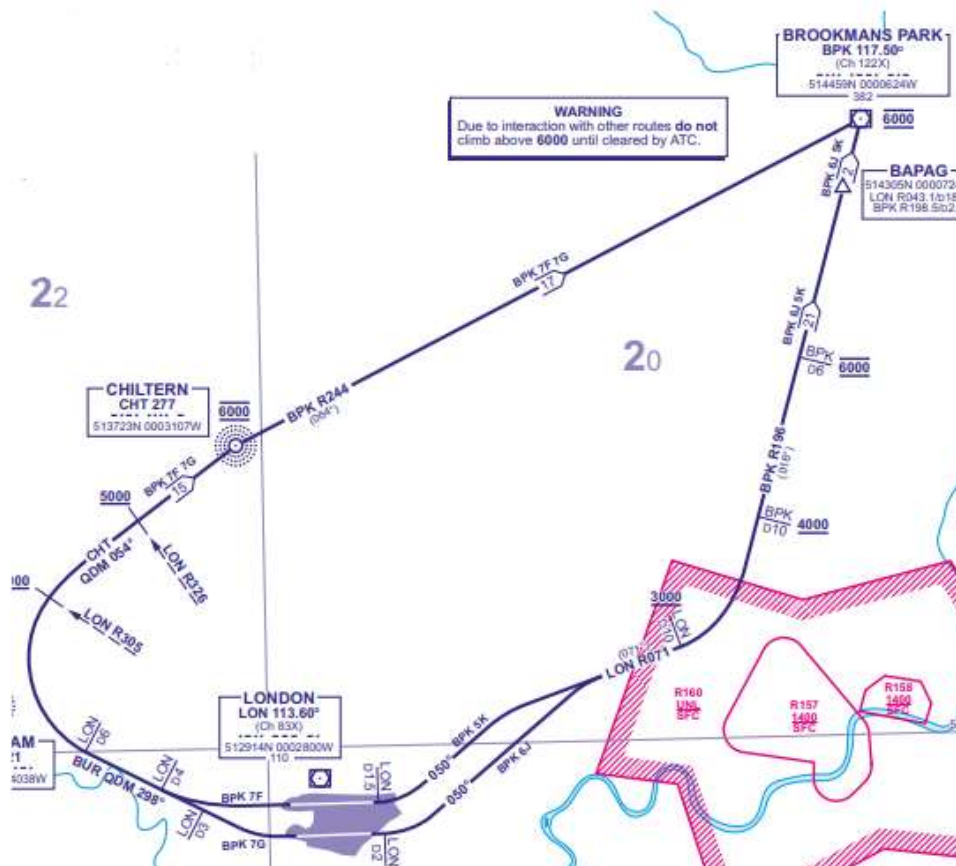
Figure 6-7. Visual description of four posible SID including BPK 7F.

The graphical results obtained from the trajectory predictor assuming a constant ROC and constant CAS type of climb, no weather influence and the Airbus A320 as the aircraft model are the following:
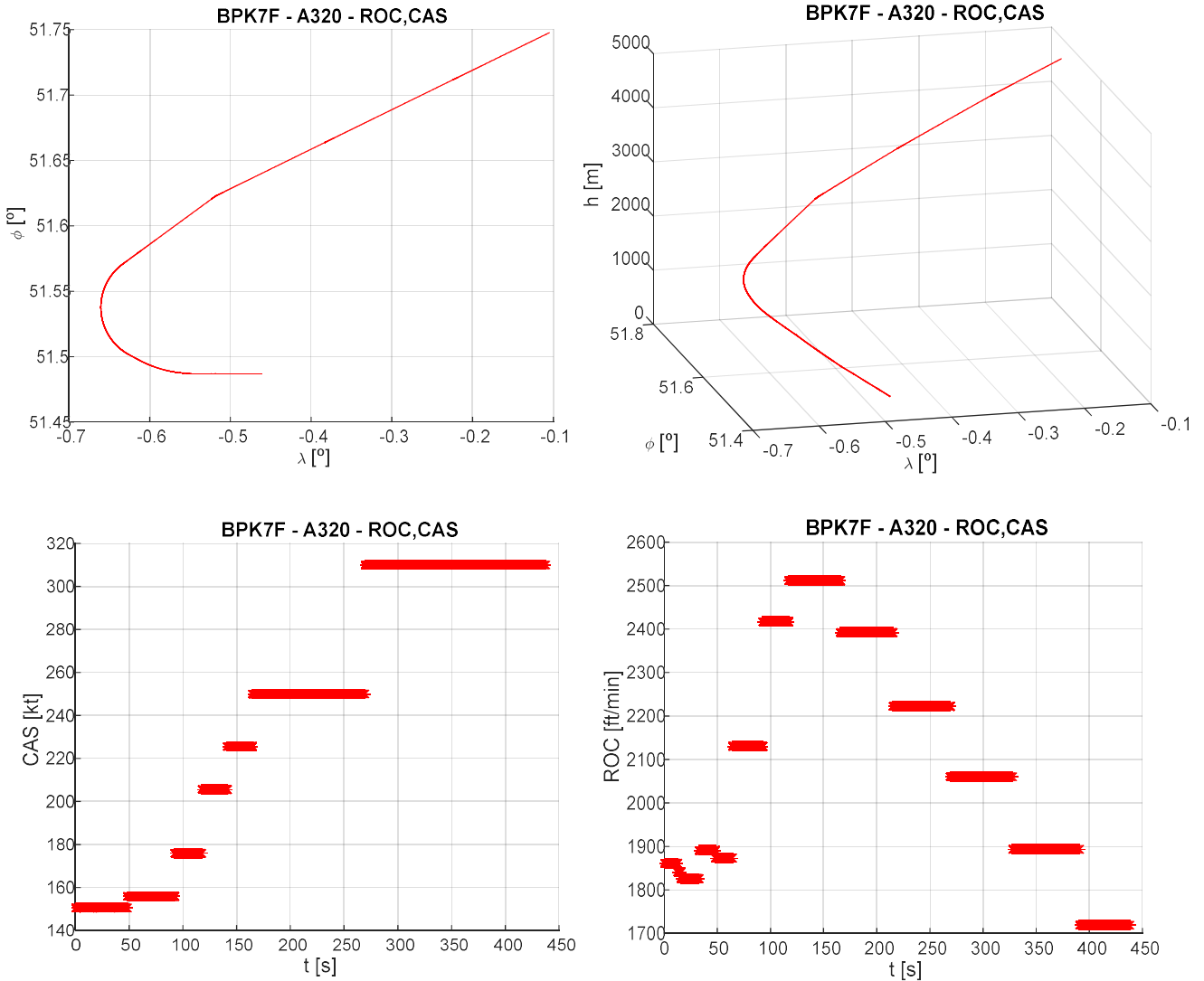
And the comparisons between aircraft models and types of climb is shown in the tables below:

Fuel consumption (kg)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	623.42	678.07	656.84
B763	1394.4	1546.7	1484.0

Table 12. Fuel consumption: BPK 7F.

Total time (s)	ROC, CAS	ROC, Tmax	CAS, Tmax
A320	435.58	380.03	415.56
B763	426.26	371.05	407.21

Table 13. Total time to complete the SID: BPK 7F.

An important conclusion of this section can be made: it exists a direct relation between the fuel consumption and the total time spent to complete the SID, that is, the less time it takes the more fuel the aircraft consumes. The introduction of the weather information and its comparison with the case shown above leads to obtain the following results. In this case, the latitude range for the wind information shown below is [51ºN, 52ºN] and the longitude one is [1ºW, 1ºE].
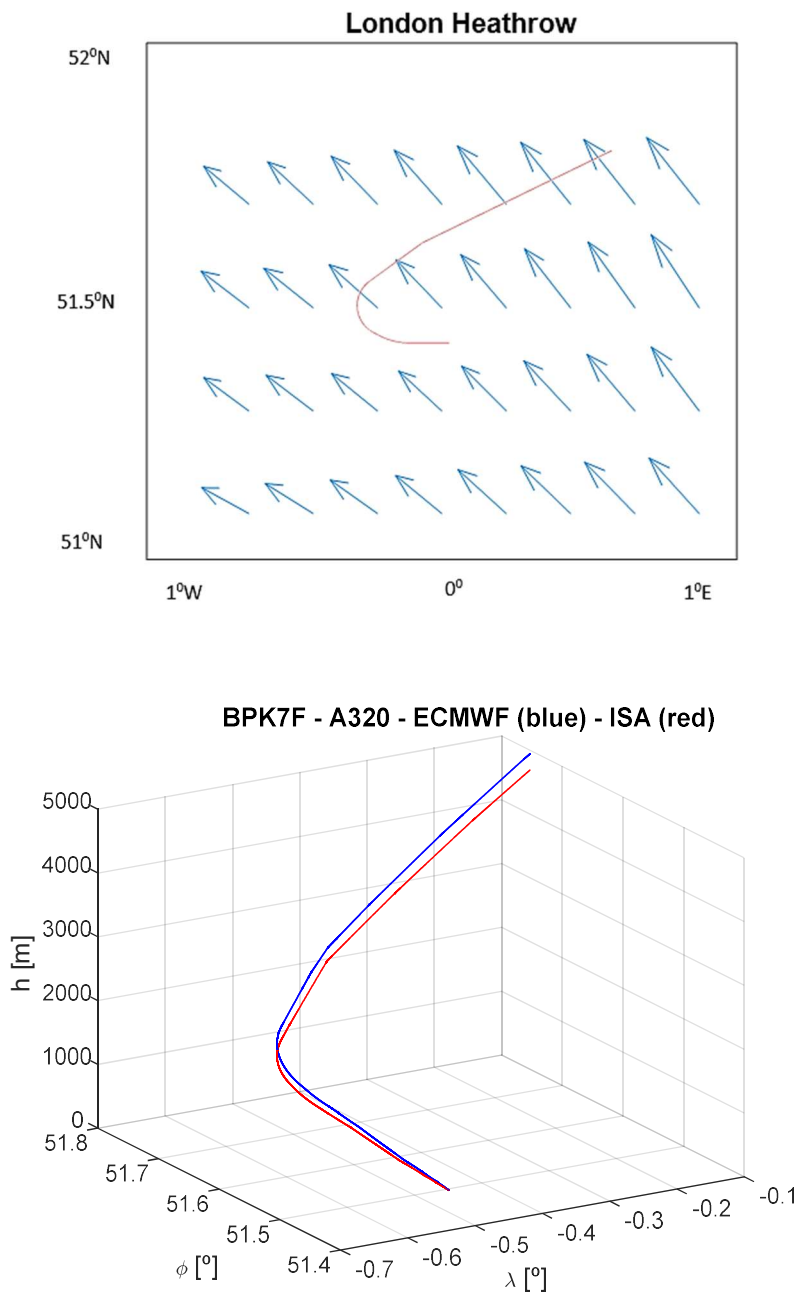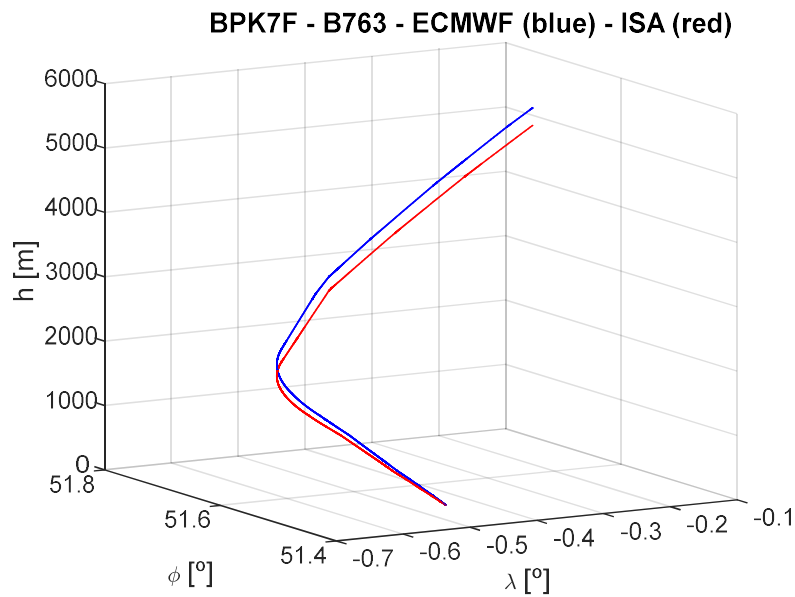
BPK7F - B763 - ECMWF (blue) - ISA (red)

In the case of the A320, the difference in the total time spent to complete the SID between both atmosohere models is 29.6 seconds. The biggest differences between both trajectories are appreciated at the beginning of the last segment of the SID. For the B763, the time difference is 29 seconds and the higher difference in the trajectories is at the end of the SID. Regarding the final altitudes reached, the variation in the case of the A320 is 253 meters and for the B763 it is 273 meters.

An immediate conclusion that can be obtained from these analyses is the fact that the weather influence supposes an increase in the time spent to complete a SID and the magnitude of this difference depends on the route analysed. This is a particular result that comes from studying these airports and the effects of the wind in a particular time, therefore, other scenarios might reveal different results.

These examples show the versatility that the predictor has to solve any given SID in multiple combinations. It must be noted that, although the interface with the user is clear, it is required from him or her some knowledge of the SID that is going to be solved because, as Figure 6-6 and Figure 6-7 demonstrate, not all of them are equally defined compared to the ones provided by ENAIRE. However, this simple pre-process is considered to be assumed by the user without problems.

7 CONCLUSIONS

This project has offered a useful and versatile tool to successfully solve any given Standard Instrumental Departure Route. To do so, the complex formulation of the Flight Mechanics equations has been included and three different types of climb studied. These configurations are similar to the real procedures that airlines follow to complete the flight segments involved in the SID problem and permit to close the problem formulation by defining the remaining degrees of freedom.

Going deeper in the problem implementation, each possible type of segment that can occur during a SID has been included in the search of making this trajectory predictor as general as possible. This has supposed the solution of many geometrical problems as well as the use of a wide range of mathematical tools to make the least of the user influence in the solving procedure and, in fact, the user interface is optimized to minimize the data required from him or her.

The software workload of the project uses a lot of resources from Matlab optimizing them as much as possible although the search of generalism in the problem formulation has made that some codes are considerable long. Anyway, the operational cost of the problem has not been significantely increased even in the cases where interpolation methods or differential equations appear.

It should also be remarked the fact that not all the SIDs are equally define. In the case of Spain, ENAIRE network is fully defined in this topic and provides the data enough. However, the AIP description of other countries are less detailed and requires from the user to do previous estimations to have the information required to close the problem.

From a more technical point of view, these comments can be made:

- Stiff problems are computationally more demanding than those who are not and, as a consequence, it has to be taken more care at the moment of implementing the Flight Mechanics equations or the stop conditions. Also the jacobian must be correctly provided to avoid the singulariy of the problem.

- The hypostheses established at the beginning of the project are demonstrated to be logical to the problem to which they are formulated. What is more, the precaution of maintaining them along all the work done is essential to the coherence of the presented tool and results.

- From the three different possible climb configurations, the constant ROC and constant CAS one and the constant CAS and maximum thrust one offer similar results. It is the proof to justify the cinematic point of view from which the problem with the weather information is solved in benefit of a more simple formulation.

- Weather information provided by ECMWF is not in the format in which the predictor needs the information. A pre-processing is necessary to include the data and indications of the meaning of each variable has to be also provided.

- There are remarkable differences between some flight configutations and others. This is not far away from reality as small differences in the way in which the initial segments of the flight are done may suppose high differences in economic terms. In fact, there are airlines that priorize the quickness a climb or a descent is done whereas other companies prefer to slow it down and in exchange the cost is lower in terms of fuel consumption.

Roughly, the problem of the lack of automatization in the SID solution is not trivial. It does not ease the problem the fact that each country present its own AIP in the format they consider more useful making more difficult the creation of a tool capable of solving any given SID. For this reason, to do the *trajectory predictor*, it has been necessary to do a sampling of possible routes in order to determine which types of segments permit to solve any of them.

What is more, it is not possible to reach a full state of automatization as the SIDs description are given in paper format and, for this reason, a minimum of knowledge about this topic is required to the person who is going to implement the SID. What it is actually possible is to minimize the information required to the user and to try to hide the bulk of the problem in the Matlab code and this is what the solution presented aims to do.

In the words of the author, the objective fixed at the beginning of the project has been achieved. A useful and versatile tool has been presented in order to contribute to the reach of the maximum level of automatization in the analysis of the Standard Instrument Departure Routes.

8 FUTURE IMPLEMENTATIONS AND IMPROVEMENTS

The project that has been developed in these sheets offers an acceptable solution to the problem of the lack of automatization in the Standard Instrumental Departures Routes. It has allowed the user to compare between diferent ways of completing the same SID with different aircraft and airline procedures. However, it is always possible to improve and optimize the solution proposed. Here are some lines of improvement for the work done:

- The formulation of the problem when the weather information is included has been limited to the kinematic point of view. As it has been conveniently justified, this is done because the dynamic formulation turns to be more complex and the analysis without the weather effects reveals the similar results between both points of view in two of the three types of climb. However, to get closer to reality, the dynamic problem should also be implemented dealing with a more complex formulation. The work done up to date is a solid base to face this challenge.

- The hypothesis of flat Earth has been considered along all the project in the nearby of the departure airport. It is something logical noticing the relative small region in which the SID is developed. Obviously, it does not describe the real shape of Earth and a conversion between Cartesian and spherical coordinates has had to be included. The reformulation of the problem considering its spherical shape will provide more accurate and close-to-reality results, but no so far from the ones already obtained. Anyway, the orography of the terrain has been considered.

- Files that describes the performances of each aircraft model are currently manually modified to provide the data in the format in which the predictor is able to read them. As a future improvement, files provided by BADA should have been left in the predictor directory in the same format and not pre-processing them by the user.

- The available weather information used in this project covers a huge range that allows to study the SIDs of airport such as Seville or London Heathrow. As a future improvement, a higher data network that covers numerous international airports could be a considerable update to the proposed solution in order to make it valid in any airport.

- During this project only one weather scenario has been considered and it is supposed to be quasi-stationary. However, reality is a bit different from this as the weather information can even change during the aircraft flight along the SID. The inclusion of this improvement can be considerably complex although the proposed tool is a good base to start the process. In this sense, a set of weather scenarios could be also provided to study and analyse which is the optimum one.

- The presented tool interface works through the Matlab command window. Next step could be the development of its own interface out of the Matlab format.

These recommendations are focused on making the best of the tool presented in terms of optimizing the design, including the more complex Flight Mechanics equations and considering several weather scenarios, among other tasks. This project aims to establish the base for all thoses improvements offering a useful and versatile solution.

9 ANNEX

9.1 Aircraft model files.

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC A306__.OPF CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC/
CC                                                                         /
CC                 AIRCRAFT PERFORMANCE OPERATIONAL FILE                   /
CC                                                                         /
CC                                                                         /
CC       File_name: A306__.OPF                                            /
CC                                                                         /
CC       Creation_date: Apr 30 2002                                       /
CC                                                                         /
CC       Modification_date: Sep 05 2008                                   /
CC                                                                         /
CC                                                                         /
CC====== Actype ==========================================================/
CD   A306__            2 engines    Jet                        H          /
CC   A300B4-622    with PW4158 engines                      wake          /
CC                                                                         /
CC====== Mass (t) ========================================================/
CC    reference      minimum       maximum     max payload   mass grad    /
CD    .14000E+03    .87000E+02    .17170E+03    .39000E+02    .15103E+00  /
CC====== Flight envelope =================================================/
CC    VMO(KCAS)       MMO        Max.Alt        Hmax       temp grad      /
CD    .33500E+03    .82000E+00    .41000E+05    .32378E+05   -.2716E+02   /
CC====== Aerodynamics ====================================================/
CC Wing Area and Buffet coefficients (SIM)                                /
CCndrst Surf(m2)     Clbo(M=0)         k            CM16                   /
CD 5   .26000E+03    .13150E+01    .84080E+00    .00000E+00               /
CC     Configuration characteristics                                      /
CC n Phase  Name     Vstall(KCAS)    CD0           CD2        unused      /
CD 1 CR    Clean     .15100E+03    .20591E-01    .51977E-01   .00000E+00  /
CD 2 IC    S15F00    .11700E+03    .33057E-01    .45362E-01   .00000E+00  /
CD 3 TO    S15F00    .11700E+03    .33057E-01    .45362E-01   .00000E+00  /
CD 4 AP    S15F15    .10900E+03    .38031E-01    .44932E-01   .00000E+00  /
CD 5 LD    S30F40    .97000E+02    .78935E-01    .44822E-01   .00000E+00  /
```

Figure 9-1. Example of an OPF file.

```
CCCC                                                                  CCCC/
CC                                                                        /
CC                    AIRLINES PROCEDURES FILE                            /
CC                                                                        /
CC      File_name: A306__.APF                                            /
CC                                                                        /
CC      Creation_date: Apr 30 2002                                       /
CC                                                                        /
CC      Modification_date: Mar 05 2009                                   /
CC                                                                        /
CC                                                                        /
CC                                                                        /
CC    LO= 087.00 to ---.--  /  AV= ---.-- to ---.--  /  HI= ---.-- to 171.70  /
CC                                                                        /
CC========================================================================/
CC  COM CO    Company name ------climb-------  --cruise--  -----descent------  --approach- model- /
CC                         mass  lo  hi         lo  hi         hi  lo          (unused)           /
CC   version engines  ma  cas cas mc xxxx xx  cas cas mc  mc cas cas xxxx xx  xxx xxx xxx  opf___ /
CC===:======:=======::==::===:===:==::====:==::===:===:==::==:===:===:====:==::===:===:===::=====:/
CD  *** **    Default Company                                                                     /
CD   B4_622          LO   310 310 79         250 310 79  79 290 290          0   0   0  A306__ /
CD   B4_622          AV   310 310 79         250 310 79  79 290 290          0   0   0  A306__ /
CD   B4_622          HI   310 310 79         250 310 79  79 290 290          0   0   0  A306__ /
CC===:======:=======::==::===:===:==::====:==::===:===:==::==:===:===:====:==::===:===:===::=====:/
CC///////////////////////////////////////////THE END/////////////////////////////////////////////
```

Figure 9-2. Example of an APF file.

```
BADA PERFORMANCE FILE                                    Apr 01 2010

AC/Type: A306__
                                Source OPF File:         Sep 05 2008
                                Source APF file:         Mar 05 2009

Speeds:   CAS(LO/HI)  Mach   Mass Levels [kg]      Temperature:  ISA
climb   - 250/310     0.79   low      - 104400
cruise  - 250/310     0.79   nominal  - 140000     Max Alt. [ft]:  41000
descent - 250/290     0.79   high     - 171700
===================================================================================
FL |       CRUISE        |           CLIMB              |        DESCENT
   |  TAS       fuel      |  TAS         ROCD     fuel   |  TAS  ROCD   fuel
   | [kts]    [kg/min]    | [kts]        [fpm]   [kg/min]| [kts] [fpm] [kg/min]
   |       lo   nom   hi  |         lo   nom   hi   nom  |        nom    nom
===================================================================================
  0 |                     | 157   2454  1925  1556  219.7 | 131   698   84.1
    |                     |                               |
  5 |                     | 158   2437  1907  1536  217.8 | 132   714   83.3
    |                     |                               |
 10 |                     | 159   2420  1889  1517  215.9 | 138   730   82.9
    |                     |                               |
 15 |                     | 166   2530  1974  1588  214.9 | 149   774   82.9
    |                     |                               |
 20 |                     | 167   2512  1955  1568  213.0 | 181   988   28.3
    |                     |                               |
 30 | 230  53.3 69.9 88.8 | 190   2940  2289  1852  212.9 | 230  1287   20.2
 40 | 233  53.4 70.1 89.0 | 225   3474  2695  2191  214.6 | 233  1306   19.9
 60 | 272  60.0 73.3 88.5 | 272   4081  2973  2285  213.7 | 272  1520   19.3
 80 | 280  60.3 73.8 89.1 | 280   3932  2846  2168  206.0 | 280  1561   18.7
100 | 289  60.5 74.2 89.7 | 357   3897  2879  2256  208.7 | 334  1984   18.0
120 | 297  60.9 74.6 90.3 | 367   3687  2706  2101  200.8 | 344  2027   17.4
140 | 378  82.2 91.8 102.8| 378   3472  2527  1941  193.0 | 354  2071   16.8
160 | 389  82.4 92.3 103.4| 389   3250  2344  1776  185.2 | 365  2075   16.1
180 | 401  82.7 92.7 104.0| 401   3023  2156  1607  177.4 | 376  2119   15.5
```

Figure 9-3. Example of a PTF file.

```
Low mass CLIMBS
===============

FL[-]  T[K]  p[Pa]   rho[kg/m3]  a[m/s]  TAS[kt]  CAS[kt]  M[-]  mass[kg]  Thrust[N]
   0   288  101325    1.225       340    136.35   136.35   0.21  104400    297160
   5   287   99508    1.207       340    137.34   136.35   0.21  104400    294268
  10   286   97717    1.190       339    138.34   136.35   0.21  104400    291385
  15   285   95952    1.172       339    144.45   141.35   0.22  104400    288510
  20   284   94213    1.155       338    145.51   141.35   0.22  104400    285643
  30   282   90812    1.121       337    168.52   161.35   0.26  104400    279935
  40   280   87511    1.088       336    202.72   191.35   0.31  104400    274260
  60   276   81200    1.024       333    272.30   250.00   0.42  104400    263011
  80   272   75262    0.963       331    280.34   250.00   0.44  104400    251895
 100   268   69682    0.905       328    345.37   300.00   0.54  104400    240914
 120   264   64441    0.849       326    355.51   300.00   0.56  104400    230066
 140   260   59524    0.796       324    366.04   300.00   0.58  104400    219352
 160   256   54915    0.746       321    376.97   300.00   0.60  104400    208772
 180   252   50600    0.698       319    388.32   300.00   0.63  104400    198326
 200   249   46563    0.653       316    400.10   300.00   0.65  104400    188013
 220   245   42791    0.610       314    412.32   300.00   0.68  104400    177835
 240   241   39271    0.569       311    425.00   300.00   0.70  104400    167790
 260   237   35989    0.530       308    438.16   300.00   0.73  104400    157879
 280   233   32932    0.493       306    451.80   300.00   0.76  104400    148102
 290   231   31485    0.475       304    458.81   300.00   0.78  104400    143263
 310   227   28745    0.442       302    463.54   293.28   0.79  104400    133687
 330   223   26201    0.410       299    459.48   280.58   0.79  104400    124245
 350   219   23842    0.380       297    455.37   268.17   0.79  104400    114936
 370   217   21663    0.348       295    453.12   256.08   0.79  104400    105761
 390   217   19677    0.316       295    453.12   244.46   0.79  104400     96720
 410   217   17874    0.287       295    453.12   233.34   0.79  104400     87813
```

Figure 9-4. Example of a PTD file.

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