$1/f^{\gamma}$ NOISE GENERATION THROUGH A CHAOTIC NONLINEAR SWITCHED-CAPACITOR CIRCUIT

Manuel Delgado-Restituto, Angel Rodríguez-Vázquez and José L. Huertas
Dept. of Design of Analog Circuits
Centro Nacional de Microelectrónica
Edificio C.I.C.A., C/ Tarfia sn, 41012-Sevilla, SPAIN

A programmable switched-capacitor circuit is reported for the generation of $1/f^{\gamma}$ noise. The circuit is described by a chaotic first-order piecewise-linear finite-differences equation which yields a hopping transition between regions of chaotic motions and hence produces $1/f^{\gamma}$ noise. Experimental results illustrating the circuit performance are included.

Introduction

Noise sources have a large number of applications [Gupt75], specially in communication and instrumentation systems. Traditionally, two different techniques have been available for the design of electronic noise sources [Horo89]. One is based on the exploitation of the intrinsic noise characteristics of electronic devices, specially that of zener diodes. The other is a digital technique based on connecting long shift registers into feedback configurations to produce very long period pseudo-random sequences.

Recently, it has been suggested that chaos can be exploited as an alternative for the design of electronic noise sources [McGo87, Chua90, Murc90]. Chaos has two basic features supporting this potential application. On the one hand it is *deterministic* (i.e., exactly describable by equations) and hence suitable for implementation by electronic means. On the other, chaos is extremely sensitive to initial conditions which, taking into account that initial conditions of an electronic circuit can be never exactly specified, means that chaotic electronic circuits are in fact *unpredictable* (i.e., *stochastic-like*). Combining these two features provide the foundations of a new technique for the design of electronic noise sources. Besides, the fact that chaos can be obtained from very simple nonlinear *switched-capacitor (SC) circuits* [Rodr87a, b, Rodr90] paves the way for the practical design of noise sources using analog integrated circuit techniques [Rodr90].

Previously reported chaotic switched-capacitor noise generators are characterized by flat frequency spectra (white noise sources) [McGo87, Rodr87b, Rodr90, Chua90]. However colored noise generators are required in some applications, and, in particular, those with power spectral density proportional to $1/f^{\gamma}$ [Cors88]. Obviously, colored noise can be obtained by appropriately filtering the output of a white noise source [Kesh82], but thus may result in large area overhead for monolithic implementations. Developing more simple models yielding colored noise is hence advisable. In this communication we are aimed to this basic purpose.

It has been demonstrated that a mechanism for the appearance of $1/f^{\gamma}$ divergencies is based in the *hopping transition* between regions of chaotic motion. This hopping mechanism, introduced by Arecchi and Lisi [Arec82], is a good candidate to be exploited for the design of our $1/f^{\gamma}$ noise generator due to the fact that divergence appears over a wide range of frequencies, and the cutoff can be pushed down to arbitrarily small frequencies by decreasing the rate of jumps from one region of chaotic motion to the other. In this paper, a very simple chaotic nonlinear model is proposed herein which give rise to strange attractors with two loci of chaotic motion, thus verifying the hopping mechanism. This model is the form of a piecewise-linear finite-differences first-order equation (*discrete map*), and hence readily realizable in SC form [Espe90]. Besides, a programmable parasitic insensitive piecewise-linear switched-capacitor circuit is reported which is demonstrated via a breadboard prototype to generate $1/f^{\gamma}$ noise. Finally, Sec. IV offers experimental results from breadboard prototypes and concludes with a discussion of the effect of changing the parameters of the map over the power spectral density.

Hopping mechanism

Let us consider the family of one-dimensional discrete maps:

$$X_{n+1} = F(m_c, m_l, B; X_n) \qquad n = 0, 1, 2, \ldots \tag{1}$$

applying the interval $[-m_cB, m_cB]$ onto itself, as shown in Fig.1. As it can be seen, different members of this family can be obtained by changing parameters m_c, m_l and B of the nonlinear function,

$$F(\cdot) = \begin{cases} m_l X + C & , B < X < m_c B \\ m_c X & , -B < X < B \\ m_l X - C & , -m_c B < X < -B \end{cases} \tag{2}$$

where $C = (m_c - m_l)B$. For our purposes, parameters will be chosen as to verify the following conditions :

1) $m_c > 1$. This means the map has no stable periodic points of any period and hence that it exhibits ergodic and even mixing character. As a consequence, a diffusive broadening of the trajectories result.

2.a) $F(m_c, m_l, B; m_cB) > -m_cB$ or, equivalently, $F(m_c, m_l, B; -m_cB) < m_cB$. This precludes the existence of divergent points, i.e., all the orbits starting in the definition interval remain confined to it.

2.b) $F(m_c, m_l, B; m_cB) < 0$ or, equivalently, $F(m_c, m_l, B; -m_cB) > 0$. This precludes the trajectories to be confined to only the left or the right side of the map.

As it has been stated above, the first condition guarantees there is no stable periodic points. On the other hand, the other conditions make the trajectories to be confined to the interval $[-m_cB, m_cB]$, the rightmost and leftmost extremes of the nonlinear function being respectively located in the fourth and in the second quadrant, as it is shown in Fig.1. Also, these conditions are necessary to guarantee that points in the first quadrant will eventually reach the third one and viceversa. By the way of example assume that, given an orbit $\{X_n\}$, exists an integer

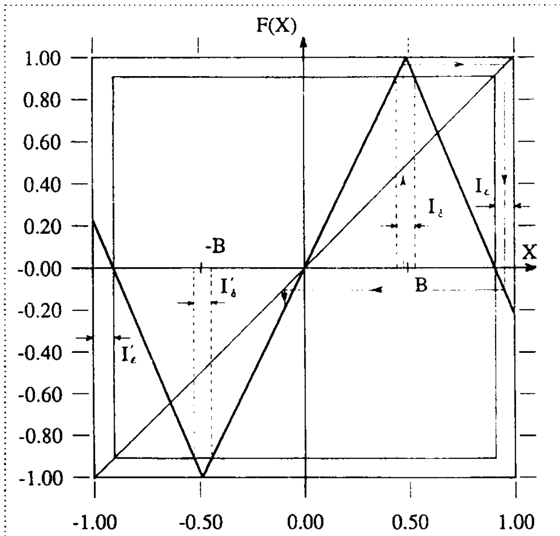

Figure 1: Graphical representation of $F(\bullet)$ for $m_c=2.043$, $m_l=-2.401$ and $B=1/m_c$. Conditions 2 may be solved in terms of the parameters giving rise to the following pair of equivalent inequalities:

$$-\frac{2m_c}{m_{c-1}}<m_l<-\frac{m_c}{m_c-1} \quad \text{or} \quad \frac{m_l}{m_l+2}<m_c<\frac{m_l}{m_l+1}$$

Notice in these expressions parameter B does not appear. Analysis shows this parameter is only a scale factor.

N such that $X_N \in I_\delta$ (in a similar way, $X_n \in I'_\delta$). From Fig.1 it becomes clear that after passing by the fourth (second) quadrant, X_{N+3} will be contained in the opposite quadrant where X_N was.

The proposed map together with previous conditions yields a strange attractor which contain two loci of chaotic motion with jumps between them. It is, on the other hand, apparent that the jumps will be more and more infrequent as the length of the interval $I_\varepsilon=F(I_\delta)$ is reduced (taking into account the map is symmetric the length of the interval $I'_\varepsilon=F(I'_\delta)$ will be correspondingly reduced). This is the "hopping mechanism" introduced by Arechi and Lisi. Observe that, because of the symmetry, there is no net drift. It is to say, hopping in either direction occurs equally often. Also note that the transition between the loci of chaotic motion is not due to noise but is related to the mechanism which yields deterministic diffusion. Numerical simulations show that this class of dynamical systems gives rise to $1/f^\gamma$ divergencies, where γ is close to unity.

Let us now consider how to model the low-frequency behavior of this class of one-dimensional maps. Due to symmetry, the map in Fig.1 can be reduced for analysis purposes to the one in Fig.2,

$$f(\cdot)=\begin{cases} m_c x & ,0<x<B \\ m_l x+C & ,B<x<-C/m_l \\ -(m_l x+C) & ,-C/m_l<x<-B \end{cases} \qquad (3)$$

Given a trajectory $\{x_n\}$ over this reduced map ($x_n \in [0,m_cB]$, ∀n) and assuming the initial value X_0 in the original map is known, we can directly calculate the sequence $\{X_n\}$ ($X_n \in [-m_cB,m_cB]$, ∀n) as follows:

$$X_n=F_n \cdot x_n$$

$$F_n=F_0 \cdot \exp\left[j\pi \sum_{i=0}^{n-1} \Delta(x_i)\right] \qquad (4)$$

$$\Delta(x)=\begin{cases} 1 & ,x \in I_\varepsilon=[-C/m_l,m_cB] \\ 0 & ,elsewhere \end{cases}$$

where $F_0=\pm 1$ according to the original position of X_0. The sequence $\{F_n\}$ may be interpreted as a bistable step waveform of uniform amplitude, with randomly distributed time intervals between steps, closely resembling a random telegraph signal, see Fig.3. Let us suppose without loss of generality that $F_0=1$. If after n iterations the number of points falling in the interval I_ε of the reduced map is even, then $F_n=1$. Otherwise, $F_n=-1$. Simulations show that the low frequency $1/f^\gamma$ divergency of the original map is in direct relation with that of the associated sequence $\{F_n\}$, as shown in Fig.4. Observe that the spectra is nearly flat up to a certain frequency, f_0. However, this cutoff frequency can be pushed down to arbitrarily small values by changing the map parameters.

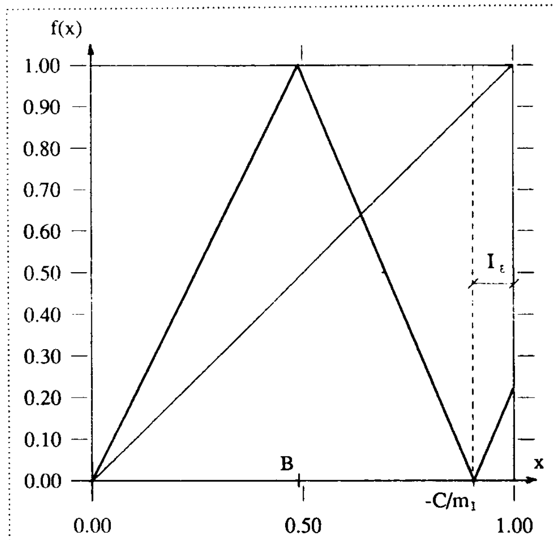

Figure 2: Reduced map.

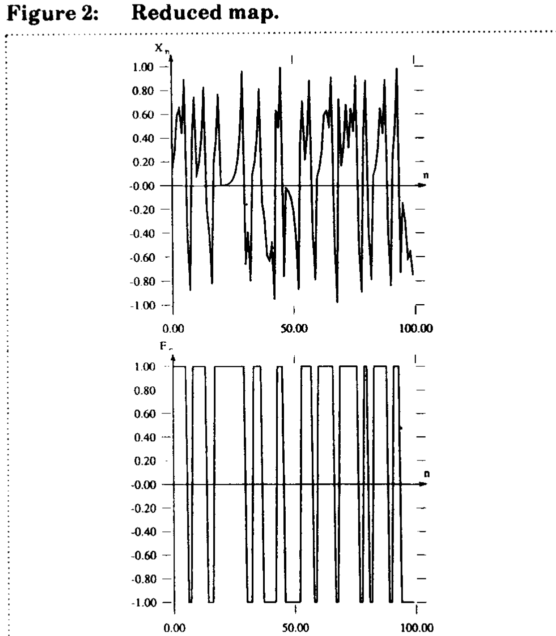

Figure 3: (a) Typical sequence $\{X_n\}$ verifying the "hopping mechanism". (b) Sequence $\{F_n\}$.

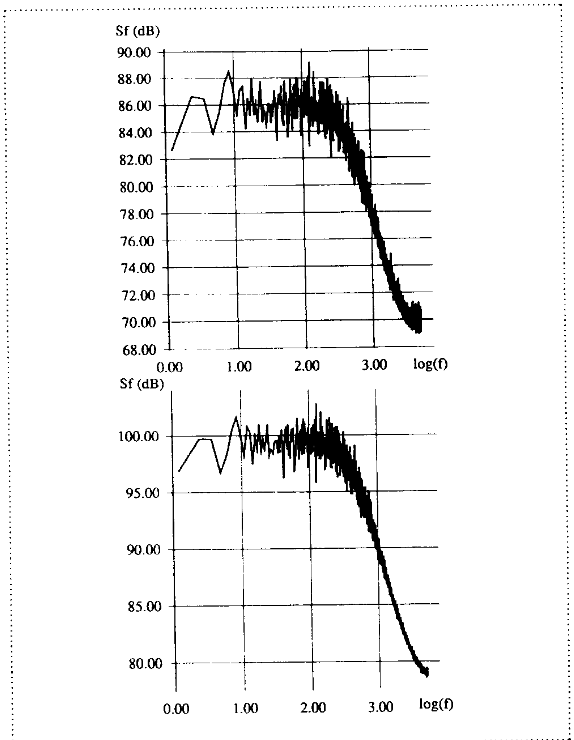

Figure 4: Power spectra of (a) $\{X_n\}$ and (b) $\{F_n\}$ for $m_c=2.043$, $m_l=-2.401$ and $B=1/m_c$.

After a few calculations the following can be found

$$f_0 \simeq \frac{\mu^* \varepsilon}{2\pi} f_c \qquad (5)$$

where μ^* is the average invariant density of $f(.)$ in the interval I_ε, ε is the length of this interval and f_c is the clock frequency. It is apparent now that by reducing ε one can push the cutoff to arbitrarily low frequencies.

Concerning to the invariant measure of $f(.)$, we can obtain a first-order solution by solving the Frobenius-Perron equation under the assumption $f(m_c, m_l, B; m_cB) > -C/m_l$. It can be demonstrated that

$$\mu(x) \simeq \begin{cases} \dfrac{\mu^*}{|m_l| - |m_l|/|m_c| - 1} & , x \in [0, -(m_c m_l B + C)] \\ \mu^* & , x \in [-(m_c m_l B + C), m_c B]] \end{cases} \qquad (6)$$

i.e., $\mu(x)$ is a staircase function (see Fig.5). After normalization, $\int \mu(x)dx = 1$, an approximated value of μ^* in terms of the parameters can be calculated.

A parasitic insensitive SC circuit

A stray insensitive SC circuit that generates the sequence $\{X_n\}$ is shown in Fig.6. The circuit is based on the use of parasitics and offset insensitive SC stages [Greg86]. The opamp labeled OA1 is used to implement a weighted summation yielding the piecewise-linear function $F(\bullet)$ of (1). Slopes in the map are controlled by the capacitors C_1, C_2, and C_3 as follows:

$$m_c = \frac{C_2}{C_3} \quad , \quad m_l = \frac{C_2 - C_1}{C_3} \qquad (7)$$

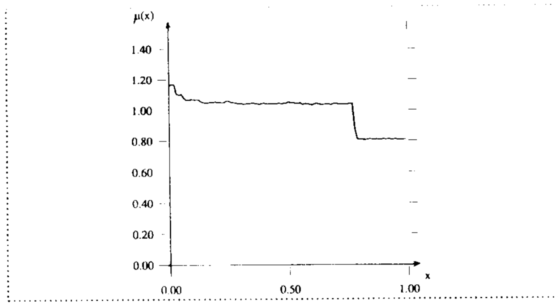

Figure 5: Numerical estimation of the invariant measure $\mu(x)$.

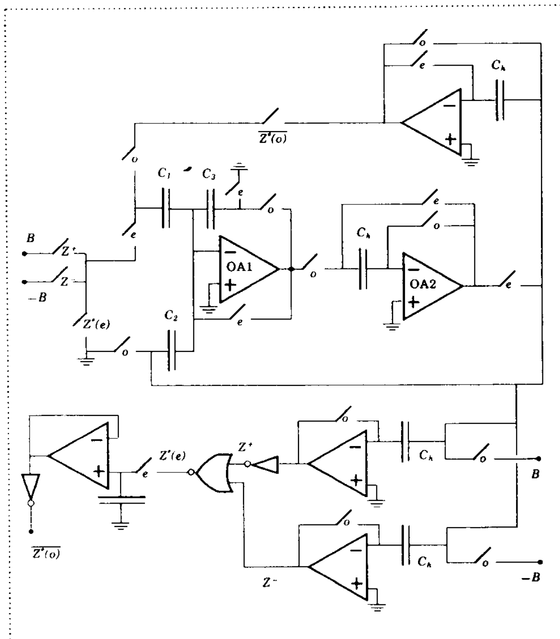

Figure 6: SC schematics for the discrete map.

For a proper practical implementation of Fig.6, design criteria should be given to prevent the circuit to be eventually locked at parasitic stable points caused by the opamp voltage saturation characteristics. A such criteria can be obtained from the previous theoretical analysis of the map (see Fig.1). Additionally, the box of length $2m_cB$ must be enclosed between the power supplies V_{DD} and V_{SS}.

These conditions guarantee that Fig.6 will, in response to the power-ON transient, self-start to generate a chaotic sequence whose values are comprised in the interval $[-m_cB, m_cB]$.

Experimental results

Fig. 6 has been breadboarded using off-the-shelves components. The measured results are in accordance to the numerical simulations. Fig.7 shows a typical waveform at the output of opamp OA1. The height of each pulse corresponds to X_n. Fig.8 shows measured power spectra obtained via numerical simulation and through the prototype in Fig. 6, for two sets of parameter values. Fig.8a is the simulation result for $m_c=2.662$, $m_l=-1.781$ and $B=2$. The corresponding experimental result is shown in Fig.8b. On the other hand, Figs.8c and d respectively show the simulation and the experimental result for $m_c=2.043$, $m_l=-2.029$ and $B=2$. A clock frequency of $f_c=10$kHz was used. The resulting noise power density spectra are confirmed by measurements to show $1/f^{\gamma}$ divergencies, with γ close to unity, in the frequency range going from f_0, see (5), to nearly $f_c/2$.

Summarizing, the results in the paper demonstrate the possibility of getting $1/f^{\gamma}$ noise by using analog IC techniques. Current work is directed towards the setting and implementation of a programmable colored noise generator using analog/digital MOS techniques.

References

[Arec82] F.T. Arecchi and F. Lisi; "Hopping Mechanism Generating 1/f Noise in Nonlinear Systems". *Phys. Rev. Lett. 49*, nº2, pp. 94-97, Jul. 1982.

[Chua90] L.O. Chua et al: "Generating Randomness from Chaos and Constructing Chaos with Desired Randomness". *Int. Journal Circuit Theory and Applications*, Vol.18, pp. 215-240, 1990.

[Cors88] G.Corsini and R. Saletti: "A $1/f^{\gamma}$ Power Spectrum Noise Sequence Generator". *IEEE Trans. Instrumentation and Measurement*, Vol.37, pp. 615-619, Dec. 1988.

[Espe90] S. Espejo-Meana et al.: "Application of Piecewise-Linear Switched-Capacitor Circuits for Random Number Generation". *Proc. of the 32nd Midwest Symposium on Circuits and Systems*, pp. 960-964.

[Greg86] R. Gregorian and G.C. Temes: "Analog MOS Integrated Circuits for Signal Processing". John Wiley 1986.

[Gupt75] M. Gupta: "Applications of Electrical Noise". *Proceedings of the IEEE*, Vol. 63, pp. 996-1010, July 1975.

[Horo89] P. Horowitz and W. Hill: *"The Art of Electronics (2nd edition)"*. Cambridge University Press 1989.

[Kesh82] M. S. Keshner: "1/f Noise", *Proc. of the IEEE*, vol. 70, pp. 212-218, Mar. 1982.

[McGo87] G.C. McGonigal and M.I. Elmasry: "Generation of Noise by Electronic Iteration of the Logistic Map". *IEEE Trans. Circuits and Systems*, vol. CAS. 34, pp. 981-983, Aug. 1987.

[Murc90] A.R. Murch and R.H.T. Bates: "Colored Noise Generation Through Deterministic Chaos". *IEEE Trans. Circuits and Systems*, vol. CAS. 37, pp. 608-613, May 1990.

[Rodr87a] A. Rodríguez-Vázquez et al.; "Chaos from Switched Capacitor Circuits: Discrete Maps". *Proceeding of the IEEE*, Vol.75, pp. 1090-1106, Aug. 1987.

[Rodr87b] A. Rodríguez-Vázquez et al.; "Chaos via a Piecewise-linear Switched Capacitor Circuit". *Electronics Letters*, Vol.23, pp. 662-663, Jun. 1987.

[Rodr90] A. Rodríguez-Vázquez et al.; "Analog Building Blocks for Noise and Truly Random Number Generation in CMOS VLSI". *Proceeding of 1990 European Conference on Solid-State Circuits*, pp. 225-228, Sept. 1990.

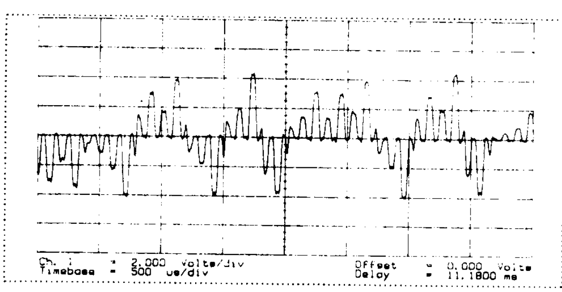

Figure 7: Signal at the output of OA1 in Fig.6 for $m_c=2.043$, $m_l=-2.203$ and $B=2v$.

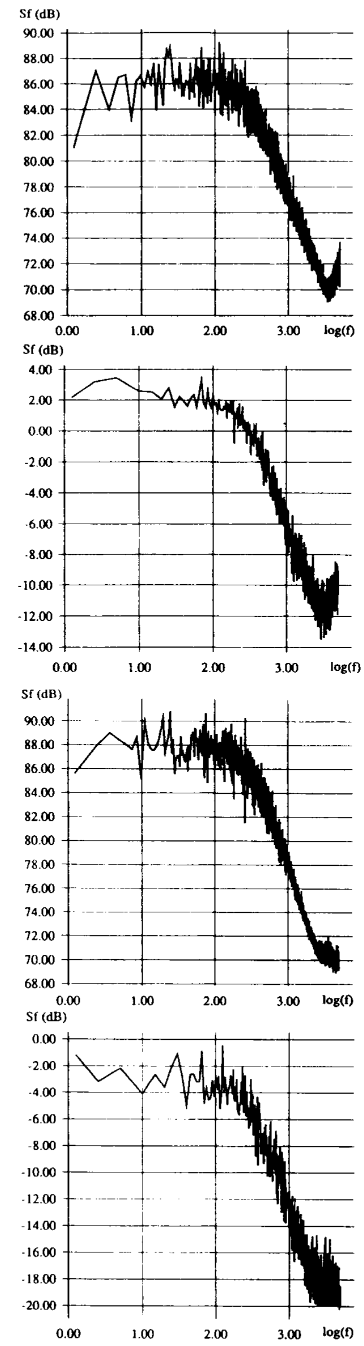

Figure 8: Measured spectra.