2016 European Control Conference (ECC)
June 29 - July 1, 2016. Aalborg, Denmark

Binary Search Algorithm for Mixed Integer Optimization: Application to energy management in a microgrid.

Paulo R. C. Mendes[1], José M. Maestre[2], Carlos Bordons[2] and Julio E. Normey-Rico[1]

Abstract— **This paper presents a binary search algorithm to deal with binary variables in mixed integer optimization problems. One example of this kind of problem is the optimal operation of hydrogen storage and energy sale and purchase into a microgrids context. In this work was studied a system composed by a microgrid that has a connection with the external electrical network and a charging station for electric cars. The system modeling was carried out by the Energy Hubs methodology. The proposed algorithm transforms the MIQP (Mixed Integer Quadratic Program) problem into a QP (Quadratic Program) that is easier to solve. In this way the overall control task is carried out the electricity purchase and sale to the power grid, maximizes the use of renewable energy sources, manages the use of energy storages and supplies the charge of the parked vehicles.**

I. INTRODUCTION

A microgrid is defined as a hybrid system which includes several sources and storage devices to fulfill the local loads [3]. Microgrids can be composed by distributed generation, renewable sources, storage devices, and local loads connected to the external network. The use of storage systems allows deciding the microgrid optimal operating point both in islanded mode and connected to grid and makes possible to manage the ideal time to exchange energy with the external network. Specifically, the hydrogen storage together with electric batteries and supercapacitors seems to be a suitable solution for renewable generation [7].

In addition, V2G systems enable to set new business models where new actors appear, such as load managers that are responsible for the recharging infrastructure, providing service to vehicles, buying or selling electrical energy. Microgrid managers have to provide the required demand to the load, decide the optimal operating point and manage the sale and purchase of energy with the external network.

In this work the case of study is composed by a microgrid connected to a external network and a V2G system. The microgrid manager and the load manager interact with each other to use cars batteries as storage and to provide the energy necessary to charge the cars batteries. On this scenery a distributed control structure becomes a good option, for sharing information about energy exchange to decide the optimal operating point for all system. In the last years many distributed model predictive control techniques were proposed on the literature [9]. In this work it will be used the DMPC (Distributed Model Predictive Control) presented in [12].

To model the system the concept of Energy hubs proposed in [6] will be used. According to the different characteristics of storages charge and discharge and the different prices of energy sale and purchase, binary variables can appear in the model characterizing a mixed integer optimization problem, which is computationally complex. The standard way to solve this kind of problem is to use a dedicated solver, for example, the solver CPLEX [1].

There are on the literature other algorithms to deal with the presence of the binary variables. In [15] and [2] the authors use a technique called TIO-MPC (Time Instant Optimization - MPC), where time instants are introduced as control inputs in place of binary variables. The time instants when the changes to the structures state should occur are optimized for a selected number of changes, resulting in a real-valued programming problem. In [14] and [13] a technique is presented that defines two new continuous manipulated variables for each binary manipulated variable, which corresponds to the duration of the activation/deactivation of the binary variables on the prediction horizon. In this way the original problem was turned into a non-linear optimization with continuous variables that can be solved using sequential quadratic programming (SQP).

The objective of this paper is to present an algorithm to deal with the binary variables in distributed mixed integer optimization problems and to illustrate its application to a microgrid. The basic idea is to provide an easy way to solve this kind of problem without a MIQP solver. The proposed technique first decides about the values of the binary variables and transforms the optimization problem into a QP.

The rest of the paper is organized as follows. Section II describes the model of the studied system. Section III presents the distributed MPC structure. Section IV proposes the algorithm to deal with mixed integer problem. Section V illustrates the potential of the proposed algorithm in a simulation study. Section VI concludes this paper.

II. SYSTEM DESCRIPTION

This work considers the model of the HyLab microgrid, located at the University of Seville, connected to a electric cars charging station. This microgrid was designed to study control strategies applied to energy management of a network that has hydrogen storage and renewable energy sources ([16], [17], [18]). The facility has special features

[1]Paulo R. C. Mendes and Julio E. Normey-Rico are with Department of Automation and Systems, Federal University of Santa Catarina, Florianpolis, Brazil paulo.mendes@ufsc.br, julio.normey@ufsc.br

[2]José M. Maestre and Carlos Bordons are with the Departamento de Ingeniería de Sistemas y Automática, University of Seville, Seville, Spain pepemaestre@us.es, bordons@us.es

that allow implementing and studying different operating modes and control strategies. Figure 1 shows the outline of the studied system. To replicate renewable energy systems,

Fig. 1. Hylab Microgrid

the microgrid has a programmable power supply that can emulate the dynamic behavior of a wind turbine and/or a photovoltaic field, for example. It also includes a battery bank, an electronic load to emulate different load profiles and, finally, includes a hydrogen storage system comprising a PEM (Proton Exchange Membrane) electrolyzer to produce hydrogen, a metal hydride system to store hydrogen and PEM fuel cell to produce energy. The electric car charging station has the capacity to charge four cars simultaneously.

From the microgrid operation point of view, usually the energy produced does not match the demand. Then, the excess energy from renewable sources can be stored in batteries or used to produce hydrogen through electrolysis. The hydrogen produced is stored in the metal hydride tank. Finally, when power from renewable sources is not available, the fuel cell uses hydrogen to supplement the lack of demand. Additionally, the microgrid has a connection to the main network allowing the energy purchase and sale. The hybrid storage allows operating strategies on two time scales: the battery can absorb/provide small amounts of energy on fast transients while hydrogen storage supplements biggest oscillations. In this sense, when the cars are parked, the cars batteries can be used by the microgrid to expand the buffer capacity of fast transients.

In this work, it was used the energy hub modeling methodology to model the system. An energy hub can be used to model the interface between energy production, consumer and the transmission line. From the standpoint of the system, an energy hub can be identified as a unit that provides the following features: (1) Input and output power; (2) energy conversion; (3) energy storage. The energy hub can be expressed as a generic MLD (Mixed Logical Dynamical) model described by

$$x(k+1) = Ax(k) + \Lambda u(k) \tag{1}$$

$$y(k) = \Gamma u(k) + \Pi_{in} w_{in}(k) \tag{2}$$

$$E_u u(k) \leq E_0 \tag{3}$$

$$w_{out}(k) = \Pi_{out} y(k) \tag{4}$$

with

$$\begin{aligned}
u(k) &= \begin{bmatrix} u^L(k) \\ u^E(k) \\ \delta(k) \\ z(k) \end{bmatrix} z(k) = u^E(k) \times \delta(k) \\
E_u &= \begin{bmatrix} \mathbf{0} & & & \\ & -E_u^E & & \\ & & E_\delta & \\ & & & E_z \end{bmatrix} \\
\Lambda &= \begin{bmatrix} e_{s,1}(k) & & \\ & \ddots & \\ & & e_{s,n_s}(k) \end{bmatrix} \\
e_s(k) &= \begin{cases} e_s^+ & \text{if} \quad u_s^E(k) \geq 0 \qquad (charging) \\ e_s^- & \qquad \text{else} \qquad (discharging) \end{cases}
\end{aligned}$$

where x is the system state; $\mathbf{u}$ is the input vector; u^L is the energy source input; u^E is the storage interface input; δ is the binary variable related with storage charge and discharge; z is an auxiliary continuous variable related with storage charge and discharge; w_{in} is the input interconnecting variable; w_{out} is the output interconnecting variable; $y(k)$ is the system output; A is the state matrix; e_s^+ and e_s^- are the efficiencies of charging/discharging interface s of the Hub; u_s^E is the storage interface output flow, where superscript E is associated to hub variables related to storage; Λ is the storage efficiency matrix; Γ is the coupling matrix, Π_{in} is the input interconnecting matrix; Π_{out} is the output interconnecting matrix; the matrices E_u,E_u^E, E_δ, E_z and the vector E_0 are used in MLD constraints. The binary variable $\delta(k)$ and the auxiliary variable $z(k)$ are used to deal with the different efficiencies of a storage charging and discharging or the different price of energy sale and purchase. More details about this modeling framework can be found in [5] and [11].

Figure 2 show the energy hub diagram of the system. The system is divided in two hubs. The first is the microgrid and the second is the charging station. There is an interconnecting variable that represents the power flow between the both hubs. To model the first hub it is necessary to

Fig. 2. Energy Hub Diagram

define the variable $z_{H2}(k)=P_{H2}(k)\delta_{H2}(k)$ that it is related to charging/discharging the hydrogen storage. As the battery bank is considered to have the same charging/discharging efficiency it is not necessary to define binary variables. To manage the purchase and sale of energy to the external network different weights for sale and purchase were used. To make this possible a new variable was defined $z_{Network}(k)= P_{Network}(k)\delta_{Network}(k)$ and introduced MLD constraints. The model is represented by the following input vector and set of matrices.

$$u = \begin{bmatrix} u^L \\ u^E \\ \delta \\ z \end{bmatrix} = \begin{bmatrix} P_{Solar} \\ P_{Network} \\ P_{aux} \\ \hline P_B \\ P_{H2} \\ \hline \delta_{H2} \\ \delta_{Network} \\ \hline z_{Network} \\ z_{H2} \end{bmatrix} \tag{5}$$

$$x = \begin{bmatrix} SOC \\ MHL \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Pi_{out} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\Lambda = \left[\begin{array}{ccc|cc|cc|cc} 0 & 0 & 0 & \eta_B & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta^{FC}_{H_2,e} & 0 & 0 & 0 & \eta^E_{e,H_2} - \eta^{FC}_{H_2,e} \end{array}\right]$$

$$\Gamma = \left[\begin{array}{ccc|cc|cc|cc} \eta^S_{rad,e} & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

where η stands for the storage and conversion efficiencies of the converters and storages; P_{Solar} is the solar power generated; $P_{Network}$ is the power of the external network; P_B is the power of the battery bank; P_{H2} is the power of the hydrogen storage. The states are the SOC (state of charge) of the batteries bank and the MHL (Metal Hydride Level) of the hydrogen storage. The conversion efficiencies values were obtained based on tests on the microgrid equipments.

The model of the second hub is defined by the following equations.

$$u = \begin{bmatrix} P_{Bc1} \\ P_{Bc2} \\ P_{Bc3} \\ P_{Bc4} \end{bmatrix} A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$\Lambda = \begin{bmatrix} \eta_{Bc1} & 0 & 0 & 0 \\ 0 & \eta_{Bc2} & 0 & 0 \\ 0 & 0 & \eta_{Bc3} & 0 \\ 0 & 0 & 0 & \eta_{Bc4} \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} -1 & -1 & -1 & -1 \end{bmatrix} \Pi_{in} = 1$$

where η_{Bc1},η_{Bc2}, η_{Bc3} and η_{Bc4} are the storage efficiencies of the car batteries; P_{BC1}, P_{BC2}, P_{BC3} and P_{BC4} are the car batteries powers. The states are the SOC (state of charge) of the cars batteries. In this case the output y will be zero because this hub don't have a external load demand, which implies in $w_{in}(k) = P_{BC1} + P_{BC2} + P_{BC3} + P_{BC4}$.

III. DISTRIBUTED OPTIMIZATON

This section gives details of the distributed predictive control structure focusing on optimizing the system. In this control structure, the global optimization problem is distributed between agents in the form of local optimization goals.

The optimization problem for the Hub 1 uses the following objective function:

$$\begin{aligned} J_{local1} = &\sum_{l=0}^{N_p-1} \hat{u}(k+l)^T Q \hat{u}(k+l) + f^T \hat{u}(k+l) + \\ &\sum_{l=0}^{N_p-1} (\hat{u}(k+l) - \hat{u}_{Solar-available}(k+l))^T Q_e \\ &(\hat{u}(k+l) - \hat{u}_{Solar-available}(k+l)) + \\ &\sum_{l=0}^{N_p-1} (\hat{x}(k+l) - \hat{x}_{ref}(k+l))^T Q_x (\hat{x}(k+l) - \hat{x}_{ref}(k+l)) \end{aligned} \tag{7}$$

subject to local dynamics (5) and the following constraints:

$$\underline{x}_i \leq \hat{x}_i(k+l+1) \leq \overline{x}_i \tag{8}$$

$$\underline{u}_i \leq \hat{u}_i(k+l) \leq \overline{u}_i \tag{9}$$

$$\hat{y}_i(k+l) = y_{dem}(k) \tag{10}$$

$$x_i(k) = \breve{x}_i(k) \tag{11}$$

for $l = 0, \ldots, N_p - 1$, where Q, Q_e, Q_x are positive definite weighting matrices, f is a vector and N_p is the prediction horizon. With respect to the notation, hat ($\hat{a}$) over variables is used to denote variables over the prediction horizon, $\underline{a}_i$ and $\bar{a}_i$ denote minimum and maximum allowed values respectively, and $\breve{a}_i$ refers to variables whose values are supposed to be known, for example, initial conditions. In this work we have assumed a bidirectional energy flow between the hub and the network so that the negative threshold is used.

The first term of the objective function is used for the management of the renewable energy sources and for purchasing power for the network. The weights Q and f are tuned according to the price of each energy source. The second term is responsible for ensuring the maximum use of solar energy source in order to minimize the error between the amount of power available ($\hat{u}_{Solar-available}$) and the amount of energy used ($u(1) = P_{Solar}$). The third term is responsible for maintaining the load of storage around 50% of the total load, and allows deviations from this value when there is need to store more energy or use the stored energy.

Analyzing only the part of the objective function in (7) related to the energy flow exchanged with the network, we have

$$\begin{aligned} J_{local1} = &\sum_{l=0}^{N_p-1} \hat{P}_{Network}(k+l)^T Q_{sale} \hat{P}_{Network}(k+l) + \\ &\hat{z}_{Network}(k+l)^T (Q_{purchase} - Q_{sale}) \hat{z}_{Network}(k+l) + \\ &f_{sale} \hat{P}_{Network}(k+l) + (f_{purchase} - f_{sale}) \hat{z}_{Network}(k+l) \end{aligned} \tag{12}$$

When power $P_{Network} > 0$ we have $\delta_{Network} = 1$ and $z_{Network}(k) = P_{Network}(k)$, which means that energy is purchased to the network and therefore the purchase weight is used. Otherwise $P_{Network} < 0$ implies $\delta_{Network} = 0$ and $z_{Network}(k) = 0$ and the sale weight is used. This makes it possible to use different weights for the same variable. The values of the weights were adjusted according to the price of energy.

The optimization problem for the Hub 2 uses the following objective function:

$$\begin{aligned} J_{local2} = &\sum_{l=0}^{N_p-1} (\hat{x}(k+l) - \hat{x}_{ref}(k+l))^T Q_x (\hat{x}(k+l) - \hat{x}_{ref}(k+l)) + \\ &(\hat{x}(k+N_p) - \hat{x}_{ref}(k+N_p))^T Q_{N_p} (\hat{x}(k+N_p) - \hat{x}_{ref}(k+N_p)) \end{aligned} \tag{13}$$

subject to local dynamics (6) and the constraints (8).

The objective function is used only to charge the batteries of electric vehicles according to the charging type, as will be explained below. The first term minimizes the error between the state and the state reference. The second term relative to the final state weights is introduced to ensure that the vehicle batteries will be fully charged at the end of the charging time.

To use the battery of electric vehicles over the network a supervisory algorithm that determines the charging type and the charging time was designed. When vehicle connect to the charge station, the user must inform the charging type (slow or fast) and the time the vehicle will be parked. If slow charge is chosen the battery charges over the parking time using low-power charge. In the case of fast charge the battery is charged with maximum power in a half hour period before the pre-set parking time, and in the rest of the time the battery is available for use as a storage for microgrid. During the period slow or fast charge the weights $\mathbf{Qx}$ and $\mathbf{Q}_{N_p}$ are tuned to a positive value in order to ensure that the load is charged on time. When the battery is used as a storage these weights are zero.

As both hubs are interconnected by the interconnection variable, both optimization problems have to satisfy the constraint $w_{out1}(k) = w_{in2}(k)$. To ensure that this constraint will be satisfied it is necessary to apply a distributed control algorithm based on communication between the agents. In this work, the Lagrange-based DMPC control structure with synchronous parallel communication presented in [12] was used. In a Lagrange-based control scheme, each control agent incorporates terms related with the interconnecting constraint in its local objective function. In order to divide the overall problem into local sub problems, the interconnecting constraints are removed from the constraint set and added to the objective function of the overall problem. This way, each local problem can be written as follows:

$$\min_{\substack{\hat{x}_i(k+1),\hat{u}_i(k),\hat{y}_i(k),\\ \hat{w}_{in,i}(k),\hat{w}_{out,i}(k)}} \begin{array}{c} J_{local,i}(\hat{x}_i(k+1),\hat{u}_i(k),\hat{y}_i(k))+ \\ J_{inter,i}(\hat{w}_{in,i}(k),\hat{w}_{out,i}(k)) \end{array} \tag{14}$$

where $J_{local,i}$ is the local objective function defined in (7) for the first hub and in (13) for the second hub, and $J_{inter,i}$ is the following interconnection function:

$$\begin{aligned} J_{inter,i}\left(\hat{w}^p_{in,i}(k),\hat{w}^p_{out,i}(k)\right) = & \begin{bmatrix} \hat{\lambda}^p_{in,i\leftarrow j}(k) \\ \hat{\lambda}^p_{out,i\rightarrow j}(k) \end{bmatrix}^T \begin{bmatrix} \hat{w}^p_{in,i\leftarrow j}(k) \\ \hat{w}^p_{out,i\rightarrow j}(k) \end{bmatrix} \\ & + \frac{\gamma_c}{2} \left\| \begin{bmatrix} \hat{w}^{p-1}_{in,j\leftarrow i}(k) - \hat{w}^p_{out,i\rightarrow j}(k) \\ \hat{w}^{p-1}_{out,j\rightarrow i}(k) - \hat{w}^p_{in,i\leftarrow j}(k) \end{bmatrix} \right\|_2^2 \\ & + \frac{\gamma_b - \gamma_c}{2} \left\| \begin{bmatrix} \hat{w}^p_{in,i\leftarrow j}(k) - \hat{w}^{p-1}_{in,i\leftarrow j}(k) \\ \hat{w}^p_{out,i\rightarrow j}(k) - \hat{w}^{p-1}_{out,i\rightarrow j}(k) \end{bmatrix} \right\|_2^2 \end{aligned} \tag{15}$$

where γ_c and γ_b are positive constants, $\hat{w}^{p-1}_{in,i\leftarrow j}(k)$ and $\hat{w}^{p-1}_{out,i\rightarrow j}(k)$ are the own information set of the previous iteration, $\hat{w}^{p-1}_{in,j\leftarrow i}(k)$ and $\hat{w}^{p-1}_{out,j\rightarrow i}(k)$ are the information set collected from the neighboring agent $j \in N_i$ at the previous iteration, the superscript p represents the actual iteration at the time instant k and $\hat{\lambda}^p_{in,i\leftarrow j}(k)$ and $\hat{\lambda}^p_{out,i\rightarrow j}(k)$ are the Lagrange multipliers.

At each time step k, each local agent must perform the iterative algorithm described below with parallel communication. One of the main drawbacks of Lagrangian methods is the slow convergence. The algorithm convergence speed can be improved through the so-called warm start by initializing the interconnecting inputs and the Lagrange multipliers every step to the values obtained from previous decision making step rather than initializing these values arbitrarily. The algorithm is implemented according to the following steps:

1) Make a measurement of current state $\hat{x}_i(k)$
2) Compute the optimal control sequence $\tilde{u}^*_i(k)$. To do so, perform the following steps:
   a) Parameter initialization:
      - for $k = 0$

$$\begin{aligned} p &= 1 \\ e_i &\gg 1 \\ \hat{\lambda}^0_{in,i\leftarrow j}(k) &= 0 \\ \hat{\lambda}^0_{out,i\rightarrow j}(k) &= 0 \\ \hat{w}^0_{in,i\leftarrow j}(k) &= 0 \\ \hat{w}^0_{out,j\rightarrow i}(k) &= 0 \end{aligned}$$

      - for $k > 0$ use warm start

$$\begin{aligned} \hat{\lambda}^0_{in,i\leftarrow j}(k) &= \hat{\lambda}^p_{in,i\leftarrow j}(k-1) \\ \hat{\lambda}^0_{out,i\rightarrow j}(k) &= \hat{\lambda}^p_{out,i\rightarrow j}(k-1) \\ \hat{w}^0_{in,i\leftarrow j}(k) &= \hat{w}^p_{in,i\leftarrow j}(k-1) \\ \hat{w}^0_{out,j\rightarrow i}(k) &= \hat{w}^p_{out,j\rightarrow i}(k-1) \\ p &= 1 \\ e_i &\gg 1 \end{aligned}$$

   for all $i \in N$.
   b) Solve the optimization problem in equation (14), subject to local dynamics and constraints.
   c) Send $\hat{w}^p_{in,i\leftarrow j}(k)$ and $\hat{w}^p_{out,i\rightarrow j}(k)$ to neighboring agents $j \in N_i$ and collect $\hat{w}^0_{in,j\leftarrow i}(k)$ and $\hat{w}^p_{out,j\rightarrow i}(k)$ from them, where $\hat{w}^p_{out,i\rightarrow j}(k)$ is calculated as:

$$w^p_{out,i\rightarrow j}(k) = \Pi_{out,i\rightarrow j} y^p_i(k) \tag{16}$$

   d) Upgrade the multipliers:

$$\hat{\lambda}^{p+1}_{in,i\leftarrow j}(k) = \hat{\lambda}^p_{in,i\leftarrow j}(k) + \gamma_c\left(\hat{w}^p_{in,i\leftarrow j}(k) - \hat{w}^p_{out,j\rightarrow i}(k)\right) \tag{17}$$

$$\hat{\lambda}^{p+1}_{out,i\rightarrow j}(k) = \hat{\lambda}^p_{out,i\rightarrow j}(k) + \gamma_c\left(\hat{w}^p_{out,i\rightarrow j}(k) - \hat{w}^p_{in,j\leftarrow i}(k)\right) \tag{18}$$

   e) Evaluate the stopping conditions:

$$p > \bar{p} \tag{19}$$

$$e_i = \left\| \hat{\lambda}^{p+1}_{in,i\leftarrow j}(k) - \hat{\lambda}^p_{in,i\leftarrow j}(k) \right\| \leq \bar{e} \tag{20}$$

   where $\bar{p}$ is the maximum number of iterations allowed and $\bar{e}$ is the maximum error allowed. If both conditions are false, move on to next iteration $p \rightarrow p+1$ and go to step $2b$. If (19) is true, go to step 3. If (20) is true, set agent i termination flag $flag_i(k) = 1$. If all neighboring agents $j \in N_i$ termination flags are equal to 1, go to step 3. If not, move on to next iteration and go to step $2b$.

3) Implement the optimal control action $\hat{u}_i^*(k)$.
4) Start a new control cycle $k \leftarrow k + 1$ and go to step 1.

IV. PROPOSED ALGORITHM

The optimization problem formulated in preview sections includes binary variables related to the hydrogen storage charging and discharging and energy sell and purchase to the external network. The presence of binary and continuous manipulated variables characterizes a mixed integer optimization, which is computationally complex and should be solved with dedicated MIQP solver. In this section a algorithm to deal with this kind of optimization problem will be presented. The basic idea is to decide the values of the binary variables and then solve the optimization problem as a QP. The algorithm is implemented at each energy hub by the following steps.

1) Define the prediction horizon (N_p) and the binary decision horizon (N_b) to test the possible combinations of each binary variable. The horizon N_b should be loser or equal than the prediction horizon.
2) Initialize a loop for $i = 1$ until N_b
3) Define the four possible combinations between the instants i and $i + 1$ on the horizon N for each binary variable. For the next instants the value should be equal to the second instant value.[1]
   a) Initialize a loop for $j = 1$ until 4.
   b) Introduce the j line of each δ_{ref} variable trough the following constraints:

$$\begin{aligned} \delta &= \delta_{ref} \\ z &= \delta_{ref} \times P \end{aligned} \tag{21}$$

   c) Solve the optimization problem with a QP solver.
   d) Compute the cost for this combination.
   e) Increment j and return to the item b.
4) Choose the combination with the minimum cost.
5) Fix the i column of the δ_{ref} with the value of j line and i column that implied in the minimum cost.[2]
6) Increment i and return to the item 2.
7) At least choose the combination that obtained the minimum cost and apply the calculated control signal to the process.

As introduced on item (b), new equality constraints relating the binary and continuous variables, it is not necessary to use anymore the MLD constrains on (3).

V. RESULTS

The proposed control strategy was applied to the system model. A simulation with a period of 24 hours and a sampling time of 5 minutes was performed using the software Matlab [10] with Yalmip toolbox[8]. A prediction horizon of $N_p = 5$ and a binary decision horizon of $N_b = 1$ were used. The performance of the proposed binary search algorithm is compared with the solver CPLEX. Both algorithms were implemented with the distributed optimization framework described on section III. The control objective is to maximize the use of renewable energy sources, make the purchase and sale management of electricity to the external network, use the storage to minimize the oscillations between the production and demand, perform the charging of electric vehicles and ensure the load demand at all periods of time.

In the Fig. 3 the graphs with dashed line are the results obtained with the proposed algorithm and the graphs with continuous line are the results obtained with CPLEX. From the analysis of Fig. 3(a), it can be verified that when there is no solar radiation the microgrid uses the storages and buys energy from external network. In the period where there is a surplus of solar energy the storages are charged and the excess energy is sold to the external network. Analyzing Fig. 3(b) the proposed algorithm had a different behavior with respect to hydrogen storage dynamic, this may be due by the choice of N_b horizon. The results of charge and discharging performed by the both algorithm were the same. Cars 1 and 3 use fast charge so that during the period of the time they are connected to the microgrid they function as storage and only in the last 30 minutes the batteries are loaded. Cars 2 and 4 use slow charge and take longer for charging. According the simulation both methods had similar results.

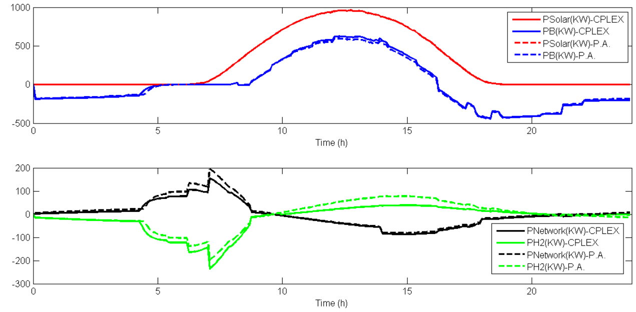
The difference between both methods is presented in Fig. 4. As the proposed algorithm computes four QP for each iteration of the distributed controller, it can be implemented in a parallel way. In this case, each QP may be computed by one core of the processor, and the total calculation time of one iteration will be approximately the time required to compute only one QP. In this scenery the proposed algorithm took 3.03 minutes to compute all simulations results versus the 6.04 minutes used by CPLEX. When analyze the total iterations and cost, the proposed algorithm obtained 1960 iterations and a cost of $6.6261e+006$ against 1982 iterations and cost of $8.2502e + 006$ reached by CPLEX. There is no guarantee that the proposed algorithm always will have smaller cost, but probably the value of N_b horizon has a important influence. According to these results the proposed algorithm is a good alternative to solve MIQP optimization problems when the sampling time is not too small.

[1] For example, using $N_p = 5$, $N_b = 2$ and $i = 1$ each binary variable will be set as:

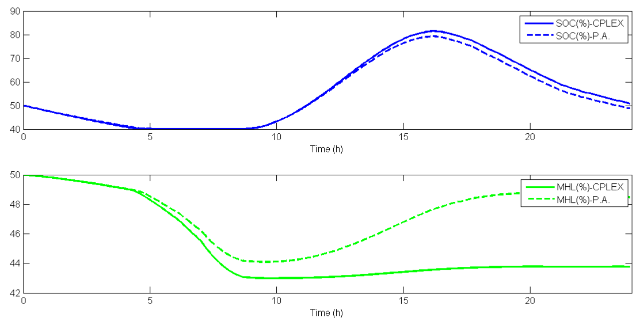
$$\delta_{ref} = \left[\begin{array}{cc|ccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

[2] For example, the small cost was obtained with third line of δ_{ref} and the value of j line and i column is 1, for the next iteration all elements of column i will have this value as:

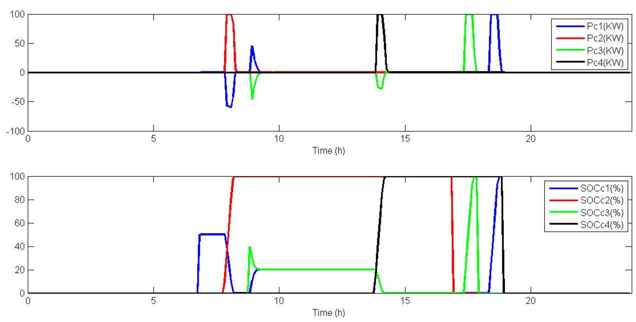
$$d = \left[\begin{array}{c|cccc} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$



(a) Energy sources



(b) Storages Level



(c) Cars

Fig. 3. Simulations results (cplex(-) and proposed algorithm (–)).

VI. CONCLUSIONS

The proposed strategy obtained satisfactory results, managing properly the purchase and sale of energy to the external network and making the charge of the electric cars batteries and ensuring the load demand. The proposed algorithm appears as a good alternative to deal with the binary variables in a distributed hybrid optimization problems. Further work will be done to implement the control framework on the real system and get some experimental results. A more exhaustive analysis will be carried out to assess the performance of the proposed approach in different operating conditions and with different systems.

ACKNOWLEDGMENT

The authors thank CNPq for financing Scholarship doctoral stay at the University of Seville and for financing the projects CNPq401126/2014-5 and CNPq303702/2011-7, and the Spanish Ministry of Economy and Competitiveness for financing the project DPI2013-46912-C2-1-R.

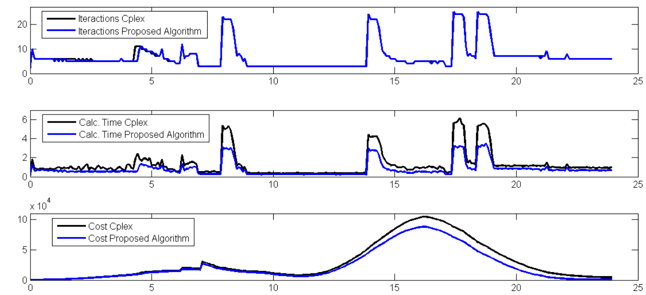


Fig. 4. Iterations, calculation time and cost (proposed algorithm and cplex).

REFERENCES

[1] ILOG, (2007) CPLEX.

[2] H. van Ekeren, R. R. Negenborn, P. J. van Overloop, and B. De Schutter, Hybrid model predictive control using time-instant optimization for the Rhine-Meuse delta, in Proc. International Conference on Networking, Sensing and Control, 2011.

[3] Q. Fu, A. Hamidi, A. Nasiri, V. Bhavaraju, S. Krstic, and P. Theisen, The Role of Energy Storage in a Microgrid Concept, Int. IEEE Electrification Magazine, 2013.

[4] M. Gautschi, O. Scheuss, C. Schluchter, Simulation of an Agent Based Vehicle-to-Grid (V2G) Implementation , Internal Report EEH Power Systems Laboratory, ETH Zurich, 2009.

[5] M. Geidl, Integrated Modeling and Optimization of Multi-Carrier Energy Systems, Ph.D. dissertation, ETH, Zurich, 2007.

[6] M. Geidl, G. Koeppel, P. Favre-Perrod, B. Klockl, G. Andersson and K. Frohlich, Energy hubs for the future, Int. IEEE Power and Energy Magazine, 2007.

[7] H.Jiayi, J.Chuanwen, X. Rongi, A review on distributed energy resources and Microgrid, Int. Renewable Sustainable Energy, 2008.

[8] Lfberg, J., (2004) YALMIP: A Toolbox for Modeling and Optimization in MATLAB, *Proceedings of the CACSD Conference*, Taiwan,

[9] J. M. Maestre and R. R. Negenborn, Distributed Model Predictive Control Made Easy, London, Springer-Verlag, 2014.

[10] Mathworks, (2009) Matlab.

[11] P.R.C. Mendes, and J. E. Normey-Rico, and C. Bordons, Economic Energy Management of a Microgrid Including Electric Vehicles, in Proc. Innovative Smart Grid Technologies Latin America ISGT-LA 2015.

[12] R. R. Negenborn, Multi-Agent Model Predictive Control with Applications to Power Networks , Ph.D. thesis, Delft University of Technology, 2007.

[13] C. de Prada, I. Grossmann, D. Sarabia, S. Cristea, A strategy for predictive control of a mixed continuous batch process, Int. Journal of Process Control, 2009.

[14] D. Sarabia, F. Capraro, L. F. S. Larsen, C. de Prada, I. Grossmann, S. Cristea, Hybrid NMPC of supermarket display cases, Int. Control Engineering Practice, 2009.

[15] B. De Schutter, B. De Moor, Optimal traffic light control for a single intersection, Int. European Journal of Control, 1998.

[16] Valverde, L., Bordons, C., Rosa, F., Power Management using Model Predictive Control in a hydrogen-based microgrid, in Proc. Annual Conference on IEEE Industrial Electronics Society, 2012.

[17] Valverde, L., Rosa, F., Bordons, C., Design, planning and management of a hydrogen-based microgrid, IEEE Transactions on Industrial Electronics, 2013.

[18] Valverde, L., Rosa, F., Del Real, A., Arce, A., Bordons, C., Modeling, Simulation and Experimental Set-up of a Renewable Hydrogen-based Domestic Microgrid, Int. Journal of Hydrogen Energy, 2013.