DYNAMIC STIFFNESSES OF FOUNDATIONS

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ABSTRACT. A general theory that describes the B.I.E.M. in
steady-state elastodynamics is developed. A comprehensive
formulation for homogeneous and heterogeneous media is presented
and also some results in practical cases as well as a general
review of several other possibilities.

1. The problem

A problem which has traditionally fascinated soil-dynamic
researchers has been the foundation design of vibrating machinery.
Several methods based either on the WINKLER idea (BARKAN -1962,
SAVINO - 1955, etc.) or in assumption of a elastic homogeneous,
isotropic halfspace (REISSNER - 1936, QUINLAN - 1953, SUNG
1953) have been developed.

Using that experience some authors have extended the
usefulness of those results to earthquake engineering problems.

The starting point was a paper published by REISSNER in
1936, in which, trying to establish a theoretical basis for the
DEGBO (Deutschen Forschungsgesellschaff fur Bodenmechanic)
experimental research, he used the LAMB'S (1904) problem
solution (half space underharmonic load to obtain the soil
response to an oscillating load applied through a rigid circular
plate).

The solution method was the integration of LAMB results in
the circular area, which means a uniform contact pressure.

REISSNER results were very different from experimental ones
mainly because of a sign error (SHECKTER 1948) in the algebra
and also because the uniform distribution does not produce the
equality of displacements required by the plate rigidity.

In the "Fifty-sixth Annual Meeting of the American Society for Testing Materials (July 1953)," P.M. QUINLAN and T.Y. SUNG presented two papers in which they repeated the analysis for "several" cases. The first was a static rigid plate distribution, the second a uniform one and the third a parabolic one more similar to a flexible plate case.

As can be seen all results were obtained through a simplification of the initial boundary problem.

For the rigid disk it is a mixed problem in which the displacement shape is known in the part under it and the tractions are nil in the rest of the half space boundary.

REISSNER and SUNG solved a different problem when they assumed a prescribed traction distribution under the plate. This leads as we said, to a series of displacements incompatible with the hypothesis of rigid plate and, consequently, the results must be used with great care.

In several reports, especially in the 1971-73 ones, VELETSOS and coworkers gave an interesting new approach and also useful numerical results. For instance, the case of rocking and horizontal displacement was treated (VELETSOS-WEI (1971)) with the following hypothesis:

a) during the horizontal imposed displacement there are no normal stresses in the half space boundary, in spite of the existence of vertical displacements.

b) in the rocking case there are no tangential stresses in the surface of the half space, in spite of the existence of horizontal displacements.

These hypotheses allow the computation of the horizontal case independently of the rocking one, but on the other hand, they produce no rigid displacements of the plate. Nevertheless through the use of a weighted reciprocity relation, this approach produced very accurate results.

The viscoelastic case (MEEK - VELETSOS (1973)) was also studied after a polynomial representation of previous elastic results.

By the way it is worth noting that the method presented by DAS GUPTA - SACKMANN (1977) using a correspondence principle for discretized set of values appears to be more promising.

The importance of the embedment of the footing was also recognized very early (RICHART 1960) and a large amount of work has been done by NOVAK and coworkers (1972, 1973...). The

fundamental idea is to admit that the soil around the footing is built by a series of elemental layers which react independently of the half space over which they are resting. The agreement with experimental results is, of course, not very good.

After these pioneering works a considerable amount of research has been done in recent years to determine the dynamic stiffness of foundations of various shapes (GAZETAS - ROESSET, 1976, WONG - LUCO, 1976, ELSABEE - MORRAY, 1977, KAUSSEL, ROESSET - CHRISTIAN, 1976...).

An interesting comparison between two and three dimensional solutions including the influence of embedment has been published recently (JAKUB, 1977).

All these results are currently applied to the cases in which the base of the structure behaves as a single rigid footing (base slab in nuclear reactor buildings, high-rise buildings on mat foundation, etc.). In order to permit analysis of other structures where this idealization is unreasonable, CHOPRA - coworkers (1969, 1974, 1975, 1976, etc.) have obtained results using a different approach.

The half space is treated as a substructure and the dynamic (frequency-dependent) stiffness matrix is of order equal to the number of connecting degrees of freedom on the structure-foundation interface.

In this way the i_j term of the matrix is defined as the force in the i mode when a harmonic unit displacement e^{iwt} is applied in the j mode, with other d.o.f. being kept fixed.

In the case of the pure plane horizontally homogeneous half space one needs the results only for two boundary problems: the vertical displacement and the horizontal one. An appropriate translation will provide the results for all the degrees of freedom. The displacement between the activated node and the neighbouring ones is assumed according to the isoparametric representation to be used in the subsequent F.E. analysis of the above ground structure.

Instead of solving this mixed problem directly CHOPRA et al. prescribed zero displacements also outside the structure-foundation interface, and after a standard condensation, they get the correct dynamic foundation stiffness matrix.

2. The numerical procedure

The elastodynamic equations are a classic in mathematical physics and several procedures have been devised in order to solve them.

In general, a weak formulation through a set of functions $\left[\psi^i\right]_n$ i.e., a projective method, is the most popular numerical approach.

Taking a member $\underline{\psi}^k$ of the family, the equilibrium equations can be put as

$$- \int_\Omega \sigma_{ij,j} \cdot \psi_i^k \cdot d\Omega = \int_\Omega X_i \cdot \psi_i^k \cdot d\Omega \qquad (2.1)$$

where σ_{ij} is the stress tensor and $\underline{X}$ the vector of body forces.

Integrating the left hand side of (2.1) and considering $\underline{\psi}^k$ as a displacement vector it is possible to write

$$\int_\Omega \varepsilon_{ij}^* \cdot \sigma_{ij} = \int_{\partial\Omega} \psi_i^k \cdot T_i^\nu + \int_\Omega X_i \cdot \psi_i^k \qquad (2.2)$$

where $\quad \varepsilon_{ij}^* = \frac{1}{2}(\psi_{i,j}^{*k} + \psi_{j,i}^{*k})$

If the "star" stresses are defined through

$$\sigma_{ij}^* = \lambda \varepsilon_{jj}^* \cdot \delta_{ij} + 2.G.\varepsilon_{ij}^* \qquad (2.3)$$

a relation, reciprocal of (2.2) would be

$$\int_\Omega \varepsilon_{ij} \cdot \sigma_{ij}^* = \int_{\partial\Omega} U_i \cdot T_i^* + \int_\Omega X_i^* \cdot u_i \qquad (2.4)$$

But the same equations (2.3) show that the left hand side of (2.2) and (2.4) are the same, so

$$\int_{\partial\Omega} \psi_i^k \cdot T_i^\nu + \int_\Omega X_i \cdot \psi_i^k = \int_{\partial\Omega} U_i \cdot T_i^{\nu *} + \int X_i^* \cdot u_i \qquad (2.5)$$

(2.2) are the starting point for the F.E.M. while (2.5) is the corresponding one for the B.I.E.M. Notice that for (2.4) to be valid ψ^k need to be a classical solution of the field equations, (2.5) is, in vectorial form

$$\int_{\partial\Omega} \underline{\psi} \cdot \underline{T}^\nu - \int_{\partial\Omega} \underline{u} \cdot \underline{T}^* = \underline{u}(\underline{x}) - \int_\Omega \rho \underline{\ddot{u}} \cdot \underline{\psi}^k \qquad (2.6)$$

In order to eliminate the time dependency CRUSE & RIZZO proposed to work in the frequency domain. The problem is then reduced to a quasi-static one with the following fundamental solution

$$\psi_i = U_{ij} \cdot e_j \qquad T_i^\nu = T_{ij} \cdot e_j \qquad (2.7)$$

$$U_{ij} = \frac{1}{\alpha\rho\Pi c_2^2} [\psi.\delta_{ij} - \chi.r_{,i}.r_{,j}]$$

$$T_{ij} = \frac{1}{\alpha\pi} \left[\left(\frac{\partial\psi}{\partial r} - \frac{1}{r}\chi\right)(\delta_{ij}\frac{\partial r}{\partial\nu} + r_{,j}\nu_i) - \frac{2}{r}\chi(\nu_j r_{,j} - \right.$$

$$\left. - 2r_{,i}\cdot r_{,j}\frac{\partial r}{\partial\nu}) - 2\frac{\partial\chi}{\partial r}r_{,i}r_{,j}\frac{\partial r}{\partial\nu} + \left(\frac{c_1^2}{c_2^2}-2\right)\left(\frac{\partial\psi}{\partial r} - \frac{\partial\chi}{\partial r} - \frac{\alpha}{2r}\chi\right)r_{,i}\nu_j \right]$$

where, for instance, in 2-D cases

$$\psi = K_o\left(\frac{kr}{C_2}\right) + \frac{C_2}{kr}\left[K_1\left(\frac{kr}{C_2}\right) - \frac{C_2}{C_1}K_1\left(\frac{kr}{C_1}\right)\right]$$

$$\chi = K_2\left(\frac{kr}{C_2}\right) - \frac{C_2^2}{C_1^2}K_2\left(\frac{kr}{C_1}\right)$$

in which
$$C_1^2 = \frac{\lambda+2\mu}{\rho}$$
$\lambda\mu$ Lame CTS
$$C_2^2 = \frac{\mu}{\rho}$$
ρ density

$$k = i.\omega$$ (2.9)

$$K_i = K_o(k.r/c)$$

ω is the chosen frequency and r is the distance between the point of load application and observation and K_o is the modified Bessel function of zero order and second kind.

In ref 1 the 3-D fundamental solution is also presented. Under these conditions eq. (2.6) reduces to

$$\underline{u}\,(\underline{x}) + \int_{\partial\Omega}\underline{u}\cdot\underline{T}^* = \int_{\partial\Omega}\underline{\psi}\cdot\underline{T}$$ $\underline{x}\in D$ (2.10)

$\underline{X}$ being the point of load application. In order to manage values of $\underline{X}$ only the boundary is necessary to take a limit process after which

$$\underline{c}\cdot\underline{u}(\underline{x}) + \int_{\partial\Omega}\underline{u}\cdot\underline{T}^* = \int_{\partial\Omega}\underline{\psi}\cdot\underline{T}$$ $\underline{x}\in D$ (2.11)

and $\underline{c}$ is a numerical matrix which reflects the geometrical properties of the boundary round $\underline{X}$. If, for instance, the boundary is smooth $\underline{c} = \frac{1}{2}\underline{I}$ where I is the identity matrix.

The numerical procedure known as Boundary Integral Equation method (B.I.E.M.) produces a set of linear equations by discretizing the boundary with n points and writing n set equations of the form assuming as family $\left[\psi^k\right]_n$ the set of fundamental solutions $\delta(\underline{X})$ where $\underline{X}$ are the chosen n points at the boundary.

As round every point $\underline{X}_i$ we have 6 unknowns (in the case of plane conditions) i.e.: the two components u,v of diplacements and

the four components of traction $[\sigma,\tau]^b$ $[\sigma,\tau]^a$ before and after
the point, it is necessary to assume an evolution of $\underline{u}$ and $\underline{T}$
between "nodes" and to establish 4n boundary conditions in order
to obtain a compatible system of equations.

In B.I.E.M. one usually takes a family of locally based
functions whose superposition represents the tractions and the
displacements

$$\underline{u} = \Sigma \ \underline{u}^i . \underline{\xi}^i$$
$$\underline{T} = \Sigma \ \underline{T}^i . \underline{\xi}^{*i}$$

(2.12)

of course ξ_j^i and $\underline{\xi}^{*i}$ need not be the same, but it is usual to
put $\underline{\xi}^j = \underline{\xi}^{*i}$. The locally based character is useful in the sense
that $\underline{u}^i$ and $\underline{T}^i$ have physical meaning, but this does not help in the
bandedness of the resultant matrix because the weighting family
$|\Psi k|_n$ is defined everywhere and the final matrix is full.

This is perhaps the most important drawback of B.I.E.M. in
face of F.E.M. Its main advantage is of course, the reduction
of the dimension by one.

A "linear element discretization", for instance, is

$$\underline{u} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} N_1\ N_2 & \vdots & 0\ \ 0 \\ \cdots\cdots\cdots & \vdots & \cdots\cdots \\ 0\ \ 0 & \vdots & N_1\ N_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ --- \\ v_1 \\ v_2 \end{bmatrix}^e$$

(2.13)

$$\underline{T} = \begin{bmatrix} \bar{X} \\ \bar{Y} \end{bmatrix} = \begin{bmatrix} N_1\ N_2 & \vdots & 0\ \ 0 \\ \cdots\cdots\cdots & \vdots & \cdots\cdots \\ 0\ \ 0 & \vdots & N_1\ N_2 \end{bmatrix} \begin{bmatrix} \bar{X}_1 \\ \bar{X}_2 \\ \cdots \\ \bar{Y}_1 \\ \bar{Y}_2 \end{bmatrix}^e$$

where
$$N_1 = -\tfrac{1}{2}(\eta - 1)$$
$$N_2 = -\tfrac{1}{2}(\eta + 1)$$

(2.14)

$$dS^e = \tfrac{1}{2} L^e \, d\eta$$
$$-1 \le \eta \le 1$$

η being dimensionless length and e representing element.
Writing (2.11) now

$$2\underline{C}\begin{bmatrix} u \\ v \end{bmatrix}_i + \sum_{j=1}^{N} L \begin{bmatrix} \int_{\partial D_j} N_1 T_{11} d\xi & \vdots & \int_{\partial D_j} N_2 T_{11} d\xi & \vdots & \int_{\partial D_j} N_1 T_{12} d\xi & \vdots & \int_{\partial D_j} N_2 T_{12} d\xi \\ \cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots & \vdots & \cdots\cdots \\ \int_{\partial D_j} N_1 T_{21} d\xi & \vdots & \int_{\partial D_j} N_2 T_{21} d\xi & \vdots & \int_{\partial D_j} N_1 T_{22} d\xi & \vdots & \int_{\partial D_j} N_2 T_{22} d\xi \end{bmatrix} . \begin{bmatrix} u_j \\ \cdots \\ u_{j+1} \\ v_j \\ \cdots \\ v_{j+1} \end{bmatrix} =$$

$$= \sum_{j=1}^{N} L_j \begin{bmatrix} \int_{\partial D_j} N_1 U_{11} d & \vdots \int_{\partial D_j} N_2 U_{11} d & \vdots \int_{\partial D_j} N_1 U_{12} d & \vdots \int_{\partial D_j} N_2 U_{12} d \\ \int_{\partial D_j} N_1 U_{21} d & \vdots \int_{\partial D_j} N_2 U_{21} d & \vdots \int_{\partial D_j} N_1 U_{22} d & \vdots \int_{\partial D_j} N_2 U_{22} d \end{bmatrix} \cdot \begin{bmatrix} \dot{X}_j \\ \cdots X_{j+1} \cdots \\ \dot{Y}_j \\ \cdots Y_{j+1} \cdots \end{bmatrix}$$

$$(2.15)$$

Finally a system of equations

$$\underset{(2n \times 2n)}{A} \cdot \underset{(2n \times 1)}{u} = \underset{(2n \times 4n)}{B} \cdot \underset{(4n \times 1)}{T}$$

is prepared to receive the boundary conditions. After they are imposed we have to solve

$$\underset{(2n \times 2n)}{K} \cdot \underset{(2n \times 1)}{X} = \underset{(2n \times 1)}{f} \qquad (2.17)$$

where $\underline{X}$ collects the unknowns and $\underline{f}$ the data, both weighted by the appropriate element from A & B. More details can be seen elsewhere (ref 2).

In the heterogeneous case the approach is also very simple.

Assume, for instance, a matrix D with boundary Γ and properties E and ν, and two inclusions $(\bar{D}, \bar{\Gamma}, \bar{E}, \bar{\nu})$, $(\bar{\bar{D}}, \bar{\bar{\Gamma}}, \bar{\bar{E}}, \bar{\bar{\nu}})$

The field equations are established in ordered fashion.
Assume respective discretizations with N, $\bar{n}$, $\bar{\bar{n}}$, linear elements. Equation (2.16) in region D can be written in a partitioned form.

$$\begin{bmatrix} A, & A_1, & A_1 \end{bmatrix} \begin{bmatrix} u \\ \bar{u}^1 \\ \bar{\bar{u}}^1 \end{bmatrix} = \begin{bmatrix} B, & B_1, & B_1 \end{bmatrix} \begin{bmatrix} T \\ \bar{T}^1 \\ \bar{\bar{T}}^1 \end{bmatrix}$$

$$(2.18)$$

$$(2n_t \times 2n_t)(2n_t \times 1) \qquad (2n_t \times 4n_t)(4n_t \times 1)$$

and similarly for $\bar{D}$ and $\bar{\bar{D}}$

$$\bar{A}_2 \cdot \bar{u}^2 = \bar{B}_2 \cdot \bar{T}^2$$

$$(2\bar{n} \times 2\bar{n})(2\bar{n} \times 1) \qquad (2\bar{n} \times 4\bar{n})(4\bar{n} \times 1) \qquad (2.19)$$

$$\bar{\bar{A}}_3 \cdot \bar{\bar{u}}^3 = \bar{\bar{B}}_3 \cdot \bar{\bar{T}}^3$$

$$(2\bar{\bar{n}} \times 2\bar{\bar{n}})(2\bar{\bar{n}} \times 1) \qquad (2\bar{\bar{n}} \times 4\bar{\bar{n}})(4\bar{\bar{n}} \times 1)$$

conditions of compatibility and equilibrium are

$$\bar{u}^1 = \bar{u}^2 = \bar{u}$$
$$\bar{\bar{u}}^1 = \bar{\bar{u}}^2 = \bar{\bar{u}}$$

$$\bar{T}^1 = -\bar{T}^2 = \bar{T}$$
$$\bar{\bar{T}}^2 = -\bar{\bar{T}}^3 = \bar{\bar{T}}$$

grouping (2.18) and (2.19) using (2.20) produces

$$n_t = N + \bar{n} + \bar{\bar{n}}$$

$$
\begin{array}{c} 2n_t \\ 2\bar{n} \\ 2\bar{\bar{n}} \end{array}
\begin{bmatrix} A & \bar{\bar{A}}_1 & \bar{\bar{A}}_1 \\ 0 & \bar{A}_2 & 0 \\ 0 & 0 & \bar{\bar{A}}_3 \end{bmatrix}
\begin{bmatrix} u \\ \bar{u} \\ \bar{\bar{u}} \end{bmatrix}
=
\begin{bmatrix} B_1 & \bar{\bar{B}}_1 & \bar{\bar{B}}_1 \\ 0 & -\bar{B}_2 & 0 \\ 0 & 0 & \bar{\bar{B}}_3 \end{bmatrix}
\begin{bmatrix} T \\ \bar{T} \\ \bar{\bar{T}} \end{bmatrix}
\qquad (2.21)
$$
$$\begin{array}{ccc} 2N & 2\bar{n} & 2\bar{\bar{n}} \end{array} \qquad\qquad 4n_t$$

which for a NEUMANN problem can be written

$$
\begin{array}{ccccc} 2N & 2\bar{n} & 4\bar{n} & 2\bar{\bar{n}} & 4\bar{\bar{n}} \end{array}
$$
$$
\begin{bmatrix} A & \bar{\bar{A}}_1 & B_1 & \bar{\bar{A}}_1 & \bar{\bar{B}}_1 \\ 0 & \bar{A}_2 & \bar{B}_2 & 0 & 0 \\ 0 & 0 & 0 & \bar{\bar{A}}_3 & \bar{\bar{B}}_3 \end{bmatrix}
\begin{bmatrix} u \\ \bar{u} \\ \bar{T} \\ \bar{\bar{u}} \\ \bar{\bar{T}} \end{bmatrix}
=
\begin{bmatrix} B & T \\ 0 \\ 0 \end{bmatrix}
\begin{array}{c} 2N-\bar{n}+\bar{\bar{n}} \\ 2\bar{n} \\ 2\bar{\bar{n}} \end{array}
\qquad (2.22)
$$

Imposing conditions on $\bar{T}$ and $\bar{\bar{T}}$, for instance

in the case of smooth boundaries one can solve a system of $2N+4(\bar{n}+\bar{\bar{n}})$ equations with the same amount of unknowns i.e.: the 2N component displacements of the outer boundary and the $4(\bar{n}+\bar{\bar{n}})$ displacements and tractions at the interior boundaries.

The generalization to other boundary conditions or number of inclusions is straightforward. Also interesting is the case in which the regions are connected in series, because then the matrix of the left hand side in eq. (2.22) takes a banded form.

This has been used in cases with a very narrow domain even with homogeneous properties.

REFERENCES

1. CRUSE & RIZZO: "A direct formulation and numerical solution of the general transient elastodynamic problem". Journ. of Math. Anal. & App. 1968.
2. ALARCON, MARTIN & PARIS: "Boundary elements in potential and elasticity theory". Computers and Structures. 1979.
3. ALARCON, DOMINGUEZ, MARTIN and PARIS: "Boundary methods in soil-structure interaction" 2nd Conference Microzonation. Dec - 78.
4. DOMINGUEZ: "Dynamic stiffness of rectangular foundations" M.I.T. Pub. R78-20. A.g. 1978.

EXAMPLES

In order to see the behaviour of the method we started analyzing the response with several elements outside the rigid foundation.

Plate No. 1 records the relative results for a surface foundation and plate No. 2 for embedded one. In abscissa we have the number of last element and in ordinates the relative values of factors (taking as reference the finer mesh).

The Oscillating shape of results reflects the modelling of waves. It is seen that for the surface case the results without discretizing the exterior boundary are good enough and that for embedded foundations we need at least six elements.

Plates 3 and 4 represent the results putting in ordinates the actual value of stiffness.

In this case we use a complex G in order to investigate the possibility of including material damping. Nevertheless the amount is small so the results can be compared with the previous ones, (as Jakub's).

Looking at this picture, the possibility of truncating the discretization not far from the footing is clear.

In plates No. 5 the results are compared with the analytical ones of KARASUDHI, KEER & LEE.

Notice the remarkable accuracy obtained without discretizing the exterior boundary.

Plate No. 6 shows the same kind of results for an embedded foundation studied with only twenty-eight constant elements.

Plate No. 7 shows the graph of surface displacements for a dimensionless frequency $a_o = wb/c_s = 0.5$ where w is the excitation frequency, b the width of the plate and c_s the celerity of S waves.

Finally plate No. 8 suggests some of the possible applications of the method.
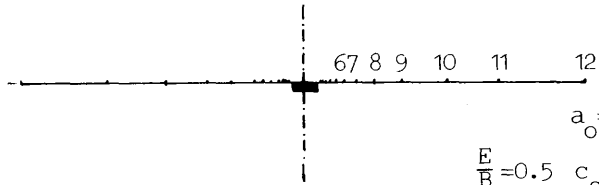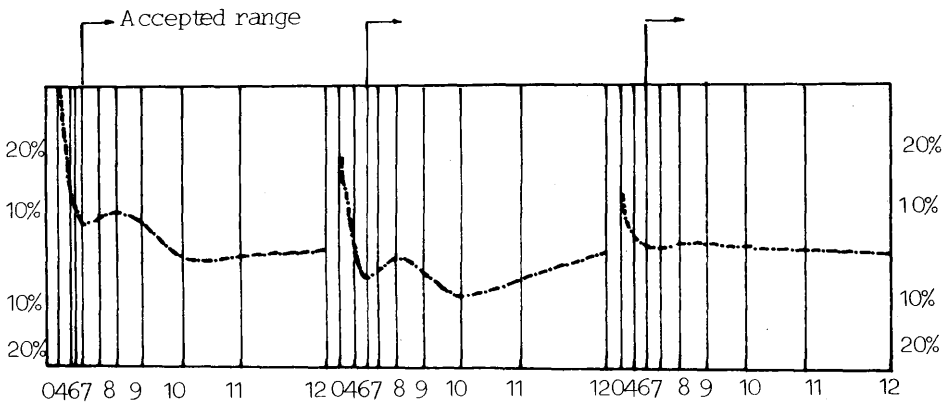
SURFACE FOUNDATION–STRIP FOOTING

16 elements uniformly spaced inside the rigid plate

DISCRETIZATION

HALF SPACE

$a_0 = 0.5$
$c_s = 1.$
$\rho = 1.$

D_9

D_{10}

$K_j = k_j + i.c_j$

$\dfrac{k_x}{(k_x)_{20}}$ $\dfrac{k_y}{(k_y)_{20}}$ $\dfrac{k_\theta}{(k_\theta)_{20}}$

20%
10%
100
10%
D/b D/b · D/b 20%

8 9 10 11 12 56 7 8 9 10 11 1 35 7 8 9 10 11 12

REAL PART

$\dfrac{c_x}{(c_x)_{20}}$ $\dfrac{c_y}{(c_y)_{20}}$ $\dfrac{c_\theta}{(c_\theta)_{20}}$

20%
10%
100
100
13%
7.4% 1.4% 10%
20%

3 56 7 8 9 10 11 35 6 7 8 9 10 11 35 6 7 8 9 10 11

IMAGINARY PART

Plate   1

EMBEDDED STRIP FOUNDATION

6 7 8 9   10   11   12

$a_o = 0.5$

$\frac{E}{B} = 0.5$ $c_s = 1.$ $c_p = 2.$

$\rho = 1.$

Accepted range

20%
10%
10%
20%

0 4 6 7 8 9   10   11   12 0 4 6 7 8 9   10   11   12 0 4 6 7 8 9   10   11   12

REAL PART

20%
10%
10%
20%

0 4 6 7 8 9   10   11   12 0 4 6 7 8 9   10   11   12 0 4 6 7 8 9   10   11   12

IMAGINARY PART

Plate   2

SURFACE FOUNDATION STRIP FOOTING

$c_s = 1.$ $c_p = 2.$
$\rho = 1.$

$k_{xo}c_1/G$

$k_{xo}k_1/G$

$K_x = k_{xo}(k_1 + ia_o c_1)$

0.1π 0.3π 0.5π a_o

$k_{yo}c_1/G$

$k_{yo}k_2/G$

$K_y = k_{yo}(k_2 + ia_o c_2)$

0.1π 0.3π 0.5π a_o

k_θ , c_θ

$k_{\theta o} = 2.399$ Luco: 2.355
                          Jakub: 2.43

$K_\theta = k_{\theta o}(k_\theta + ia_o c_\theta)$

k .1–0.2a_o Jakub

$c = a_o^2/(1+2a_o^2)$

0.1π 0.3π 0.5π a_o

Plate   3

BOUNDARY DISCRETIZATION
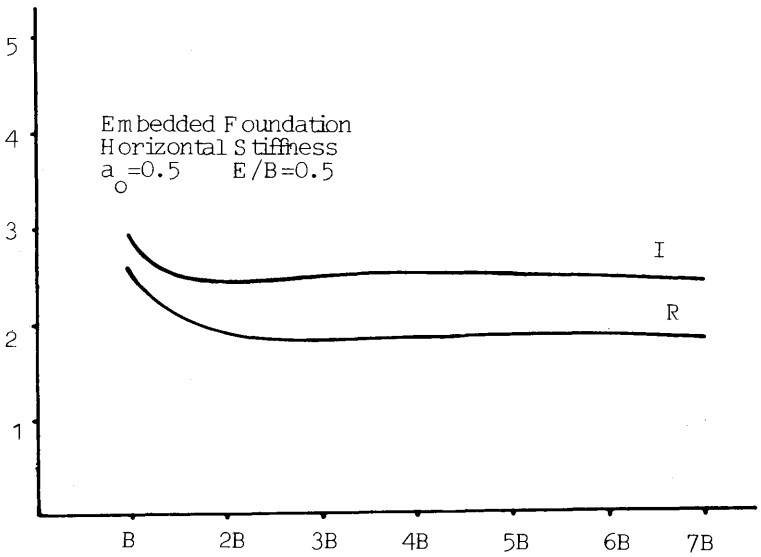
EMBEDDED STRIP FOUNDATION

$c_s = 1.$ $\rho = 1.$

$c_p = 2.$ $E/B = 0.5$

$$\frac{K_{xo} \cdot c_1}{G}$$

$$\frac{K_{xo} \cdot K_1}{G}$$

$$-K_x = K_{xo}(K_1 + iac_1)-$$

$$\frac{K_{yo} \cdot c_2}{G}$$

$$\frac{K_{yo} \cdot K_2}{G}$$

$$-K_y = K_{yo}(K_2 + iac_2)-$$

$$\frac{K_{\theta o} \cdot K_\theta}{G}$$

$$\frac{K_{\theta o} \cdot c_\theta}{G}$$

$$-K_\theta = K_{\theta o}(K_\theta + ia_o c_\theta)-$$

Plate   4

SURFACE STRIP FOOTING

Vert.   Horiz.

Vert.   Horiz.

Vert.   Horiz.
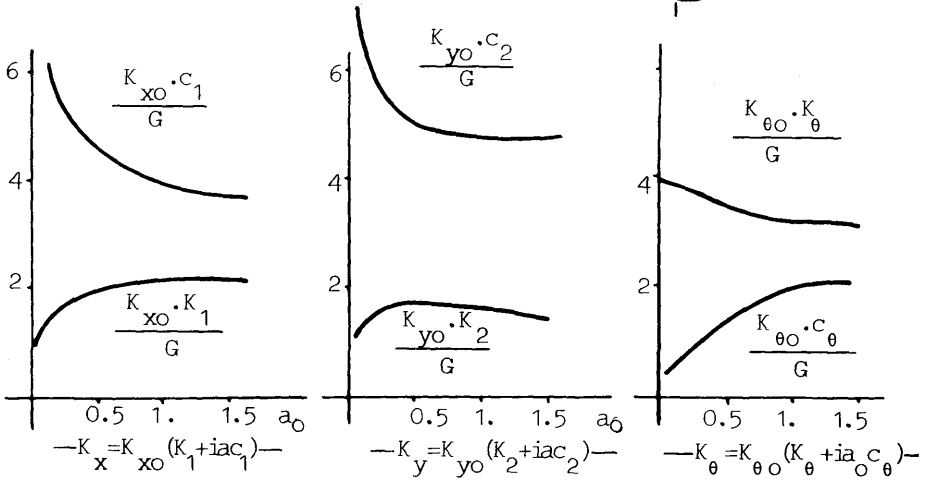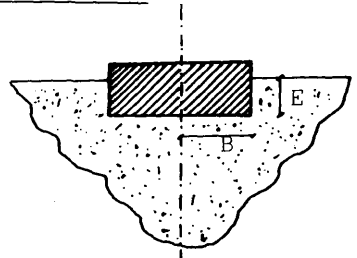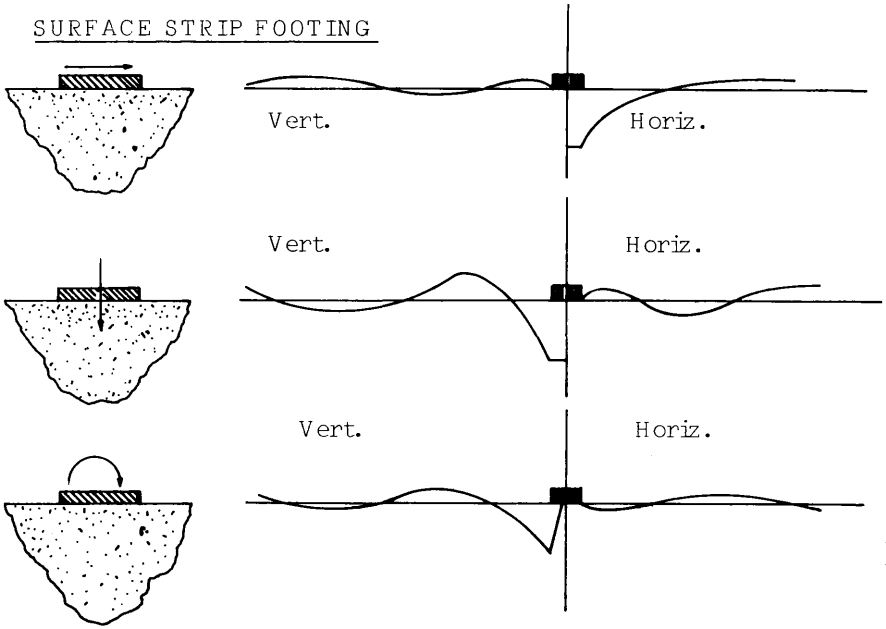
SURFACE DISPLACEMENTS $a_o=0.5$
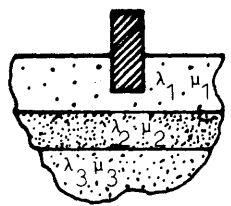
EMBEDDED STRIP FOOTING

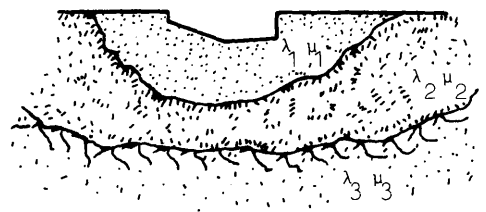Vert.   Horiz.

Vert.   Horiz.

Vert.   Horiz.

Plate   5

Embedded Foundation
Horizontal Stiffness
$a_o = 0.5$ $E/B = 0.5$

I

R

B   2B   3B   4B   5B   6B   7B

Embedded Foundation
Horizontal Stiffness
$a_o = 1.$
$E/B = 0.5$

I

R

B   2B   3B   4B   5B   6B   7B

Plate   6

Surface Foundation
Rocking Stiffness
$G=1+0.1i$
$\nu=1/3$
$a_o = \omega B/c_s = 0.01$

- - - M. Jakub

R

I

B   2B   3B   4B   5B   6B   7B

Embedde Foundation
Rocking Stiffness
$a_o=0.01$
$E/B=0.5$

- - - - M. Jakub

R

I

B   2B   3B   4B   5B   6B   7B

Plate   7

(a)

(b)

$v=1$
$\sigma_x=0$

$\wedge v=1$
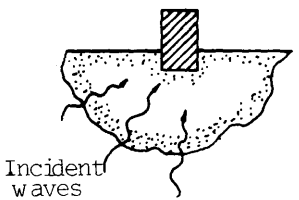$\sigma_x=0$

$v=1$
$\tau_{xy}=0$
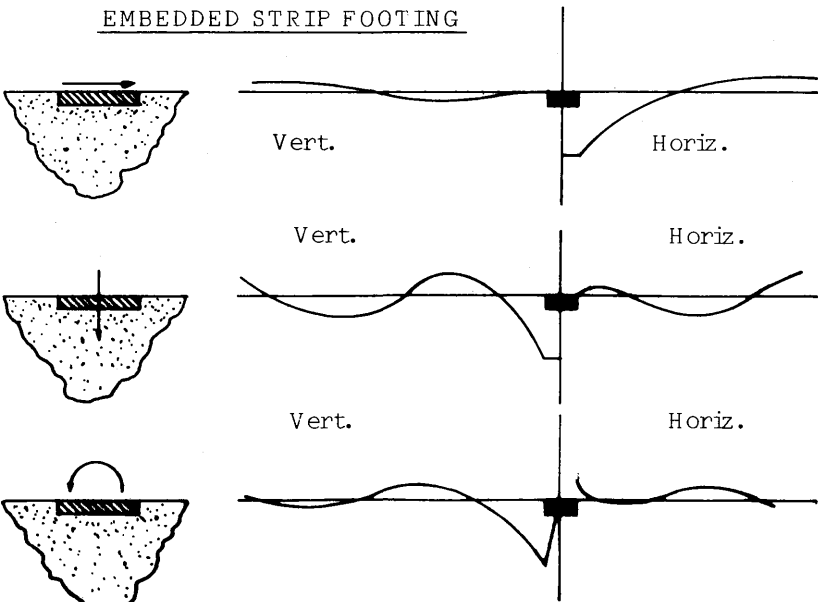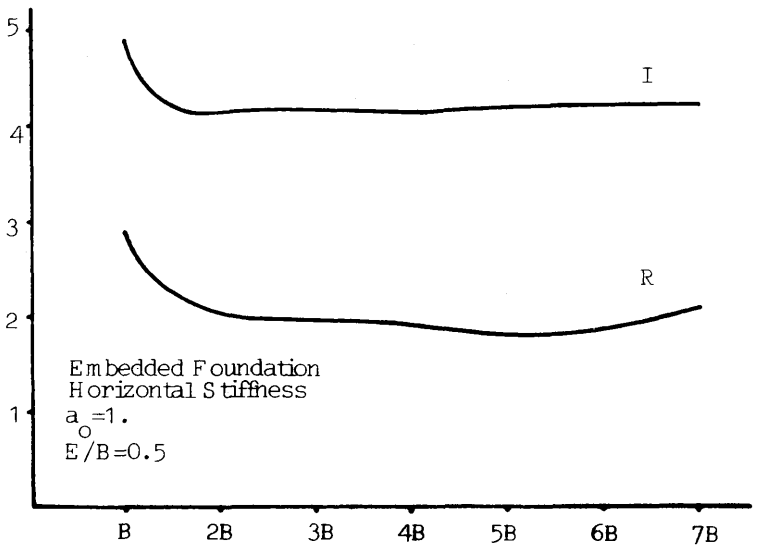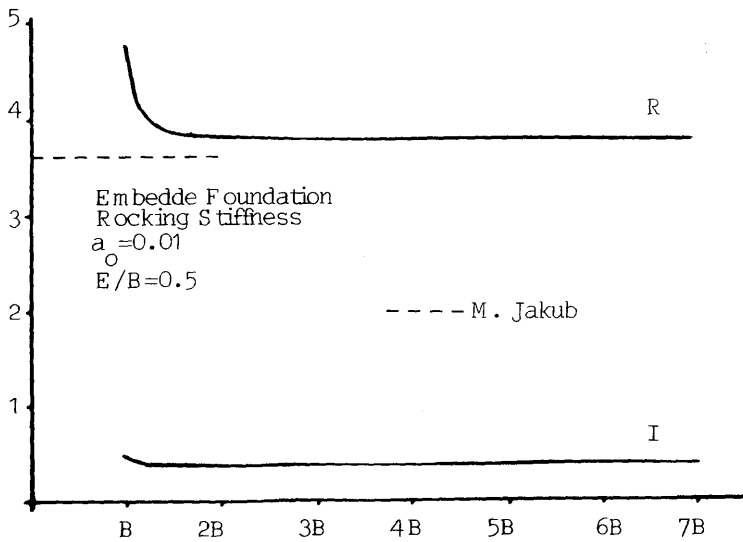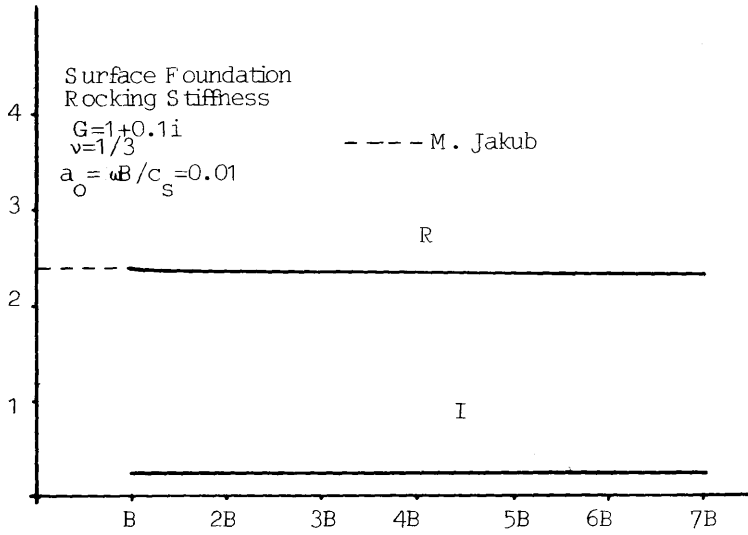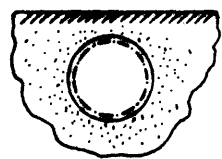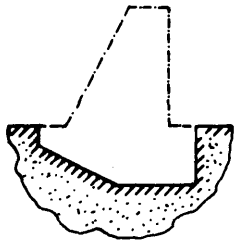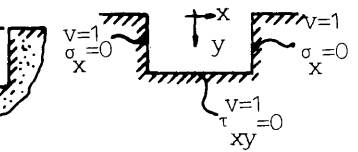
Rigid-smoot bonding
(Vertical displacements)
(c)

Rigid
λ
μ

(d)

$\lambda_1 \mu_1$
$\lambda_2 \mu_2$
$\lambda_3 \mu_3$

(e)

$\lambda_1 \mu_1$
$\lambda_2 \mu_2$
$\lambda_3 \mu_3$

(f)

Incident
waves

(g)

Plate 8