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A motivation for the study of reduced models like one-dimensional systems in Solid State Physics is the complexity of the full problem. In recent years our group has studied theoretically, numerically and experimentally wave propagation in lattices of nonlinearly coupled oscillators. Here, we present the dynamics of magnetically coupled pendula lattices. These macroscopic systems can model the dynamical processes of matter or layered systems. We report the results obtained for harmonic wave propagation in these media, and the different regimes of mode conversion into higher harmonics strongly influenced by dispersion and discreteness, including the phenomenon of acoustic dilatation of the chain, as well as some results on the propagation of localized waves i.e., solitons and kinks.

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1. INTRODUCTION

Lattices of nonlinearly coupled oscillators have been used to model a broad range of problems at different scales. At nanoscale, the dynamics of ionic crystals in a trap have been presented.[1] These systems are formed by charged particles, e.g. atomic ions, interacting by means of the Coulomb repulsion and confined by external electromagnetic potentials to keep the system at equilibrium. At mesoscale, lattices of non-linearly coupled oscillators are reduced models of granular media.[2] Here, Hertz potential governs the interaction among the hard spherical particles that conform the lattice. At macroscale, phononic crystals are examples of periodic media where nonlinearity usually comes from the nonlinear elastic response of the acoustic media.[3, 4] Other systems include lattices of nonlinearly coupled oscillators with curved geometry.[5,6] In all of these examples, due to the presence of periodicity, waves suffer strong dispersion at wavelengths comparable with the lattice constant, i.e., the size of the unit cell of the crystal or lattice. The balance between dispersion and nonlinearity governs the propagation of waves as we will see below.

In this work, we present the dynamics of a lattice of magnetically coupled pendula in harmonic and transient regime. Chains of magnetic pendulums were used in the past to simulate localized travelling excitations in some natural layered silicate crystals[7] and to experimentally study solitary waves.[8] It has been numerically proven the existence of nonlinear localized travelling waves in a system with realistic interatomic and substrate potentials of mica muscovite.[9] Here, we present the harmonic generation processes in a chain of magnetically coupled oscillators with internal resonances.[10,11]

2. NONLINEAR EFFECTS IN REPULSIVE LATTICES

Our general setting is an infinite chain of identical particles with mass M aligned along the x-axis and separated a distance a. Every particle interact with its nearest neighbors by means a repulsive potential, V_{int}. Due to the fact that forces are repulsive, the equilibrium in finite systems is obtained adding an external potential V_{ext} that keeps the particles confined. This effect can be provided by a periodic on-site potential, in our case the gravity in pendula. The equation of motion is written as:

$$M\ddot{u}_n = V'_{\text{int}}\left(u_{n+1} - u_n\right) - V'_{\text{int}}\left(u_n - u_{n-1}\right) + V'_{\text{ext}}, \tag{1}$$

where u_n is the displacement of the n-th particle measured with respect to its equilibrium position and V' are derivatives of the potential with respect to the spatial coordinate, i.e., the forces acting on each particle. If we consider an $\alpha-$power-law model for the interparticle interaction, i.e., $V'(r) = \beta r^{-\alpha}$ and a harmonic approximation for the pendula motion, then:

$$\ddot{u}_n = \frac{\beta}{(a - u_{n+1} + u_n)^\alpha} - \frac{\beta}{(a - u_n + u_{n-1})^\alpha} + \Omega_0 u_n, \tag{2}$$

where $\omega_m = \sqrt{4\alpha\beta/Ma^{\alpha+1}}$ is the maximum frequency of propagating waves (upper cutoff frequency of the dispersion relation), $\Omega_0 = \sqrt{g/L}/\omega_m$ is the on-site potential characteristic frequency, $\Omega = \omega/\omega_m$, g is the gravitational acceleration, L is the length of the pendula. Depending on the monopole/dipole character of the interaction between magnets, $\alpha \in [2, 4]$. In our case, a fit with experimental data gives $\alpha = 3.6$. The experimental setup is composed by a set of 80 pendula (T-shape aluminium bar with a magnet in the extreme), all of them equispaced with $a = 0.2$m. All the details about the setup and the acquisition system can be found in.[10,11]

A. NONLINEAR MONOCHROMATIC WAVES

We excite the chain by driving the first magnet with a sinusoidal motion. Due to the nonlinear interaction between magnets the propagation of the perturbation give rise to harmonic generation. In view of the

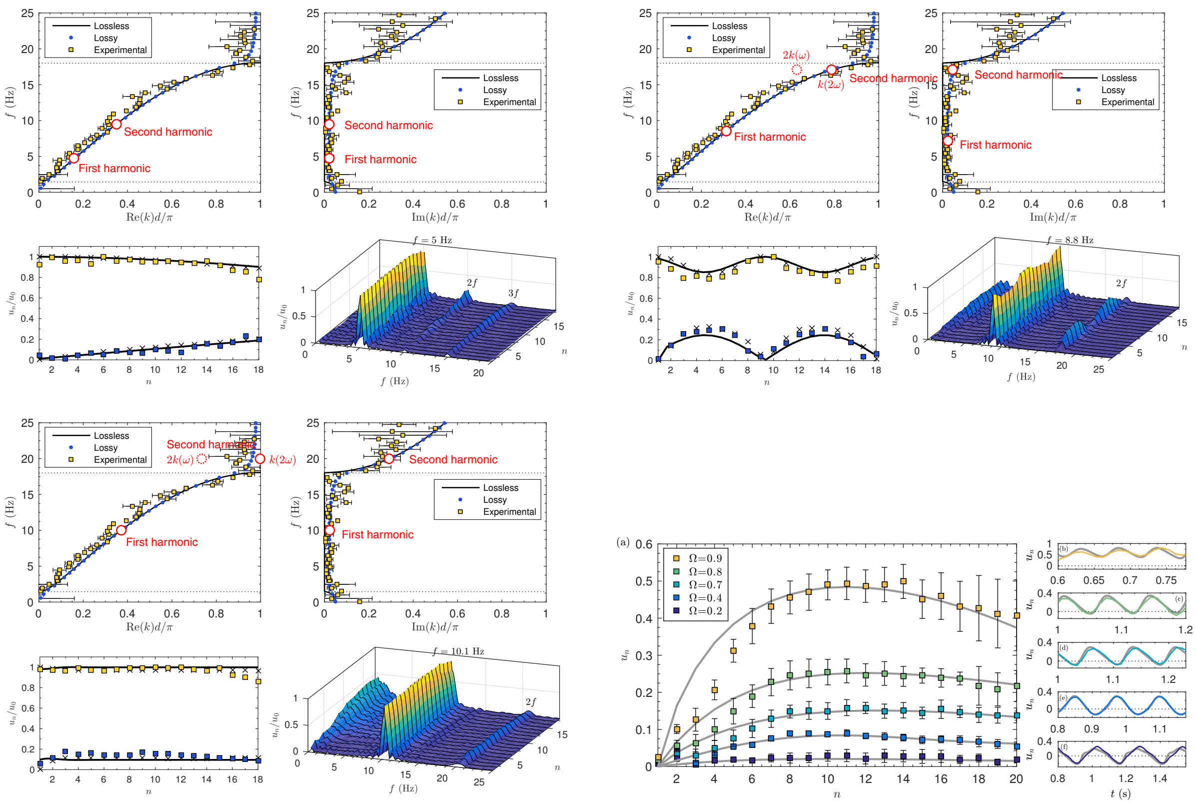

Figure 1: Real and imaginary part of the dispersion relation of a mono-atomic chain obtained analytically (continuous line), by the experimental measurements (squares), and numerically including damping (circles). Plots in upper left and right corners and lower left corner show the position of first and second harmonic in the dispersion relation and the evolution of the relative amplitude. Plot in the lower right corner shows DC mode and wave forms

dispersion relation, shown in Fig. 1, we can distinguish three different regimes. Regime (*i*): Low frequency excitation. In this regime, the frequency of the fundamental wave lies in the lower part of the passband while the generated second harmonic, also in the passband, lies in the region of weak dispersion, i.e., $k(2\omega) \approx 2k(\omega)$. Both waves propagate with nearly the same phase velocity and the amplitude of the second harmonic increases roughly linearly with distance. The first harmonic amplitude decreases due to the energy transfer from the fundamental component to the higher harmonics, as shown in the group of four plots in the upper left corner of Fig.1. Regime (*ii*): Dispersive second harmonic generation. In this regime the frequency of the first harmonic approaches the half of the pass-band and second-harmonic frequency falls in the high dispersive region, i.e., $k(2\omega) \neq 2k(\omega)$, as shown in the group of four plots in the upper right corner of Fig.1. Note second harmonic still lies in a propagative band. Here, the amplitude of the second harmonic first increases with distance and then, at a particular distance given by the coherence length $2lc = 2\pi/|k(2\omega) - 2k(\omega)|$ it decreases, producing a characteristic beating. Regime (*iii*): Second harmonic in bandgap. In this case the frequency of the second harmonic falls in band-gap, i.e., $\mathrm{Im}\, k(2\omega) \neq 0$. The second harmonic component is generated locally by the fundamental wave, but due to its evanescent character cannot and becomes locally trapped. The result is that the field of the second harmonic becomes constant with space, as shown in Fig.1 lower left corner. Finally, an expansion of the chain (zero frequency mode) is also observed, as shown in Fig.1 lower right corner. Such phenomenon has been reported for acoustic waves propagating in a solid. The amplitude of the static displacement mode is a function of the space. The agreement between numerical integration of the equation of motion (continuous lines) and experimental measurements (markers) at different frequencies is very good. Due to the resonance of the pendula DC mode is evanescent. Therefore, DC mode do not cumulate with distance, after a given distance it saturates and remains constant.

B. NONLINEAR LOCALIZED TRAVELLING WAVES

Our system also supports nonlinear localized travelling waves as solution of (Eq. 2).[9,11] Experimental results were obtained driving the chain at the first oscillator with a pulsed waveform excitation (half-sinusoidal wave). The obtained temporal waveforms for the displacement and the compression are shown in Fig. 2 for an amplitude of $A = 0.15$. This figure shows the experimental results and the numerical simulations of Eq. 2 with $\alpha = 3.6$, and a harmonic substrate potential. Experimental data are in good agreement with the numerical simulation. It can be observed the formation of a kink at the front of the displacement wave leading to a compression soliton as can be clearly seen in the plot corresponding to the strain $v_n = u_n - u_{n-1}$ on Fig. 2.

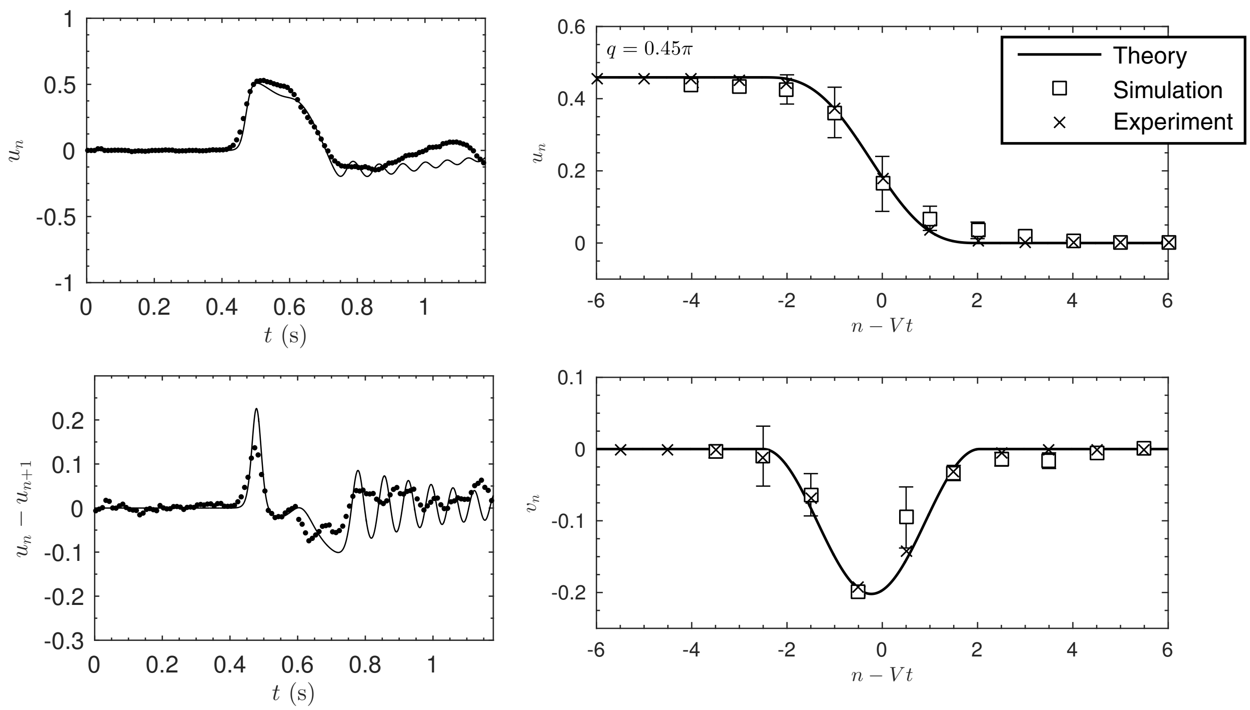

Figure 2: (a) Kink profile in time measured at $n = 23$. Numerical solution including the harmonic substrate (solid line), and experimental data (back dots). (b) Corresponding compression $-v_{n+1} = u_n - u_{n+1}$.

As $\Omega_0 \neq 0$, kinks cannot exist, they are transient solutions and pendula returns to its initial position. After the kink passes, a phonon tail is observed, being its arrival time given by the linear dispersion relation.

3. CONCLUSION

In this work we experimentally and numerically report nonlinear wave propagation in a chain of magnetically coupled pendula. Due to the nonlinear interaction between magnets, harmonic generation is produced locally. The evolution of the amplitudes of the different harmonics along the chain is explained in terms of the dispersion relation. A DC mode also appears that can be related to the chain dilation. Finally, the chain supports nonlinear localized traveling waves that are unstable (transient) kinks. Measurements show that low-frequency band-gap avoids the existence of kinks, i.e., KdV like compression solitons. This simple system that present internal resonances is a reduced model of a nonlinear acoustic metamaterial, therefore, it can be used to model the dynamics of other complex structured media.

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