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Remarks on shape phase transitions in nuclei

L Fortunato[1,2], A Vitturi[1,2], C E Alonso[3] and J M Arias[3]

[1] Dipartimento di Fisica "G.Galilei", via Marzolo 8, I-35131 Padova, Italy
[2] I.N.F.N., via Marzolo 8, I-35131 Padova, Italy
[3] Departamento de Física Atómica, Molecular y Nuclear, Facultad de Física, Universidad de Sevilla, Apartado 1065, E-41080 Sevilla, Spain

Abstract. Various aspects of shape phase transitions in even as well as odd systems are reviewed. Firstly, the case of an odd $j = 3/2$ particle coupled to an even-even boson core that undergoes a transition from spherical limit (U(5)) to the γ-unstable limit (O(6)) is illustrated. Energy spectrum and electromagnetic transitions, in correspondence of the critical point, display behaviors qualitatively similar to those of the even core and they agree qualitatively with the model based on the E(5/4) boson-fermion symmetry. Secondly, we describe a study on two-particle transfer reactions: the evolution of the transfer spectroscopic intensities within the interacting boson model is analyzed as a possible signature of shape phase transitions. In correspondence to the critical points characterizing the phase transitions, the two-particle transfer matrix elements to both ground and excited 0^+ states display a rapid discontinuity that might help validating the experimental search for the critical point.

1. Introduction

We will report in the following on various results obtained by our collaboration during recent years [1, 2, 3] on the theme of shape phase transitions in even and odd systems. In particular we will concentrate on i) a brief introduction to shape phase transitions in the collective model and in the IBM , ii) the analysis of the γ-unstable case in odd systems and iii) two-particle transfer intensities and other signatures of the shape phase transition.

The collective model of Bohr and Mottelson treats quantistically the collective properties of nucleons in a nucleus as vibrations and rotations of an ellipsoidal surface (namely the quadrupole d.o.f.) within a single formalism that is based on a Hamiltonian operator (Bohr hamiltonian) expressed in the deformation coordinates β and γ. The Bohr hamiltonian can be solved analytically in a few cases (reviewed in Ref. [5]) depending on the potential term $V(\beta,\gamma)$ and analytic solutions correspond to algebraic structures that have their common origin in the U(6) underlying symmetry of the collective model. In particular the γ-independent harmonic oscillator potential generates a solution that has U(5) has its underlying dynamical symmetry, while the γ-unstable rotor has O(6). A third case, the axial symmetric rotor is associated with SU(3). Each of the three mentioned cases is also associated with certain spectral properties (energy spectra, electromagnetic transitions, etc.) and with a given ellipsoidal shape that characterize a given phase. Although good examples of each paradigm can be found, most nuclei do not strictly match these ideal limits, but sit somewhere in the middle between them, therefore people has studied transitions from one phase to another. Critical point for this shape phase transitions can be found.

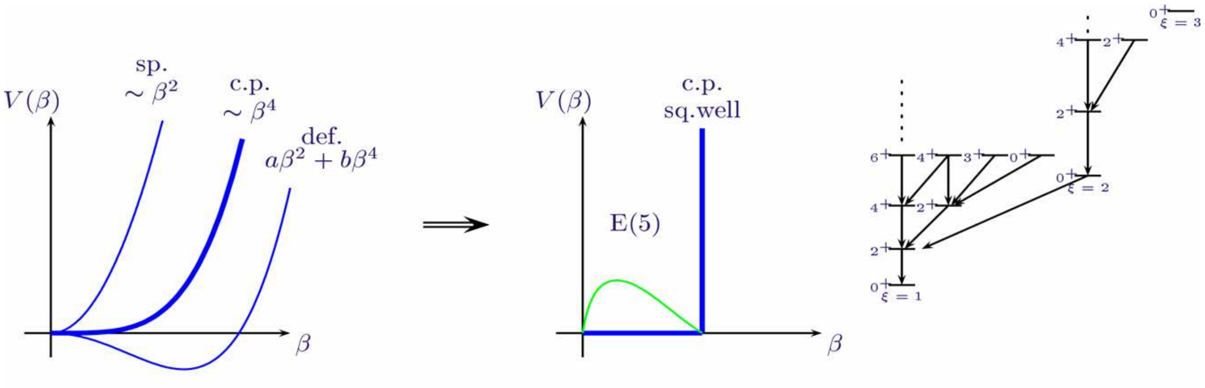

Figure 1. Evolution of the potential $V(\beta)$ along the spherical to γ-unstable shape phase transition, passing through the critical point β^4 potential. This is approximated with a square well that is analytically solvable and gives a novel dynamical symmetry, E(5) that is associated with an energy spectrum with precise rules.

Recently three papers of Iachello introduced the so-called critical point symmetries in the framework of the collective model for even nuclei. Namely he has identified the path from one phase to another with a change in the parameters of the potential and he has approximated the potential at the critical point with a solvable one that can be associated with a new algebraic structure. As an example we discuss the E(5) case (see figure 1): in the transition from the U(5) spherical limit (harmonic oscillator) to the O(6) $\gamma-$unstable limit (champagne bottle or mexican hat potential) one sees that the evolution of the potential goes from a pure β^2 to a combination of β^2 and β^4 that has a deformed minimum. At the critical point of this second order transition the potential is a pure β^4 that is not analytically solvable. This potential is approximated with a square well that is analytically solvable and is associated with a E(5) structure, the Euclidean group in five dimensions.

2. Odd-even phase transitions

Until now most of the work has been carried out for even-even nuclei, using either the Bohr Hamiltonian and the surface collective variables or algebraic approaches like the interacting boson model. Review articles with references to the original works can be found in Refs. [6, 7, 8]. In the case of odd-even nuclei, with an odd particle coupled to an even core undergoing a phase transition, attention has been put on the shape transition from spherical to deformed gamma-unstable shapes. In correspondence to the critical point in the even core two new boson-fermion critical point symmetries have been proposed, in the case of an odd particle moving in a single $j = 3/2$ shell (E(5/4) symmetry [9]) or in the $j = 1/2, 3/2, 5/2$ shells (E(5/12) symmetry [10]). Characteristic sequences of levels and ratios of electromagnetic transitions are predicted in both cases.

We consider here, within the Interacting Boson Fermion Model, the particular case of an odd $j = 3/2$ particle coupled to an even-even boson core that undergoes a transition from spherical U(5) to γ-unstable O(6) situation. The choice of the $j = 3/2$ orbital preserves in the odd case the condition of gamma-instability of the system.

Within the IBM this can be obtained, for example, from the Hamiltonian of the even-even part, as

$$H_B = x\hat{n}_d - \frac{1-x}{N}\hat{Q}_B \cdot \hat{Q}_B \ , \tag{1}$$

which produces, varying the parameter x from 1 to 0, a transition between the two extreme situations characteristic of U(5) and O(6) symmetries with a second order phase transition. The operators appearing in the Hamiltonian above are given by

$$\hat{n}_d = \sum_\mu d^\dagger_\mu d_\mu \ , \tag{2}$$

$$\hat{Q}_B = (s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s})^{(2)} \ , \tag{3}$$

and N is the total number of bosons. For any value of x this Hamiltonian maintains the typical degeneracies of the O(5) symmetry. At the critical point, the energy surface acquires a β^4 behaviour, which is approximated by an infinite square well in the E(5) critical point symmetry [4] within the framework of the collective Bohr Hamiltonian.

The odd-even system is described by

$$H = H_B + H_F + V_{BF} \ , \tag{4}$$

where the term V_{BF} couples the bosonic and fermionic parts, namely

$$V_{BF} = -2\frac{1-x}{N}\hat{Q}_B \cdot \hat{q}_F \ , \tag{5}$$

where $\hat{Q}_B$ (taken of the form given in Eq.(3)) and $\hat{q}_F = (a^\dagger_{3/2} \times \tilde{a}_{3/2})^{(2)}$ are the boson and fermion quadrupole operators, respectively.

The part V_{BF}, has to be diagonalized and its eigenvalues, doubly degenerate, turn out to be γ-independent,

$$E_\pm(\beta,\gamma) = E_\pm(\beta) = \pm 2\frac{(1-x)\beta}{1+\beta^2} \ . \tag{6}$$

In other words, the addition of the odd particle does not destroy the γ-instability of the system, giving rise to energy surfaces for the different odd intrinsic states that are still γ-independent.

The resulting energy spectra in the odd system are shown in the right panel of figure 3 as a function of the control parameter $1-x$. The total number of bosons, N, has been assumed to be equal to 7. For a better comparison, we also show in the left panel of the figure the corresponding evolution of the spectrum in the even core.

The level evolution in the odd case shows a behaviour qualitatively similar to that of the even case. The group structure of $Spin^{BF}(5)$ with respect to O(5) simply leads to a richer pattern for the fermion case and slightly different ratios for the energy levels.

The position of the 3/2 state (or better the $(\tau_1 = 1/2, \tau_2 = 1/2)j = 3/2$ state) is the key element to characterize the particular situation and its position along the transitional path. The position of this state plays the same role as the key position of the first excited 0^+ state in even nuclei.

Transition probabilities, state by state, for the odd nucleus at the critical point situation are shown in figure 2. It can be observed that E2 transitions are stronger between states with the same ξ value and $\Delta\tau_1 = 1$. Transitions between states pertaining to families with different ξ are one or two orders of magnitude smaller. Transitions between states with the same ξ and τ_1 values, but different spins, corresponding to the same multiplets are also one or two orders of magnitude smaller than the ones between different multiplets in the same band.

To summarize, we have considered, within the Interacting Boson Fermion Model, the coupling of an odd $j = 3/2$ particle to a boson core that undergoes a transition from spherical U(5) to γ-unstable O(6) character. The particular choice of the Hamiltonian and of the $j = 3/2$ orbital preserves in the odd case the condition of $\gamma-$instability of the system, and it is reflected in the preservation of the degeneracies associated with the $Spin^{BF}(5)$ symmetry. As a consequence, the energy spectrum and the electromagnetic transitions for the odd nucleus with a critical core

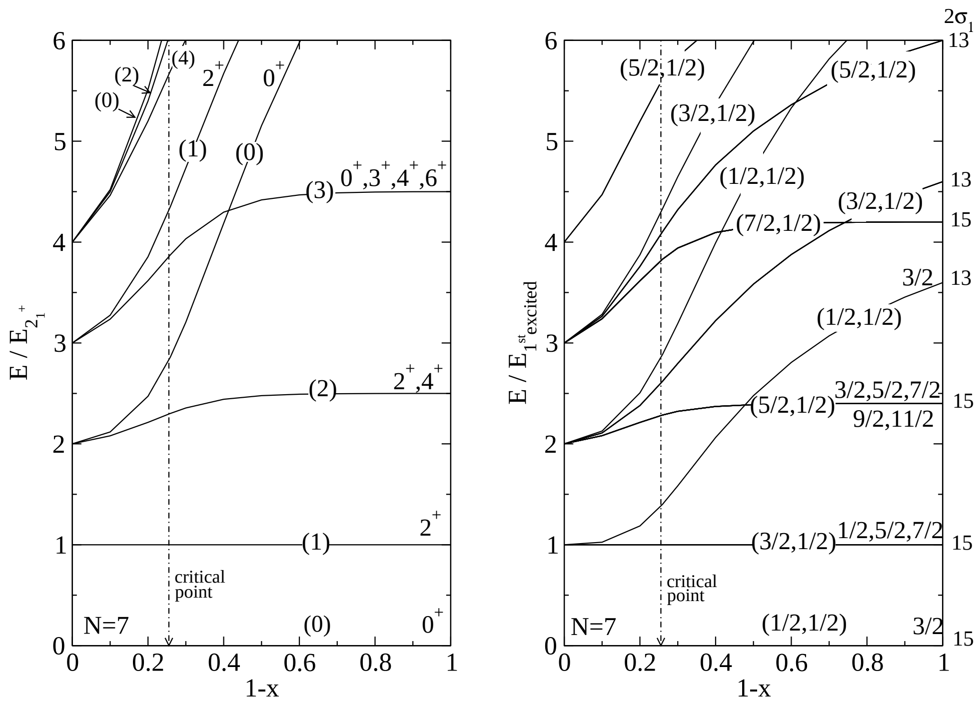

Figure 2. Energy levels (normalized to the energy of the first excited state) for the even and odd systems are displayed as a function of the parameter $(1 - x)$ for the boson (1) and boson-fermion (4) Hamiltonians. A number $N = 7$ of bosons has been assumed in both cases, while the odd particle has been taken in the $j = 3/2$ orbital. In the left panel (even case) we indicate for each level the τ quantum number (in parenthesis), spin and parity. In the right panel (odd case) we quote the (τ_1, τ_2) quantum numbers (in parenthesis) and spin. In the extreme $x = 0$ case we also indicate the σ_1 quantum number. The position of the even critical point is marked.

display behaviours qualitatively similar to those characterizing the phase transition in the even core. We have compared our results with the recently proposed E(5/4) approach, based on the Bohr hamiltonian. Both approaches display similar qualitative pictures, although we evidence a number of quantitative differences, that can be traced back to the different nature of the two schemes.

3. Two-neutron pair transfer in even-even nuclei

Pair-transfer reactions are also useful in the study of nuclear shape phase transitions, i.e., the rapid evolution of nuclear structure with mass number, such as from sphericity to axial-symmetric deformed or from sphericity to deformed γ-unstable nuclei. In our study, we have used the original version of the interacting boson model, the IBM-1 [9], which describes nuclei by assuming nucleon pairs as basic building blocks and treats them as bosons (without distinction between protons and neutrons). The transfer of one boson to a nucleus thus corresponds to the transfer of a nucleon pair, which makes the IBM model exceptionally well suited to describing two-nucleon transfer reactions.

Arima and Iachello defined the most general boson equivalent of the $L = 0$ pair-transfer operator $P^{(0)}_{+,0}$, taking into account up to cubic terms, that can be put in the form

$$P^{(0)}_{+,0} = a_1 s^\dagger + a_2[\hat{N} \times s^\dagger]^{(0)} + a_3[\hat{Q} \times d^\dagger]^{(0)} \qquad (7)$$

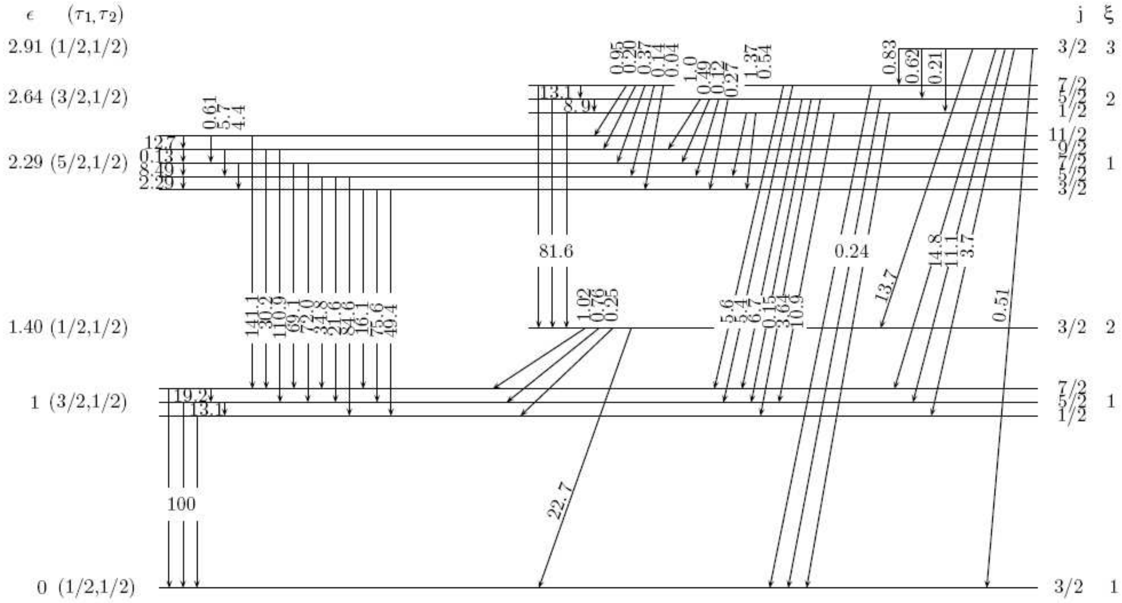

Figure 3. Energy levels and quadrupole transition rates B(E2↓) for the odd system at the core critical point. For illustration purposes the various multiplets, labeled by the (τ_1, τ_2) quantum numbers in parenthesis, have been arbitrarily split according to their j quantum number. The label on the extreme left is the energy in relative units. B(E2↓)'s have been normalized to the value 100 for the transitions (with equal strengths) between the states of the first multiplet and the ground state.

where $\hat{Q}$ is the quadrupole operator. We calculate transfer matrix elements and intensities (see [2] for details) in the Boson Coherent State formalism, obtaining exact results in terms of simple algebraic formulas that only need as input the quadrupole deformation β and β' of the nuclei between which the pair transfer takes place. We first carried out our study in a general way between each of the three dynamic limits of the IBM symmetry triangle: U(5), O(6), and SU(3). Next, we applied our results to some specific series of isotopes that are known to display phase transitions of first and second order (the ^{60}Nd, ^{62}Sm, ^{64}Gd and ^{66}Dy series, and the ^{44}Ru series, respectively). Some of our results are shown in figure 4 for the $Z \sim 60-66$ region. In this region, nuclear shapes evolve from spherical to axial-symmetric deformed Both transfer intensities from ground-to-ground and from ground-to-quasi-β bandheads are plotted. Solid lines give the exact IBM calculationswhile dashed lines give the results produced by the boson coherent-state framework (mean field). In all the isotope chains studied, a drop down is observed in ground-to-ground pair-transfer intensities at the mean-field level for a given value of N, reflecting that the structures of the isotopes involved are changing rapidly. A complementary increase in the ground-to-quasi-β pair-transfer intensity at the mean-field level is also observed. Abrupt drops in ground-to-ground transfer intensity with the complementary increments in the ground-to-β intensities clearly mark the phase transition in the case of the Nd, Sm, Gd, and Dy isotope series.

In conclusion, the intensity for pair transfer between two ground states is in general much larger than the intensity for transfer between the ground state and the β-vibrational or higher phonon 0^+ excited states. At the critical point, however, for U(5) to O(6) and U(5) to SU(3) (second-order and first-order transitions, respectively) both the IBM and the boson coherent-state framework predict sudden changes in the evolution of the transfer intensity: the gs to gs

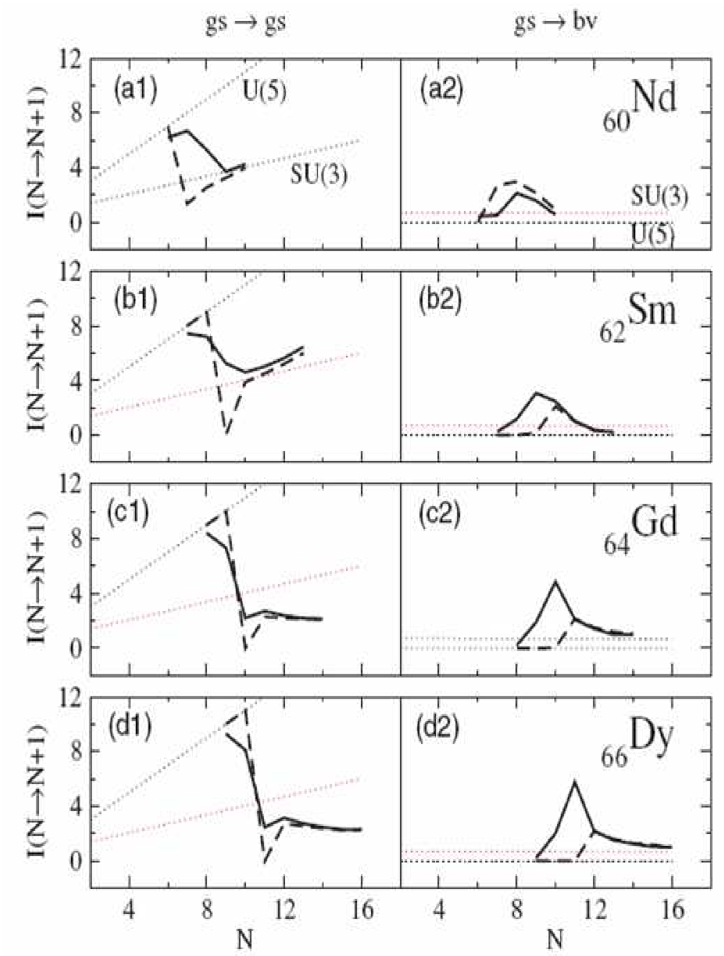

Figure 4. Two-particle transfer intensities for the $^{144-154}$Nd (with 5 proton bosons and 1 to 6 neutron bosons), $^{146-160}$Sm (p = 6 and n = 1-8), $^{148-162}$Gd (p = 7 and n= 1-8), and $^{150-166}$Dy (p = 8 and n = 1-9) isotope chains vs the total number of bosons N, for the gs to gs and gs to bv transfers.

transfer loses in strength, whereas the intensity for gs to bv and gs to dbv transfer show a peak (gs means ground state, bv means beta vibrational state and dbv means double beta vibrational state). This feature is especially present in the first-order phase transition, where in some cases the population of the ground states becomes even smaller than those to the β-vibrational 0^+ state.

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