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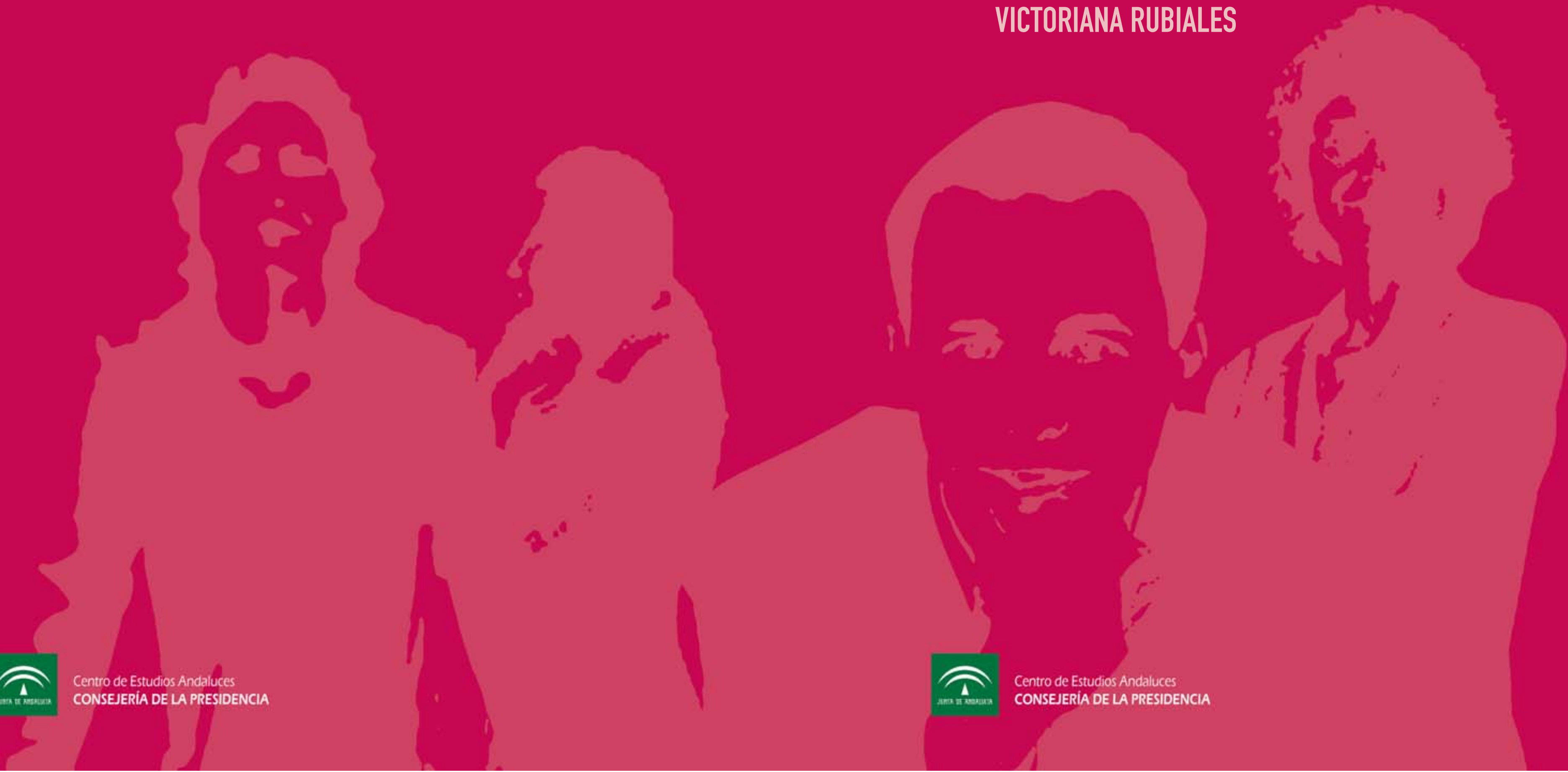

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A bargaining model for finite n-person multi-criteria games

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RESUMEN

En este trabajo consideramos un modelo de juego multicriterio en el que se tienen en cuenta las interacciones entre jugadores. El problema se analiza como un juego cooperativo para alcanzar soluciones de consenso que se valoran con respecto a varios criterios simultáneamente. La idea fundamental consiste en estudiar los juegos finitos *n*-personales multicriterio como juegos de negociación multicriterio. Para ello, establecemos el concepto de pagos garantizados Pareto-óptimos como una generalización de los valores maximin de los juegos escalares y, a continuación, proponemos dos conceptos de solución diferentes que se caracterizan como soluciones de problemas lineales multicriterio. Esto permite la incorporación de información adicional sobre las preferencias de los agentes en el proceso de obtención de una solución final de consenso.

Palabras clave: Juegos finitos multicriterio. Juegos de negociación. Análisis multicriterio.

ABSTRACT

In this paper we consider a multi-criteria game model which allows interactions between players. The problem addressed is considered as a cooperative game in order to achieve consensus solutions which are evaluated with respect to several criteria simultaneously. The main idea consists of analyzing finite multi-criteria *n*-person games as multi-criteria bargaining games. The notion of Pareto-optimal guaranteed payoffs as a generalization of the maximin values of scalar games is proposed, together with two different solution concepts which can be characterized as the solutions of multi-criteria linear programming problems. A procedure to incorporate additional information about the agents' preferences in order to reach a final consensus is also provided.

Keywords: Finite multi-criteria games. Bargaining games. Multi-criteria analysis.
JEL classification: C72, C78, C61.

1 Introduction

The theory of multi-criteria games is concerned with situations in which a number of players must take into account several criteria, each of which depends on the decision of all players. This situation arises in many economic, social and political contexts.

The first publication on multi-criteria normal form games was Blackwell (1956) and different formulations of multi-criteria games have since been introduced, (Zeleny (1976), Bergstresser an Yu (1977), Li(1998), Cubiotti (2000)). Multi-criteria finite n-person games are usually analyzed in the framework of noncooperative game theory. Two different notions are considered in order to establish a solution for these games: equilibrium concepts and security strategies. In the literature, the solutions proposed (such as Wierzbicki (1990), Zhao, (1991), Ghose and Prasad (1989), Fernández and Puerto (1996), Fernández et al. (1998a), (1998b), Voorneveld (1999), Puerto et al.(1999), Borm et al. (2003), Allevi et al. (2003)) are vector extensions of the solutions for the single criterion case. Therefore, they exhibit the same inconveniences inherited from the classic game theory, together with the additional difficulties derived from the multi-dimensional nature of the vectors representing the outcomes.

Due to these problems, it would be interesting to try a different analysis which takes into account the fact that the players may decide together what is a reasonable outcome of the game and then agreeing to implement that outcome. In this paper, the application of bargaining procedures to find a consensus solution for this cooperative situation is proposed. Thus, we analyze the n-person finite multi-criteria game as a multi-criteria bargaining game.

A multi-criteria bargaining game is a generalization of the classic bargaining problem where each player has a set of criteria to value any decision. In these situations, there are two decision problems to be jointly considered: one related to the preferences of the players with respect to their own criteria and the other related to the problem of selecting a solution that could be accepted by all the rational players.

The literature on multi-criteria bargaining is scarce. Hwang and Lin(1987) reduce the multi-criteria bargaining game to a single criterion game by considering the number of agents equal to the sum of the number of criteria of all the agents. In the analysis proposed by Krus and Bronisz(1993) and Krus(2002), each agent, by first solving his multi-criteria problem in an earlier stage, establishes a utopian outcome and then an n-person conventional bargaining game is derived by aggregating a utility function for each agent. In these approaches, the solution concepts for classic bargaining games can be applied, however, there is the possibility that some information in the multi-criteria game may be lost.

In Hinojosa et al.(2004), Hinojosa et al.(2005), Mármol et al.(2005), a more general framework is presented which differs from the existing literature in that the analysis proposed maintains the multi-dimensional nature of each agents' payoff.

The purpose of this paper is to model the multi-criteria bargaining game derived from the finite n-person multi-criteria game and to introduce solution concepts which lead to consensus results that may be accepted by the agents in a cooperative context.

The set of feasible outcomes of the model proposed is a polyhedron, and as a consequence, the solutions can be computed by solving multi-criteria linear problems. Furthermore, the characterization of the solutions as the efficient outcomes of multi-criteria linear problems enables the introduction of information on the preferences of the agents so that a final consensus solution can be obtained.

Finally, we show how our model can be applied to formalize and analyze the consequences of the interactions of economic agents.

The paper is organized as follows. Section 2 provides the mathematical model for the non-cooperative multi-criteria finite n-person game. In Section 3 a brief outline of multi-criteria bargaining games is presented. In Section 4 we establish the multi-criteria game derived from the finite n-person game and propose and characterize the solutions to solve these problems. In Section 5 we present a strategic decision problem in publicity in order to illustrate the model and the methodology proposed. Section 6 is devoted to the conclusions.

2 Multi-criteria finite n-person games

The class of n-person non-zero sum finite games in normal form models a decision making process similar in nature to that modelled by bimatrix games, but with $n(> 2)$ interacting decision makers. Each player desires to maximize his own payoff, without regard for the welfare of others, and under the assumption that all other players will behave similarly. Decisions are made independently and out of a finite set of alternatives for each player, nevertheless they act in an environment where other players' decisions influence their outcomes. In scalar games the payoff is represented by an n-dimensional vector whose components are the payoffs to each player. For multi-criteria games, the payoff to each player is a vector with as many components as the number of criteria considered by the player. Thus, the outcome of a multi-criteria game is represented by a set of vectors. Although a matrix formulation on the plane is not possible, a precise formulation can be established.

For multi-criteria finite n-person games, $N = \{1, \ldots, n\}$ denotes the set of players, and each player is assumed to value the same m criteria. There is a finite number of alternatives or pure strategies, r_i, $i \in N$, for each player to choose from. The set of pure strategies of player i is denoted by $E_i =$

$\{e_i^1, \ldots, e_i^{r_i}\}$. A pure strategy combination is $(e_1^{k_1}, \ldots, e_n^{k_n})$, that is, a pure strategy $e_i^{k_i} \in E_i$ for each player $i \in N$, determines an outcome given by an $m \times n$ matrix, $A = (a_{ij}^{k_1,\ldots,k_n})$, $i = 1, \ldots, n$, $j = 1, \ldots, m$, where $a_{ij}^{k_1,\ldots,k_n}$ represents the payoff to player i in the criterion j.

We denote the mixed strategy space for player $i \in N$ by $\mathcal{Y}_i$,

$$\mathcal{Y}_i = \{y_i \in I\!R^{r_i}, \sum_{k=1}^{r_i} y_i^k = 1, y_i^k \geq 0, \forall k = 1, \ldots, r_i\}$$

where y_i^k, $k = 1, \ldots, r_i$ are the components of vector y_i, i.e., $y_i = (y_i^1, \ldots, y_i^{r_i})$.

If players do not cooperate, each player selects a strategy from his mixed strategy space, $y_i \in \mathcal{Y}_i$, and thus the payoff function is defined in the cartesian product $\times_{i=1}^n \mathcal{Y}_i \subseteq I\!R^r$, where $r = \sum_{i=1}^n r_i$. An element in $\times_{i=1}^n \mathcal{Y}_i$ is represented as $(y_1, \ldots, y_n)$, where $y_i = (y_i^1, \ldots, y_i^{r_i})$. The payoff function is a multilinear vector function, $f^{NC} : \times_{i=1}^n \mathcal{Y}_i \to I\!R^{m \times n}$, given by

$$f^{NC}(y_1, \ldots, y_n) = \left(\sum_{k_1=1}^{r_1} \ldots \sum_{k_n=1}^{r_n} y_1^{k_1} y_2^{k_2} \ldots y_n^{k_n} a_{ij}^{k_1,\ldots,k_n} \right)_{\substack{i=1,\ldots,n \\ j=1,\ldots,m}}$$

The solution concepts established for these games are based on notions of equilibrium and on security strategies. Unfortunately, these solutions have some unattractive and problematical properties. However, despite these difficulties, it is still possible to deal with this problem by using an alternative analysis. If all the players are willing to cooperate in order to pursue potentially better results, then bargaining procedures can be applied in order to obtain solutions which are, in some sense, "fair" to all the players.

In the next section we introduce multi-criteria bargaining games as a framework for the cooperative analysis.

3 Multi-criteria bargaining games

Formally, an n-person multi-criteria bargaining game is described as the set of players $N = \{1, 2, \ldots, n\}$ such that each player considers the same m criteria to value the possible agreements, and a pair (S, D) where $S \subseteq I\!R^{m\times n}$ is the set of all feasible outcomes and D is an outcome in $\mathcal{D} \subseteq I\!R^{m\times n}$ where $\mathcal{D}$ is the set of the players' possible disagreement points.

The outcomes in S are obtained as the result of a joint decision of all the players. Therefore, if $X = (x_1, \ldots, x_n) \in S$, there exists an agreement, that gives player $i \in N$ an outcome vector $x_i = (x_i^1, x_i^2, \ldots, x_i^m)^t \in I\!R^m$, where x_i^j is the payoff for player i in criterion j, $j = 1, \ldots, m$. Thus, an outcome in S can be represented by an $m \times n$ matrix, where $x_i \in I\!R^m$ denotes the payoff for player i in each of the m criteria, $i = 1, \ldots, n$, and $x^j \in I\!R^n$ denotes the payoff for each player in the jth criterion, $j = 1, \ldots, m$.

The points in $\mathcal{D}$ are the obtainable results if the players fail to reach an agreement. For conventional bargaining games, there is a unique disagreement point. However, in the multi-criteria case, due to the multi-dimensional nature of each agent's payoff, several disagreement points may exist. That is to say, the players can consider different results from which only higher levels of outcomes are acceptable.

For a multi-criteria bargaining game (S, D), a bargaining solution specifies a non-empty subset of the possible outcome set, S, that would be accepted by all the players under certain reasonable principles. There are two basic principles which a bargaining solution should satisfy: individual rationality and Pareto optimality.

In a multi-criteria game, individual rationality establishes that each player will only negotiate at or above those outcomes that improve upon the disagreement point D. Thus, the set of outcomes where players will negotiate is

$D^{\geq} = \{ X \in I\!R^{m\times n},\, X \geqq D\}$.[1]

In relation to Pareto optimality, the bargaining solution must provide an outcome that cannot be simultaneously improved for all players. The terms Pareto-optimality and weak Pareto-optimality will be used for the following extensions to matrices of these concepts for vectors.

Definition 3.1 *$X \in I\!R^{m\times n}$ is Pareto-optimal in $S \subset I\!R^{m\times n}$ if there does not exist $Y \in S$ such that $Y \geq X$.*

Definition 3.2 *$X \in I\!R^{m\times n}$ is weakly Pareto-optimal in $S \subset I\!R^{m\times n}$ if there does not exist $Y \in S$ such that $Y > X$.*

Moreover, the following non-dominance concept is proposed between matrices which is specific for the analysis presented in this paper.

Definition 3.3 *$X \in I\!R^{m\times n}$ is Pareto-optimal by criteria in $S \subset I\!R^{m\times n}$ if there does not exist $Y \in S$ such that $Y \geq X$ with $y^j > x^j$ for some j.*

Observe that this last non-dominance condition is stronger than that of weak Parto-optimality and weaker than that of Pareto-optimality.

Note that a bargaining solution assumes that only the set of feasible outcomes S and the disagreement outcome D matter in order to obtain the final payoffs to the players.

4 The cooperative approach

To apply bargaining procedures to a n-person finite multi-criteria game in normal form, it is necessary to obtain the associated bargaining model. Thus, the set of possible outcomes and the set of disagreement points that can be derived from the finite game, have to be specified.

[1]Given $X, Y \in I\!R^{m\times n}$, we denote $X \geqq Y$ if $x_i^j \geq y_i^j$, $\forall i, j$; $X \geq Y$ if $x_i^j \geq y_i^j$, $\forall i, j$ and $X \neq Y$; $X > Y$ if $x_i^j > y_i^j$, $\forall i, j$.

4.1 The feasible set

Assuming cooperation between all the players, they agree on coordinate their different actions. In this case the game has $R = \prod_{i=1}^{n} r_i$ joint pure strategies consisting of

$$E = \times_{i=1}^{n} E_i = \{(e_1^{k_1}, \ldots, e_n^{k_n}),\ k_i = 1, \ldots, r_i,\ \forall i = 1, \ldots, n\}.$$

and therefore a jointly mixed strategy is a probability distribution on the cartesian product $\times_{i=1}^{n} E_i$. The joint decision space for the finite n-person cooperative game becomes

$$\mathcal{Y} = \{y \in I\!R^R, \sum_{k=1}^{R} y^k = 1, y^k \geq 0\}.$$

Note that each component of $y \in \mathcal{Y}, y^{k_1,\ldots,k_n}, k_i \in \{1, \ldots, r_i\}$ represents the probability that the group of players selects the joint pure strategy $(e_1^{k_1}, e_2^{k_2}, \ldots, e_n^{k_n})$.

The payoff function in the jointly randomized space, $\mathcal{Y}$, is a vector linear function $f^C : \mathcal{Y} \to I\!R^{m\times n}$ that can be written as

$$f^C(y) = \left(\sum_{k_1,\ldots,k_n} y^{k_1,\ldots,k_n} a_{ij}^{k_1,\ldots,k_n} \right)_{\substack{i=1,\ldots,n \\ j=1,\ldots,m}}$$

where $\sum_{k_1,\ldots,k_n}$ is the sum for all jointly mixed strategies of the players, $k_s \in \{1, \ldots, r_s\}$, $s = 1, \ldots, n$.

As the jointly randomized space, $\mathcal{Y}$, is a closed and convex set and the payoff function, f^C, is a vector linear function, then by using jointly mixed strategies, any convex combination of pure strategy payoff vectors can be generated in the game. Thus, the convexification of the payoff space can be obtained and $f^C(\mathcal{Y})$ is a convex set. In fact, it is a convex polyhedron whose extreme points are the payoffs corresponding to the pure strategies. Therefore, the feasible set of the bargaining game associated with the multi-criteria finite n-person game is $S = f^C(\mathcal{Y}) \subseteq I\!R^{m\times n}$.

4.2 The set of disagreement points

In order to extend the idea of the maximin values to the multi-criteria bargaining problem, we determine which are the best outcomes that the agents can guarantee themselves by analysing the problem as a non-cooperative multi-criteria game.

For $i \in N$ the set $N - \{i\}$ is considered as a unique player that acts against player i. Therefore, this is a two-person non-cooperative game where the set of pure strategies, E_{-i}, and the mixed strategy space, $\mathcal{Y}_{-i}$, for $N - \{i\}$ are

$$E_{-i} = \times_{\substack{j=1 \\ j \neq i}}^{n} E_j$$

$$\mathcal{Y}_{-i} = \{y_{-i} \in I\!R^{q_i} / \sum_{k=1}^{q_i} y_{-i}^k = 1, y_{-i}^k \geq 0, \forall k = 1, \ldots, q_i\}$$

where $q_i = \prod_{k \neq i} r_k$ is the number of pure strategies for $N - \{i\}$. That is to say, $\mathcal{Y}_{-i}$ is the set of jointly mixed strategies for all players except i.

As the game is a finite game, the payoff for player i can be represented by m matrices, $A_i = (A_i(1), \ldots, A_i(m))$, where $A_i(j) = (a_i^{st}(j))_{\substack{s=1,\ldots,r_i \\ t=1,\ldots,q_i}}$ is a matrix of order $r_i \times q_i$, with r_i as the number of pure strategies of player i and $q_i = \prod_{k \neq i} r_k$ as the number of joint pure strategies of the players in $N - \{i\}$. The element $a_i^{st}(j)$ is the payoff to player i in the j-criterion when strategies $e_i^s \in E_i$ and $e_{N-\{i\}}^t \in E_{-i}$ are played, where $a_i^{st}(j) = a_{ij}^{k_1,\ldots,k_n}$ for $s = k_i$ and $t = (k_1, \ldots, k_{i-1}, k_{i+1}, \ldots, k_n)$.

The payoff function for this two-person non-cooperative game is a bilinear function $f^{NC} : \mathcal{Y}_i \times \mathcal{Y}_{-i} \rightarrow I\!R^m$ given by

$$f^{NC}(y_i, y_{-i}) = (y_i^t A_i(1) y_{-i}, \ldots, y_i^t A_i(m) y_{-i}).$$

In order to determine the disagreement points for player i, the notion of Pareto Optimal Security Strategies is applied for two-person multi-criteria finite games (Ghose and Prassad(1989), Fernández and Puerto(1996)) to this particular game. To this end, each strategy $y_i \in \mathcal{Y}_i$ is valued by a vector whose components are the lowest payoff in each one of the criteria that the player could possibly obtain by choosing y_i.

Definition 4.1 *The guaranteed payoff vector for each strategy of player* $i \in N$, $y_i \in \mathcal{Y}_i$, *is* $V_i(y_i) = \left(\min_{y_{-i}\in\mathcal{Y}_{-i}} y_i^t A_i(1) y_{-i}, \ldots, \min_{y_{-i}\in\mathcal{Y}_{-i}} y_i^t A_i(m) y_{-i}\right)$.

The best payoffs that player i can guarantee for himself, irrespective of the actions of the other players, are obtained from strategies which maximize the guaranteed payoff vector in the multi-criteria sense, that is, those that cannot be improved componentwise.

Definition 4.2 $V_i = (V_i^1, \ldots, V_i^m)$ *is a Pareto optimal guaranteed payoff vector for player* $i \in N$, *if* $V_i = V_i(y_i)$, *where* $y_i \in \mathcal{Y}_i$, *and there does not exist* $y_i' \in \mathcal{Y}_i$ *such that* $V_i(y_i') \geq V_i(y_i)$.

The set of all Pareto optimal guaranteed payoff vectors for player $i \in N$ is denoted by $\mathcal{D}_i$. Vectors in $\mathcal{D}_i$ provide lower bounds of the payoffs that the player will obtain independently from the actions of the other players, by using the corresponding strategy. This set of assured payoffs for each of the players plays the role of the maximin values in the conventional bargaining game, that is, when each player values a unique criterion.

To characterize the set of Pareto optimal guaranteed payoff vectors for player $i \in N$ we consider the following vector linear problem associated to the two-person non-cooperative game, whose players are $\{i,\, N - \{i\}\}$, denoted by $(P(i))$,

$$\begin{aligned} max \quad & V_i^1, V_i^2, \ldots, V_i^m \\ s.t. \quad & y_i^t\, A_i(1) \geq (V_i^1, \ldots, V_i^1) \\ & \qquad \vdots \\ & y_i^t\, A_i(m) \geq (V_i^m, \ldots, V_i^m) \\ & y_i \in \mathcal{Y}_i \end{aligned}$$

The following proposition establishes the relationship between the set of

Pareto optimal guaranteed payoff vectors and the set of efficient solutions of $(P(i))$.

Proposition 4.1 *V_i^* is a Pareto optimal guaranteed payoff vector for player $i \in N$ and $y_i^* \in \mathcal{Y}_i$ is the corresponding strategy, if and only if (V_i^*, y_i^*) is an efficient solution of problem $(P(i))$.*

Proof: From Definition 4.2, it follows that the set of Pareto optimal guaranteed payoff vectors for agent $i \in N$ can be obtained by solving the following vector maximization problem:

$$\begin{array}{ll} max & V_i^1(y_i), \ldots, V_i^m(y_i) \\ s.t. & \qquad y_i \in \mathcal{Y}_i \end{array}$$

which is equivalent to $(P(i))$. $\square$

Due to the linearity of problem $(P(i))$, existing algorithms valid for the determination of efficient solutions of vector linear problems and the corresponding software, such as ADBASE (Steuer, 1995), can be applied to obtain the set of Pareto-optimal guaranteed payoffs associated to each player.

Once the set of disagreement points for each player, $\mathcal{D}_i$, is determined, we obtain the disagreement set for the bargaining game associated to the multi-criteria finite n-person games as $\mathcal{D} = \times_{i=1}^n \mathcal{D}_i$. The outcomes in this set are those outcomes that the agents can achieve if a consensus solution is not reached. Therefore, cooperation should lead them to an improved outcome with respect to the selected disagreement point from this set.

4.3 Multi-criteria bargaining solutions

Two solution concepts will be applied in order to solve the bargaining problem proposed, the multi-criteria maximin solution and the multi-criteria Kalai-Smorodinsky solution which can be considered as a generalization of the Kalai-

Smorodinsky solution for conventional bargaining games, (Kalai and Smorodinsky, 1975).

First, we introduce the concept of multi-criteria maximin solution and the results which characterize this family of solutions are established.

Consider the multi-criteria bargaining game (S, D), where $S = f^C(\mathcal{Y}) \subseteq I\!R^{m\times n}$ and $D \in \mathcal{D} = \times_{i=1}^{n}\mathcal{D}_i$, $D = (d_i^j), i = 1, \ldots, n, j = 1, \ldots, m$. For each feasible outcome $X \in S$, we denote $\tilde{X} = (\tilde{x}_i^j)$ $i \in N, j = 1, \ldots, m$, where $\tilde{x}_i^j = x_i^j - d_i^j$, $i \in N$, $j = 1, \ldots, m$, represents the utility gains from the disagreement point, obtained by player i in criterion j.

To establish the multi-criteria maximin solution for this class of games, each feasible outcome is going to be valued by a vector in terms of its worst utility gains with respect to each one of the criteria.

Definition 4.3 *In the multi-criteria bargaining game* (S, D), *for each feasible outcome* $X \in S$, $Z(X) = (Z^1(X), Z^2(X), \ldots, Z^m(X))$, *is the minimum utility gains vector, where* $Z^j(X) = \min_{1\leq i\leq n}\{\tilde{x}_i^j\}$.

$Z^j(X)$ is the guaranteed minimum utility gains to the agents in the jth criterion and vector $Z(X)$ represents the minimum utility gains that all the agents can attain in each criterion. This vector can be obtained from different feasible outcomes in S.

The multi-criteria maximin solution concept is based on the idea that the agents jointly agree on those outcomes whose minimum utility gain levels cannot be simultaneously improved with respect to all the criteria. Therefore, the players will choose an outcome such that the associated minimum utility gain vector is as good as possible, in the sense that there is no other outcome whose minimum utility gain vector is better componentwise.

Definition 4.4 *For the multi-criteria bargaining game* (S, D), *a feasible outcome* $X \in S \cap D^{\geq}$ *is a multi-criteria maximin solution if there does not exist*

$Y \in S \cap D^{\geq}$ *such that* $Z(Y) \geq Z(X)$.

It is now shown that, in general, multi-criteria maximin solutions verify a condition which is stronger than that of weak Pareto-optimality and weaker than that of Pareto-optimality in $S \cap D^{\geq}$.

Proposition 4.2 *The multi-criteria maximin solutions for* (S, D) *are Pareto-optimal by criteria in* $S \cap D^{\geq}$.

Proof: Consider a multi-criteria maximin solution X^* and suppose that is not Pareto-optimal by criteria in $S \cap D^{\geq}$, then $\exists Y \in S \cap D^{\geq}$ such that $Y \geq X^*$ and $\exists j = 1, \ldots, m$ such that $y^j > x^{*j}$. Therefore, $Z(Y) \geq Z(X^*)$, and $Z^j(Y) = \min_{1 \leq i \leq n}\{\tilde{y}_i^j\} > \min_{1 \leq i \leq n}\{\tilde{x}_i^j\} = Z^j(X^*)$. This is a contradiction to X^* being a multi-criteria maximin solution of (S, D). $\square$

In order to obtain the multi-criteria maximin solutions and the associated minimum utility gain vector, the following multiobjective problem is considered, and is denoted by (PM)

$$
\begin{array}{rlll}
max & z^1, \ldots, z^m & & \\
s.t. & \tilde{x}_i^1 & \geq z^1 & \forall i = 1, \ldots, n \\
 & & \vdots & \\
 & \tilde{x}_i^m & \geq z^m & \forall i = 1, \ldots, n \\
 & X \in S \cap D^{\geq} & &
\end{array}
$$

The following result characterizes multi-criteria maximin solutions as efficient solutions of problem (PM).

Proposition 4.3 *If* (X^*, z^*) *is a nondominated solution of* (PM), *then* X^* *is a multi-criteria maximin solution for* (S, D) *and* z^* *its associated minimum utility gains vector. Conversely, if* X^* *is a maximin solution for* (S, D), *then* $(X^*, Z(X^*))$ *is a nondominated solution of* (PM).

Proof: From Definition 4.5, it follows that multi-criteria maximin solutions are the nondominated solutions of the vector maximization problem

$$\begin{array}{ll} max & Z^1(X), \ldots, Z^m(X) \\ s.t. & X \in S \cap D^{\geq} \end{array}$$

where $Z^j(X) = \min_{1\leq i\leq n}\{\tilde{x}_i^j\}\, \forall j = 1, \ldots, m$. This problem is equivalent to the problem (PM). $\square$

As the feasible set, $S = f^C(\mathcal{Y})$, is a convex polyhedron, the bargaining game associated to the finite game is a linear bargaining game and (PM) is a multi-criteria linear problem. Thus, the concepts and tools of multi-criteria linear programming can be applied in order to solve these games.

On the other hand, this characterization of the multi-criteria maximin solutions enables us to obtain them in terms of optimal solutions of appropriated scalar linear optimization problems. In this process, weights λ_j, $j = 1, \ldots, m$ are introduced on players' minimum utility gain levels z^j, $j = 1, \ldots, m$ and a real-valued function is formed by summing the m weighted minimum levels. The parametric problem denoted by $P(\lambda)$ is

$$\begin{array}{lll} max & \sum_{j=1}^m \lambda_j z^j & \\ s.t. & \tilde{x}_i^1 \geq z^1 & \forall i = 1, \ldots, n \\ & \vdots & \\ & \tilde{x}_i^m \geq z^m & \forall i = 1, \ldots, n \\ & X \in S \cap D^{\geq} & \end{array}$$

where $\lambda \in \Lambda = \{\lambda \in I\!R^m, \sum_{j=1}^m \lambda_j = 1, \lambda > 0\}$.

Proposition 4.4 *X^* is a maximin solution for (S, D) and z^* is its associated minimum utility gain vector if and only if there exists $\lambda^* \in \Lambda$ such that (X^*, z^*) is an optimal solution of the problem $P(\lambda^*)$.*

Proof: The result follows from the characterization of maximin solutions given in Proposition 4.3 and the equivalence between nondominated solutions of a multiobjective linear problem and the solutions of the associated weighted-sum problems, (Zeleny, 1976). □

If the players are able to specify a vector of weights λ for their minimum utility gain levels, then the scalar function is determined and the solution, according to this information, can be computed. Unfortunately, precisely establishing the weights of the criteria is not an easy task. However, the players can provide partial information by estimating a range of the weights for the criteria. When this information can be incorporated into the model, the set of maximin solutions is reduced and the players will find it easier to choose a consensus solution in accordance with their preferences.

Mármol et al.(1998) propose a procedure to incorporate partial information on the importance of the criteria in multi-criteria linear problems which is based on the extreme points of different information sets. Furthermore, it is possible to perform an analysis of the sensitivity of the solutions with respect to changes in the weights that generate the result. This analysis is based on the reduced-cost matrix associated to the optimal basic solution of the multi-criteria linear problem and permits to decompose the set of all possible weights into a finite number of subsets such that weights corresponding to a certain subset generate the same solutions.

The second solution concept that we propose in this paper, the multi-criteria Kalai-Smorodinsky solution, can be derived by a similar process. In this case, we take into account the players' most optimistic expectations with respect to the criteria, which are represented by the ideal outcome of the game. This ideal outcome, denoted as $I = (I_i^j)$, $i \in N$, $j = 1, \ldots, m$, is obtained by solving the following linear optimization problems, $I_i^j = max\{x_i^j,\, X \in S \cap D^{\geq}\}$.

The multi-criteria Kalai-Smorodinsky solution is achieved by replacing the utility gains of the agents by the proportion with respect to their most optimistic expectations. Therefore, if for each feasible outcome $X \in S$ we consider the quotients $\dfrac{x_i^j - d_i^j}{I_i^j - d_i^j}$, $i \in N$, $j = 1, \ldots, m$, and let $K(X)$ denote the minimum proportional utility vector whose components are $K^j(X) = \min_{1 \leq i \leq n}\{\frac{x_i^j - d_i^j}{I_i^j - d_i^j}\}$, $\forall j = 1, \ldots, m$, the following definition emerges.

Definition 4.5 *For the multi-criteria bargaining game (S, D), a feasible outcome $X \in S \cap D^{\geq}$ is a multi-criteria Kali-Smorodinsky solution if there does not exist $Y \in S \cap D^{\geq}$ such that $K(Y) \geq K(X)$.*

Similar results as those stated for the multi-criteria maximin solutions can be established for the multi-criteria Kali-Smorodinsky solution. Hence, this solution is Pareto-optimal by criteria in $S \cap D^{\geq}$ and the efficient solutions of the following multi-criteria problem

$$
\begin{array}{rll}
max & z^1, \ldots, z^m & \\
s.t. & \frac{x_i^1 - d_i^1}{I_i^1 - d_i^1} \geq z^1 & \forall i = 1, \ldots, n \\
 & \vdots & \\
 & \frac{x_i^m - d_i^m}{I_i^m - d_i^m} \geq z^m & \forall i = 1, \ldots, n \\
 & X \in S \cap D^{\geq} &
\end{array}
$$

provide the set of multi-criteria Kali-Smorodinsky solutions for the multicriteria bargaining game (S, D).

5 Strategic advertising decisions

This section is devoted to illustrating the concepts and results obtained in previous sections, showing that the cooperation between agents permits them to obtain outcomes that improve their individual expectations.

A company is established in two regions. Three different departments, which can decide on their own advertising policies, want to promote their products in the two regions. Each department can place two advertisements on television which focus on different aspects of the products. It has been observed that the effect of the adverts is different in the two regions and that the publicity of each product has an indirect effect on the sales of the others.

The estimation of the increase in the net profits generated under the two advertising campaigns are shown in Table 1.

$$\begin{array}{ccc}
 & e_3^1 & e_3^2 \\
e_1^1\ e_2^1 & \begin{pmatrix} 5 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix} & \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix} \\
e_1^1\ e_2^2 & \begin{pmatrix} 3 & 1 & 4 \\ 2 & 1 & 4 \end{pmatrix} & \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 6 \end{pmatrix} \\
e_1^2\ e_2^1 & \begin{pmatrix} 2 & 3 & 5 \\ 0 & 3 & 5 \end{pmatrix} & \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 5 \end{pmatrix} \\
e_1^2\ e_2^2 & \begin{pmatrix} 5 & 4 & 1 \\ 2 & 1 & 3 \end{pmatrix} & \begin{pmatrix} 1 & 0 & 5 \\ 0 & 0 & 4 \end{pmatrix}
\end{array}$$

Table 1: Increase in the net profits

For instance, if the three departments place their first advertisement, the effect in the first region consists of an increase in the net profits corresponding to the first product of 5 monetary units, of 1 monetary unit for the second product and of 2 monetary units corresponding to the third product. In the second region the increases in the net profit corresponding to the three products are respectively 1 monetary unit, 2 monetary units and 4 monetary units.

The company aims to achieve an equitable increase in the profits generated by the three departments in both regions. On the other hand, each department wishes to maximize the effects of the advertising campaign on the sales of its own products. For this reason, they are willing to negotiate with the other departments to determine the most appropriate combinations of adverts.

As a consequence of the interdependency between the departments, this strategic situation can be analyzed as a game with three players (the departments), each of whom has two strategies (to place the first or the second advertisement). The results of the decisions, i.e., the payoffs they attain when they play these strategies against each other, are measured in relation to two criteria corresponding to the increase in the net profits obtained in each of the two regions.

Cooperation between the players guarantees that the results they obtain will improve on the payoffs attained under non-cooperation. If it is possible to consider different combinations of the joint strategies of the three departments, this problem can be formalized as a multi-criteria bargaining game, and solutions are obtained by applying bargaining procedures.

For the cooperative situation the set of joint pure strategies for the three departments is

$$E = \times_{i=1}^{3} E_i = \{(e_1^{k_1}, e_2^{k_2}, e_3^{k_3}),\ k_i = 1, 2\, \forall i = 1, 2, 3\}.$$

Each of these strategies represents a combination of advertisements of the three departments. A jointly randomized strategy is a probability distribution on these different combinations.

The set of feasible payoffs of the associated bargaining game is a polyhedron whose extreme points are the payoffs corresponding to the joint pure strategies:

$$S = \{X = \sum_{k=1}^{8} y^j P^k,\ \sum_{k=1}^{8} y^k = 1, y^k \geq 0,\ \forall k = 1, \ldots, 8\}$$

where $P^1, \ldots, P^8$ denote the payoff matrices defining the game, and the payoffs corresponding to the jointly randomized strategies are the convex combination of these matrices.

5.1 The set of disagreement points

In the first step of the analysis of the problem, the set of disagreement points has to be determined. The set of Pareto optimal guaranteed payoff vectors for each department is obtained by solving a finite multi-criteria non-cooperative game.

For Department 1 the matrices for this game are

$$A_1(1) = \begin{pmatrix} 5 & 1 & 3 & 1 \\ 2 & 2 & 5 & 1 \end{pmatrix} \quad A_1(2) = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 3 & 2 & 0 \end{pmatrix}$$

and represent the payoff to Department 1 when the two other departments consider their joint pure strategies. It follows from Proposition 4.1 that the set of Pareto optimal guaranteed payoff vectors is obtained by solving the following multi-criteria linear programming problem.

$$\begin{array}{rl} max & V_1^1, V_1^2 \\ s.t. & (y_1^1, y_1^2) \begin{pmatrix} 5 & 1 & 3 & 1 \\ 2 & 2 & 5 & 1 \end{pmatrix} \geq (V_1^1, V_1^1, V_1^1, V_1^1) \\ & (y_1^1, y_1^2) \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 3 & 2 & 0 \end{pmatrix} \geq (V_1^2, V_1^2, V_1^2, V_1^2) \\ & y_1 \in \mathcal{Y}_1 \end{array}$$

This problem has a unique efficient solution, that is, a unique Pareto-optimal guaranteed payoff vector, $\mathcal{D}_1 = \{(1, 1)\}$. Therefore, if Department 1 does not cooperate then the guaranteed increase in the net profit in each of the two regions is 1 unit, independently of the joint actions of the other departments.

The Pareto-optimal guaranteed payoff vectors for the other departments are $\mathcal{D}_2 = \{(1, 1)\}$, $\mathcal{D}_3 = \{(2, 4)\}$. It follows that the disagreement point for this game is

$$D = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 4 \end{pmatrix}.$$

In the second step of the analysis, the focus is on determining the consensus solutions on which the departments will agree and the corresponding jointly mixed strategies. The multi-criteria maximin solutions and the multi-criteria Kalai-Smorodinsky solutions are analyzed for this situation as two representative solutions which enables the consideration of different aspects of the cooperation.

5.2 Multi-criteria maximin solutions

The maximin criterion is considered the most appropriate to seek out equitable solutions. In this context, the multi-criteria maximin solution will provide results such that the minimum level of increase in the profits of the three departments cannot be improved in both regions simultaneously. It follows from Proposition 4.3 that maximim solutions are obtained by solving the following multi-criteria linear programming problem.

$$\begin{aligned} max \quad & z^1, z^2 \\ s.t. \quad & \tilde{x}_i^1 \geq z^1 \quad \forall i = 1,2,3 \\ & \tilde{x}_i^2 \geq z^2 \quad \forall i = 1,2,3 \\ & X \in S \cap D^{\geq} \end{aligned}$$

where $\tilde{x}_i^1 = x_i^1 - d_i^1$ and $\tilde{x}_i^2 = x_i^2 - d_i^2 \quad \forall i = 1,2,3$

The problem has been solved with the software package ADBASE (Steuer, 1995). Table 2 shows the efficient extreme solutions and the jointly mixed strategies corresponding to these results[2].

[2] $ch\{A\}$ we denotes the convex hull of A.

	z	S(z)	Strategy	Probability
(1)	$\begin{pmatrix} 1.66 \\ 0.75 \end{pmatrix}$	$\left\{ \begin{pmatrix} 2.66 & 2.66 & 4.31 \\ 1.75 & 1.75 & 4.75 \end{pmatrix} \right\}$	e_1^1, e_2^1, e_3^1	0.19
			e_1^2, e_2^1, e_3^1	0.28
			e_1^2, e_2^1, e_3^2	0.5
			e_1^2, e_2^2, e_3^1	0.03
(2)	$\begin{pmatrix} 1.6 \\ 0.76 \end{pmatrix}$	$\left\{ \begin{pmatrix} 2.6 & 2.6 & 4.4 \\ 1.76 & 1.76 & 4.8 \end{pmatrix} \right\}$	e_1^1, e_2^1, e_3^1	0.2
			e_1^2, e_2^1, e_3^1	0.28
			e_1^2, e_2^1, e_3^2	0.52
(3)	$\begin{pmatrix} 1 \\ 0.8 \end{pmatrix}$	$\left\{ \begin{pmatrix} 2 & 3 & 5 \\ 1.8 & 1.8 & 5 \end{pmatrix} \right\}$	e_1^2, e_2^1, e_3^1	0.4
			e_1^2, e_2^1, e_3^2	0.6
(4)	$\begin{pmatrix} 1.91 \\ 0.52 \end{pmatrix}$	$ch \left\{ \begin{pmatrix} 2.91 & 2.91 & 3.91 \\ 1.52 & 1.83 & 4.52 \end{pmatrix}, \begin{pmatrix} 2.91 & 2.91 & 3.91 \\ 1.99 & 1.52 & 4.52 \end{pmatrix} \right\}$	e_1^1, e_2^1, e_3^1	0.13
			e_1^2, e_2^1, e_3^1	$0.2 + 0.15t, t \in [0, 1]$
			e_1^2, e_2^1, e_3^2	$0.5 - 0.15t, t \in [0, 1]$
			e_1^2, e_2^2, e_3^1	0.17

Table 2: Multi-criteria maximin solutions.

However, the resolution of the multi-criteria decision problem must finish with a unique solution. At this point, it is necessary to incorporate additional information about the agents' preferences into the process.

In this case, agents may be able to provide imprecise *a priori* information about the weights that represent the importance of the minimum levels of increase in profits in both regions and this information can be incorporated into the model. If the agents agree on the exact values of the weights, λ_1, λ_2, then a maximin solution is obtained by solving the corresponding weighted problem. However, it is not always the case that the agents are able to achieve a consensus about these values. Nevertheless, even if only partial information about the weights that each agent would accept is available, then the set of solutions could be reduced so that the selection of a result becomes easier. Table 3 shows the effect of different assumptions about the relative importance of the minimum levels on the profits in both regions. Numbers in brackets correspond to the solutions in Table 2.

Note that the set of solutions obtained when the same importance is at-

$\lambda_1 = \lambda_2$	$\lambda_1 \geq \lambda_2$	$\lambda_2 \geq \lambda_1$	$3\lambda_1 \geq \lambda_2 \geq \lambda_1$
(4)	(4)	(1),(2),(3),(4)	(1), (4)

Table 3: Maximin solutions for different preference information assumptions.

tached to the minimum levels of benefits in both regions coincides with the set of solutions obtained for the case in which the importance of the first region is not less than the importance of the second region. On the other hand, if the weight associated to the minimum level of profits is not less for the second region than for the first region, the whole set of maximin solutions is still obtained. Nevertheless, if the information is refined so that the importance attached to the second region does not exceed three times the importance attached to the first region, a significant reduction in the set of consensus solutions is achieved.

An analysis of the sensitivity of the solutions with respect to changes in the weights have also been performed. Table 4 shows how the set of information weights is decomposed in this example, and the maximin solutions associated to the different sets of weights.

Weights	Solutions
$\lambda_2 \leq 1.125\lambda_1$	(4)
$1.125\lambda_1 \leq \lambda_2 \leq 5.625\lambda_1$	(1)
$5.625\lambda_1 \leq \lambda_2 \leq 15\lambda_1$	(2)
$\lambda_2 \geq 15\lambda_1$	(3)

Table 4: Sets of weights and solutions

5.3 Multi-criteria Kalai-Smorodinsky solutions

The concept of maximin multi-criteria solutions is aimed at obtaining equitable results without taking into account the most optimistic expectations of the agents. The concept of multi-criteria Kalai-Smorodinsky solution enables these expectations to influence the final results.

To obtain the multi-criteria Kalai-Smorodinsky solutions we compute the ideal point of the game, $I = \begin{pmatrix} 5 & 3.5 & 5 \\ 3 & 2.3 & 6 \end{pmatrix}$, and the disagreement point $D = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 4 \end{pmatrix}$, thus $I - D = \begin{pmatrix} 4 & 2.5 & 3 \\ 2 & 1.3 & 2 \end{pmatrix}$.

By solving the corresponding multi-criteria linear problem, (PM),

$$\begin{array}{rl} max & z^1, z^2 \\ s.t. & \frac{x_i^1 - d_i^1}{I_i^1 - d_i^1} \geq z^1 \quad \forall i = 1,2,3 \\ & \frac{x_i^2 - d_i^2}{I_i^2 - d_i^2} \geq z^2 \quad \forall i = 1,2,3 \\ & X \in S \cap D^{\geq} \end{array}$$

the set of multi-criteria Kalai-Smorodinsky solutions is obtained. This set consists of those payoffs such that the proportions with respect to the most optimistic expectations of the agents cannot be improved simultaneously. The extreme points of the set of minimum level of increase profits and the corresponding outcomes are shown in Table 5.

In order to compare the results obtained when applying the concept of multi-criteria maximin solution with those for the concept of multi-criteria Kalai-Smorodinsky solutions, we consider, for instance, the case in which the agents agree on importance weights such that $\lambda_2 \geq 20\lambda_1$. The maximin and the Kalai-Smorodinsky solutions obtained are:

$$X_{max-min} \begin{pmatrix} 2 & 3 & 5 \\ 1.8 & 1.8 & 5 \end{pmatrix} \quad X_{K-S} = \begin{pmatrix} 1.99 & 2.99 & 4.99 \\ 2.01 & 1.66 & 5.01 \end{pmatrix}.$$

	z	S(z)
(1)	$\begin{pmatrix} 0.42 \\ 0.45 \end{pmatrix}$	$\left\{ \begin{pmatrix} 2.69 & 2.05 & 3.76 \\ 1.89 & 1.58 & 4.89 \end{pmatrix} \right\}$
(2)	$\begin{pmatrix} 0.25 \\ 0.5 \end{pmatrix}$	$\left\{ \begin{pmatrix} 1.99 & 2.99 & 4.99 \\ 2.01 & 1.66 & 5.01 \end{pmatrix} \right\}$
(3)	$\begin{pmatrix} 0.55 \\ 0.26 \end{pmatrix}$	$\left\{ \begin{pmatrix} 3.22 & 2.39 & 3.70 \\ 2.25 & 1.33 & 4.52 \end{pmatrix} \right\}$
(4)	$\begin{pmatrix} 0.56 \\ 0.25 \end{pmatrix}$	$ch \left\{ \begin{pmatrix} 3.24 & 2.4 & 3.68 \\ 1.5 & 1.83 & 4.5 \end{pmatrix}, \begin{pmatrix} 3.24 & 2.4 & 3.68 \\ 2.24 & 1.33 & 4.5 \end{pmatrix} \right\}$
(5)	$\begin{pmatrix} 0.52 \\ 0.32 \end{pmatrix}$	$ch \left\{ \begin{pmatrix} 3.06 & 2.29 & 3.93 \\ 1.64 & 1.78 & 4.64 \end{pmatrix}, \begin{pmatrix} 3.06 & 2.29 & 3.93 \\ 2.19 & 1.41 & 4.64 \end{pmatrix} \right\}$

Table 5: Multi-criteria Kalai-Smorodinsky solutions.

Note that both results provide nearly the same increase in the profits that each department obtains in the two regions. However, while the maximin solution equates the levels of profits corresponding to the first and second department in the second region, the Kalai-Smorodinsky solution provides a higher increase in the department with more optimistic expectations.

6 Conclusions

In this paper we show that finite multi-criteria n-person games can be analyzed as multi-criteria bargaining games when the agents cooperate in order to achieve consensus outcomes which improve on the individual outcomes. The two main points in this study are the determination of the set of disagreement points and the proposal of solutions for these games. In both cases, multi-criteria linear optimization techniques have been proved to be an effective tool in order to characterize the solutions and obtain the outcomes of multi-criteria games.

The model and procedures that we propose can also be applied to scalar games with uncertain payoffs. In these cases, the payoffs associated to different states of nature can be identified with the different criteria of our model.

Finally, it is interesting to mention that the methodology proposed requires information from the agents in two phases of the process. They have to provide individual information about their preferences with respect to the criteria in order to determine the disagreement point, and collective information to direct the procedure towards that solution which is most in accordance with the joint preferences of the agents.

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