Analysis of the Foreign Sector as an Endogenous Variable in SAM Linear Models: An Empirical Proposal

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ABSTRACT

The traditional consideration of the Foreign Sector as an exogenous variable in Input-Output or Social Accounting Matrices (SAM) linear models has led to biased estimations of hypothetical demand shocks when applying those models. The objective of this work is to analyse the possibility of including operations with foreign countries as an endogenous variable while solving the difficulties associated to this option. Thus, the methodology herein proposed makes it possible to prevent the influence and impact of the Foreign Sector from being overvalued, such as they are when estimated by other methodologies. A Social Accounting Matrix for Spain has been used as the database for this work.

Keywords: Foreign Sector, Social Accounting Matrix, Linear Multiplier Model, Impact Analysis.

Análisis del sector exterior como variable endógena en los modelos lineales SAM: Una propuesta empírica

RESUMEN

La consideración tradicional del sector exterior como una variable exógena en los modelos lineales Input-Output y de Matrices de Contabilidad Social, ha dado lugar a estimaciones sesgadas de hipotéticas perturbaciones de la demanda en la aplicación de esos modelos. El objetivo de este trabajo es analizar la posibilidad de incluir las operaciones con el extranjero como una variable endógena, proponiendo una solución a las dificultades asociadas a esta opción. Por lo tanto, la metodología aquí propuesta hace posible evitar la sobrevaloración en el impacto del sector exterior, como ocurre cuando se estima con otras metodologías. Una Matriz de Contabilidad Social de España se ha utilizado como base de datos para este trabajo.

Palabras clave: Sector Exterior, Matriz de Contabilidad Social, Modelo lineal de multiplicadores, Análisis de impacto.

JEL Classification: C67, D57

1. INTRODUCTION

In Input-Output linear models and in those models that have developed from them, in particular in Social Accounting Matrices (SAM), the Foreign Sector is traditionally considered as an exogenous factor and the income generation process is described as a circular flow typical of a closed economy. However, this involves leaving aside the impact of the Foreign Sector on the income generation process. This consideration of the external sector as an exogenous variable when we want to analyse the effect of a shock produced in another component of final demand, such as household consumption, can generate some distortion in the effects Measured by the multipliers of the linear model, since both variables will act indistinctly on the endogenous ones. But, in fact a major bias occurs when the foreign sector is introduced as an endogenous variable, since that means assuming that the economy analysed is capable of influencing the decisions of external agents, which is not usually true in most the cases. Thus, an overvaluation of the real multiplier effects occurs which, in addition, affects the hypothetical distribution of sectors that are influential in the economic activity. The first solution that may be contemplated is the introduction of the Foreign Sector into the set of endogenous variables, but this entails a risk concerning the interpretation of the resulting multipliers. The inclusion of foreign relations into the circular flow and their subsequent interactions imply that those multipliers will indicate that the national economy has an influence on that of the rest of the world, something unsustainable under the small country assumption required for the analysis of the economy of a specific country or region.

Therefore, it is necessary to look for ways to introduce the interaction effects of the Foreign Sector into Social Accounting Matrix analysis but avoiding those overvaluation risks. Such is the objective of this work, in which we propose an alternative method to develop this type of analysis and prove its validity and consistency. This alternative method is based in the framework of Miyazawa models (Miyazawa, 1976), using an adapted version of the methodology used in the endogeneization of final consumption. This proposal is compared with simplest, but biased, approximation based in include foreign trade directly as an exogenous variable. In this sense, we complete here the research started in Mainar *et al.* (2012) and Fuentes *et al.* (2015), comparing alternative methodology referring this issue.

Other alternative methods could be the endogeneization of Foreign Sector using domestic Input-Output tables (Duarte *et al.,* 2015), or the use of activities by commodities type Social Accounting Matrices (Eurostat, 2008), allowing the consideration of domestic and imported production in the same model and database.

This paper is structured as follows: first, we will describe the methodology generally used in SAM linear models, as well as the database used in this work, which is the one for the Spanish economy in year 2000; in the following two sections we will propose two alternative methods for the endogenization of the Foreign Sector; finally, we will present the main results and conclusions.

2. LINEAR MULTIPLIERS BASED ON SOCIAL ACCOUNTING MATRICES

The description, explanation and analysis of the economy allowed by the traditional Input - Output model make it a key tool. National accounting systems have been developed and generalized for most economies, forming a set of basic benchmarks of the economic situation. However, multisectoral analysis with Input-Output model remains limited, in the attempt to reflect the full functioning of the economic system, because it does not incorporate all economic transactions that occur in the system. It does not take into account how the so-called institutional sectors (households, enterprises, governments, etc.) perform transactions and reallocations of resources and income. For example, the distribution of production factors' income among institutional sectors is not recorded. I-O models do not fully contemplate the circular flow of income, the subsequent feedback processes of income reallocation and its influence on the composition of demand.

To address this limitation, a Social Accounting Matrix can be defined (in a simplified form) as an extension of a traditional Input-Output table with a more disaggregated structure of expenditure and income, integrating the relationships between institutional sectors, estimated with information from national accounting systems. This way, the objective of closing the full economic flow is achieved. Thus, a SAM is a consistent framework gathering national income data, product accounts, input-output table, and which reflects the monetary flows between institutions. Therefore, a SAM is a matrix representing in a comprehensive, flexible, and disaggregated way all the transactions of a socioeconomic system. It reflects the process of income generation by activities, of production, and the distribution and redistribution of income between institutional groups. The various interdependencies in the economic system are taken as a whole by recording the actual and imputed transactions and transfers between the agents in the system. The key point that distinguishes a SAM from alternative accounting systems is the importance of the factors, household and institutional dimensions, compiled in a comprehensive way (Pyatt & Round, 1985), (Pyatt & Thorbecke, 1976).

A SAM provides an appropriate framework for analysing the main socioeconomic matters such as employment, poverty, growth and income distribution, etc. By integrating data from household survey into national

accounts, a SAM captures the macro transactions of an economic system on the base of the micro level transfers between all economic agents in the economy (Pyatt & Round, 1985), (Reinert & Roland-Holst, 1997). Also, a SAM may incorporate income distribution dimensions by disaggregating households per socio-economic characteristics (e.g.: income level, rural-urban split etc.).

2.1. Basics of SAM and its development

As described above, the SAM is an economy-wide data framework representing the real economy of a territory. It is represented by a square matrix in which each account is represented by a row and column. Each cell shows the payment from the account in column to the account in row. Accordingly, the income of an account is shown along its row and its expenditures along its column. By double-entry accounting principle, total receipts equal total payments for each account in a SAM, and as a consequence, the total revenue of the economy (total of rows) equals the total expenditure (total of columns).

A SAM provides information on how the different agents in an economy interact with the rest of the system. Therefore, the compilation of a SAM implies access to data and estimates going beyond what is included in the standard national accounting practice. Such data are never available from one single source; information from various sources must be compiled and made consistent. This process is itself valuable since inconsistencies between statistical sources and areas where data reliability could be improved are detected. Additionally, SAMs are the necessary analytical framework for modelling through direct fixed-price multiplier models and Computable General Equilibrium (CGE) models. Finally, the interest of SAMs also resides in the possibilities of temporal or spatial comparison between socio-economic structures: environment, employment, productivity, poverty, development, technological change, etc.

Usually, a SAM has six basic groups of accounts: Activities/Commodities, Factors of production, Private institutions, Government, Combined capital accounts and Rest of the World. The dimensions of the matrix are determined by the level of disaggregation of these six basic groups. Table 1 shows the structure of a basic SAM with Private institutions disaggregated in Household and Enterprises and taking separately Activities and Commodities.

This simplified structure of Table 1 (see Round 2003) is enough to show the complex work necessary in compilation of data for a SAM. Only in few developing countries National Accounts are calculated on the basis of the income or expenditure methods. Also, until recent time, it was not usual to find commodity balances (e.g. input-output tables) or integrate data on the incomes and expenditure of households, enterprises or government. In this work, we have used the Social Accounting Matrix for Spain in year 2000 (SAMESP00), which can be consulted in Cardenete and Fuentes (2009).

The formulation of SAM linear models (see Stone 1978, Pyatt and Round 1979, among others) allows obtaining matrix **M**, an accounting multiplier matrix the components of which (m_{ij}) reflect the impact generated by an exogenous income unit of endogenous account j on the income of endogenous account i. The addition of the columns of the accounting multiplier matrix reveals the total effect of an exogenous shock received by an endogenous account on the rest of the economic activity (*backward linkage effect*).

Table 1
Structure of a Social Accounting Matrix

	Commodities	Margins	Activities	Factors	Households	Enterprises	Government	Savings-Investment	Rest of the World	*Total*
Commodities		Transaction cost (trade and transport)	Intermediate consumption (inputs)		Household consumption		Government consumption	Fixed capital formation and change in stock (Investment)	Exports	*Total demand*
Margins	Transaction cost (trade and transport)									*Total margins*
Activities	Marketed output / Domestic sales									*Activity income*
Factors			Factor income from activities						Factor income from ROW	*Factor income*
Households				Labour and mixed income	(Inter Households transfers)	Distributed benefits to Households / Other transfers	Current transfers to Households		Current transfers to Households from ROW	*Household income*
Enterprises				Operating surplus / Capital income			Current transfers to Enterprises		Current transfers to Enterprises from ROW	*Enterprises income*
Government	Net taxes on products		Net taxes on production	Factor income to Government	Direct taxes	Surplus to Government / Enterprises taxes			Current transfers to Government from ROW	*Government income*
Savings-Investment					Household savings	Enterprises savings	Government savings	(Capital accounts transfers)	Capital transfers from ROW	*Savings*
Rest of the World	Imports			Factor income to ROW	Households transfers to ROW	Surplus to ROW	Government transfers to ROW	Current external balance		*Foreign exchange outflow*
Total	*Total supply*	*Total margins*	*Cost of production activities*	*Factor income payments*	*Households expenditures*	*Enterprises expenditures*	*Government expenditures*	*Investment*	*Foreign exchange inflow*	

Source: Round (2003) and own elaboration.

As mentioned before, if the Foreign Sector account is considered to be exogenous, the real multiplier effect of hypothetical demand shocks becomes biased (not all the effects take place in the domestic sectors), but its simple incorporation as an endogenous variable involves the substitution of this bias by another more detrimental one, according to which the Foreign Sector reacts to an increase in income due to imports by intensifying its demands from the economy. We will now suggest some solutions to this problem.

3. SOLVING THE PROBLEM THROUGH MULTIPLIER DECOMPOSITION ANALYSIS

The starting point for the analysis is the well-knowed equilibrium equation:

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y} \Leftrightarrow \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = \mathbf{M}\mathbf{y} \tag{1}$$

where $\mathbf{x}$ is the vector of total gross output of endogenous accounts (N) in the economy, and $\mathbf{y}$ is the vector of final demand of these accounts; $\mathbf{A}$ is the matrix of coefficients in the SAM framework, where the representative element $a_{ij} = \dfrac{X_{ij}}{x_j}$, $i, j = 1, ..., $ N, shows the participation that the payment X_{ij} of a sector j in other sector i has on the total payments of sector j (x_j); $\mathbf{M}$ is the matrix of SAM multipliers.

One way to endogenize the Foreign Sector and avoid the multiplier effect overvaluation problem would be to apply the multiplier decomposition technique. The decomposition technique as such is widely known, the works by Stone (1978), Pyatt and Round (1979) and Defourny and Thorbecke (1984) being the pioneers of this type of analysis, while in Spain we can especially refer to the research made by Polo *et al.* (1991) and Cardenete and Sancho (2003). The first application of this technique to issues concerning the Foreign Sector can be seen in the works by Llop and Manresa (2007) and Fuentes *et al.* (2015).

This methodology allows the decomposition of the ($\mathbf{M}$-$\mathbf{I}$) matrix of net multipliers into three addends (N_1, N_2 and N_3). To do this, $\mathbf{A}$ is decomposed into two matrices, in this case collecting one of them foreign sector and the other one the other endogenous accounts

$$\mathbf{M} = \left(\mathbf{I} - \mathbf{B}_1 - \mathbf{B}_2\right)^{-1} = \left(\mathbf{I} - \mathbf{D}^2\right)^{-1}\left(\mathbf{I} + \mathbf{D}\right)\left(\mathbf{I} - \mathbf{B}_1\right)^{-1} = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1, \tag{2}$$

with $\mathbf{D} = (\mathbf{I} - \mathbf{B}_1)^{-1}\mathbf{B}_2$.

Then

$$
\begin{aligned}
d\mathbf{Y} - d\mathbf{x} = \left(\mathbf{M} - \mathbf{I}\right)d\mathbf{x} = \left(\mathbf{M}_3\mathbf{M}_2\mathbf{M}_1 - \mathbf{I}\right)d\mathbf{x} = \\
\left(\mathbf{M}_3\mathbf{M}_2\mathbf{M}_1 + \left(\mathbf{M}_2\mathbf{M}_1 - \mathbf{M}_2\mathbf{M}_1 + \mathbf{M}_1 - \mathbf{M}_1\right) - \mathbf{I}\right)d\mathbf{x} = \\
\left((\mathbf{M}_3 - \mathbf{I})\mathbf{M}_2\mathbf{M}_1 + (\mathbf{M}_2 - \mathbf{I})\mathbf{M}_1 + \mathbf{M}_1 - \mathbf{I}\right)d\mathbf{x} = \left(\mathbf{N}_3 + \mathbf{N}_2 + \mathbf{N}_1\right)d\mathbf{x}
\end{aligned} \tag{3}
$$

and

$$\mathbf{N}_1 = \left(\mathbf{M}_1 - \mathbf{I}\right), \mathbf{N}_2 = \left(\mathbf{M}_2 - \mathbf{I}\right)\mathbf{M}_1, \mathbf{N}_3 = \left(\mathbf{M}_3 - \mathbf{I}\right)\mathbf{M}_2\mathbf{M}_1 \tag{4}$$

resulting

$$\mathbf{M} - \mathbf{I} = \mathbf{N}_1 + \mathbf{N}_2 + \mathbf{N}_3 \tag{5}$$

where N_1 represents the *own net effects*, including the effects that exogenous accounts have on the economy that are due only to the relations and interactions between domestic accounts in that economy, while N_3 refers to the *circular net effects*, that is, those that are neither included in N_1 nor in N_2. The N_2 component deserves special attention. When considering foreign sector as endogenous, it reflects the *open net effects* and contains those effects derived from the contribution of the Foreign Sector to the income generation system, illustrating the relation between domestic economy and Foreign Sector.

Through this decomposition, it is possible to omit N_3 results and thus eliminate the feedback effect of the Foreign Sector account which causes the bias in the multipliers. Subsequently, by adding N_1 and N_2 the real effects will be obtained free from that bias.

4. AN ALTERNATIVE SOLUTION: THE ENDOGENIZATION OF THE FOREIGN SECTOR

In this work, we propose to analyse the impact of the Foreign Sector account in a way that the small country assumption can be maintained and which consists in endogenizing the effect of imports only and keeping exports as an exogenous factor. Indeed, the introduction of the imports made by productive and institutional sectors as a linear function of their respective total employment or resources values, while exports remain constant, allows analysing the real influence of the Foreign Sector. Consequently, it is possible to avoid the overvaluation of its effects caused by the fictitious feedback (export increase due to the increase of foreign income through imports) that takes place when this variable is considered as endogenous. This is precisely the novelty of this work and its main contribution, being also the underlying idea susceptible of being modified or varied depending on the needs of the analysis or the availability of data.

This approach requires the reformulation of the usual linear model, especially of its components. First, it is necessary to specify the way in which each sector's imports, which are now taken as endogenous, will depend on the sector's total employment or resources volume. Within the context of linear models, we assume that the imports made by sector i (z_i) can be expressed as the product of the sector's total resources net of imports (x^n_i) by a fixed coefficient, h_i.

$$h_i = z_i / x^n_i \qquad (6)$$

Therefore, the vector of coefficients $\mathbf{h} = \{h_i\}$ will reveal to what extent each sector's resources derive from the Foreign Sector and are not generated by the domestic production process. Higher values of these coefficients define sectors whose expansion, directly as a consequence of exogenous demand shocks or

indirectly in the form of input, is dissipated by a high percentage through the demand of imports.

This method, which is designed to endogenize the imports, makes it necessary, in the context of using a Social Accounting Matrix, to modify the traditional matrix of technical coefficients. The elements of the matrix (a^n_{ij}) will now show the expenditure of account i for each monetary unit of total expenditure or employment of account j, but net of imports.

Hence, we can write the usual linear model (Leontief type) as follows:

$$\mathbf{x}^n = \mathbf{A}^n\mathbf{x}^n + \mathbf{y} - \mathbf{z} = \mathbf{A}^n\mathbf{x}^n + \mathbf{y} - \mathbf{H}\mathbf{x}^n \Leftrightarrow \mathbf{x}^n = (\mathbf{I} - \mathbf{A}^n + \mathbf{H})^{-1}\mathbf{y} \qquad (7)$$

where:

$\mathbf{x}^n$ is the vector of total employment or total resources net of imports

$\mathbf{A}^n$ is the matrix of coefficients of endogenous variables, calculated over $\mathbf{x}^n$

$\mathbf{y}$ is the matrix of exogenous variables (including exports)

$\mathbf{z}$ is the imports vector

$\mathbf{H}$ is the diagonal matrix with elements m_i (coefficients of imports)

From the previous expression we can easily derive a new version of the traditional Leontief inverse matrix (Input-Output frame), $(\mathbf{I}-\mathbf{A}^n+\mathbf{H})^{-1}$.

The elements of the inverse matrix (analogous to the one used, in an Input-Output context, by Miyazawa (1976) to analyse the effects of endogenizing consumption in an open economy, or by Mainar *et al.* (2012) in a SAM model) reflect the impact ultimately generated by an exogenous income unit (even if derived from exports) of endogenous account j on the income of endogenous account i, but now showing as well the effect of the Foreign Sector caused by the imports required for the production processes and the generation of subsequent incomes and discounting the effect of the external sector due to the necessary imports in the processes of production and generation of subsequent incomes.

5. MAIN RESULTS

The application of the above mentioned techniques, using as database the Social Accounting Matrix for Spain[1] in year 2000, is summarised in Table 2. The columns of this table show the total multipliers (*backward linkage effects*) obtained for unitary exogenous shocks of the final demand in the productive

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[1] The matrix used can be considered outdated in order to analyse the current economic reality, but the empirical results of the article are not so much aimed at explaining the situation of the Spanish economy or the role of the foreign sector in it, as to illustrate the effects of the application of one or the other method addition, there have been repeated analogous experiments with similar results, as a first step in other works with more recent data (for example Duarte *et al.,* 2015), but its presentation will force restructure what already presented here, without providing additional information on the methodology itself.

sectors of the economy. For each case, the *Savings/Investment* and *Public Sector* accounts are taken as exogenous whereas the *sectors*, *production factors* and *Private Consumption* accounts are considered endogenous. Columns 2 and 3 present the results obtained by using the usual multiplier matrix, **M**, in the case of, respectively, exogeneity and endogeneity of the Foreign Sector account. Column 4 shows the N_1+N_2+I values resulting from the decomposition analysis, while on column 5 the multipliers that correspond to matrix $(I-A^n+H)^{-1}$ are displayed. Columns 6 to 9 present the resulting sector rankings according to each criterion.

Table 2
Total backward linkage coefficients and resulting distributions according to endogeneity criteria and models used

Sector	Addition by columns (*backward linkage* coefficients of each sector)				Position of each sector in the sector rankings according to the *backward linkage* effect			
	Foreign Sector Exogenous	Foreign Sector Endogenous			Foreign Sector Exogenous	Foreign Sector Endogenous		
	$(I-A)^{-1}$	$(I-A)^{-1}$	N_1+N_2+I	$(I-A^n+H)^{-1}$	$(I-A)^{-1}$	$(I-A)^{-1}$	N_1+N_2+I	$(I-A^n+H)^{-1}$
1.- AGRICULTURE, LIVESTOCK AND FORESTRY	5,72	9,36	6,12	5,27	6	12	4	5
2.- FISHING	4,61	9,11	5,11	4,08	18	17	18	17
3.- COAL	3,41	8,86	4,01	2,78	26	20	26	26
4.- PETROLEUM AND NATURAL GAS	1,06	10,03	2,05	0,06	30	3	30	30
5.- NON-ENERGY EXTRACTIVE INDUSTRIES	3,81	9,30	4,41	3,17	23	14	23	23
6.- COKING PLANTS, REFINERY AND NUCLEAR FUELS	2,81	10,21	3,63	1,98	29	1	29	29
7.- ELECTRIC ENERGY PRODUCTION AND DISTRIBUTION	5,72	9,25	6,11	5,29	5	15	5	4
8.- GAS PRODUCTION AND DISTRIBUTION	3,26	10,03	4,00	2,49	28	2	27	28
9.- WATER COLLECTION, TREATMENT AND DISTRIBUTION	5,39	8,69	5,76	4,99	9	22	9	10
10.- FOOD, BEVERAGES AND TOBACCO	5,76	10,02	6,23	5,25	4	4	2	6
11.- TEXTILES, LEATHER AND FURS	4,41	9,58	4,98	3,81	19	10	19	20
12.- WOOD MANUFACTURES	4,65	9,57	5,19	4,07	17	11	17	18
13.- CHEMICAL INDUSTRY	3,67	9,66	4,33	2,99	24	6	24	24
14.- BUILDING MATERIALS	5,29	9,02	5,71	4,85	10	18	10	14
15.- MINING AND IRON AND STEEL INDUSTRY	4,24	9,60	4,83	3,62	21	8	20	21
16.- METAL MANUFACTURES	4,91	9,21	5,39	4,41	15	16	15	15
17.- MACHINERY	3,27	9,59	3,97	2,55	27	9	28	27
18.- VEHICLES	3,52	9,97	4,23	2,78	25	5	25	25
19.- TRANSPORT ELEMENTS	3,85	9,62	4,49	3,19	22	7	21	22
20.- OTHER MANUFACTURES	4,88	9,31	5,37	4,35	16	13	16	16
21.- CONSTRUCTION	5,96	8,86	6,28	5,60	1	19	1	2
22.- SALE AND REPAIR OF MOTOR VEHICLES; AUTOMOTIVE FUEL TRADE	5,59	8,77	5,94	5,20	7	21	7	7
23.- OTHER TRADES	5,92	8,35	6,19	5,60	2	25	3	1
24.- TRANSPORT AND COMMUNICATIONS	5,45	8,67	5,81	5,05	8	23	8	8
25.- OTHER SERVICES	5,24	8,36	5,59	4,85	14	24	11	13
26.- SALE-ORIENTED SERVICES	5,85	8,07	6,09	5,55	3	26	6	3
27.- NON-SALE ORIENTED SERVICES	5,28	7,29	5,50	5,01	11	27	12	9

Source: Own elaboration.

The estimations thus obtained provide three main results. First of all, the excessive increases that values in column 3 show in relation to those in column 2 illustrate how the simple endogenization of the Foreign Sector generates an overvaluation of the multiplier effect of exogenous shocks that exceeds the effect produced by the mere inclusion of an additional endogenous variable and significantly alters the real distribution of sectors according to their impact capacity (see columns 6 and 7). Second, columns 4 and 5 prove that the suggested endogenization methods eliminate the above mentioned overvaluation and produce multipliers that are more real and that make it possible, not only to quantify the effects, but also to generate rankings of those sectors with greater growth potential in the economy that are better adjusted and, at the same time, consistent with the distribution obtained before considering the Foreign Sector as an endogenous variable (see columns 6, 8 and 9).[2]

It is interesting to note that activities such as *Petroleum and Natural Gas* collapse their multiplier effect when imports are endogenized. In general, it can be understood that those activities whose production comes mainly from abroad are more sensitive to the change of methodology, since when eliminating the effect duo to imports, its multiplier effect is reduced to its domestic effect, which in these cases is very small.

Finally, we can also prove the consistency and validity of the proposed methods through the existing correlation between the multipliers resulting from the application of these methods, as well as between them and the initial estimation in which the Foreign Sector was not endogenized (Figure 1). This correlation is almost perfect and provide almost identical sectoral distributions (see columns 8 and 9 in Table 2).

6. CONCLUSIONS

The endogenization method proposed herein allows introducing the Foreign Sector as an endogenous variable in the analysis of SAM linear models, so that no overvaluation effects are produced due to the breaking of the small country assumption in international trade. With this method, multipliers are obtained that not only quantify the effects in a more precise manner but generate better adjusted distributions of the sectors with greater growth potential in the economy. This results in an improvement that is both quantitative and qualitative in relation to the simple inclusion of the Foreign Sector as an endogenous factor. It is

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[2] In a first analysis, both the results (not only those presented here, but in other similar experiments), and the intuition after the proposed methodology, appear that the values of the matrix $(I-A^n + H)$ reflect better the endogenization of imports, since these are introduced directly, whereas in $N1 + N2 + I$ the component $N3$ has been eliminated, which may have a more diffuse effect, even slightly overestimating the true effect. Anyway, it would be necessary to go deeper into the reason of this, probably derived from mathematical questions too.

showed, also, that including imports, multiplier effect of activities dependent on foreign production is reduced to its domestic effect, which in these cases is very small. Concluding, the method proposed proves as well its consistency in relation to other methodologies, for it achieves an isolation of the multiplier effect of demand shocks that matches that of other techniques, and it may be used in various ways depending on the research objective.

Figure 1

Dispersion graph and regression line of sectors *backward linkage* coefficients, in ratio with the average of $(I-A^n+H)^{-1}$ (x-axis) and (N_1+N_2+I) (y-axis) ($R^2=0.99$)
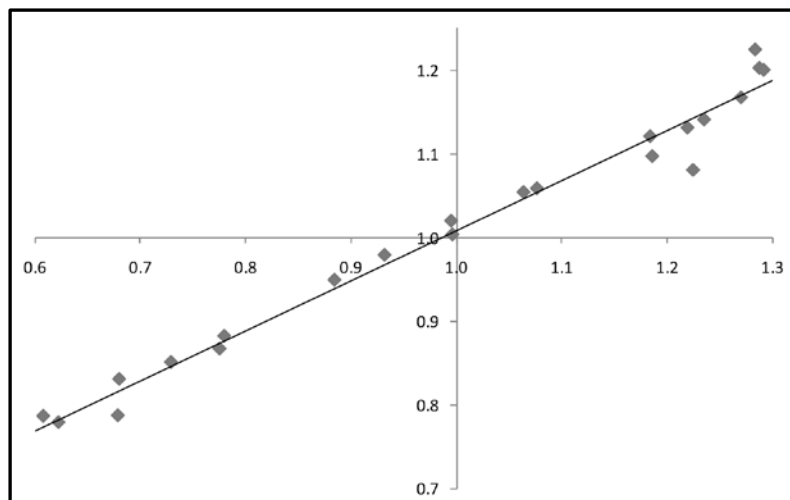
Source: Own elaboration.

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