A HARTREE-BOSE MEAN-FIELD APPROXIMATION FOR IBM-3*)

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A Hartree-Bose mean-field approximation for the IBM-3 is presented. A Hartree-Bose transformation from the spherical to the deformed bosons with charge-dependent parameters is proposed which allows bosonic pair correlations and includes higher angular momentum bosons. The formalism contains previously proposed IBM-2 and IBM-3 intrinsic states as particular limits.

With the advent of radioactive nuclear beam (RNB) facilities unexplored regions of the nuclear chart will become available for spectroscopic studies. Nuclei with roughly equal numbers of protons and neutrons ($Z \approx N$) and with masses in between ^{40}Ca and ^{100}Sn are of particular interest. Although recent breakthroughs [1, 2] have made shell-model calculations possible for this mass region, they are still of a daunting complexity and alternative approximation schemes are required that yield a better intuitive (e.g. geometric) insight.

One of the possible alternatives is the Interacting Boson Model (IBM) [3]. It has been shown [4] that nuclei with protons and neutrons filling the same valence shell require an extended boson model, IBM-3. In IBM-3 three types of bosons are included: proton–proton (π), neutron–neutron (ν), and proton–neutron (δ). The π, ν, and δ bosons are the three members of a $T = 1$ triplet, and their inclusion is necessary to obtain an isospin-invariant formulation of the IBM.

The mean-field formalism has been an important tool to acquire a geometric understanding of the IBM ground state and of the vibrations around the deformed equilibrium shape [5–7]. Moreover, a treatment based on mean-field techniques generally leads to a considerable reduction in the complexity of the calculation, allowing the introduction of additional degrees of freedom if needed.

An intrinsic-state formalism for the IBM-3 was recently presented by Ginocchio and Leviatan (GL) [8]. Here the charge-independent deformation parameters are imposed in the Hartree-Bose transformation from spherical to deformed bosons and

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the trial wavefunction is taken to have good isospin and isospin $SU(3)$ symmetry. We therefore present in this work a generalization of the treatment of GL in which none of the above symmetries (isospin $SU(3)$ and $SU(2)$) is imposed on the trial wavefunction and which includes bosons of angular momenta higher than $\ell = 2$.

Deformed bosons (Γ) are defined in terms of spherical ones (γ) by means of a unitary Hartree-Bose transformation

$$\Gamma^\dagger_{p\tau} = \sum_{\ell m} \eta^{p\tau}_{\ell m} \gamma^\dagger_{\ell m \tau}, \qquad \gamma^\dagger_{\ell m \tau} = \sum_{p} \eta^{*p\tau}_{\ell m} \Gamma^\dagger_{p\tau}, \tag{1}$$

and their hermitian conjugates. Note the explicit dependence on the isospin component τ of the transformation η, allowing different structures for the different condensed bosons π, ν, and δ. The index p labels different possible deformed bosons. We choose $p = 0$ for the fundamental deformed bosons. Since in this work we only treat the ground-state condensed boson, the Hartree superscript p is always zero here and in the following it will be omitted.

Following Ref. 8, the trial wavefunction for the ground state of an even–even system with a proton excess is of the form (the trial wavefunction for an even–even system with a neutron excess is obtained by interchanging the role of protons and neutrons)

$$|\phi(\alpha)\rangle = {\Lambda^\dagger}^{N_n}(\alpha) {\Gamma^\dagger_1}^{N_p - N_n} |0\rangle\,, \tag{2}$$

where the operator $\Lambda^\dagger$ creates a correlated bosonic pair in isospin space

$$\Lambda^\dagger(\alpha) = \Gamma^\dagger_1 \Gamma^\dagger_{-1} + \alpha \Gamma^\dagger_0 \Gamma^\dagger_0. \tag{3}$$

In Eq. (2) $N_{\rm p}$ ($N_{\rm n}$) is the number of proton (neutron) pairs in the valence space. The trial wavefunction (2) is not the most general but it is the simplest one that contains the isospin-conserving formalism of GL and the IBM-2 as natural limits.

The Hartree-Bose equations for the orbital variational parameters η are obtained by minimizing the ground-state energy constrained by the norm of the transformation. Assuming a charge-conserving Hamiltonian the following Hartree-Bose equations result:

$$\sum_{\ell_2 m_2} h^\tau_{\ell_1 m_1, \ell_2 m_2} \eta^\tau_{\ell_2 m_2} = E_\tau \eta^\tau_{\ell_1 m_1}, \tag{4}$$

where the Hartree-Bose matrix h^τ is

$$\begin{aligned} h^\tau_{\ell_1 m_1, \ell_2 m_2} &= \epsilon_{\ell_1 \tau} f_1(\alpha, \tau) \delta_{\ell_1 \ell_2} \delta_{m_1 m_2} \\ &+ 2 \sum_{\ell_3 m_3 \ell_4 m_4 \tau_2 \tau_3 \tau_4} \frac{V_{\ell_1 m_1 \tau, \ell_3 m_3 \tau_3, \ell_4 m_4 \tau_4, \ell_2 m_2 \tau_2}}{\sqrt{1 + \delta_{\ell_1 \ell_3} \delta_{m_1 m_3} \delta_{\tau \tau_3}} \sqrt{1 + \delta_{\ell_4 \ell_2} \delta_{m_4 m_2} \delta_{\tau_4 \tau_2}}} \\ &\times \frac{\eta^{*\tau_3}_{\ell_3 m_3} \eta^{\tau_4}_{\ell_4 m_4} \eta^{\tau_2}_{\ell_2 m_2}}{\eta^\tau_{\ell_2 m_2}} f_2(\alpha, \tau \tau_3 \tau_4 \tau_2). \end{aligned} \tag{5}$$

and

$$f_1(\alpha, \tau) = \frac{\langle \phi(\alpha) | \Gamma^\dagger_\tau \Gamma_\tau | \phi(\alpha) \rangle}{\langle \phi(\alpha) \mid \phi(\alpha) \rangle}, \quad f_2(\alpha, \tau_1 \tau_2 \tau_3 \tau_4) = \frac{\langle \phi(\alpha) | \Gamma^\dagger_{\tau_1} \Gamma^\dagger_{\tau_2} \Gamma_{\tau_3} \Gamma_{\tau_4} | \phi(\alpha) \rangle}{\langle \phi(\alpha) \mid \phi(\alpha) \rangle}. \tag{6}$$

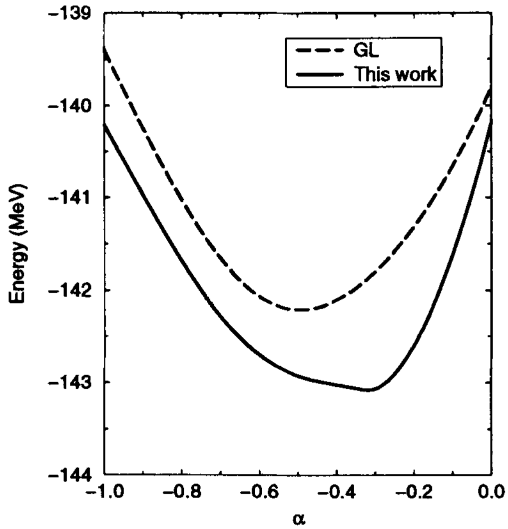

Fig. 1. Ground-state intrinsic energy as a function of α for a system with 5 proton and 3 neutron pairs interacting through the Ginocchio Hamiltonian (7) with $\kappa = 1$ MeV.

The coefficients $V_{\ell_1 m_1 \tau_1, \ell_2 m_2 \tau_2, \ell_3 m_3 \tau_3, \ell_4 m_4 \tau_4}$ are the interaction matrix elements between normalized two-boson states and $\varepsilon_{\ell\tau}$ are the one-boson energies.

To test the present formalism and to compare it with the one by GL, we used a simple Hamiltonian recently proposed by Ginocchio [9],

$$H = -\kappa \sum_{T=0,1,2} (s^\dagger \tilde{\tilde{d}} + (-1)^T d^\dagger \tilde{\tilde{s}})^{L=2,T} : (s^\dagger \tilde{\tilde{d}} + (-1)^T d^\dagger \tilde{\tilde{s}})^{L=2,T} . \qquad (7)$$

In these equations the symbol : denotes a scalar product in orbital and isospin spaces and $\tilde{\tilde{\gamma}}_{\ell m \tau} = (-1)^{\ell - m + 1 - \tau} \gamma_{\ell - m - \tau}$. The Hamiltonian (7) is clearly isospin invariant.

Figure 1 shows, for a system with 5 proton pairs and 3 neutron pairs, the ground-state energy for the Hamiltonian (7) as a function of α. The dashed line is calculated with τ-independent deformation parameters; the GL minimum energy is reproduced for $\alpha = -\frac{1}{2}$. The full line is calculated with the present formalism. The latter calculation always gives a lower energy and, in particular, the minimum is not obtained for $\alpha = -\frac{1}{2}$ but for $\alpha \approx -0.32$. In addition, the corresponding deformation parameters are τ dependent. We note that for a system with equal number of protons and neutrons the present formalism recovers exactly the GL results; differences occur for $Z \neq N$. In all our calculations we found that the Ginocchio Hamiltonian (7) leads to a γ-independent energy surface.

In summary, we have extended the intrinsic-state formalism of Ginocchio and Leviatan [8] for IBM-3 in three different ways. First, the Hartree-Bose transformation is chosen to depend on the isospin component τ; this allows a richer-structured energy surface, contrary to the case of GL where it has the same functional form as in IBM-1. Second, variable isospin bosonic pair correlations are introduced through the parameter α. Finally, higher-order bosons, other than the usual s and d bosons, are included in the Hartree-Bose transformation. This formalism contains the IBM-2 and GL intrinsic states as particular limits. Substantial differences in the deformation parameters are obtained when $N_\mathrm{p} \neq N_\mathrm{n}$.

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