
COMMENTS

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Comment on “Pairing interaction and Galilei invariance”

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(Received 16 December 1997)

A recent article by Dussel, Sofia, and Tonina studies the relation between Galilei invariance and dipole energy weighted sum rule (EWSR). The authors find that the pairing interaction, which is neither Galilei nor Lorentz invariant, produces big changes in the EWSR and in effective masses of the nucleons. They argue that these effects of the pairing force could be realistic. In this Comment we stress the validity of Galilei invariance to a very good approximation in this context of low-energy nuclear physics and show that the effective masses and the observed change in the EWSR for the electric dipole operator relative to its classical value are compatible with this symmetry. [S0556-2813(99)00805-5]

PACS number(s): 21.60.Jz, 21.30.-x

In a recent paper [1], Dussel, Sofia, and Tonina presented a detailed study of the effect of using the pairing force for calculating the energy weighted sum rule (EWSR) for a mass dipole operator. In that work they developed a very useful formalism based on the coupled angular momentum scheme and found that the EWSR changes as much as 18% for medium and heavy nuclei. This result is in agreement with previous calculations for the electric dipole EWSR [2]. These changes are attributed, in both works, to the violation of the Galilei invariance by the pairing interaction. However, while in [2] it is argued that these changes are spurious, in [1] it is claimed that they may be physical and indicate a genuine breaking of Galilei invariance in the nuclear Hamiltonian. They mentioned two main arguments to put into question the requirement of Galilei invariance of the nuclear Hamiltonian: (i) the dynamical effective mass of the nucleon inside the nucleus is considerably smaller than its free mass, and (ii) the experimental data for the EWSR for the $E1$ operator are systematically larger than its classical value. In this Comment we show that Galilei invariance should be a good symmetry for the study of nuclei at low excitation energies and that the observed deviations of the electric dipole EWSR from its classical value and the effective masses of nucleons can be achieved within Galilei invariant interactions.

Galilei invariance of a system as a whole implies that its intrinsic properties do not depend on the velocity of the system of reference that one uses to describe it. It implies that the Hamiltonian can be written as

$$H = \frac{P_{\text{c.m.}}^2}{2M_t} + H_i \quad (\text{intrinsic variables}), \quad (1)$$

where $\vec{P}_{\text{c.m.}}$ is the center-of-mass linear momentum and M_t is the total mass of the system. When the velocity of the reference system is large, relativistic effects such as Lorentz con-

traction can induce deviations from Galilei invariance. Lorentz invariance should be considered instead. Lorentz invariance and Galilei invariance are equivalent when $P_{\text{c.m.}} \ll M_t c$. One can always take a reference frame where the nucleus is initially at rest. The nucleus can acquire momentum and energy, for example, by absorbing a γ ray, but the momentum and/or excitation energy involved in low-energy nuclear physics (about 10 MeV/c and 10 MeV, respectively) will be negligible compared to its mass (about 100 GeV/c²). The fact that M_t is in general less than the sum of the mass of the free nucleons is a relativistic effect, but it does not affect the validity of Galilei invariance as far as M_t can be taken as a constant, which is the case of low-energy nuclear physics. Thus, Galilei invariance will be satisfied to a great degree of accuracy. It does not mean that relativistic effects may not be important for the intrinsic variables (for example, through spin-orbit forces that depend on the intrinsic moments), but the dependence on the center-of-mass momentum should be as in Eq. (1).

The presence of effective masses of the nucleons in a nucleus and the experimental deviations of the electric dipole EWSR with respect to its classical value do not imply necessarily violations of Galilei invariance. They are associated with the dependence of the Hamiltonian on the intrinsic momenta. To illustrate this, we can consider a Hamiltonian of two particles of mass m without spin. It will in general depend on the coordinate and momentum of the center of mass $\vec{R}_{\text{c.m.}}, \vec{P}_{\text{c.m.}}$, and the relative coordinate and momentum \vec{r}, \vec{p} . Translational invariance means that the Hamiltonian should not depend on $\vec{R}_{\text{c.m.}}$, and Galilei invariance means that the dependence on $\vec{P}_{\text{c.m.}}$ should be as in Eq. (1). However, Galilei invariance implies no restriction on the dependence on \vec{p} . Thus, a general translational and Galilei invariant Hamiltonian can be written as

$$H = \frac{P_{c.m.}^2}{2M_t} + \frac{p^2}{2\mu} + V(\vec{r}, \vec{p}), \quad (2)$$

where $M_t = 2m$ and $\mu = m/2$. To illustrate the appearance of effective masses and the changes in the sum rule, we will take $V(\vec{r}, \vec{p}) = V_0(r) + \vec{p}V_2(r)\vec{p}$. This makes it such that the Hamiltonian can be written as

$$H = \frac{P_{c.m.}^2}{2M_t} + \vec{p} \frac{1}{2\mu_{\text{eff}}(r)} \vec{p} + V_0(r), \quad (3)$$

where

$$\frac{1}{2\mu_{\text{eff}}(r)} = \frac{1}{2\mu} + V_2(r). \quad (4)$$

Thus, we see that the dependence of the interaction on the relative momentum generates an effective reduced mass. We could obtain an effective particle mass from the effective reduced mass as $m_{\text{eff}}(r) = 2\mu_{\text{eff}}(r)$. However, the total mass of the system is unaffected and it is not correct to identify it with the sum of the effective masses of the constituents.

Let us now calculate the energy weighted sum rules EWSR_{rel} and $\text{EWSR}_{\text{c.m.}}$ associated with the operators \vec{r} and $\vec{R}_{\text{c.m.}}$, respectively. We get

$$\begin{aligned} \text{EWSR}_{\text{rel}} &= \sum_n \langle \text{g.s.} | \vec{r} | n \rangle (E_n - E_{\text{g.s.}}) \langle n | \vec{r} | \text{g.s.} \rangle \\ &= \frac{1}{2} \langle \text{g.s.} | [\vec{r}, [H, \vec{r}]] | \text{g.s.} \rangle, \end{aligned} \quad (5)$$

where $|\text{g.s.}\rangle$ stands for the ground state and $|n\rangle$ for the excited states. A similar expression can be obtained for $\text{EWSR}_{\text{c.m.}}$:

$$\text{EWSR}_{\text{c.m.}} = \frac{1}{2} \langle \text{g.s.} | [\vec{R}_{\text{c.m.}}, [H, \vec{R}_{\text{c.m.}}]] | \text{g.s.} \rangle. \quad (6)$$

Evaluating the double commutators, we get

$$\begin{aligned} \text{EWSR}_{\text{rel}} &= 3\hbar^2 \left\langle \text{g.s.} \left| \frac{1}{2\mu_{\text{eff}}(r)} \right| \text{g.s.} \right\rangle \\ &= \frac{3\hbar^2}{2\mu} + 3\hbar^2 \langle \text{g.s.} | V_2(r) | \text{g.s.} \rangle \end{aligned} \quad (7)$$

and

$$\text{EWSR}_{\text{c.m.}} = \frac{3\hbar^2}{2M_t}. \quad (8)$$

Thus, we see that the EWSR associated with a relative coordinate can indeed be modified by the presence of a

momentum-dependent interaction, and this change is closely related to the change in the effective mass for that coordinate. However, the EWSR related to the center of mass is not modified.

These results can be extended to a system with A particles. The center-of-mass coordinates can always be completely decoupled from the intrinsic ones using Jacobi coordinates or the redundant variable method ([3], pp. 454 and 455) having a Hamiltonian similar to Eq. (1). The potential energy of H_i will depend in general on the coordinates \mathbf{r}_i and momenta \mathbf{p}_i of the particles in the center-of-mass frame. Part of this dependence on \mathbf{p}_i can be written in terms of effective masses of the particles ([3], [p. 214, [4]]) in a similar way as in the simple example above. Therefore, the effective masses that appear in mean field calculations refer to the mass associated with the relative coordinate from the nucleon to the center of mass of the nucleus and are originated from momentum-dependent (i.e., nonlocal) interactions. Whereas those interactions do not modify the mass dipole mode (whose associated operator is proportional to the center-of-mass coordinate), the velocity dependence of the interactions does contribute to the mass parameter of the isovector dipole mode, and so to its EWSR, through the neutron-proton interaction ([5], p. 484). Such dependence originates an oscillator strength in the dipole resonance about 20% larger than the classical value ([5], p. 486), compatible with the available empirical evidence in heavy nuclei. Thus, the deviation of the electric dipole EWSR from the classical value is not an indication of violation of Galilei invariance. It indicates that an interaction between neutrons and protons depending on the relative momentum exists.

The pairing potential usually used does not include interactions between protons and neutrons. Therefore, both EWSR's, for the center-of-mass operator and for the isovector dipole operator, have the same value. In a nonrelativistic scheme, the first one should coincide with the classical dipole EWSR; so the whole calculated increase of around 20% over the classical value is due to a breaking of the same order of Galilei invariance. Such a big violation, as stated above, is not plausible in low-energy nuclear physics.

In conclusion, the increase in the sum rule induced by pairing interactions is due, as is shown in this Comment and in Ref. [2], to the fact that it is not Galilei invariant. In a nonrelativistic description, the observed increase should be studied using a Galilei invariant Hamiltonian, including interactions depending on the proton-neutron relative momenta.

This work has been supported in part by the Spanish DGICYT under Contract No. PB95-0533 and by the European Commission under Contract No. CII*-CT94-0072.

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