

# Connection between decoherence and excited state quantum phase transitions

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## Abstract.

In this work we explore the relationship between an excited state quantum phase transition (ESQPT) and the phenomenon of quantum decoherence. For this purpose, we study how the decoherence is affected by the presence of a continuous ESQPT in the environment. This one is modeled as a two level boson system described by a Lipkin Hamiltonian. We will show that the decoherence of the system is maximal when the environment undergoes a continuous ESQPT.

**Keywords:** Quantum decoherence, excited quantum phase transition

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Real quantum systems always interact with the environment. This interaction leads to decoherence, the process by which quantum information is degraded and purely quantum properties of a system are lost.

The connection between decoherence and environmental quantum phase transitions has been recently investigated. In this contribution and in references [1, 2] we analyze the relationship between decoherence and an environmental excited state quantum phase transition (ESQPT).

We will consider our system composed by a spin 1/2 particle coupled to a spin environment by the Hamiltonian  $H_{SE}$ :

$$H_{SE} = I_S \otimes H_E + |0\rangle\langle 0| \otimes H_{z_0} + |1\rangle\langle 1| \otimes H_{z_1}.$$

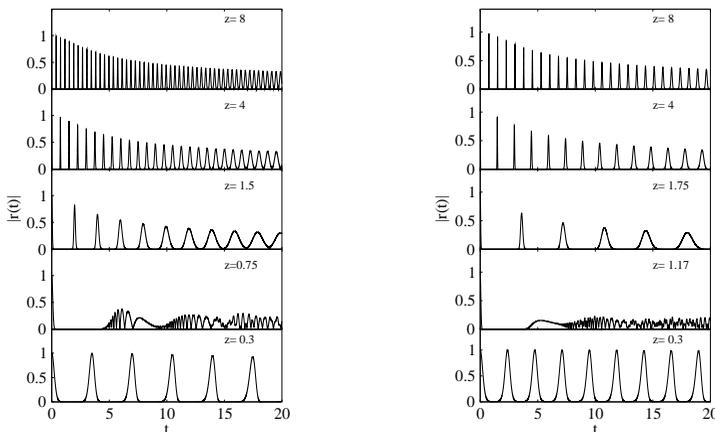
With this kind of coupling, the environment evolves with an effective Hamiltonian depending on the state of the central spin  $H_j = H_E + H_{z_j}$ ,  $j = 0, 1$  giving rise to a decoherence factor (up to an irrelevant phase factor)  $r(t) = \langle g_0 | e^{-iH_1 t} | g_0 \rangle$ , where  $|g_0\rangle$  is the ground state of  $H_0$ . To be specific, we introduce a two-level boson model in terms of two species of scalar bosons  $s$  and  $t$  and whose Hamiltonian is

$$H_E = x n_t - \frac{1-x}{N} Q^y Q^y, \quad \text{with } Q^y = s^\dagger t + t^\dagger s + y t^\dagger t,$$

where  $n_t$  is the number of  $t$  bosons and  $N$  the total number of bosons. This Hamiltonian has a second order ground state quantum phase transition (QPT) along the line  $y = 0$ ,

and a first order ground state QPT for  $y \neq 0$ . Choosing  $z_0 = 0$  and  $z_1 = z$  the coupling Hamiltonian is  $H_{\text{Coup}} = z n_t$ , which results into the effective Hamiltonians for each component of the system  $H_0 = H_E$ ,  $H_1 = (x+z) n_t - \frac{1-x}{N} Q^y Q^y$ .

A semiclassical calculation shows that  $H_E$  experiments a continuous ESQPT at  $E_c = 0$ , if  $z < z_*$ . Where  $z_*$  provides the value at which  $H_1$  suffers a ground state QPT [1, 2]. In Fig. 1 we show the modulus of the decoherence factor  $|r(t)|$  for  $x = 0.5$  and several  $z$  values for two  $y$  values:  $y = 0$  (left panel) and  $y = 1/\sqrt{2}$  (right panel). In each case, four of the five cases of  $z$  show a similar patterns for the decoherence factor, fast oscillations plus a smooth decaying envelope. When  $z$  drives the environment to the critical energy  $E_c$  corresponding to a continuous ESQPT ( $z = 0.75$  for  $y = 0$  and  $z = 1.17$  for  $y = 1/\sqrt{2}$ ) then  $|r(t)|$  quickly decays to zero and randomly oscillates around a small value. We have tested that this behaviour is similar for other  $x$  and  $y$  values. Therefore, we conclude that **if the system-environment coupling drives the environment to the critical energy  $E_c$  of a continuous ESQPT, the decoherence induced in the coupled qubit is maximal** [1]. We have also studied the behaviour of the decoherence in the thermodynamic limit in references [1, 2].



**FIGURE 1.**  $|r(t)|$  for  $x = 1/2$  and five different values of  $z$  for two selections of the coupling system-environment parameter  $y$ . In all cases  $N = 1000$ . The left panel is for  $y = 0$ , and the right one for  $y = 1/\sqrt{2}$ .

In conclusion, we have found that the decoherence is maximal when the system-environment coupling introduces in the environment the energy required to undergo a continuous ESQPT either we set  $y = 0$  or  $y \neq 0$ .

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## REFERENCES

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