Stabilization method in two-body systems with core excitations

J.A. Lay*, J.M. Arias*, J. Gómez-Camacho*,† and A.M. Moro*

*Departamento de FAMN, Facultad de Física, Universidad de Sevilla, Apdo. 1065, E-41080 Sevilla, Spain
†Centro Nacional de Aceleradores, Avda. Thomas A. Edison, E-41092, Sevilla, Spain

Abstract. The validity of the stabilization method in core+valence systems including the possibility of exciting the core is studied. A pseudostate method, based on the transformed harmonic oscillator basis, is extended to include the core degrees of freedom. The method is applied to the case of ^{11}Be structure considering the 0^+ ground state and the 2^+ first excited state of the ^{10}Be core. The stabilization method is defined in terms of one parameter that can be chosen either discrete or continuous. In the application to ^{11}Be, both cases are analyzed.

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INTRODUCTION

Recently, the study of reactions involving loosely bound exotic nuclei have been one of the most active fields in Nuclear Physics. This kind of reactions is known to be strongly influenced by the coupling to the unbound states of the weakly bound nucleus. Therefore there has been a great interest in finding a realistic description of the continuum of these nuclei.

In reaction calculations, one-neutron halo nuclei are usually described using a two-body model, comprising an inert core and one valence particle. This Hamiltonian can be solved using pseudostates (PS). The PS method consists in diagonalizing the Hamiltonian in a square-integrable basis. The eigenstates obtained are a finite approximation of the bound and unbound states of the nucleus. Moreover, the positive energy eigenstates can help to understand the structure of the continuum and its influence in reactions involving these nuclei. Several bases have been tested in the PS method. Here we use a transformed harmonic oscillator (THO) basis, which is formed by applying the following transformation

$$s(r) = \left[\frac{1}{\left(\frac{1}{r}\right)^4 + \left(\frac{1}{\gamma\sqrt{r}}\right)^4} \right]^{\frac{1}{4}}, \tag{1}$$

to the solutions of the isotropic harmonic oscillator. This transformation was first introduced in [1]. This basis is easy to calculate and, in most of the cases, is able to reduce the minimum number of functions needed for convergence of the scattering observables with respect to other bases [2].

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As mentioned above, in the description of halo nuclei usually an inert core is considered. However, recent experimental and theoretical developments suggest that possible excitations of the core may affect not only to the structure but also to the reaction dynamics. In a recent work [3] we have generalized the THO basis in order to include the excitations of the core in a simple particle-rotor model.

Here we study in more detail the so-called stabilization method that allows to identify resonances in the continuum using PS basis [4]. In this method, we assume there is a parameter related to the extension of the basis. The resulting eigenvalues depend on this parameter, so that they typically decrease in energy when the basis extension is increased. When an eigenvalue crosses the energy of a resonance, this behaviour changes. The energy of the eigenvalue is stabilized for a certain range of values of the parameter giving rise to a plateau around the energy of the resonance. Moreover, the pseudostate corresponding to this stabilized energy is found to reproduce well the properties of the resonance.

THE THO APPROACH TO PARTICLE-ROTOR MODEL

In the weakly coupling limit, when we consider a core+valence system including different excitations of the core, we have a Hamiltonian of the form:

$$H = T_r + V_{vc}(\vec{r},\vec{\xi}) + h_{\text{core}}(\vec{\xi}), \tag{2}$$

where $h_{\text{core}}(\vec{\xi})$ represents the internal Hamiltonian of the core, with eigenstates $\phi_I(\vec{\xi})$. $V_{vc}(\vec{r},\vec{\xi})$ is the interaction between the particle and the core and it is responsible for the coupling between the valence particle motion and the excitations of the core. It depends on the model assumed for the excitations of the core. In a particle-rotor model, we suppose the core to be permanently deformed. In this case the interaction with the valence particle would depend on the state of rotation of the deformed core and the relative orientation between particle and rotor.

If the coupling is weak we can consider the total wavefunction as a combination of core states and valence configurations coupled to the corresponding total angular momentum J:

$$\Psi_{\varepsilon;JM}(\vec{r},\vec{\xi}) = \sum_{\alpha} R^{J}_{\varepsilon,\alpha}(r) \left[\mathscr{Y}_{\ell s j}(\hat{r}) \otimes \phi_I(\vec{\xi})\right]_{JM}. \tag{3}$$

Each term of this sum is called channel and it is characterized by a given set of quantum numbers $\alpha = \{\ell, s, j, I\}$.

If we now construct a THO function for each channel in the form:

$$\Phi^{\alpha}_{n,JM}(\vec{r},\vec{\xi}) = R^{THO}_{n,\alpha}(r) \left[\mathscr{Y}_{\ell s j}(\hat{r}) \otimes \phi_I(\vec{\xi})\right]_{JM}, \tag{4}$$

we can readily diagonalize the Hamiltonian in a truncated basis of N THO functions so that the final PS will be:

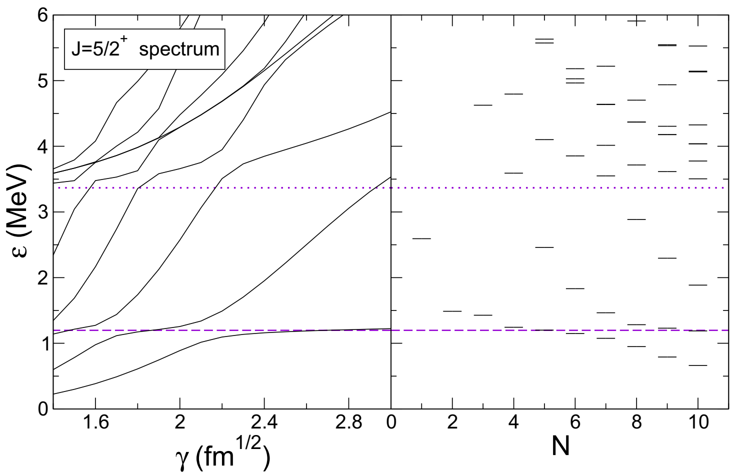

FIGURE 1. (Color online) Eigenvalues obtained from the diagonalization of the ^{11}Be Hamiltonian in a THO basis, as a function of the LST continuum parameter (γ) in the left panel, and as a function of the number of oscillator states included in the basis (N) in the right panel. The dashed line indicates the energy of the 5/2$^+$ resonance and the dotted line, the energy of the ^{10}Be(2$^+$)+n threshold.

$$\Psi^{(N)}_{i,JM}(\vec{r},\vec{\xi}) = \sum_{n=1}^{N}\sum_{\alpha} c^{i}_{n,\alpha,J}\Phi^{\alpha}_{n,JM}(\vec{r},\vec{\xi}). \tag{5}$$

APPLICATION TO ^{11}BE

Parameters γ and N control the radial extension of the THO basis. Therefore, the stabilization method can be applied with both parameters. In previous works we have used N for this purpose, whereas γ was already fixed in order to tailor the density of states to the energy region of interest.

The main drawback is that N is a discrete variable. There is no guarantee that the point where the energy is stabilized coincides with a value of N. That is the main reason to use also γ, a continuous variable. However, the first derivative of the evolution of the eigenvalue is related to the width of the resonance. This implies that, for narrow resonances, the energy does not vary in a region near the stabilized point and the nearest value of N is a good approximation to the stabilized energy.

In order to analyze the validity of γ and N as stabilization parameters in two-body systems with excitations of the core, we apply the PS THO basis to ^{11}Be considering a particle-rotor model. Here we consider that the core ^{10}Be has a quadrupole deformation with $\beta = 0.67$. We include only the first excited state of the core ^{10}Be, 2^+, in addition to its 0^+ ground state. Further details of the potential can be found in [5]. The first resonance in ^{11}Be has spin and parity $5/2^+$. In figure 1, we show the spectrum obtained varying both γ and N parameters. We see that the stabilization appears at the energy of the resonance (1.2 MeV above the ^{10}Be(0^+)+n threshold), so that the stabilization method remains valid in this excited core framework.

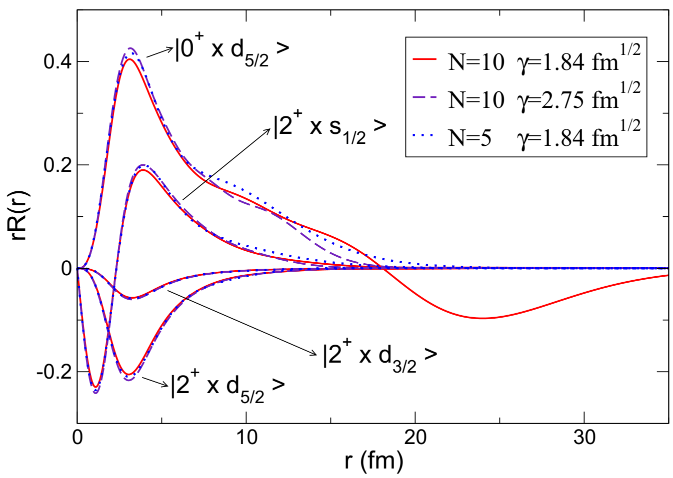

FIGURE 2. (Color online) Radial wavefunctions of the $5/2^+$ resonance in ^{11}Be for three different stabilized set of parameters N and γ.

However it is necessary to check how representative are the corresponding eigenstates when we vary N and it is not possible to obtain the exact stabilized point. In order to do so, we show in figure 2, two different wavefunctions for two different values of N close to this point. We also compare with a wavefunction where we vary γ for a similar N where we can get the *exact* parameter for the stabilization. We see that the main difference between the three wavefunctions is its extension, so that the interior is always the same, thus retaining the properties of the resonance. Therefore, the selection of the exact parameter for stabilization is not crucial to obtain a pseudostate representative of the resonance.

This first resonance has mainly a $d_{5/2}$ configuration for the halo neutron with the ^{10}Be core in its 0^+ ground state. Therefore it is a case very similar to the two-body case with an inert core. A more complicated case are resonances whose main configuration is bound. This is the case of the first $3/2^+$ resonance in ^{11}Be. The main contribution of this resonance is an $s_{1/2}$ wave with the ^{10}Be core in its 2^+ excited state. This resonance is below the ^{10}Be(2^+)+n threshold so that this configuration is effectively bound. Nevertheless, we find in figure 3 that the eigenvalues are also stabilized for the energy of this resonance (3.0 MeV above the ^{10}Be(0^+)+n threshold).

CONCLUSIONS

We have seen how we can construct a THO basis for two-body systems with excited core in order to apply the PS method. The stabilization method is used to identify the resonant energies of a two-body halo nucleus, including the effect of core excitations. As an example, the method has been applied to the $5/2^+$ resonance in ^{11}Be, using a particle-rotor model to describe this nucleus. The stabilization in the evolution of the eigenenergies provides the value of the resonance, as seen in two-body systems with an

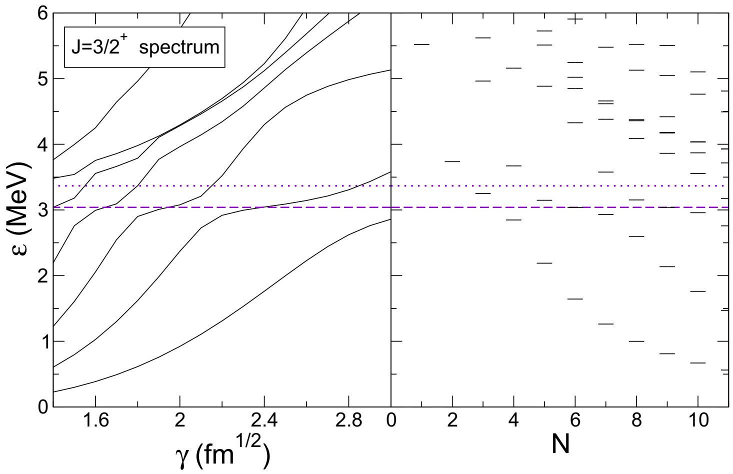

FIGURE 3. (Color online) Eigenvalues obtained from the diagonalization of the ^{11}Be Hamiltonian in a THO basis, as a function of the LST continuum parameter (γ) in the left panel, and as a function of the number of oscillator states included in the basis in the right panel. The dashed line indicates the energy of the 3/2$^+$ resonance and the dotted line, the energy of the ^{10}Be(2$^+$)+n threshold.

inert core. We also found that the corresponding pseudostates for different stabilized sets of parameters are representative of this resonance. Even using the discrete parameter *N* where we cannot select the the *exact* value for stabilization, we obtain good results.

For the $3/2^+$ resonance in ^{11}Be we found the same stabilization pattern. Therefore the stabilization method applies also for resonances whose main configuration is bound with respect to the corresponding core+valence threshold. Moreover, we note that the $3/2^-$ resonance predicted by the particle-rotor model used here gives similar results, although we have not include it in the discussion [3].

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