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ORIGINAL PAPER

A computational comparison of several formulations for the multi-period incremental service facility location problem

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Abstract The Multi-period Incremental Service Facility Location Problem, which was recently introduced, is a strategic problem for timing the location of facilities and the assignment of customers to facilities in a multi-period environment. Aiming at finding the strongest formulation for this problem, in this work we study three alternative formulations based on the so-called impulse variables and step variables. To this end, an extensive computational comparison is performed. As a conclusion, the hybrid impulse–step formulation provides better computational results than any of the other two formulations.

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1 Introduction

Given a time horizon, a set of customers and a set of facilities (e.g., production plants), the *Multi-period Incremental Service Facility Location Problem* (MISFLP) is concerned with locating the facilities within a given discrete set of potential sites and assigning the customers to the facilities along given periods in a time horizon.

In actual world, it is usual to look for sequential decisions that ensure a certain level of coverage at each time period. In the case of essential services, it is required that the full population demand be serviced at all time periods. Different works have addressed situations of this type since the late sixties. Most of these approaches have been used for the design of supply structures by deciding which existing facilities should be closed and where new facilities should be opened. The pioneering work by Ballou (1968) on the single-facility version of the problem was continued by the works of Lodish (1970), Wesolowsky (1973), Drezner and Wesowlosky (1991), Bastian and Volkmer (1992), Andreatta and Mason (1994). On the other hand, the multi-facility dynamic problem has been addressed by Scott (1971), Warszawski (1973), Wesolowsky and Truscott (1975), Khumawala and Whybark (1976), Kelly and Marucheck (1984), Rosenblatt (1986), Frantzeskakis and Watson-Gandy (1989), Chardaire et al. (1996), Saldanha da Gama and Captivo (1998), and Hinojosa et al. (2008). van Roy and Erlenkotter (1982) extend the adjustment procedure developed in Erlenkotter (1978) for the static problem to solve the dynamic problem. The dynamic p-median problem has been studied by Galvão and Santibañez-Gonzalez (1990, 1992) and Drezner (1995). One of the earliest attempts to incorporate capacity constraints is due to Sweeney and Tatham (1976), whereas Melachrinoudis et al. (1995) studied a multi-objective version of the capacitated multi-period location problem. Some authors have addressed the problem of deciding in which periods the capacity of a given set of facilities should be extended to meet increasing demand patterns (see, for instance, Antunes and Peeters 2001; Erlenkotter 1981; Fong and Srinivasan 1981, 1986; Jacobsen 1977; Lee and Luss 1987; Shulman 1991) or allow the possibility of resorting to external suppliers (see Hinojosa et al. 2000). Recently, Melo et al. (2006) have studied the mathematical modeling framework for dynamic multi-commodity facility location within the context of strategic supply chain planning.

However, in the case of nonessential services, full coverage needs to be attained only at the end of the planning horizon. Instead, strategic and organizational reasons require that a pre-specified level of coverage be reached at each time period and that the demand of covered customers remains satisfied in the subsequent periods. It is clear that these types of problems represent actual applications. The simplest application is the sequential optimal location plan and distribution pattern of a good or service to reach full coverage of a set of potential customers in a finite time horizon. This is, for instance, the case of libraries, nursing homes, kindergartens, parking lots, supermarkets, banks, etc., that are planed to provide service to the entire set of potential customers. The nature of these applications requires that service to users can not be interrupted in the remaining planning horizon once it has been started. Nevertheless, budgetary constraints avoid to achieve complete coverage in one single period, and therefore, an optimal plan is needed to fulfill the full coverage goal while ensuring partial achievements at minimum cost over the whole planning horizon. In a recent paper, Albareda-Sambola et al. (2009) have proposed the MISFLP to minimize the total cost throughout a finite time horizon while ensuring at each single period t the service of a minimum number of customers, say n^t. In the MISFLP it is accepted that the allocation of any customer to the servers might change in different periods. Nevertheless, once a customer is served (allocated) in a time period, it must be served at any subsequent period. Moreover, it is assumed that p^t new facilities are opened in each time period, at least, and that once a facility is opened, it remains open until the end of the time horizon. Notice that the value of p^t can be zero, but it can also be a positive number due to strategic reasons of the decision maker.

In Albareda-Sambola et al. (2009) it is seen that the MISFLP is quite general, and it has as particular cases problems with apparently different characteristics. When the planning horizon consists of one single period, the MISFLP reduces to the well-known p-median problem. For a general planning horizon, when the assignment costs are nonnegative and there is a pre-specified coverage level at each intermediate period ($n^t > 0$), the resulting problem can be seen as a cost-minimizing problem. On the other hand, the profit-maximizing version of the problem where there is no required coverage level at each intermediate period ($n^t = 0$), but there is a profit associated with serviced customers, also fits within the MISFLP. Moreover, one can introduce penalties to require that all customers are served at all time periods or to allow customers with service demand not to be served. The MISFLP belongs to the class NP-hard; notice that it reduces to the well-known p-median problem.

One question that arises naturally when addressing the MISFLP is whether or not the problem that we consider could be also modeled by means of a series of independent decoupled models (one for each time period). This point was analyzed in Albareda-Sambola et al. (2009), where the authors compared empirically the results obtained with the integrated formulation and the solution of series of decoupled models. Their results enlighten the differences between both models. On the one hand, the deviations of the optimal values of the decoupled model with respect to the MISFLP were relatively large. On the other hand, the differences between the two types of models affected not only to the optimal values but also to the structures of the optimal solutions, specially for larger instances.

In this paper we address a particular case of the MISFLP where each customer needs to be serviced only in a subset of the periods of the time horizon. In this version, we assume that this set of periods is known for each customer and that at each period, customers with demand are not necessarily served. In this case, a penalty cost is incurred to reduce the effect of loss of customers goodwill. Alternatively, these penalties can be seen as the pricing for resorting to external suppliers. As it will be seen when we formulate this problem, this particular case fits within the MISFLP model.

Multi-period location problems are related to the evolution of dynamic organizations. In this context, there are various elements that evolve with time, like costs, availability of resources, demands, etc., which define the framework for the development of the system. Typically, the organizations have historical data on the evolution of such elements that are used to forecast their values in the near or medium-term future. Therefore, it is natural to assume that estimations on the set of periods when each customer will have demand are available. One can argue, however, that, in practice, the actual behavior of the system is not deterministic. This leads to consider stochastic programming counterparts of multi-period location problems, where historical data are used to estimate the probability distribution that governs the customers behavior. This is the actual goal of a future research, in which a multi-period location problem where some parameters are not known with certainty is studied, and Stochastic Integer Programming is used. It is well known that stochastic integer programs are extremely difficult to solve exactly, so that using appropriate formulations is very important. For instance, if branch-and-fix-coordination (Alonso-Ayuso et al. 2003) is used, many deterministic instances of the problem need to be solved exactly. Therefore, finding a strong formulation for the deterministic version of the problem becomes a major concern. This issue is specifically the goal of the current paper. In particular, our objective is to propose different formulations for the MISFLP and to compare them computationally for deciding which is the most suitable formulation to be used when addressing the stochastic version of the MISFLP.

Nevertheless, the interest of finding out the strongest formulation for the deterministic MISFLP does not reduce to the possibility of using it in a stochastic framework. Notice that the MISFLP has been recently proposed, but it is a very flexible model, which has many applications. In particular, there are many potential extensions of the MISFLP that have not been studied so far like, for instance, multiple products, capacitated facilities, and more than one facility layer, among others. Knowing which is the best formulation for the basic model is an important step towards the possibility of solving these extensions efficiently.

In this work, we present three 0–1 formulations of the model based on the methodology of impulse variables and step variables introduced in Bertsimas and Stock-Patterson (1998) in a different context. The first formulation we propose basically coincides with the formulation proposed in Albareda-Sambola et al. (2009). An extensive computational comparison is made to assess the performance of the three formulations and to select the formulation to use in the stochastic programming setting. The computations have been performed using a state-of-the-art optimization system.

The rest of the paper is organized as follows: Sect. 2 is devoted to the presentation of the problem. Section 3 presents the impulse variables, the impulse-step variables, and the step variables based formulations. Section 4 is devoted to the computational comparison of the formulations. Finally, Sect. 5 concludes the paper.

2 Problem description

The MISFLP is defined on a network induced by a set of *customers*, a set of potential locations for *facilities*, and a *time horizon* described in terms of a set of consecutive time periods (see Fig. 1). At each time period, a given number of facilities must be opened. It is assumed that, once a facility is opened, it remains open throughout

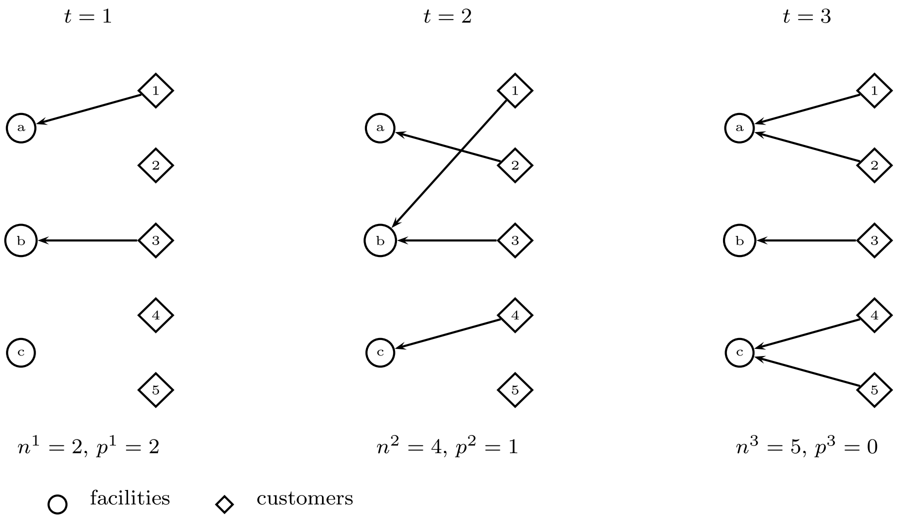

Fig. 1 Facility location and customer assignment

the time horizon. We further assume that the subset of time periods in which each customer has demand of service is known. The demand of service of a customer is not necessarily satisfied at all time periods. At the periods when it is satisfied, the customer is *serviced* from the open facility it is *assigned* to. The assignment of a customer can change in different time periods, and the requests for service of unassigned customers will be neglected. Once a customer has been assigned, we will service it in all the subsequent periods when it has demand until the end of the planning horizon. We require that at each time period, a minimum number of customers have to be assigned and that all customers are assigned at the end of the planning horizon.

The opening of a facility at a given time period has associated a depreciation cost. This depreciation cost includes a fixed setup cost for opening the facility, and its maintenance cost for the remainder of the planning horizon. On the other hand, we differentiate between assignment and service costs for customers. For a given customer, assignment costs will be incurred at all time periods after it has been assigned for the first time. We can consider such cost as the price for keeping the service available to the customer. On the contrary, service cost is only incurred in those periods in which the customer is assigned and it has demand. As usual, we can consider such cost as the actual cost for providing the service. Additionally, we assume that, when a customer is served, all its demand is shipped from one single facility. Thus, service costs already take into account both the distance from the customer to its assigned facility, and the quantity of demand. Finally, every time an unassigned customer has demand, a penalty cost is incurred to reduce the effect of loss of customers goodwill. Alternatively, this penalty can be seen as the pricing for resorting to external suppliers. The goal of the MISFLP is to find the sets of facilities to open and the assignment pattern for each time period that minimize the total cost along the time horizon. As we have just mentioned, the total cost includes setup depreciation costs of open plants, assignment and service costs of customers, and penalties for neglected requests for service.

In the remainder of this paper, the following notation will be used:

Sets

$\mathcal{I}$, Index set of facilities, $m = |\mathcal{I}|$.
$\mathcal{J}$, Index set of customers.
$\mathcal{T}$, Index set of time periods.
$\mathcal{T}_j$, Index set of time periods in which customer j has demand, for $j \in \mathcal{J}$.

Parameters

f_i^t, setup depreciation cost for facility i in time period t, for $i \in \mathcal{I}, t \in \mathcal{T}$.
p^t, minimum number of facilities to be opened at time period t, for $t \in \mathcal{T}$.
n^t, minimum number of customers to be assigned in time period t, for $t \in \mathcal{T}$.
c_{ij}^t, assignment cost of customer j to facility i in time period t, for $i \in \mathcal{I}$, $t \in \mathcal{T}$, $j \in \mathcal{J}$.
s_{ij}^t, service cost of customer j from facility i in time period t, for $i \in \mathcal{I}$, $t \in \mathcal{T}_j$, $j \in \mathcal{J}$.
ρ_j, penalty for not timely servicing customer j, for $j \in \mathcal{J}$.

For all $i \in \mathcal{I}$ and $t \in \mathcal{T}$, let $\overline{c}_{ij}^t = c_{ij}^t + s_{ij}^t - \rho_j, \forall t \in \mathcal{T}_j$, and $\overline{c}_{ij}^t = c_{ij}^t, \forall t \in \mathcal{T} \setminus \mathcal{T}_j$. For a pair (j,t), we define $\mathcal{I}_{jt}$ as the index set of facilities ordered according to the nondecreasing $\overline{c}$–cost coefficients. For $r \leqslant s$, we will use the following notation:

$$i_r \preceq i_s \quad \Longleftrightarrow \quad \overline{c}_{i_r j}^t \leq \overline{c}_{i_s j}^t.$$

Notice that, while r and s denote the rth and sth positions in $\mathcal{I}_{jt}$, respectively, i_r and i_s denote the corresponding elements in the ordered set $\mathcal{I}_{jt}$.

3 Formulation of MISFLP

We next present three different formulations for the MISFLP. They differ one from the other in the type of variables that are used to formulate the location and allocation decisions. In some cases they are impulse variables, whereas in other cases they are step variables. The former are set at value one for indicating changes in the status of the system at a given period, whereas the values of the latter define the actual status of the system at a given period.

3.1 Impulse variables based formulation F1

Variables

They are 0–1 variables such that

$y_i^t = 1$ if facility i is opened *for the first time at* time period t and 0 otherwise, for $i \in \mathcal{I}, t \in \mathcal{T}$.
$x_{ij}^t = 1$ if customer j is *assigned to facility* i at time period t and 0 otherwise, for $i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}$.

Pure 0–1 Formulation

$$\text{(F1)} \quad \min \sum_{i\in\mathcal{I}}\Big[\sum_{t\in\mathcal{T}}\Big(f_i^t y_i^t + \sum_{j\in\mathcal{J}} c_{ij}^t x_{ij}^t\Big) + \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}_j} s_{ij}^t x_{ij}^t\Big]$$

$$+ \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}_j}\rho_j\Big(1-\sum_{i\in\mathcal{I}} x_{ij}^t\Big)$$

$$= \sum_{j\in\mathcal{J}}|\mathcal{T}_j|\rho_j + \min\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\Big[f_i^t y_i^t + \sum_{j\in\mathcal{J}}\overline{c}_{ij}^t x_{ij}^t\Big], \tag{1}$$

subject to

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} x_{ij}^t \ge n^t \quad \forall t\in\mathcal{T}, \tag{2}$$

$$\sum_{i\in\mathcal{I}} x_{ij}^t \le 1 \quad \forall j\in\mathcal{J}, t\in\mathcal{T}, \tag{3}$$

$$\sum_{i\in\mathcal{I}} x_{ij}^t \ge \sum_{i\in\mathcal{I}} x_{ij}^{t-1} \quad \forall j\in\mathcal{J}, t\in\mathcal{T}: t>1, \tag{4}$$

$$\sum_{i\in\mathcal{I}} x_{ij}^{|\mathcal{T}|} = 1 \quad \forall j\in\mathcal{J}, \tag{5}$$

$$x_{ij}^t \le \sum_{0\le k\le t} y_i^k \quad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}, \tag{6}$$

$$\sum_{i\in\mathcal{I}} y_i^t \ge p^t \quad \forall t\in\mathcal{T}, \tag{7}$$

$$\sum_{t\in\mathcal{T}} y_i^t \le 1 \quad \forall i\in\mathcal{I}, \tag{8}$$

$$x_{ij}^t \in \{0,1\} \quad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}, \tag{9}$$

$$y_i^t \in \{0,1\} \quad \forall i\in\mathcal{I}, t\in\mathcal{T}. \tag{10}$$

The objective function (1) consists of minimizing the total cost. Constraints (2) require that there are at least n^t customers assigned to facilities at time period t. Constraints (3) ensure that each customer is assigned to at most one facility at each time period. Constraints (4) demand that a customer is assigned at time period t if it is already assigned at period $t-1$, although the facility can change. Constraints (5) require that all customers are assigned to some facility at the end of the time horizon. Notice that, when $n^{|\mathcal{T}|} = |\mathcal{J}|$, these constraints can be obtained from (2) and (3). Constraints (6) ensure that the customers are assigned to open facilities. Constraints (7) require that a given number p^t of facilities be opened at each time period, at least. Given that at each time period we know the number of customers n^t that must be served, we also know the required system service capacity at each time period. One way to guarantee this overall capacity at each time period is by including a cardinality constraint on the set of plants to open at each time period. Constraints (8) ensure that the facilities are opened at most once along the time horizon.

Observe that formulation F1 also includes the possibility of forcing to serve all demand customers at each time period, by setting $\rho_j = +\infty$ for all $j \in \mathcal{J}$. It also allows the possibility of not assigning customers after the last time period in which they have demand, by setting zero assignment costs for each customer for all the periods subsequent to the last one when he has demand.

In (Albareda-Sambola et al. 2009) it was proven that when the set of open facilities is fixed in F1, i.e., when the vector y is fixed, the coefficient matrix of the resulting assignment subproblem is totally unimodular, and, thus, it has the integrality property. As a consequence, we can substitute the binary conditions on the x variables by bounding constraints of the form $0 \le x_{ij}^t \le 1$, $\forall i, j, t$.

3.2 Impulse-Step variables based formulation F2

In the formulation we present next, the x-variables are impulse variables as in F1, but the y-variables have been replaced by step variables.

Variables

$\bar{y}_i^t = 1$ if facility i is open *by* time period t and 0 otherwise, for $i \in \mathcal{I}, t \in \mathcal{T}$.
$x_{ij}^t = 1$ if customer j is *assigned to facility* i at time period t and 0 otherwise, for $i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}$.

Notice that now $\bar{y}_i^t = 1$ whenever plant i has been opened at time period t or before. Therefore, $y_i^t = \bar{y}_i^t - \bar{y}_i^{t-1}$, $\forall i \in \mathcal{I}, t \in \mathcal{T}$, and $\bar{y}_i^0 = 0$, $\forall i \in \mathcal{I}$.

Pure 0–1 Formulation

$$
\begin{aligned}
\text{(F2)} \quad \min \sum_{i\in\mathcal{I}} \Bigg[\sum_{t\in\mathcal{T}} \Bigg(f_i^t (\bar{y}_i^t - \bar{y}_i^{t-1}) + \sum_{j\in\mathcal{J}} c_{ij}^t x_{ij}^t \Bigg) + \sum_{j\in\mathcal{J}} \sum_{t\in\mathcal{T}_j} s_{ij}^t x_{ij}^t \Bigg] \\
+ \sum_{j\in\mathcal{J}} \sum_{t\in\mathcal{T}_j} \rho_j \Bigg(1 - \sum_{i\in\mathcal{I}} x_{ij}^t \Bigg) \\
= \sum_{j\in\mathcal{J}} |\mathcal{T}_j| \rho_j + \min \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{T}} \Bigg[(f_i^t - f_i^{t+1}) \bar{y}_i^t + \sum_{j\in\mathcal{J}} \bar{c}_{ij}^t x_{ij}^t \Bigg], \quad (11)
\end{aligned}
$$

subject to

$$\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} x_{ij}^t \ge n^t \quad \forall t \in \mathcal{T}, \tag{12}$$

$$\sum_{i\in\mathcal{I}} x_{ij}^t \le 1 \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, \tag{13}$$

$$\sum_{i\in\mathcal{I}} x_{ij}^t \geq \sum_{i\in\mathcal{I}} x_{ij}^{t-1} \quad \forall j\in\mathcal{J}, t\in\mathcal{T}: t>1, \tag{14}$$

$$\sum_{i\in\mathcal{I}} x_{ij}^{|\mathcal{T}|} = 1 \quad \forall j\in\mathcal{J}, \tag{15}$$

$$x_{ij}^t \leq \bar{y}_i^t \quad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}, \tag{16}$$

$$\sum_{i\in\mathcal{I}} \left(\bar{y}_i^t - \bar{y}_i^{t-1}\right) \geq p^t \quad \forall t\in\mathcal{T}, \tag{17}$$

$$\bar{y}_i^{t-1} \leq \bar{y}_i^t \quad \forall i\in\mathcal{I}, t\in\mathcal{T}, \tag{18}$$

$$x_{ij}^t \in \{0,1\} \quad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}, \tag{19}$$

$$\bar{y}_i^t \in \{0,1\} \quad \forall i\in\mathcal{I}, t\in\mathcal{T}. \tag{20}$$

The objective function (11) consists of minimizing the total cost (where we define $f_i^{|\mathcal{T}|+1} = 0$, $\forall i\in\mathcal{I}$). The constraint system (12)–(15) in $F2$ is as the system (2)–(5) in $F1$. Constraints (16) ensure that the customers are assigned to open facilities. Constraints (17) require that a given number p^t of facilities be opened for the first time at each time period, at least. Constraints (18) ensure all facilities open at time period $t-1$ continue open at time period t.

Given that when the set of $\bar{y}$ variables is fixed, the resulting assignment subproblem is exactly the same as in the case of F1, again the assignment subproblem has the integrality property, and we can substitute the binary conditions on the x variables by bounding constraints of the form $0 \leq x_{ij}^t \leq 1$, $\forall i, j, t$.

3.3 Step variables based formulation F3

In this last formulation, both the x-variables and y-variables are step variables.

Variables

$\bar{y}_i^t = 1$ if facility i is opened *by* time period t and 0 otherwise, for $i\in\mathcal{I}, t\in\mathcal{T}$.
$\bar{x}_{ij}^t = 1$ if customer j is *assigned to a facility* i' *for* $i' \preceq i$ at time period t and 0 otherwise, for $i\in\mathcal{I}$, $j\in\mathcal{J}, t\in\mathcal{T}$. Recall that, as defined at the end of Sect. 2, $i' \preceq i$, $i', i \in \mathcal{I}_{jt}$, means that at time period t, facility i' has a smaller service cost for customer j than facility i. So, $\bar{x}_{ij}^t = 1$ means that the customer j is assigned at time period t to either facility i or a facility, say, i' with a smaller cost. Notice that $x_{i_r j}^t = \bar{x}_{i_r j}^t - \bar{x}_{i_{r-1} j}^t$, $\forall i_r \in \mathcal{I}_{jt}$, $j\in\mathcal{J}, t\in\mathcal{T}$, and $\bar{x}_{i_0 j}^t = 0$, $\forall j\in\mathcal{J}, i\in\mathcal{T}$.

Pure 0–1 Formulation

$$\text{(F3)} \quad \sum_{j\in\mathcal{J}} |\mathcal{T}_j|\rho_j + \min \sum_{t\in\mathcal{T}} \left[\sum_{i\in\mathcal{I}} \left(f_i^t - f_i^{t+1}\right)\bar{y}_i^t + \sum_{j\in\mathcal{J}} \sum_{i_r\in\mathcal{I}_{jt}} \left(\bar{c}_{i_r j}^t - \bar{c}_{i_{r+1}, j}^t\right)\bar{x}_{i_r j}^t \right], \tag{21}$$

subject to

$$\sum_{j \in \mathcal{J}} \bar{x}^t_{i_m j} \geq n^t \quad \forall t \in \mathcal{T}, \tag{22}$$

$$\bar{x}^t_{i_{r-1},j} \leq \bar{x}^t_{i_r j} \quad \forall i_r \in \mathcal{I}_{jt}, j \in \mathcal{J}, t \in \mathcal{T}, \tag{23}$$

$$\bar{x}^{t-1}_{i_m j} \leq \bar{x}^t_{i_m j} \quad \forall j \in \mathcal{J}, t \in \mathcal{T} : t > 1, \tag{24}$$

$$\bar{x}^{|\mathcal{T}|}_{i_m j} = 1 \quad \forall j \in \mathcal{J}, \tag{25}$$

$$\bar{x}^t_{i_r j} - \bar{x}^t_{i_{r-1},j} \leq \bar{y}^t_{i_r} \quad \forall i_r \in \mathcal{I}_{jt}, j \in \mathcal{J}, t \in \mathcal{T}, \tag{26}$$

$$\sum_{i \in \mathcal{I}} \left(\bar{y}^t_i - \bar{y}^{t-1}_i\right) \geq p^t \quad \forall t \in \mathcal{T}, \tag{27}$$

$$\bar{y}^{t-1}_i \leq \bar{y}^t_i \quad \forall i \in \mathcal{I}, t \in \mathcal{T}, \tag{28}$$

$$\bar{x}^t_{ij} \in \{0, 1\} \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, t \in \mathcal{T}, \tag{29}$$

$$\bar{y}^t_i \in \{0, 1\} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \tag{30}$$

Formulation (21)–(30) is directly derived from formulation (11)–(20), given the redefinition of the x-variables and the definition of set $\mathcal{I}_{jt}$, except for constraints (23). This is so, since for each customer and time period, the last term in the objective function (21) accounts for the assignment cost of one facility, at most. Constraints (23) come out directly from the nonnegativity of $x^t_{i_r j}$, as it has correctly been pointed out by a referee.

Observe that now, when the set of $\bar{y}$ variables is fixed, the structure of the resulting assignment subproblem is no longer the same as in the case of F1. However, given the relationship between the x and the $\bar{x}$ variables, it is easy to see that the coefficient matrix of the assignment subproblem in the $\bar{x}$ variables can be transformed into the coefficient matrix of the assignment subproblem in the x variables by means of elementary transformations of its columns. In this transformation, constraints (23) are associated with the nonnegativity constraints of the x variables in formulation F1. Therefore, the integrality property of the assignment subproblem in the $\bar{x}$ variables also holds, and we can substitute the binary conditions on the x variables by bounding constraints of the form $0 \leq \bar{x}^t_{ij} \leq 1, \forall i, j, t$.

4 Computational comparison

In order to compare the quality of the proposed formulations, we have solved a series of instances derived from some of the MISFLP instances in Albareda-Sambola et al. (2009). In particular, we have considered three sets of instances, all with 30 facility locations; the ones with 4 time periods, the ones with 8 time periods, and the ones with 12 time periods, all with the variants for 50 and 100 customers.

In Albareda-Sambola et al. (2009) the instances are generated as follows:

– The setup depreciation costs are the sum of two terms:

  - A fixed opening cost which is drawn from a uniform distribution in [3000, 5000].
  - Maintenance costs associated with each period, which are added to the setup cost over all the periods, since a facility is opened until the end of the time horizon, and which are drawn from a uniform distribution in $[50\frac{|\mathcal{J}|}{|\mathcal{T}|}, 100\frac{|\mathcal{J}|}{|\mathcal{T}|}]$.
- Assignment costs are drawn from a uniform distribution in [10, 100].
- The values of n^t follow a uniform distribution in $[n^{t-1}, |\mathcal{J}|]$ for $t = 1, \dots, |\mathcal{T}| - 1$ $(n^0 = 1)$, whereas $n^{|\mathcal{T}|} = |\mathcal{J}|$.
- For each instance, the value p^t, $t \in \mathcal{T}$ was generated as follows:
  - We first obtain a number p from a discrete uniform distribution in $[|\mathcal{T}|, |\mathcal{I}|]$. This number is an approximation of the total number of facilities to be opened $\sum_{t\in\mathcal{T}} p^t$, and $\frac{p}{|\mathcal{T}|}$ is the average number of facilities to be opened per period.
  - For $t = 1, \dots, |\mathcal{T}|$, generate p^t from a discrete uniform distribution in $[a, b]$ with $a = 1$ and $b = \lceil 2\frac{p}{|\mathcal{T}|} - a \rceil$.
  - If $\sum_{t\in\mathcal{T}} p^t \geq |\mathcal{I}|$, we reject the instance and restart the process again.

For the variant of the MISFLP considered in this paper, some extra information had to be generated to account for customer service costs in the periods when they have demand, s_{ij}^t, and penalty costs for neglecting customers demands, ρ_j. To this end, requests for service of each customer at each time period, d_j^t, have been generated following independent Bernoulli distributions with probability $p = 0.5$. Observe that since we are dealing with single source, each service cost can also be seen as a fixed cost which is either fully accounted for, or not accounted at all. Thus, the magnitude for such costs will depend both on the type of service which is given and on the quantity of demand that is serviced. As a consequence, some suitable interval for the service costs is difficult to estimate, since one can figure out different types of applications where these costs are similar in magnitude to assignment costs, and other applications where the magnitudes would be rather different. For our computational experiments, service costs have been randomly taken from the interval [10, 100], which is the same interval as for assignment costs. Penalties for not servicing the demand of a customer have been defined as $\rho_j = 0.35(\sum_{i,t}(c_{ij}^t + s_{ij}^t))/(|\mathcal{I}||\mathcal{T}|)$.

Finally, to obtain instances where the different parts of the objective function are equally relevant in the optimal solutions, we have rescaled the facility setup depreciation costs by dividing them by $95|\mathcal{I}|/|\mathcal{J}|$.

Our modeling approach has been implemented in a C code. We use the CPLEX v.11.1 optimization engine as the solver. The computations were carried out under a Pentium IV, 512 RAM, Windows XP, having a CPU speed of 1.8 GHz.

We have generated 10 instances for each set referred above. Each instance is named as follows: Et-Py-Cx-w, where t indicates the number of time periods, $|\mathcal{T}|$; y, the number of facilities, $|\mathcal{I}|$; x, the number of customers, $|\mathcal{J}|$; and w, the instance correlative number into its set. Table 1 shows the model dimensions for the impulse, impulse–step, and step formulations for each set. The headings are as follows: *nv*, number of 0–1 variables; *nr*, number of constraints; *dens%*, matrix density.

For all formulations, there were instances that could not be optimally solved within the elapsed time limit that we set to six hours. Table 2 gives the optimality *GAP* for the formulations. The headings are as follows: Z_{LP}, solution value of the *LP* relaxation of the original problem; $\overline{Z}_{IP}$, solution value found by CPLEX for the original problem;

Table 1 Formulations' dimensions

Instance	Impulse formulation F1			Impulse–step formulation F2			Step formulation F3		
	nv	*nr*	*dens* (%)	*nv*	*nr*	*dens* (%)	*nv*	*nr*	*dens* (%)
E4-P30-C50	6120	6388	0.108	6120	6448	0.085	6120	12048	0.041
E8-P30-C50	12240	12796	0.071	12240	12976	0.044	12240	24176	0.021
E12-P30-C50	18360	19204	0.058	18360	19504	0.030	18360	36304	0.014
E4-P30-C100	12120	12738	0.055	12120	12798	0.043	12120	23998	0.021
E8-P30-C100	24240	25546	0.036	24240	25726	0.022	24240	48126	0.010
E12-P30-C100	36360	38354	0.029	36360	38654	0.015	36360	72254	0.006

GAP, optimality gap defined as $(\overline{Z}_{IP} - \underline{Z})/\underline{Z}\%$, where $\underline{Z}$ is the best known lower bound of the optimal solution; nn, number of explored branch-and-bound nodes; tt, elapsed time (secs.).

We can observe in Table 2 that the impulse–step formulation F2 (11)–(20) gives better results than the impulse formulation F1 (1)–(10) and the step formulation F3 (21)–(30) for the elapsed time when the optimal solution has been obtained. It also gives better optimality *GAP* when the time limit is reached, see also Table 3 below.

Additionally, we can observe that the time limit is reached for the instances with 100 customers. The computational effort is usually affordable for instances with 50 customers.

Furthermore, among the 40 instances solved up to optimality by formulation F2, see Table 2, there are 4 instances that have been solved only by this formulation. But, all instances solved up to optimality by the impulse or step formulations have also been solved by the impulse-step one, except the outlier case E4-P30-C100-2 whose optimality is only obtained by the step formulation.

The superiority of formulation F2 is twofold. On the one hand, observing the instances that could be solved exactly with all three formulations, we can see that, in general, fewer nodes of the branch and bound tree and smaller elapsed time are needed when this formulation was used. On the other hand, a smaller GAP is obtained by formulation F2 for the cases that are solved without proving the optimality.

Table 3 shows the following average results for the instances: incumbent solution value ($\overline{\overline{Z}}_{IP}$), optimality gap ($\overline{GAP}$), number of explored branch-and-cut nodes ($\overline{nn}$), and elapsed time ($\overline{tt}$) for all the instances, namely, those instances where the optimal solution has been obtained by the impulse–step formulation and the rest of instances (i.e., those where the time limit has been reached without obtaining/proving the optimality). Observe that the *deviation from the best* is defined as

$$\sum_{j=1,2,3} \frac{Z_{IP}^{j} - \min_{i=1,2,3}\{Z_{IP}^{i}\}}{\min_{i=1,2,3}\{Z_{IP}^{i}\}}\%,$$

where Z_{IP}^{j} is the incumbent solution value for formulation Fj. We can notice that, clearly, the impulse–step formulation gives better results than the other two formulations. Notice that the ratios *F2 is better* versus *F1* and *F2 is better* versus *F3* are

Table 2 Formulation's performance

Instance	Z_{LP}	Impulse formulation F1				Impulse–step formulation F2				Step formulation F3			
		Z_{IP}	GAP	nn	tt	Z_{IP}	GAP	nn	tt	Z_{IP}	GAP	nn	tt
E4-P30-C50-1	3342.98	3345.64	0.00	0	1	3345.64	0.00	0	3	3345.64	0.00	5	6
E4-P30-C50-2	2247.29	2264.57	0.00	11	8	2264.57	0.00	6	8	2264.57	0.01	79	19
E4-P30-C50-3	2763.37	2820.11	0.00	153	34	2820.11	0.01	98	35	2820.11	0.01	734	187
E4-P30-C50-4	3867.35	3874.70	0.00	0	6	3874.70	0.00	10	7	3874.70	0.00	0	9
E4-P30-C50-5	3133.93	3257.72	0.01	2967	411	3257.72	0.01	2157	305	3257.72	0.01	1864	423
E4-P30-C50-6	3292.64	3307.04	0.00	28	9	3307.04	0.00	24	10	3307.04	0.00	48	14
E4-P30-C50-7	3975.75	3979.51	0.00	0	5	3979.51	0.00	0	5	3979.51	0.01	0	5
E4-P30-C50-8	3568.19	3652.69	0.00	723	159	3652.69	0.01	309	56	3652.69	0.01	749	227
E4-P30-C50-9	3976.49	4098.33	0.01	1413	262	4098.33	0.01	2174	334	4098.33	0.01	2338	447
E4-P30-C50-10	3053.63	3054.02	0.00	0	0	3054.02	0.00	0	1	3054.02	0.00	0	2
E8-P30-C50-1	4212.72	4301.79	0.00	743	290	4301.79	0.00	200	116	4301.79	0.01	367	291
E8-P30-C50-2	5658.76	5704.84	0.00	182	76	5704.84	0.00	99	82	5704.84	0.01	167	145
E8-P30-C50-3	4535.72	4543.95	0.01	10	15	4543.95	0.00	5	14	4543.95	0.00	14	27
E8-P30-C50-4	5073.82	5109.34	0.01	541	82	5109.34	0.01	295	67	5109.34	0.01	489	226
E8-P30-C50-5	4674.94	4779.39	0.01	1338	589	4779.39	0.01	986	513	4779.39	0.01	1870	1177
E8-P30-C50-6	5228.77	5275.15	0.01	1099	270	5275.15	0.00	219	95	5275.15	0.01	332	155
E8-P30-C50-7	5877.43	5994.89	0.01	4737	1635	5994.89	0.01	8541	1705	5994.89	0.01	3994	2057
E8-P30-C50-8	5536.29	5581.72	0.00	50	54	5581.72	0.00	49	74	5581.72	0.01	156	162
E8-P30-C50-9	5980.75	6066.23	0.00	781	374	6066.23	0.00	243	97	6066.23	0.01	3564	1344
E8-P30-C50-10	5792.29	5865.78	0.01	1146	395	5865.78	0.00	341	157	5865.78	0.01	2319	688

Table 2 (*Continued*)

Instance	Z_{LP}	Impulse formulation F1				Impulse–step formulation F2				Step formulation F3			
		Z_{IP}	GAP	*nn*	*tt*	Z_{IP}	GAP	*nn*	*tt*	Z_{IP}	GAP	*nn*	*tt*
E12-P30-C50-1	7229.12	7254.15	0.00	63	57	7254.15	0.01	231	28	7254.15	0.01	183	244
E12-P30-C50-2	8145.52	8388.46	0.80	34297	21600	8380.65	0.13	46864	21600	8384.50	0.68	14883	21600
E12-P30-C50-3	7329.46	7370.57	0.00	203	125	7370.57	0.01	212	98	7370.57	0.01	327	354
E12-P30-C50-4	8030.42	8093.44	0.00	723	518	8093.44	0.01	886	193	8093.44	0.01	350	365
E12-P30-C50-5	7137.60	7150.41	0.00	26	36	7150.41	0.00	11	30	7150.41	0.01	138	207
E12-P30-C50-6	7803.91	7862.61	0.01	248	239	7862.61	0.01	237	108	7862.61	0.00	266	516
E12-P30-C50-7	6934.33	6982.07	0.01	166	117	6982.07	0.00	47	55	6982.07	0.00	81	206
E12-P30-C50-8	7222.42	7269.51	0.01	309	234	7269.51	0.01	376	149	7269.51	0.01	1856	1584
E12-P30-C50-9	6789.69	6868.30	0.01	1847	1018	6868.30	0.01	6843	1471	6868.30	0.01	2823	2973
E12-P30-C50-10	8111.43	8227.11	0.01	7622	3542	8227.11	0.01	2473	971	8227.11	0.01	7410	6118
E4-P30-C100-1	3342.98	3345.64	0.00	0	1	3345.64	0.00	0	2	3345.64	0.00	5	6
E4-P30-C100-2	10398.54	10706.64	0.32	56079	21600	10706.64	0.35	106628	21600	10706.64	0.01	22888	11707
E4-P30-C100-3	10569.93	10674.94	0.00	263	126	10674.94	0.01	323	164	10674.94	0.01	333	212
E4-P30-C100-4	12830.61	13004.08	0.00	658	489	13004.08	0.00	139	132	13004.08	0.00	1073	732
E4-P30-C100-5	6376.14	6688.37	0.54	84767	21600	6688.35	0.01	5154	1381	6688.35	0.01	35621	18648
E4-P30-C100-6	10685.03	10786.09	0.01	245	163	10786.09	0.00	349	152	10786.09	0.00	733	537
E4-P30-C100-7	8574.37	9035.83	2.09	64828	21600	8989.32	0.01	3997	1744	9007.73	1.00	25959	21600
E4-P30-C100-8	9572.51	9752.27	0.01	2246	1063	9752.27	0.01	2119	946	9752.27	0.01	5976	2748
E4-P30-C100-9	9819.83	10134.88	0.01	37347	14368	10134.88	0.01	7179	2859	10134.88	0.01	24820	13048
E4-P30-C100-10	5787.66	6248.22	2.61	48510	21600	6224.40	0.09	64191	21600	6296.48	4.79	28166	21600

Table 2 (*Continued*)

Instance	Z_{LP}	Impulse formulation F1				Impulse–step formulation F2				Step formulation F3			
		Z_{IP}	GAP	nn	tt	Z_{IP}	GAP	nn	tt	Z_{IP}	GAP	nn	tt
E8-P30-C100-1	13829.83	14460.44	2.98	20422	21600	14390.02	1.21	21160	21600	14393.91	2.06	7053	21600
E8-P30-C100-2	12921.26	13363.92	2.08	19716	21600	13322.92	1.00	21180	21600	13326.00	1.73	10183	21600
E8-P30-C100-3	11769.66	12332.90	2.92	18772	21600	12212.82	0.01	7861	10462	12214.98	1.16	6603	21600
E8-P30-C100-4	11535.78	11675.96	0.02	38365	21600	11675.96	0.01	21534	9231	11675.96	0.01	9217	21600
E8-P30-C100-5	15828.89	16258.27	1.62	26017	21600	16234.17	0.79	28334	21600	16287.12	1.65	9691	21600
E8-P30-C100-6	14602.97	14981.98	1.00	22428	21600	14975.98	0.06	43224	21600	14977.02	0.42	10300	21600
E8-P30-C100-7	14885.42	15261.44	1.29	18101	21600	15261.44	0.33	16708	21600	15261.44	0.51	6894	21600
E8-P30-C100-8	9940.52	10304.01	1.92	19780	21600	10290.22	0.08	28155	21600	10339.59	2.17	7675	21600
E8-P30-C100-9	10743.58	11042.60	1.69	21249	21600	11052.84	0.83	21506	21600	11060.72	1.92	7517	21600
E8-P30-C100-10	15976.37	16486.71	2.11	20317	21600	16382.50	0.01	17177	12095	16382.97	1.28	11299	21600
E12-P30-C100-1	17009.22	17344.95	0.67	13060	21600	17341.74	0.37	17170	21600	17374.09	0.91	4772	21600
E12-P30-C100-2	17921.46	18375.74	1.32	17823	21600	18354.75	0.78	23852	21600	18383.16	1.03	4213	21600
E12-P30-C100-3	20153.10	20737.25	2.13	14569	21600	20605.47	0.49	11469	21600	20742.44	1.85	4689	21600
E12-P30-C100-4	17569.13	18090.43	1.18	10390	21600	18134.84	0.93	17563	21600	18143.99	0.92	4711	21600
E12-P30-C100-5	19542.36	20013.15	1.00	12990	21600	19992.39	0.30	17029	21600	19986.95	0.91	3887	21600
E12-P30-C100-6	19726.06	20482.72	2.78	11003	21600	20404.00	1.94	8742	21600	20423.88	2.38	2734	21600
E12-P30-C100-7	20085.75	21076.13	3.63	10690	21600	20922.12	2.49	9506	21600	21059.54	4.05	4090	21600
E12-P30-C100-8	17832.80	18473.35	1.93	11595	21600	18406.06	0.78	13630	21600	18442.88	1.45	3961	21600
E12-P30-C100-9	20196.44	20946.23	2.46	11896	21600	20817.28	1.67	14613	21600	20889.84	2.42	3617	21600
E12-P30-C100-10	20279.63	20710.94	1.24	12075	21600	20669.49	0.31	15175	21600	20683.03	0.85	3887	21600

Table 3 Average performance

Instances	# of instances	Formulation	$\overline{\overline{Z}}_{IP}$	Deviation from the best (%)	$\overline{GAP}$	$\overline{nn}$	$\overline{tt}$
All	60	impulse	9752.34	0.12	0.71	11794	9446.18
		impulse–step	9734.83	0.01	0.25	10660	7965.92
		step	9746.44	0.09	0.61	5333	9410.27
Solved up to optimality by F2	40	impulse	6696.83	0.05	0.20	7373	3369.28
		impulse–step	6690.06	0.00	0.01	2323	1148.88
		step	6690.59	0.01	0.09	3854	3562.73
Not solved up to optimality by F2	20	impulse	15863.34	0.24	1.73	20635	21600.00
		impulse–step	15824.37	0.02	0.75	27335	21600.00
		step	15858.16	0.25	1.64	8291	21105.35

about 2.5:1. More specifically, formulation F2 gives the best results for the cases solved up to optimality and the instances where the elapsed time is reached before guaranteeing the optimality of the incumbent solution. Notice also that the elapsed time of formulation F2 for the cases where the optimality is reached is only 34% and 32% of the time required by the formulations F1 and F3, respectively. On the other hand, the average GAP of formulation F2 for the cases not solved up to optimality in the time limit is only 43% and 45% of the average GAP obtained for the formulations F1 and F3, respectively. Finally, we can see that the average elapsed time of formulation F3 in those cases is influenced by the time required by the outlier case E4-P30-C100.2.

4.1 Graphical comparison of the results obtained with the different formulations

Since formulation F2 allows one to solve the problem most efficiently among the tested instances, we next compare the results obtained by formulations F1 and F3 with respect to the results obtained by formulation F2. See Figs. 2 and 3. In all graphics, each dot of the graph represents an instance, and its coordinates are taken as $(\frac{z-z_{2IP}}{z_{2IP}}\%, \frac{t-t_{2IP}}{t_{2IP}}\%)$, where z_{2IP} and t_{2IP} give the solution value and the elapsed time obtained by CPLEX, respectively, for formulation F2, and z and t give the same values for the formulation considered in each graph. That is, the x coordinate compares the values of the obtained solutions with the formulations, whereas the y coordinate gives the percent deviation of the time taken by CPLEX to solve that instance with the formulations with respect to the time taken with formulation F2.

Notice that for formulation F3, all instances have nonnegative coordinates but two that have coordinates $(-0.03, 0)$ and $(0, -45.8)$. This means that formulation F3 was only better than formulation F2 twice. Once it was slightly better with respect to the solution quality, and once it was better with respect to the computational effort. In the case of formulation F1, there are some instances with one negative coordinate, but most of them appear represented with positive values of time deviation and solution

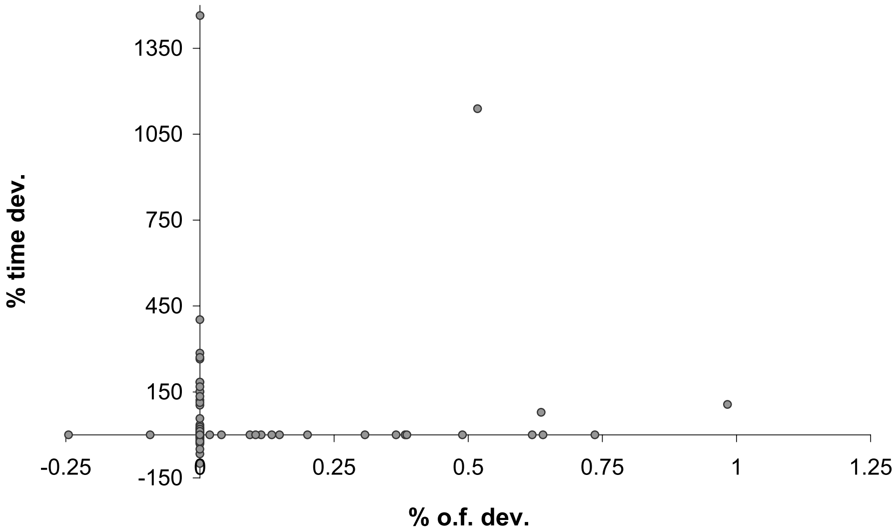

Fig. 2 Formulation F1 with respect to formulation F2

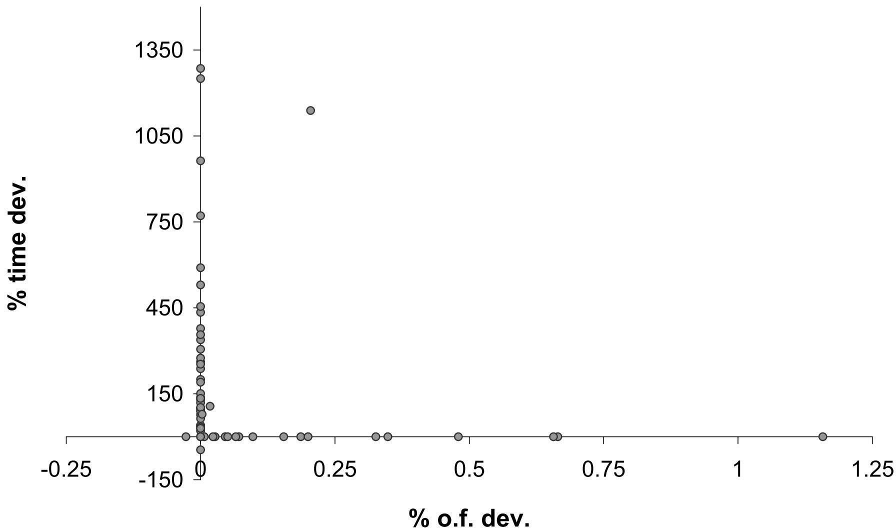

Fig. 3 Formulation F3 with respect to formulation F2

value deviation. Indeed, the positive deviations in all cases are larger in absolute value than the negative ones. These figures give further evidence of the superiority of formulation F2 over the others, specially in the case of formulation F3.

5 Conclusions

In this paper we have presented a variation of the multi-period incremental service facility location problem (MISFLP) where each customer needs to be serviced only in a subset of periods along the time horizon. Three 0–1 equivalent formulations are proposed, based on the impulse and step variables approaches. An intensive computational experimentation has been performed to assess the tightness of the formulations. The first conclusion to be withdrawn is that the problem is very difficult to solve. The second conclusion is that the formulation considering the x-variables (i.e., the assignment variables) based on the impulse formulation and the y-variables (i.e., the facility site location variables) based on the step formulation (i.e., formulation F2, see Sect. 3.2) gives better results, and, then, it is considered for future research to treat the uncertainty in the customer availability and the minimum number of customers to be serviced at each time period.

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