Model Predictive Control for Power Converters and Drives: Advances and Trends

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Abstract—Model Predictive Control (MPC) is a very attractive solution for controlling power electronic converters. The aim of this paper is to present and discuss the latest developments in MPC for power converters and drives, describing the current state of this control strategy and analyzing the new trends and challenges it presents when applied to power electronic systems. The paper revisits the operating principle of MPC and identifies three key elements in the MPC strategies, namely the prediction model, the cost function and the optimization algorithm. The paper summarizes the most recent research concerning these elements, providing details about the different solutions proposed by the academic and industrial communities.

I. INTRODUCTION

MODEL Predictive Control (MPC) has been a topic of research and development for more than three decades. Originally, it was introduced in the process industry, but a very innovative and early paper proposed that predictive control be used in power electronics [1]. In the recent years, thanks to technological advances in microprocessors, it has been proposed and studied as a promising alternative for the control of power converters and drives [2], [3]. MPC presents several advantages. For instance, it can be used in a variety of processes, is simple to apply in multivariable systems and presents a fast dynamic response. Further, it allows for nonlinearities and constraints to be incorporated into the control law in a straightforward manner, and it can incorporate nested control loops in only one loop [4], [5].

In particular, power electronic applications require control responses in the order of tens to hundreds of microseconds to work properly. However, it is well known that MPC has a larger computational burden than other control strategies. For this reason, most of the works focused on this issue at the initial research stages of MPC for power electronic systems [6]. Currently, MPC approaches can be found in the literature for almost all power electronic applications [7]. The main reason is that the computational power of modern microprocessors has dramatically increased. This has made it possible to implement more complex and intelligent control strategies, like MPC, in standard control hardware platforms [8]–[11]. At this point, MPC for power converters and drives can be considered as a well established technology in the research and development stages. However, further research and development efforts are still necessary in order to bring this technology to the industrial and commercial level [12].

The aim of this paper is to summarize the current state and analyze the most recent advances in the application of MPC for power converters and drives. Thus, the work presents the current advances and challenges of MPC for power electronic applications and addresses possible future trends.
TABLE I
Most used MPC strategies for power electronics applications

<table>
<thead>
<tr>
<th>Item Description</th>
<th>GPC</th>
<th>EMPC</th>
<th>OSV-MPC</th>
<th>OSS-MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block diagram</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>Modulator</td>
<td>SVM or PWM</td>
<td>SVM or PWM</td>
<td>Not required</td>
<td>Not required</td>
</tr>
<tr>
<td>Fixed switching frequency</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Optimization</td>
<td>Online</td>
<td>Offline (Parametric search)</td>
<td>Online</td>
<td>Online</td>
</tr>
<tr>
<td>Constraints</td>
<td>Can be included but increases the computational cost</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Long prediction horizon</td>
<td>Yes</td>
<td>Yes</td>
<td>Can be used but requires special search algorithm</td>
<td>Can be used but requires special search algorithm</td>
</tr>
<tr>
<td>Formulation</td>
<td>Complex</td>
<td>Complex</td>
<td>Very intuitive</td>
<td>Intuitive</td>
</tr>
<tr>
<td>References</td>
<td>[13], [14]</td>
<td>[15], [16]</td>
<td>[17], [18]</td>
<td>[19], [20]</td>
</tr>
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</table>

II. MODEL PREDICTIVE CONTROL: OPERATING PRINCIPLE

MPC is a family of controllers that explicitly uses the model of the system to be controlled. In general, MPC defines the control action by minimizing a cost function that describes the desired system behavior. This cost function compares the predicted system output with a reference. The predicted outputs are computed from the system model. In general, for each sampling time, the MPC controller calculates a control action sequence that minimizes the cost function, but only the first element of this sequence is applied to the system. Although MPC controllers solve an open-loop optimal control problem, the MPC algorithm is repeated in a receding horizon fashion at every sampling time, thus providing a feedback loop and potential robustness with respect to system uncertainties.

To illustrate the use of MPC for power electronics, a basic MPC strategy with a prediction horizon equal to 1 applied to the current control of a voltage source inverter (VSI) with output RL load is shown [17]. The basic block diagram of this control strategy is presented in Fig. 1, where the reference and predicted currents at instant $k+2$ are used in order to compensate for the digital implementation delay [21]. The algorithm is repeated for each sampling time and performs the following steps:

1) The optimal control action $S(t_k)$ computed at instant $k-1$ is applied to the converter.
2) Measurement of the current $i_k$ is taken at instant $k$. The reference current $i^*_k$ for instant $k+2$ is also defined.
3) The prediction model of the system is used to make a prediction of the current value $\hat{i}_{k+2}$ at instant $k+2$.
4) A cost function is evaluated using $i^*_k$ and $\hat{i}_{k+2}$. The optimal control action $S(t_{k+1})$ to be applied at instant $k+1$ is chosen as the one that minimizes the cost function's value.

Several MPC methods have been successfully implemented for a variety of power electronic applications [6], [7]. Fig. 2 shows the most common MPC strategies applied to power converters and drives, and Table I summarizes the structure and main features of these MPC strategies. Variables $i$, $\hat{i}$ and $i^*$ denote a set of current measurements, predictions and references. $u_k$ is the control signal calculated at instant $k$ and $S_k(t)$ are the firing pulses for the power switches, these values can change from instant $k$ to $k+1$. $S(t_k)$ are the firing pulses for the power switches, these values are constant from instant $k$ to $k+1$.

The MPC methods are classified based on the type of the optimization problem, i.e., if it is an integer optimization problem or not. On one hand, Continuous Control Set MPC (CCS-MPC) computes a continuous control signal and then uses a modulator to generate the desired output voltage in the power converter. The modulation strategy can be any one that is valid for the converter topology under consideration [75]. The main advantage of CCS-MPC is that it produces a fixed switching frequency. The most-used CCS-MPC strategies for power electronic applications are Generalized Predictive Control (GPC) and Explicit MPC (EMPC). GPC is useful for linear and unconstrained problems. EMPC allows the user to work with non-linear and constrained systems. The main problem of GPC and EMPC when applied to power converters is that both present a complex formulation of the MPC problem. On the other hand, Finite Control Set MPC (FCS-MPC) takes into account the discrete nature of the power converter to formulate the MPC algorithm and does not require an external modulator. FCS-MPC can be divided into
two types: Optimal Switching Vector MPC (OSV-MPC) and Optimal Switching Sequence MPC (OSS-MPC). OSV-MPC is currently the most popular MPC strategy for power electronic applications. OSV-MPC was the first FCS-MPC technique used for power electronics. For this reason, it can be found in the literature referred to as FCS-MPC. It uses the possible output voltage vectors of the power converter as the control set. OSV-MPC only calculates predictions for this control set, and it reduces the optimal problem to an enumerated search algorithm. This makes the MPC strategy formulation very intuitive. The main disadvantage of OSV-MPC is that only one output voltage vector is applied during the complete switching period. Furthermore, unless an additional constraint is added, the same output voltage vector can be used during several consecutive switching periods. Therefore, in general, it generates a variable switching frequency. OSS-MPC solves this problem by considering a control set composed of a limited number of possible switching sequences per switching period. In this way, OSS-MPC takes the time into account as an additional decision variable, i.e., the instant the switches change state, which in a way resembles a modulator in the optimization problem.

In general, MPC algorithms require a significant amount of computations. CCS-MPC usually has a lower computational cost than FCS-MPC because it computes part or all of the optimization problem offline. For this reason, CCS-MPC can
address long prediction horizon problems. For instance, GPC uses an expression to calculate the control action that can be computed beforehand, thus limiting the online computation burden [9]. On the other hand, EMPC computes and stores the optimal problem solution offline, so the online computations are limited to a search algorithm. By contrast, FCS-MPC requires that the optimization problem, which involves a large amount of calculations, be solved online. For this reason, FCS-MPC is usually limited to short prediction horizons in power electronic applications. Comparing OSS-MPC and OSV-MPC, the former has a greater computational cost.

Table II summarizes the most relevant applications of MPC for power converters and drives [7]. Other uses of MPC for power electronics can be found in the literature. Among them are predictive control strategies for quasi z-source inverters or dc/dc converters [76]–[79]. Table II includes a block diagram representing the use of OSV-MPC for each one. Other MPC strategies could be used for these applications, but the purpose of the control scheme is to show the basic concept. Therefore, OSV-MPC has been chosen for its clarity.

An analysis of MPC algorithms when applied to power converters and drives reveals that the key elements for any MPC strategy are the prediction model, cost function and optimization algorithm. Research efforts have been made in all of these topics, and several problems and limitations have been found. The existing research work have solved some of them while others are still open issues to be investigated. Among the most important studied aspects are [80]:

- Prediction model discretization.
- Frequency spectrum shaping.
- Cost function design.
- Reduction of computational cost.
- Increasing prediction and control horizon.
- Stability and system performance design.

The most recent research for all of these topics will be addressed in the following sections.

### III. PREDICTION MODEL

MPC performance is influenced by an adequate quality of the prediction model which depends on the specific application under consideration [7]. For this reason, most power converters are connected to the load through passive filters in order to minimize the effects of the commutations or distortions in the supply. First-order passive filters composed of an inductor and its parasitic resistor can be used [20], [51]. However, high order passive filters like LC or LCL are also applied in VSC-AFE [15], [27], medium voltage (MV) motor drives [81], VSC-UPS [39], [44], matrix converters [59], [61], etc. MPC can work with any passive filter topology as long as its mathematical model is incorporated in the prediction model.

Despite the fact that mathematical model of the filter is included in the prediction model, basic MPC strategies must mitigate the effects of resonance problems when a high-order passive filters are used. This is especially critical in FCS-MPC due to the variable switching frequency \( f_{sw} \) that is present in this control strategy, even though \( f_{sw} \) is limited to half of the sampling frequency. Several solutions have been proposed to deal with this problem. For instance, it is possible to mitigate the resonance effects by considering a hybrid control strategy, mixing predictive control and an active damping filter [61], [82], [83]. In addition, FCS-MPC can address the resonance issues without requiring a passive/active damping loop by increasing the prediction horizon [81], [84]. On the other hand, the design of the input filter can be simplified and the risk of resonances avoided by considering MPC strategies with fixed switching frequencies [15], [16], [27].

The MPC algorithms are usually implemented in digital hardware platforms like DSPs or FPGAs. For this reason, the prediction model of the system needs to be discretized. For linear systems, the discretization is simple and can be done as described in [39], [80]. However, non-linear systems require a more complex approach [85]. A trade-off between the model quality and complexity defines several discretization techniques, the most common being Euler approximation and Taylor series expansion [86]. Another approach consists of a first step where the system is discretized using a one-step or multiple-step Euler approximation. Then, the arising discretization error is explicitly bound to take it into consideration for the implementation of the predictive controller [87].

### IV. COST FUNCTION ISSUES

The cost function in the MPC strategy defines the desired system behavior. For this purpose, it compares the predicted and reference values. The cost function can have any form, but in general, it can be written as

\[
g = \sum_{t=k+1}^{k+N_p} \tilde{x}_t^T Q \tilde{x}_t + \sum_{r=k}^{k+N_r-1} u_r^T R u_r
\]

where \( \tilde{x}_t = \hat{x}_t - x^*_t \) is a vector in which each component represents the difference between the predicted, \( \hat{x}_t \), and the

<table>
<thead>
<tr>
<th>Application</th>
<th>Cost function</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSC-AFE [23]</td>
<td>( g =</td>
</tr>
<tr>
<td>[24]</td>
<td>( g = (q)^2 + \lambda (\hat{i}_L - i^*_L)^2 )</td>
</tr>
<tr>
<td>VSC-AFE [17]</td>
<td>( g =</td>
</tr>
<tr>
<td>[88]</td>
<td>( g =</td>
</tr>
<tr>
<td>[89]</td>
<td>( g = (\hat{i}_k - i^*_k)^2 )</td>
</tr>
<tr>
<td>Motor drive [36]</td>
<td>( g = (\bar{T} - T^<em>)^2 + \lambda (\hat{\psi} - \psi^</em>)^2 )</td>
</tr>
<tr>
<td>VSC-UPS [39]</td>
<td>( g = (\hat{v}_o - v^*_o)^2 )</td>
</tr>
<tr>
<td>Statcom [50]</td>
<td>( g = (\hat{i}_k - i^*_k)^2 )</td>
</tr>
<tr>
<td>Matrix converter [54]</td>
<td>( g =</td>
</tr>
<tr>
<td>[65]</td>
<td>( g = (\hat{i}_k - i^<em>_k)^2 + \lambda (Q - Q^</em>)^2 )</td>
</tr>
<tr>
<td>HVDC [69]</td>
<td>( g = g_1 + g_2 + g_3 )</td>
</tr>
<tr>
<td></td>
<td>( g_1 =</td>
</tr>
<tr>
<td></td>
<td>( g_2 = \lambda C \sum l</td>
</tr>
<tr>
<td></td>
<td>( g_3 = \lambda L \sum</td>
</tr>
</tbody>
</table>
A particular case is using a cost function to achieve a desired spectrum shape of an output variable. This occurs when the switching frequency is fixed or Selective Harmonic Elimination (SHE) or Selective Harmonic Mitigation (SHM) techniques are used [94]–[97]. CCS-MPC strategies do not need any special cost function because the power converter output voltage is generated using a modulator stage. The modulation technique produces a predefined spectrum content depending on the modulation strategy [75]. On the other hand, OSV-MPC needs to include this control objective in the controller design.

The first approach to solve this problem was to use
\[
g = |F \left( \hat{i}_L - i^*_L \right) | \tag{7}
\]
as the cost function, where \( F \) is a narrow band-stop filter. In this way, defined harmonic components do not contribute to the cost function value, and a concentrated switching frequency is obtained around the band-stop frequency [94]. A second procedure for OSV-MPC was to maintain (3) as the cost function but to include virtual vectors in the control set [98]. These virtual vectors are modulated using a pulse width modulation (PWM) - space vector modulation (SVM) that provides a fixed switching frequency. A more recent technique proposes to obtain the low frequency components of the control action computed by the OSV-MPC controller using (3). These components are used as the control input for the converter and are generated by a PWM-SVM modulator [91]. Finally, new approaches include the modulation stage in the optimization process. Therefore, the outputs of the FCS-MPC controller are the output voltage vectors and their application times [20], [25], [92]. Table IV summarizes these methods and shows their basic control schemes.

B. Weighting Factor Design

MPC can handle several control objectives simultaneously. In order to do so, the variables to be controlled should be included in the cost function. As a result, the cost function can contain variables of differing natures. The most common example is MPC for controlling the torque and flux in a motor drive. The usual cost function used for this application is
\[
g = \left( \hat{T} - T^* \right)^2 + \lambda \left( \hat{\psi} - \psi^* \right)^2 \tag{8}
\]
Here, \( \hat{T} \) and \( T^* \) are the predicted and reference torque values, \( \hat{\psi} \) and \( \psi^* \) are predicted and reference flux values, and \( \lambda \) is a weighting factor which defines a trade-off between the torque and flux tracking.

In general, the differing natures of the variables hinder the selection of the weighting factors. This is because these variables usually have different orders of magnitude. Therefore, they do not equally contribute to the cost function’s value. A common approach for solving this problem is to work in per
TABLE IV
MPC WITH FIXED SWITCHING FREQUENCY ALGORITHMS

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Operation principle</th>
<th>Control Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPC [14, 42, 43]</td>
<td>The GPC algorithm as presented by Clark et al. [90] is used for controlling the power converter. The algorithm allows one to work with long prediction horizons keeping a limited computational cost.</td>
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</tr>
<tr>
<td>Hybrid OSV-MPC [91]</td>
<td>The scheme is based on obtaining the low frequency components of the control action computed by the OSV-MPC. These values are computed through low-pass filters with cut-off frequency small with respect to the sampling frequency. The resulting signals are modulated by using a SVM-PWM technique.</td>
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<tr>
<td>Modulated MPC (M2PC) [92]</td>
<td>A modulation scheme is part of the minimization process. The Modulated MPC (M2PC) block defines a sequence of two voltage vectors $S_1$ and $S_2$ and two values, $G_1$ and $G_2$, proportional to their application times. A second stage calculates the final application times using the information from the M2PC block.</td>
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<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
</tr>
<tr>
<td>Optimal Switching Sequence MPC (OSS-MPC) [20, 25]</td>
<td>The MPC algorithm incorporates the modulation strategy evaluating all the possible switching sequences $\text{Seq}_j$, with $j = 1 \ldots n$, using a FCS-MPC fashion approach. The application times are calculated minimizing the selected cost function.</td>
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<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
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</tbody>
</table>

unit values in the cost function [99]. Using this method, one can modify the expression (8) which results in

$$g = \frac{1}{T_n^2} \left( \hat{T} - T^* \right)^2 + \frac{\lambda}{\psi_n^2} \left( \hat{\psi} - \psi^* \right)^2,$$

where $T_n$ and $\psi_n$ are the rated values for the torque and flux, respectively [29].

The weighting factor values have a direct influence on the system’s performance. It is not easy to define the suitable weighting factor values to achieve a desired system behavior. Usually, the procedure consists in a heuristic approach. In this way, figures of merit are defined depending on the application, and a set of simulations or experiments are performed to find the best value [99]. In general, a large number of simulations or experiments are needed, and thus the process requires a considerable development time period. To reduce this time, branch and bound techniques can be used to search for suitable weighting factor values [80].

Another approach used to avoid adjusting the weighting factor values consists in transforming the multi-objective optimization (MO) with a single cost function into a MO with multiple cost function problem (MOMCF). The last one can be solved through a Fuzzy Decision-Making (FDM) technique [64]. The MOMCF can be set out following these steps [100]:

1) The cost function is split into functions that define the desired behavior for each variable of interest. For instance, in the motor drive application, (9) is divided as

$$g_1 = \frac{1}{T_n^2} \left( \hat{T} - T^* \right)^2,$$

$$g_2 = \frac{1}{\psi_n^2} \left( \hat{\psi} - \psi^* \right)^2,$$

2) Membership functions are specified from the new functions. In the example, (10) and (11) lead to membership functions
depends on their values, some guidelines for selecting values as parallel can be found in [101]. Usually, the priority vector is chosen as \(||k||_1 = 1\). Using this rule, \(k_1 = k_2 = 0.5\) can be chosen for the motor drive application [8]. Other values can be used and lead to a different performance.

The heuristic method and the MOMCF problem approach work well. However, they do not allow one to define a desired system behavior, such as the settling time for a variable, nor do they ensure system stability. A method to solve this problem designs the cost function based on Lyapunov stability concepts [102]. As a result, the system performance can be established and sufficient conditions for local stability are ensured. The main problem is that the method can only be applied to one class of power converters, so more research is still necessary to generalize this approach for other applications.

Another possibility is to define the MPC optimization problem using cost functions without any weighting factors [38], [51], [110]. Two different proposals can be found in the literature. For certain applications, it is possible to define the set of variable of interest as a function of one of them [38], [51]. For instance, in the motor drive application, the flux reference can be constructed from the torque reference [38], and thus (9) can be simplified to

\[
\mu_D = \mu_1 \mu_2. \tag{14}
\]

Finally, the MOMCF problem is solved, and the control action is computed as the one with the maximum value of the decision function. It should be noted that priority coefficients are used instead weighting factors. In (12) and (13), the priority coefficients are \(k_1\) and \(k_2\). The system’s behavior depends on their values, some guidelines for selecting values can be found in [101]. Usually, the priority vector \(k\) is chosen as \(||k||_1 = 1\). Using this rule, \(k_1 = k_2 = 0.5\) can be chosen for the motor drive application [8]. Other values can be used and lead to a different performance.

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\[
g = |\hat{\psi}_r - \psi_s|. \tag{15}
\]

On the other hand, the problem can be addressed by using an MO ranking-based approach when FCS-MPC is considered as the control strategy [110]. This method transforms the single cost function into a MOMCF problem. To this end, the behavior of each variable of interest is described in a separate cost function. As an example, (10) and (11) can be used for the motor drive application. Then, each function is evaluated for each possible control action. The outputs are sorted and a ranking value is assigned to each of them. For instance, control actions with lower cost function values are assigned a lower ranking. In the case of the motor drive

\[
g_1 \rightarrow r_1, \tag{16}
\]

\[
g_2 \rightarrow r_2, \tag{17}
\]

The ranking value is a dimensionless variable, and therefore an average criterion can be used to select the control action. For the motor drive application,

\[
AV_{\text{ranking}} = \frac{r_1 + r_2}{2} \tag{18}
\]

represents the average ranking value. Finally, the control action is defined as the one with the minimum average value of its rankings. It should be noted that this method provides the same result as (9) when weighting factor \(\lambda = 1\) and the MOMCF problem is defined by (10) and (11). However, \(\lambda\) can be different from 1 and \(g_1\) and \(g_2\) could be defined using other expressions. Therefore, (9) can be considered as a particular case of this method.

V. OPTIMIZATION ALGORITHM ISSUES

MPC solves an optimization problem to obtain the control input to the system. Once the prediction model and cost function are defined, an optimization algorithm is used to compute the control action. This algorithm is executed online each sampling time. Usually, the algorithm requires a large amount of computation so it is time consuming. A characteristic of power electronic applications is that the sampling period tends to be short. This issue limits the algorithms that can be used to solve the MPC strategy and has motivated the search for computationally efficient optimization algorithms for these particular applications.

A. Computational Cost Reduction

The computational cost of MPC depends on the algorithm used to solve the optimization problem. The algorithm is related to the MPC method applied to control the system. Table V summarizes some of the methods that have been proposed to reduce the computational cost and shows their control scheme. For power electronic applications, CCS-MPC and FCS-MPC are the main MPC strategies.

Of the CCS-MPC, EMPC solves the optimization problem offline for all possible states. This solution is stored in a lookup table (LUT), and the control action is defined by a search algorithm, which is a function of the system state. Therefore, the online computations are limited to the search algorithm which can be done very fast using a binary search tree technique [15], [111]. On the other hand, EMPC requires significant memory to store the generated LUTs. Thus, it is limited to small-scale problems since the size of the LUTs depend on the size of the problem as defined by the number of the optimization variables and the steps of the prediction horizon.

GPC is the other CCS-MPC technique applied to power electronic problems. GPC provides an analytical solution to the optimization problem. This analytical expression can be computed beforehand, so the online computation burden is limited [9], [43].

On the other hand, FCS-MPC requires that the optimization problem be solved online. This involves a large amount of calculations, which is a drawback for its implementation in standard control hardware platforms. Different solutions have
been proposed to address this problem. A first approach consists of transforming the cost function to an equivalent optimization problem where the variables involved are an equivalent output voltage reference, $u^*$, and the possible output voltage vectors, $u_n$. For instance, the cost function for the current control (4) is replaced by

$$g = (u^* - u_n)^2.$$  \hspace{1cm} (19)

The calculation of $u^*$ depends on the system model, as an example, for a converter connected to the grid through a smoothing inductor, this can be done as

$$u^*(k) = v_s(k) - Ri(k) - L \frac{i^*(k+1) - i(k)}{T_s}. \hspace{1cm} (20)$$

where $v_s(k)$, $i(k)$, $i^*(k+1)$ and $T_s$ are the grid voltage, output and reference current at instant $k$, and $T_s$ is the sampling period. Conventional FCS-MPC requires a variable prediction for each possible output vector. The simplified FCS-MPC replaces all the predictions with the calculation of $u^*$, which is done just one time per sampling period. Therefore, the dimension of the prediction model is reduced, which implies that computational burden is lower than that of the conventional approach. This method is useful for short prediction horizons. However, it only results in a marginal reduction of the computational cost when a long prediction horizon is considered.

The second proposal also reformulates an equivalent cost function, but the optimal problem is stated as a function of a new variable $U_{uc}^{opt}$ and the possible output voltage vectors, $u_n$, [18], [105], [106]. $U_{uc}^{opt}$ depicts the unconstrained solution of the optimal problem, $u_{uc}^{opt}$, in a new space, which is calculated as

$$U_{uc}^{opt} = Hu_{uc}^{opt}, \hspace{1cm} (21)$$

where $H$ is a triangular matrix, as demonstrated in [18]. Thus, the new cost function is written as

$$g = \|Hu_n - U_{uc}^{opt}\|_2^2, \hspace{1cm} (22)$$

and the unconstrained optimal solution $u_{uc}^{opt}$ can be computed as explained in [112]. Minimizing the cost function (22), turns out to be equivalent to looking for the $Hu_n$ closest to $U_{uc}^{opt}$. This search can be done with the Sphere Decoding Algorithm (SDA) [113]. The SDA should be adapted to power electronic applications [18], but the method is very efficient and reduces the computational burden of the optimization algorithm. Further developments on this method than aim to reduce of the computational complexity can be found in [114]–[116]. It should be noted that SDA is a branch-and-bound algorithm. Other techniques belonging to this family have been used in power electronics [117], the most common being the reduction of the computational complexity (at least on average) of integer programs like FCS-MPC.
A particular optimization method can be applied when multilevel power converter topologies are considered [118]. Multilevel converters are characterized by several output voltage vectors producing the same output voltage level, these are known as redundancy vectors. For instance, in a conventional single-phase Two-Cell Cascaded H-Bridge Converter (2CCHB), there are 16 possible output voltage vectors, but they produce only five voltage levels. Usually, the redundancies are exploited to balance dc-link capacitor voltages or reduce the switching losses. Conventional FCS-MPC handles these problems through the cost function. For example, the cost function (6) allows one to track a desired current and reduce the number of commutations. Taking into account the redundancies, the FCS-MPC problem can be defined reducing the computational burden. The method was presented in [107] and is called hierarchical FCS-MPC [72], [108], [109]. It consists of the following steps:

1) The cost function is split into two functions. The first one defines the desired behavior for those variables that can be predicted as a function of the output voltage level. The second one includes the rest of the variables of interest. For instance, (6) is divided as

\[
g_1 = \left( \hat{i}_L - i_L^* \right)^2, \quad g_2 = n_c, \tag{23}
\]

2) The first cost function is minimized. For this purpose, the first cost function’s value is calculated for each one of the possible output voltage levels. The optimal output voltage level is chosen as the one that minimizes the cost function’s value.

3) The optimal output voltage level is associated with a set of redundant output voltage vectors. This set is used to minimize the second cost function. Then, the optimal control action is chosen as the one that minimizes the second cost function’s value.

B. Long Prediction Horizon

MPC with a long prediction horizon improves the system’s performance and stability as compared to short prediction horizons [4]. However, using long prediction horizons increases the optimization algorithm’s computational burden. EMPC and GPC can be formulated with long prediction horizons for power electronic applications. The main reason is that the computational costs of both algorithms are almost independent of the prediction horizon. On the other hand, the FCS-MPC optimization problem is usually solved by an exhaustive search algorithm (ESA) that computes the cost function’s value for each of the possible switching vectors or sequences. As a result, when the prediction horizon increases, the computational burden of the ESA grows exponentially [40]. The optimization problem must be solved for each sampling time, but power electronic applications use short sampling periods. Thus, the ESA usually cannot be solved in a standard hardware control platform. Therefore, FCS-MPC with a long prediction horizon needs specific optimization algorithms in order to be implemented [119].

One technique that achieves a long prediction horizon is the move-blocking strategy (MBS) [77], [120], [121]. The main idea behind the MBS is to divide the prediction horizon into two parts, \( N = N_1 + N_2 \). The prediction model in the first \( N_1 \) steps of the horizon is computed using a small sampling interval, \( T_{s1} = T_s \). The second \( N_2 \) steps of the model is computed with a bigger sampling period, i.e. \( T_{s2} > T_{s1} \). In this way, the prediction horizon can be increased while limiting the computational cost.

A second approach that achieves long prediction horizon is the extrapolation strategy [30], [122], [123]. The method introduces the concept of switching horizon as the number of steps within which the power converter switches can change. The extrapolation strategy evaluates the prediction model over the switching horizon for all possible control input sequences. Then, it determines a set of valid sequences and calculates the evolution of the variables of interest for this set by extrapolating their trajectories from the previous step. The extrapolation strategy presents a variable prediction horizon. It depends on the considered sequence and is limited by the time step where the first controlled variable hits a bound.

A third method used to achieve a long prediction horizon is the multistep FCS-MPC [18]. As explained in Section V-A, this strategy uses an SDA to solve the optimization problem instead of the ESA. A modified SDA operated in a recursive manner allows one to limit the computational burden and solve the optimal problem using a long prediction horizon.

VI. RECENT ADVANCES OF MPC FOR POWER CONVERTERS AND DRIVES IN INDUSTRY

MPC provides some different benefits for power electronic converters and their applications. However, a varying degree of effort is required in order to integrate such technologies into industrial products. A discussion of MPC development steps across the spectrum of research, technology and product development can be found in [12]. The work contributes to the understanding of the challenges that need to be addressed in order to adopt such technologies into industrial products.

The application of MPC for power converters and drives at the industrial level is not new. For instance, an early proposal was a predictive current controller with an active damping strategy for a medium voltage drive with an LC filter [81]. The strategy avoids the excitation of the filter resonance while achieving fast current control and a low switching frequency. Breakthroughs of MPC can also be found in recent literature. In [124], MPC is applied for the torque regulation of a variable-speed synchronous machine fed by current source converters. The torque and system state are stabilized by controlling the rectifier and inverter angles. This idea was tested in a 11.6 kW prototype, later, the concept was evaluated in a 48 MW industrial-scale pilot plant, where the dc-link current as well as the rectifier and inverter firing angles were controlled [125]. A new MPC strategy called Model Predictive Pulse Pattern Control (MP³C) was presented in [126] for industrial applications with medium voltage drives. The technique combines MPC with Optimized Pulse Patterns (OPP) and considers the penalization of flux error and changes.
of switching instants in the cost function. The idea was applied to a five-level power converter from ABB with a rated apparent power of 1.14 MVA [127], [128]. The results demonstrated the MPC strategy’s superior, high dynamic performance. The method could be enhanced with an active damping method based on Linear Quadratic Regulator (LQR) theory to attenuate resonances caused by an output LC filter included in medium voltage converters [129].

REFERENCES


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