# 10. Seeking Sharper Frontiers of Efficiency in Tissue P Systems

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In a P system, there are several ingredients which influence their efficiency. Varying them, one can get efficient systems (able to solve computationally hard problems in polynomial time) or non-efficient systems (e.g., solving **NP**-hard problems in an exponential time). The borderline between efficiency and non-efficiency is thus a problem of a central interest. This issue is explored here for tissue P systems.

**Required Notions:** tissue P systems, complexity classes, cell division, cell separation, symport/antiport rule

A *tissue P system with symport/antiport rules* $\Pi = (\Gamma, \mathcal{E}, \mathcal{M}_1, \ldots, \mathcal{M}_q, \mathcal{R}, i_{out})$, of degree $q \geq 1$ can be viewed as a set of $q$ cells, labeled by $1, \ldots, q$, with an environment labeled by 0 which initially have an arbitrary number of copies of some kind of objects, and a set of rules which can be of several types: communication, division or separation (see [3, 4] for details).

For each natural number $k \geq 1$, $\mathbf{TDC}(k)$ (respectively, $\mathbf{TDS}(k)$ or $\mathbf{TDA}(k)$) is the class of recognizer tissue P systems with cell division and communication rules (allowing only **s**ymport or **a**ntiport rules, respectively) of length at most $k$. Similarly,

by considering separation rules instead of division rules, we obtain $\mathbf{TSC}(k)$, $\mathbf{TSS}(k)$ and $\mathbf{TSA}(k)$ respectively. We denote by $\mathbf{PMC_R}$ the set of all decision problems which can be solved in a uniform way and polynomial time by means of families of systems from a class $\mathbf{R}$ of recognizer tissue P systems.

**(A) Tissue P systems with cell division and with cell separation**

By using the dependency graph technique, it has been proved that $\mathbf{P} = \mathbf{PMC}_{TDC(1)} = \mathbf{PMC}_{TSC(1)}$ [2, 3]. Furthermore, efficient and uniform solutions to the SAT problem by using systems from $\mathbf{TDC}(3)$ [1] and from $\mathbf{TSC}(8)$ [3] have been given. Recently, the last result has been improved to $\mathtt{SAT} \in \mathbf{PMC}_{TSC(3)}$ [6].

**Problem 10.1.** Assuming $\mathbf{P} \neq \mathbf{NP}$, in the framework of tissue P systems with cell division/cell separation, a frontier of the tractability is obtained when passing from communication rules with length 1 to communication rules with length at most 3. Does passing from 1 to 2, amounts to passing from non–efficiency to efficiency?

**Conjecture 10.1:** $\mathbf{NP} \cup \mathbf{co}\text{-}bfNP \subseteq \mathbf{PMC}_{TDC(2)}$.

**(B) The role of direction in communication rules**

Next, we deal with complexity aspects of tissue P systems with cell division/cell separation where only symport or antiport rules are allowed. We have: $\mathbf{P} = \mathbf{PMC}_{TDA(1)} = \mathbf{PMC}_{TSA(1)}$, and $\mathbf{NP} \cup \mathbf{co} - \mathbf{NP} \subseteq \mathbf{PMC}_{TDA(3)} \cap \mathbf{PMC}_{TSA(3)}$. Thus, assuming $\mathbf{P} \neq \mathbf{NP}$, a first frontier between efficiency and non-efficiency is obtained in the above framework when passing from communication rules with length 1 to communication rules with length at most 3.

**Problem 10.2.** What about the complexity classes $\mathbf{PMC}_{TDA(2)}$, $\mathbf{PMC}_{TSA(2)}$, $\mathbf{PMC}_{TDS(k)}$ and $\mathbf{PMC}_{TSS(k)}$, for all $k \geq 1$?

**Conjecture 10.2:** $\mathbf{P} = \mathbf{PMC}_{TSA(2)}$, and for all $k \geq 1$, $\mathbf{P} = \mathbf{PMC}_{TSS(k)}$.

If this conjecture is true, then passing from symport rules to antiport rules with length at least three, amounts to passing from non–efficiency to efficiency, in the framework of tissue P systems with cell separation.

**(C) The role of the environment**

Classical tissue P systems have a special alphabet associated with the environment, whose elements appear at the initial configuration of the system, in an arbitrary large amount of copies. What may happen if this property is removed, that is, if we assume that the alphabet associated to the environment is empty? We use a "hat" to indicate the case when the environment is initially empty.

Recently it has been proved that, for each $k \geq 1$, $\mathbf{PMC}_{\mathcal{TDC}(k)} = \mathbf{PMC}_{\widehat{\mathcal{TDC}}(k)}$ [5], that is, in the framework of tissue P systems with cell division the role of the environment is not relevant from the complexity point of view.

**Conjecture 10.3:** For each $k \geq 1$, $\mathbf{P} = \mathbf{PMC}_{\widehat{\mathcal{TSC}}(k)}$.

If this conjecture is true, then in the framework of tissue P systems with cell communication the following holds: (a) passing from separation rules to division

rules (length at least three) amounts to passing from non–efficiency to efficiency; and (b) the environment provides a new borderline of efficiency.

## References


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