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Presentation

NoLineal 2016 (Seville, 7-10th June, 2016) is the 10th in a series of conferences previously held in Ávila (1997), Almagro (2000), Cuenca (2002), Toledo (2004), Ciudad Real (2007), Barcelona (2008), Cartagena (2010), Zaragoza (2012), and Badajoz (2014).

The aim of this conference is to offer senior and young researchers of different areas, such as Physics, Mathematics, Biology, Economics, Social Sciences, etc, the possibility to share their latest results in this interdisciplinary meeting.

This international conference is open to researchers from all around the world. All lectures will be given by leading scientists. Participants are strongly encouraged to present and discuss their own research, especially during oral and poster sessions.

The congress will be held in memoriam of Prof. Antonio Castellanos Mata, Full Professor of Electromagnetism at the University of Seville, and Director of the group of Electrohydrodynamics and Cohesive Granular Media, who was member of the Scientific Committee and passed away during the organization of the event. At the meeting we will commemorate his life and work.

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In Memoriam: Antonio Castellanos Mata

Antonio Castellanos Mata

Personal life

Antonio Castellanos Mata was born on March 7, 1947, in Antoñanes del Páramo, a small austere village near León, Spain. A few years later, the family moved to León. He was the third (and last) child of Manuel Castellanos Berjón (1910-1993), a school teacher, and Fidela Mata Sarmiento (1915-2013). A love for reading and admiration for science were always part of family life. Antonio's parents separated when he was a small child, but his father stayed in touch with his children and taught little Antonio mathematics. Relations with his mother and his siblings, Domingo José and Aurora, were very important for Antonio during all his life.

As a little child, Antonio was very religious and decided to become a priest. While studying in a seminary, he changed his opinion, became rebellious, and started to study mathematics and physics on his own. Several months before graduating he was expelled from the seminary. However, he studied himself all the necessary subjects, including math and sciences, passed the exams, and entered the University of Valladolid. Some friendships formed in that period lasted to the very end of his life. Antonio was awarded the first PhD in physics at the University of Valladolid. He was granted a Fulbright scholarship and spent a year at the Ohio State University, concluding his stay by a journey through almost the whole of Latin America, going to the most dangerous places and talking to people of all classes. He wanted to see how people lived and wanted to change the world for the better. His character and worldview never ceased to have this revolutionary streak. He liked to travel and visited many countries of the world. In many of them, he established long-lasting scientific collaborations. One of the important events in his life was meeting Pierre Atten, who became his close friend and inspired in him a love for electrohydrodynamics.

The first spouse of Antonio was María Elena Navarrete Sandoval. They married in 1975 and had a son, Antonio Castellanos Navarrete, and a daughter, Dayeli Anahí Castellanos Navarrete. In 2006, Antonio married Elena Grekova, with whom he had two sons, León Antonievich Castellanos Grekov and Iván Antonievich Castellanos Grekov.

In 2014, several months after the death of his mother, Antonio was incidentally diagnosed with kidney cancer at an early stage, but of a rare and aggresive type. In 2015, it gave metastases, despite their very low probability, and after a year of fighting the disease, Antonio died on January 27, 2016.

To his very last days, Antonio worked, gave classes, continued his research, and directed scientific projects. He kept his sense of humour, his enchanting smile, his interest and love for science, generosity, care for people around him, fortitude and courage, his open, sincere nature, for which he was admired and loved by his colleagues, friends, and family.

Scientific achievements

Antonio defended his PhD thesis *Dispersion theory and its application to the reaction* $^{11}B(d,\alpha)Be^9$ at the University of Valladolid in 1972. He worked at many universities: Universidad de Valladolid, Universidad del País Vasco, Universidad Autonoma de Madrid; since 1983, he was a full professor at the University of Seville. Antonio made long-term scientific visits to the USA, France, Nicaragua, and Russia, collaborated with researchers from the UK, Netherlands, China. He worked in various fields of science:

- electrohydrodynamics,
- gas discharges at atmospheric pressure,
- cohesive granular materials.

Antonio founded a scientific school at the University of Seville. His research group of Electrohydrodynamics and Cohesive Granular Media included more than 20 researchers. Many of his former students are now full professors and continue the scientific tradition, seeing Antonio as their teacher and a dear friend. He always treated his students as equals, with deep respect, discussed scientific problems with them, gave them freedom, and cared about their progress in science more than about anything else. He lectured physics at various universities during all his career. For the last 33 years, he taught electrodynamics and electromagnetism at the Faculty of Physics of the University of Seville, a task which he performed with enthusiasm and passion. Antonio directed research projects for more than 30 years, and this made it possible for him to organize two laboratories at the University. He always had bright ideas, scientific intuition, and creative mind. Dedicating a lot of efforts to pure science, Antonio was also interested in practical problems and collaborated with industry (Xerox Corporation, Novartis, Dow Corning, IFPRI). Antonio signed only those papers to which he indeed contributed, but nevertheless authored more than 350 papers, with more than 7800 citations, though he himself did not give importance to these numbers. He believed that only important contributions matter. We may cite, for instance, the following ones:

- A. Castellanos, P. Atten, M. G. Velarde. Oscillatory and steady convection in dielectric liquid layers subjected to unipolar injection and temperature gradient. Physics of Fluids, vol. 27, pp. 1607-1615, 1984.
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- A. Castellanos, A. González. Nonlinear electrohydrodynamics of free surfaces. IEEE Transactions on Dielectrics and Electrical Insulation, vol. 5, pp. 334-343, 1998.
- A. Ramos, H. Morgan, N. G. Green, A. Castellanos. AC electrokinetics: a review of forces in microelectrode structures. Journal of Physics D: Applied Physics, vol. 31, no. 18, p. 2338-2353, 1998.
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- A. Castellanos, A. Ramos, J. M. Valverde. Device and method for measuring cohesion in fine granular media (Dispositivo y procedimiento para medir la cohesión de medios granulares finos). Patent PCT/ES98/00325. 1997. Publication: 2000. Organization: University of Seville.
- A. Castellanos, M. A. S. Quintanilla, J. M. Valverde. Method and device for measuring the angle of friction and the cohesion of granular media (Dispositivo para medir el ángulo de fricción y la cohesión de medios granulares.) N P200502533. 11-10-2005 N. International Patent PCT/ WO2007042585A3. Publication: 2007. University of Seville.

Antonio belonged to a generation that played an important role in the revival of physics in Spain. In 2013, he was awarded the Prize FAMA for the research career by the University of Seville.

Among his scientific results, we can mention the following:

- Galilean limits of electromagnetism.
- Temperature equation and entropy production in electrohydrodynamics.
- Seminal works on numerical simulation of electrohydrodynamic flows.
- Physical mechanism of electrothermohydrodynamic instabilities.
- Energy cascade in electrohydrodynamic turbulence.
- Stabilization of dielectric liquid bridges by ac electric fields.
- Absence of inertial (collisional) regimes in fine powders for negligible intersticial gas interaction.
- Automated apparatus to characterize fine powders (Sevilla Powder Tester).
- Apparatus to characterise the cohesive properties of grains (Triana Powder Tester).
- Model of elastoplastic contact between two powder particles.
- Microstructure characterization of fluidized bed of fine particles: aggregation, solidlike-fluidlike transition, fluctuations, influence of electromagnetic fields.
- Experimental setup for measuring acoustic properties of fine (including magnetic) powders.

In his last years, Antonio worked on thermodynamics in relativity (but he did not have time to complete this work) and on triboelectricity in fine powders (not published due to contract restrictions). His work on wave propagation in powders at low pressure, as well as other research lines initiated by him, will be continued by his colleagues. As a researcher, Antonio combined a strong theoretical mind, experimental intuition, profound understanding of physics of phenomena, and passionate love for science.

PLENARY TALKS

The folding of a small protein

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Proteins are the macromolecules that mediate most of the processes that occur in living cells. In order to function properly, after being synthesized, they must reach a well defined average structure, known as the native state, which is quite specific for each protein. According to Anfinsen's thermodynamic hypothesis [**1**], protein folding is an equilibrium process and the native state of a protein is uniquely defined by its amino acid sequence.

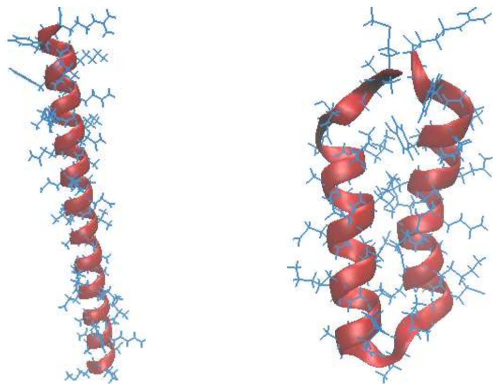

FIGURE 1. This figure shows the *same* protein in two conformations. The left panel shows the protein in a full α-helix conformation, assumed to be the structure the protein has immediately after synthesis, and the right panel shows the native state of the protein, obtained by solution nuclear magnetic resonance (NMR) methods [**5**].

However, in spite of more than four decades of study, the protein folding problem, i.e., the problem of determining the protein structure from its amino acid sequence alone, remains unsolved. In the first part of the lecture, results will be presented that suggest that one main reason for this lack of success is that each protein can assume many, very different structures that are as thermodynamically stable as the native state, as first proposed by Levinthal [**2**]. Indeed, following Levinthal, it has been suggested [**3, 4**] that protein folding is a non-equilibrium, kinetic process in which the initial structure for all proteins is helical, as shown in the left panel of figure 1. In the second part of the lecture, the idea that the initial structure of all proteins is helical is applied to the folding of PDB2HEP [**5**], a small protein with just 42 amino acids whose native structure is constituted by two α-helices joined by a loop, as seen in the right panel of figure 1. Molecular dynamics (MD) simulations will be presented, in which the initial structure of the protein is the helix in the left panel of figure 1. The aim of these MD simulations is to fold this protein. The ultimate aim of these investigations is to solve the protein folding problem by determining the conditions under which the native state of all proteins can be obtained, in a reproducible manner, from such an initial condition.

Keywords: protein folding, VES hypothesis, molecular dynamics.

Acknowledgments

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Simplifying canard theory with piecewise-linear systems. Applications to neuronal dynamics

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Canards are special solutions of dynamical systems with multiple timescales. They are special for at least two reasons: first, they follow repelling dynamical objects (slow manifolds) for long time intervals; second, they are associated with exponentially narrow parameter variations typically called *explosions*. Since they were discovered in the van der Pol oscillator at the end of the 1970s, they have been the subject of intense mathematical research, giving rise to fine analytical studies (e.g. using matched asymptotics or geometric desingularisation techniques). They are also very challenging numerically as they arise in stiff systems. Canard dynamics has become increasingly popular over the years also because these objects naturally appear in mathematical models that are relevant to a plethora of application areas, including neuroscience, chemistry, plasma physics and population dynamics, to name a few.

In this talk, I will give an overview of recent results aiming to revisit canard theory from the perspective of slow-fast piecewise-linear (PWL) dynamical systems. This framework is well known to retain all salient features of nonlinear systems while being particularly amenable to analysis, including precise quantitative estimates. Taking several examples, I will attempt to show how the PWL approach helps to simplify canard theory without losing any of its rich dynamics. I will present application of these results to neuron models, where the PWL approach is relevant since neurons are modelled in first approximation as circuits and PWL systems are notoriously very efficient to capture circuit dynamics. Starting with planar systems, where canards organise the transition between rest and spiking states, I will then consider three-dimensional cases where complex oscillatory behaviours mixing subthreshold oscillations and spikes (referred to as mixed-mode oscillations or MMOs) can also be understood by means of canard solutions and captured in a minimal PWL setting.

Acknowledgements

This work is part of a large research program which has benefited over the years from precious collaborations with a number of experts in PWL dynamics and mathematical neuroscience: Soledad Fernández-García, Emilio Freire, and Enrique Ponce from Sevilla (US); Antoni Guillamon from Barcelona (UPC); Martin Krupa from Inria Sophia Antipolis; Rafel Prohens and Antonio E. Teruel from Palma (UIB); Serafim Rodrigues from Plymouth University (UK).

Dynamical regulation in living systems

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Interactions among the biochemical components that constitute and regulate living systems are frequently nonlinear. This allows for nontrivial dynamical behaviors such as limit-cycle oscillations and pulses, which arise even in the presence of stationary environmental conditions. In this talk I will review recent work on the dynamical regulation of cells and cellular populations, discussing a variety of cell types, regulation modes and environmental conditions. In particular, both gene expression and metabolic regulation will be considered, with a special focus on the behavior of bacterial populations under nutritional and energy stress. In all the cases studied, dynamics provides a significant survival advantage with respect to alternative stationary behaviors, by allowing for instance the periodic release of stress that enables a population to maintain its viability under limiting conditions, by balancing conflicting needs such as nutrient access and protection against external attacks.

Keywords: dynamical regulation, living systems.

Droplet formation in living cells

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Cells exhibit a complex spatial organization, often involving organelles that are surrounded by a membrane. However, there exist many structures that are not membrane bounded. Examples are the centrosome, meiotic and mitotoc spindles as well as germ granules. An interesting question is how such structures are assembled in space inside the cytoplasm which is essentially a fluid where all components usually mix? I will highlight the importance of phase coexistence and phase separation of fluid phases in the cell cytoplasm as a basic mechanism of the spatial organization of cells. Droplets in the cytoplasm can represent microreactors with different composition and chemistry as compared to the surrounding cytosol. I will discuss how such concepts shed light on the structure and organization of centrosomes and of germ granules.

Complexity in discrete-time population models: other bifurcation diagrams are possible

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It is well-known that simple deterministic models governed by one-dimensional maps can display chaotic behavior. Pioneering work in this direction has been made based on discrete-time population models with overcompensatory growth, where increasing the growth rate leads to a period-doubling bifurcation route to chaos which is represented by usual bifurcation diagrams [**8**].

In many population models, it is more interesting the response of population abundance to changes in other parameters, such as harvesting effort in exploited populations or culling intensity in the control of plagues. Managers can control these parameters at some extent, searching for desirable outcomes (for example, a maximum sustainable yield in exploited populations, or preventing the risk of extinction in endangered species). It has been observed that an increasing mortality rate may give rise to new phenomena, sometimes counterintuitive, such as sudden collapses [**5, 10**], stability switches [**2, 6**], and the hydra effect (a population increasing in response to an increase in its per-capita mortality rate) [**1, 4, 6**].

In this talk, we review these phenomena in simple population models subject to different harvest strategies, and we highlight the importance of several often underestimated issues that are crucial for management, such as census timing [**4**], intervention time [**3, 9**], and carry-over effects [**7, 9**].

Keywords: population dynamics, bifurcations, stability, overcompensation.

Acknowledgments

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Shepherding intrinsic localized modes in microscopic and macroscopic nonlinear lattices

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The possibility that large amplitude, localized vibrational excitations can exist in periodic physical lattices with nonlinear intersite forces was discovered thirty years ago. The energy profiles of these intrinsic localized modes (ILMs) - also called "discrete breathers" or "lattice solitons" - resemble those of localized vibrational modes at defects in a harmonic lattice but, like solitons, they can propagate; however, in contrast with solitons they loose energy as they move through the lattice - the more localized the excitation the faster the energy loss.

First we review the experimental E&M generation of countable intrinsic localized modes in a 1-D atomic spin lattice, where countable ILMs and their controlled switching is observed.[1] Next we demonstrate that a reexamine of the inelastic neutron scattering measurements of the thermal generation of localized vibrational modes in an NaI crystal is in order.[2]

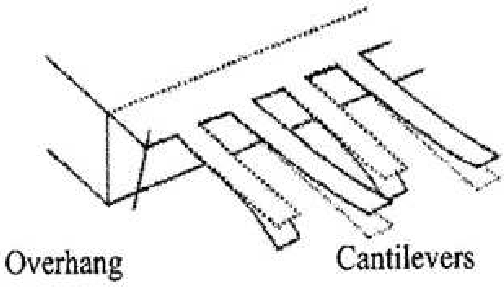

FIGURE 1. Micromechanical array.

Our most detailed ILM studies have involved the production and manipulation of localized energy along micromechanical arrays. Such a mode will stay in resonance as the driver frequency is changed adiabatically until a bifurcation point is reached. One such study involves steady state locking of ILMs, and their interactions with impurities. By measuring the linear response spectra of a driven array containing an ILM both the dynamics of bifurcation transitions and the hopping of vibrational energy have been connected to the transition properties of soft modes.[3]

Recently the search for a completely mobile ILM has focused attention on minimizing the resonance interaction that occurs between the localized excitation and small amplitude plane wave modes. Via simulations we demonstrate that when more than one type of nonlinear force is present their Fourier components can often be designed to cancel against each other in the k-space region of the plane wave dispersion curve, removing the resonance. The end result is a supertransmission channel[4] for an ILM in a discrete physical lattice. Such an engineered, intrinsic, low loss channel may prove to be a very useful property for other physical lattices treated within a tight binding approximation.

Keywords: intrinsic localized modes (ILMs), supertransmission channel.

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From macrosurf (hydrodynamics) to nanosurf (electron transfer in crystals): a common line of nonlinear thinking with useful consequences

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Soliton-assisted transport (with commercial purposes) was invented in "modern" times by the shipbuilding architect engineer J. S. Russell who in mid-XIXth Century made the discovery of the "solitary" wave at Union Canal near Edinburgh. Indeed, solitary waves, e.g., at the sea shore, and bores in rivers, permit matter transport and surfing (upstream!). Bores had been observed and used by peasants in China several centuries ago. They used them to transport goods surfing upstream, with small boats, and getting back to origin with the downstream river flow. In the later quarter of the XIXth century, such waves received a good theoretical explanation by Boussinesq and Lord Rayleigh (who acknowledged Boussinesq's earlier achievements). Later on came Korteweg and de Vries who rediscovered the solitary wave evolution equation (fifteen years after Boussinesq). Their achievement was to also provide besides the solitary wave, another solution in the form of periodic cnoidal wave-train (their stability analysis was not correct). It was not until mid-XXth Century that the curiosity of the "solitary" wave attracted the interest of Zabusky and Kruskal who numerically explored the solutions of the BKdV equation, and their (overtaking) collisions, and coined the soliton word and concept. Their research was motivated by earlier work of Fermi, Pasta and Ulam (FPU) on heat transfer and equipartition in (anharmonic) lattices. One of the lattice cases treated by FPU, i.e., with cubic potential interactions, was shown to have as continuum equivalent the BKdV equation. Eventually, subsequent mathematical work about BKdV and other soliton-bearing equations led to a new area in Applied Mathematics and General Physics (of conservative and, mostly, Hamiltonian integrable systems).

The soliton concept has been a powerful paradigm to provide a unifying understanding of a disparate collection of phenomena found in several branches of Science and not just Physics (Fluid Physics, Nonlinear Optics and Lasers, Optical Fiber transmission, Acoustics, Plasmas, Neuro-dynamics, etc).

The BKdV equation is peculiar in the sense that it possesses a (local) balance between nonlinearity (velocity depends on amplitude) and dispersion (velocity depends on wavelength/color) that permits maintaining "alive" the "solitary" wave (or the cnoidal wave-train) as time proceeds. Dissipation or damping alone generally tends to destroy solitons, mostly through a leaking (linear) "radiation", a capillary-gravity wave-tail/head where viscosity eventually kills all wave motion. In the 20s of past century, Taylor and Burgers argued that waves could survive damping if an appropriate nonlinear-dissipation balance existed (another possibility is an input-output dynamic energy balance, much later studied). The TB equation has "heteroclinic" solutions in the form of (supersonic) shocks in compressible gases (with local Mach number above unity) and bores (with corresponding Froude number above unity) in hydraulics. This (1D)-compressible gases-(2D)-hydraulics similarity had been known and exploited by Mach in the late XIXth century. Shocks, bores (mascarets, in French), hydraulic jumps, kinks, are also called "topological" solitons (to discriminate from e.g. Sech2-like solitons of the BKdV equation).

In the second half of the XXth century, room temperature soliton-assisted electron transport (ET and positive hole) was experimentally observed and theoretical described in polymers like trans-polyacetylene (tPA). Their discovery led to the Nobel Prize for Chemistry awarded to A. J. Heeger, A. G. MacDiarmid and H. Shirakawa, in 2000. The supporting theory is based on a harmonic backbone Hamiltonian (as relative displacements are of the order of 0.04 Angstroms while the equilibrium inter-atomic lattice distance is about 1.22 Angstroms). As the ground state of tPA is degenerate, in the theory this is accounted by means of an additional double-well quadratic potential. This together with the nonlinear electron-lattice interaction brings the possibility of solitons in the form of kinks. Most important feature of tPA and relatives is that they are easily doped thus offering very attractive features for electronic devices.

At about the same time, Davydov introduced the concept of electro-soliton to describe ET along bio-(macro)-molecules. He also used a harmonic backbone Hamiltonian (though he studied, albeit fragmentarily, anharmonic cases). His clever use of the nonlinearity of the electron-phonon interaction together with suitable way of transition to the continuum description, permitted him to building a soliton-bearing system. Many other scientists followed Davydov's ideas but, apparently, his predictions were shown not to survive above 10K and have not yet found support by experiment.

I shall present another possibility for soliton-assisted ET arising from starting with a soliton-bearing anharmonic lattice (using e.g. Morse interactions) and, as in the above mentioned theories, treating the electron in the standard quantum mechanical "tight-binding" approximation. The interaction between the electron and the lattice vibrations provides the dependence of the hoping electron transfer-matrix elements on the relative, time-dependent distance between neighboring lattice units (of particular interest are strong enough compressions, say, about half the equilibrium inter-atomic distance). It appears that when adding an excess electron there is electron trapping by the, generally supersonic moving, lattice soliton. This has been called a solectron which, depending on parameter values, provides (sub- and supersonic) electron surfing at the nano-level. The solectron appears as a natural extension to anharmonic lattices of both the Landau-Pekar polaron (for harmonic lattices) and the Davydov's electro-soliton. It has also been shown that such lattice solitons (in the absence of added electron and solectrons) survive up to ambient temperatures (ca. 300K) for parameter values typical of bio-(macro)-molecules. I shall discuss the role of an external electric field thus showing features of a novel form of ET that appears valid to support thirty-year old outstanding experimental results obtained by K. Donovan and E. G Wilson on highly crystalline polymers like poly-diacetylene (PDA) crystals which behave quite differently from tPA (no doping allowed).

I shall also present work on controlling ET from say a source to a drain, like in a transistor, along "natural channels" (crystallographic axes) in e.g. triangular lattices. This ET bears similarity to electron surfing on surface acoustic waves in piezoelectric materials, like GaAs layers and other systems and I shall also comment on this item.

Finally, I shall comment on the relationship between lattice solitons and DB/ILM (kind of unification with differences).

Keywords: lattice solitons, electron transport, solectrons.

Gravitational waves as nonlinear waves

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Abstract

The interferometers of Advanced LIGO have detected gravitational waves generated by the fusion of two black holes for the first time [**1**]. On September 14, 2015, the instruments in Livingston (Louisiana, USA) and Hanford (Washington, USA) detected practically the same signal with a signal-to-noise ratio of 24. The origin was the fusion of two black holes with about 36 and 29 solar masses, resulting in new one with about 62 solar masses. In the process the energy of 3 solar masses was emitted in gravitational waves, the most violent astrophysical event recorded to date.

Gravitational waves solve the weak-field approximation of the Einstein equations in vacuum [**2**]. In this limit, they evolve as linear waves. Since the energy-momentum tensor is the source of Einstein's gravitation and gravitational waves propagate energy and momentum, gravitational waves are intrinsically nonlinear waves [**3, 4**]. In fact, the frequency of the signal observed by Advanced LIGO detectors changes, a nonlinear phenomenon.

Black holes are soliton solutions (or self-similar solutions) of Einstein's equations in vacuum [**5**]. Their fusion can be interpreted as that of two (non-integrable) solitons, hence this phenomenon emits low-amplitude radiation. Such gravitational waves propagate in spacetime like linear waves, but their generation requires the inclusion of nonlinear effects.

Keywords: nonlinear waves, advanced LIGO, gravitational waves.

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ORAL COMMUNICATIONS

Part 1

THEORY AND COMPUTATION

On difference equations with predermined forbidden sets

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Let $f:\mathbb{R}^k \to \mathbb{R}$ be of the form $f(x_n,...,x_{n-k+1}) = \frac{P(x_n,x_{n-1},...,x_{n-k+1})}{Q(x_n,x_{n-1},...,x_{n-k+1})}$ that is, a *rational function*. The corresponding *rational difference equation* of order k is given by

$$x_{n+1} = f(x_n, x_{n-1},..., x_{n-k+1})$$

which can be seen as the following discrete dynamical system associate to the iteration function.

$$F(x_n, x_{n-1},.., x_{n-k+1}) = (x_{n+1}, x_n,.., x_{n-k+2})$$

A *solution* of the equation is the sequence of numbers $(x_n)_{n=0}^{\infty}$ where $(x_0,..., x_{k-1}) \in \mathbb{R}^k$ is the given vector of *initial conditions*.

Such difference equations in the autonomous and non-autonomous cases appear in models of populations dynamics. Also in the treatment of Riccatti difference equations [**1**]. Some vectors of initial conditions do not allow to construct a solution because there is a member x_{n+1} of the solution that can not be defined, usually because $Q(x_n,..., x_{n-k+1}) = 0$. It can also happen by the effect of negative parameters in the equation. We will call *forbidden set* of the equation to the set of vectors $X \in \mathbb{R}^k$ for which the solution taking them as initial conditions are not defined. It will be denoted by $\mathfrak{F}$. Given a rational difference equation, it is a classical problem to construct its forbidden set.

In this talk we will deal with the converse problems. Given a set $\mathfrak{F}$, find an iteration function in such a way that its associate $\mathfrak{F}$ has been previously specified. For example, given an arbitrary closed set $C \subset \mathbb{R}$, we are able to construct even a non-autonomous rational difference equation with a forbidden set holding $\overline{\mathfrak{F}} = C$.

On other hand, in some families of difference equations and for all its members, $\mathfrak{F}$ always contains non-bounded hypersurfaces and it is impossible to use the constructions used in the cases considered in the former paragraph.

We will also deal with some generalizations of the same problems on the forbidden sets in the setting of systems of difference equations, equations with complex parameters and equations outside of the rational frame. Some type of universal behavior will be also presented.

Keywords: rational difference equations, forbidden sets, Riccatti equations.

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When chaos meets hyperchaos: a Computer-assisted proof

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It has recently been reported that it is quite difficult to distinguish between chaos and hyperchaos in numerical simulations which are frequently "noisy". In this presentation we show that, for the classical 4D Rössler model, the coexistence of two invariant sets with different nature (a global hyperchaotic invariant set and a chaotic attractor) and the homoclinic and heteroclinic connections between their unstable periodic orbits give rise to long hyperchaotic transient behavior, and therefore it provides a mechanism for noisy simulations [**1**]. Moreover, the existence of several hyperchaotic sets provides an explanation of the smooth change from chaotic to hyperchaotic attractors due to the appearance of new heteroclinic connections among them, and so the joining of the different sets gives rise to slightly bigger and slightly more hyperchaotic attractors in the sense that the second Lyapunov exponent grows a little. The same phenomena are expected in other 4D and higher dimensional systems.

The Computer-assisted proof of this coexistence of chaotic and hyperchaotic behaviors combines topological and smooth methods with rigorous numerical computations [**2**]. The existence of (hyper)chaotic sets is proved by the method of covering relations [**3**]. We extend this method to the case of a nonincreasing number of unstable directions which is necessary to study hyperchaos to chaos transport. The cone condition [**4**] is used to prove the existence of homoclinic and heteroclinic orbits between some periodic orbits which belongs to both hyperchaotic and chaotic invariant sets. In particular, the existence of a countable infinity of heteroclinic orbits linking hyperchaos with chaos justifies the presence of long transient behaviour.

Keywords: chaos, hyperchaos, computer-assisted proofs, covering relations, hyperchaotic saddle.

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Using Extreme Learning Machines to cluster supervised data before classification

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In supervised classification, neural network classifiers return an output function based on the heuristics that attempt to accommodate the input data. This function is often a vector function with as many elements as classes in the data. Hence, the way to interpret the neural network classification criterion is to simply look at which of these elements in the output vector has the highest value. The position of the element that meets this requirement represents the class proposed by the neural network.

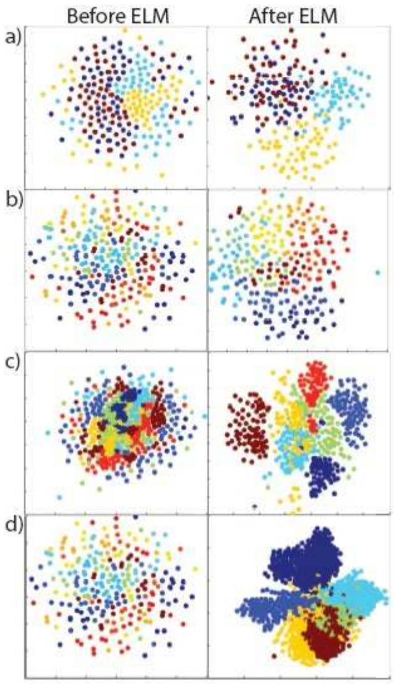

FIGURE 1. Results for UCI datasets, a) Vehicle, b) Vowel, c) Segmentation, d) Satellite.

This means that, in terms of the class output space, the classifier tries to cluster data within the hyperplanes that the prior condition suggests. If the output vector is made of $q_1, \ldots, q_k$ elements (where k is the number of classes), the region for a specific class q_i in the k-dimensional space is that in which $q_i > q_{j \neq i}$.

In this work, Curvilinear Component Analysis (CCA) [**1**] is proposed as a method to visualise the quality with which these clusters are made. CCA can be applied to any classification output in IR^q. We propose Extreme Learning Machines (ELM) [**2**] as the method to obtain such outputs. Since the heuristics of ELM propose random weights before the hidden layer, a large number of ELMs is run, in order to obtain a convergent vector $q_1, \ldots, q_k$. This vector is then dimensionally reduced to an IR^2 vector, using CCA, for visualisation purposes. The real analysis on how well are classes clustered will be performed directly on the IR^q.

Figure 1 shows CCA dimensional reduction applied to known UCI repository datasets before and after the averaged ELM is applied to them. Each dot represents and instance in the dataset. The color of the dot represents the class of that instance. The sorting of data in clusters is apparent according to these results. However a criticism of how classification is then applied, is necessary.

Keywords: supervised clustering, extreme learning machines (ELM), curvilinear component analysis (CCA).

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On autonomous and non-autonomous discrete versions of the Goodwin's model

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It is well known that the model on economical cycles introduced in 1967 by R.Goodwin [**2**], is one of the first combining behavior and economic growth. It can be interpreted like a prey-predator model similar to that formulated in the thirties of the former century by Lotka and Volterra in the setting of population dynamics.

Pohjola in [**3**] analyzed a discrete-time version of a slightly modified Goodwin system. They obtained a non-linear firt order difference equation possessing a rich dynamical behavior.

In the talk we will present another autonomous and non-autonomous discrete dynamical versions of the former. The dynamics of such versions are complicate and what it is interesting is the interpretation of the results that under the point of view of economic modelling can be don

New versions in the continuous dynamical system setting have been introduced in order to complete the original Goodwin model,[**1**]. One key idea in it was that the equilibrium point is globally stable in the positive quadrant of $\mathbb{R}^2$ which is foliated with invariant curves having the equilibrium point as a center. In new models, more than one equilivrium points can be reached. Even new equilibrium points could be asymptotically stable and the orbits can approach such points in an spiral way which means that the economy can fluctuate in a weaker way to them, which transforms the system of differential equations of Goodwin model from autonomous to non-autonomous.

Keywords: Goodwin's model, autonomous and non-autonomous systems, stability, economical cycles.

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Analytic integrability of some degenerate centers

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We study the analytic integrability around the origin inside a family of degenerate centers or perturbations of them. For this family analytic integrability does not imply formal orbital equivalence to a Hamiltonian system.

This work handles with determining the existence of analytic first integrals in a neighborhood of a degenerate center singular point.

We consider a vector field $\mathbf{F} = \sum_{j\geq r} \mathbf{F}_j$, $\mathbf{F}_j \in \mathcal{Q}^{\mathbf{t}}_j$. In [**2**], the integrability problem is solved in the case that $\mathbf{F}_r = \mathbf{X}_h$ where all the irreducible factors of h over $\mathbb{C}[x, y]$ are simple. The case $\mathrm{div}(\mathbf{F}_r) \neq 0$ with $\mathbf{F}_r$ reducible or $\mathbf{F}_r = \mathbf{X}_h$ where h has multiple factors is not solve and it is an open problem.

A necessary condition in order that $\mathbf{F}$ be integrable is that $\mathbf{F}_r$ be also integrable. The integrability problem for $\mathbf{F}_r = \mathbf{X}_h + \mu\mathbf{D}_0$ with $\mathbf{D}_0 = (t_1 x, t_2 y)^T$, $\mu \neq 0$ is solved in [**3**]. There it is shown the necessity of certain resonances in the parameters of the vector field in order that $\mathbf{F}_r$ be integrable.

On the other hand if $\mathbf{F}_r = \mathbf{X}_h + \mu\mathbf{D}_0$ is integrable and I is a first integral, there exists a quasi-homogeneous function f such that $I = \frac{1}{i}\mathbf{D}_0 \wedge \mathbf{X}_I = \frac{1}{i} f\, \mathbf{D}_0 \wedge \mathbf{F}_r = \frac{r+|\mathbf{t}|}{i} f\, h$. Hence the integrability problem of a quasi-homogeneous vector field with not null divergence is equivalent to the integrability problem of a quasi-homogeneous Hamiltonian vector field where its Hamilton function has multiple factors.

The family we are going to study have h with multiple factors. We consider degenerate systems of the form

$$(1) \qquad \dot{x} = -y(x^2 + y^2) + \cdots, \quad \dot{y} = x(x^2 + y^2) + \cdots,$$

which corresponds to $\mathbf{X}_h + \cdots$ whose $h = (x^2 + y^2)^2/4$. This type of systems was studied in [**4**] where it was proved that there exist centers inside this family without an analytic first integral.

We will see that there exist a blow-up and a scale of time that transforms system (1) into a nondegenerate system of the form

$$(2) \qquad \dot{u} = -u + \cdots, \quad \dot{v} = v + \cdots.$$

Using this transformation the center problem of system (1) reduce to the center problem for a nondegenerate system. However the integrability problem of the systems of the form (1) is open. It is clear that if system (1) is analytic integrable then system (2) is also analytic integrable but the converse is not true. Hence the analytic integrability of system (1) is not solved and in this work we will see that even for such simple family it is a difficult problem. We solve the analytic integrability problem for the generic case and only a degenerate case remains open.

Keywords: integrability problem, degenerate center problem, first integral, blow-up.

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Analysis of the Hopf-zero bifurcation and their degenerations in a quasi-Lorenz system.

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We analyze the Hopf-zero bifurcation in the clasical Lorenz system ([**1**]), which is degenerate due to the non-existence of an isolated equilibrium point. So, we study the system

$$(3)\quad \left\{\begin{array}{rcl} \dot{x} &=& \sigma(y-x),\\ \dot{y} &=& \rho x - y - xz + Byz,\\ \dot{z} &=& bz + xy + Dz^2, \end{array}\right.$$

with $\sigma, \rho, b, B, D \in \mathbb{R}, B \neq 0, D \neq 0, \rho > 1$, in which the Lorenz system is embedded, and we can study the Hopf-zero singularity at the origin and their degenerations. For this, we reduce (3) to the following normal form in cylindrical coordinates

$$(4)\quad \left\{\begin{array}{rcl} \dot{\bar{\rho}} &=& a_1\bar{\rho}z + a_2\bar{\rho}z^2 + a_3\bar{\rho}^3 + \ldots,\\ \dot{z} &=& b_1\bar{\rho}^2 + b_2 z^2 + b_3 z^3 + b_4\bar{\rho}^2 z + \ldots,\\ \dot{\theta} &=& 1 + c_1 z + \ldots, \end{array}\right.$$

The coefficients a_1, b_1, b_2, are assumed to be different from zero.

If $B = 0$, the coefficient a_1 vanishes, and we have a three-parameter unfolding that is studied in [**3**]:

$$\left\{\begin{array}{rcl} \dot{r} &=& r(\mu_1 + \mu_3 z + r),\\ \dot{z} &=& \mu_2 + cr^2 - z^2, \end{array}\right.$$

where, $c = -sgn(D)$,$\mu_1 = \frac{-1}{2}[\sigma + 1 + (2+\rho)b]$,
$\mu_2 = \frac{b^2}{4}(\rho+1)^2$ and $\mu_3 = \frac{B}{2\omega_0}$.
The analysis of this unfolding shows interesting bifurcations of periodic orbits as well as global bifurcations of equilibrium to equilibrium and of equilibrium to periodic orbit. The local results achieved are extended by means of numerical continuation methods, and they are applied to the study of the system (3) when the parameters B y D approach zero, in order to understand the dynamic of the Hopf-zero singularity in the Lorenz system.

Keywords: Lorenz, bifurcations, Hopf-zero, unfolding, normal form.

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Saddle-node bifurcation of canard solutions in planar piecewise linear systems

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In [**3**], we revisited the canard phenomenon in a piecewise-linear (PWL) framework using the singular perturbation approach that was developed in the context of smooth slow-fast systems [**2**]. To this aim, we used a 3-piece fast nullcline (critical manifold) to approximate the cubic nullcline of the van der Pol system near one of its fold points. This is a known necessary condition to generate canard cycles in PWL systems. By studying the attracting and repelling slow manifolds, we could prove the existence of a family of hyperbolic limit cycles undergoing a canard explosion up to the maximal canard, which arises when two slow manifolds are connected.
In the present work, we have added one regular parameter to the system so as to obtain a saddle-node bifurcation of cycles, which then breaks the hyperbolicity of the canard cycles. This scenario of subcritical canard explosion with a fold-of-cycle bifurcation has been studied in the smooth case in e.g. [**1**]. Furthermore, we are interested in studying a complete PWL caricature of the van der Pol system and not just the canards without head which lie across 3 linearity zones. Therefore, we have added a fourth zone to the PWL system considered in [**3**] to allow for canards with head as well. Within this setup, we analyse the existence of the non-hyperbolic canard cycle that exists at the saddle-node bifurcation, paying special attention to its location with respect to the maximal canard. Our aim is two: first, to find similarities and differences between the PWL case and the smooth one; then, to take advantage of the simplifications provided by the PWL analysis to better understand the smooth case.

Keywords: piecewise linear systems, singular perturbations, saddle-node bifurcation, canard solutions.

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Normal forms for a class of tridimensional vector fields with free-divergence in its first component.

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In the context of dynamical systems modeled by systems of nonlinear differential equations, the theory of normal forms focuses on the identification the simplest expressions. This theory is a basic tool for the study of various problems in differential equations, such as: bifurcations, stability analysis, among others. The main idea of this theory is the use of near-identity changes of variables to eliminate non-essential terms, from the dynamic point of view, in the analytical expression of the vector field. In this work, we generalize to quasi-homogeneous tridimensional vectors fields, the theory developed for quasi-homogeneous planar vector fields and we use it for calculating, to infinite order, a normal form of vectors fields with Hopf-zero singularity. In this talk, we work with systems of the form, $\dot{\mathbf{x}} = (\mathbf{X}_h, f(x,y))^T + \cdots$, being $\mathbf{x} = (x,y,z)^T$ and $\mathbf{X}_h = (-\frac{\partial h}{\partial y}, \frac{\partial h}{\partial x})$ a hamiltonian planar vector field. These systems can be described as,

$$\dot{\mathbf{x}} = \mathbf{F}_r + \cdots \tag{5}$$

where $\mathbf{F}_r$ is a quasi-homogeneous vector field in $\mathbb{R}^3$, $\mathbf{F}_r$ is independent on z and $\text{div}(\mathbf{F}_r) = 0$.

In the first part of this work, we show two broad results where the normal form of this class of vectors fields is described under $\mathscr{C}^\infty$-conjugation and $\mathscr{C}^\infty$-equivalence. Noteworthy that this theory is applicable to a wide class of singularities, one of them is the Hopf-Zero singularity, that can be described using system (5) where the first quasi-homogeneous term is of the form,

$$\mathbf{F}_r = \mathbf{F}_0 = \begin{pmatrix} -y \\ x \\ x^2 + y^2 \end{pmatrix} \tag{6}$$

In the second part of this work, the normal form of system with the Hopf-zero singularity, up to infinite order, is calculed.

The first works published about this last singularity, were obtained by Ushiki [**1**]. Later Algaba *et al.* [**2**] described the hypernormal form of vector fields having a Hopf-zero singularity and Chen *et al.* [**3, 4**] obtained the unique normal forms, under conjugancy and orbital equivalence for this singularity. More recently, Gazor & Mokhtari [**5**] provide a normal form for free divergence systems with a Hopf-zero singularity. This normal form agrees with the one obtained by us in our main result imposing divergence of the vector field equals to zero, i.e., our work shows a generalization of these results.

Keywords: normal form, Hopf-zero, equivalence, conjugation, bifurcations.

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Potential singularities for the Navier-Stokes equations

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The Cauchy problem of the Navier-Stokes equations for the flow of a viscous, incompressible, Newtonian fluid can be written as

$$\text{(7)}\quad \begin{cases} \partial_t \boldsymbol{v} - \Delta \boldsymbol{v} + \nabla p + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = \boldsymbol{0} & \text{in} \quad \mathbb{R}^3, \\ \nabla\cdot\boldsymbol{v} = 0 & \text{in} \quad \mathbb{R}^3. \end{cases}$$

Here $\boldsymbol{v}$ represents the velocity and p the pressure, and $T > 0$ is a final time of observation. It should be noted that the density and the viscosity have been normalized, as is always possible.

To these equations we add an initial condition

$$\text{(8)}\quad \boldsymbol{v}(0) = \boldsymbol{v}_0 \quad \text{in} \quad \mathbb{R}^3,$$

where $\boldsymbol{v}_0$ is a smooth, divergence-free vector field.

THEOREM 1. *Let $T_1 > 0$ be given. Then there exist initial data $\boldsymbol{v}_0$ arbitrarily large under any critical norm such that their corresponding solution $(\boldsymbol{v}(t), p(t))$ to (7) is smooth on $[0, T]$.*

THEOREM 2. *Let $0 < T_2 < 1$ be given. Then there exist initial data $\boldsymbol{v}_0$ arbitrarily large under any critical norm such that their corresponding solution $(\boldsymbol{v}(t), p(t))$ to (7) is smooth on $[T, \infty)$.*

The mathematical interpretation of these two results is that if a potential scenario of developing singularities would occur, this would have to take place either after T_1, being arbitrarily large, or before T_2, being arbitrarily small, respectively. The former implies that an enough amount of kinetic energy would be preserved so that the solution could blow up in finite time after a very long evolution time. The latter, instead, implies that the system would spends the most kinetic energy creating singularities in a very short evolution time after which the solution would become smooth.

Leray [**4**] referred as turbulent solutions to what today is known as weak solutions to the Navier-Stokes equations. He conjectured the relationship between turbulence and the breakdown of smoothness for Navier-Stokes solutions. In this setting, the turbulent phenomena in our two scenarios would take place either after a long evolution time or at the very beginning in a short evolution time.

The proof is relied on the following ingredients: (a) an *ad hoc* decomposition of (1) [**1**], (b) Kato's technique [**3**] for proving existence of mild solutions, and (c) a result of Escauriaza, Seregin, and Šverák [**2**], proving that Leray-Hopf weak solutions being $\boldsymbol{L}^3(\mathbb{R}^3)$-solutions are smooth.

Keywords: Navier-Stokes equations, weak solutions, strong solutions, blow-up, smoothness and regularity of solutions.

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Discrete and Continuous Lagrangian Descriptors for Hamiltonian systems.

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The goal of this talk is to discuss the generalization of the method of Lagrangian descriptors [**3**]. This method visualizes the phase space structure of Hamiltonian systems, in particular the stable and unstable manifolds of hyperbolic trajectories, in the case of both discrete [**1**] and continuous [**2**] dynamical systems. Such a method consists of the sum of the p-norm of the velocity field evaluated on the trajectory of points. In this work we discuss formal proofs on why this method highlights invariant manifolds.

Figure 1 displays the output of the evaluation of Lagrangian descriptors on altimeter data sets on an ocean region close to the Gulf Stream where an interesting hyperbolic trajectory has been detected.

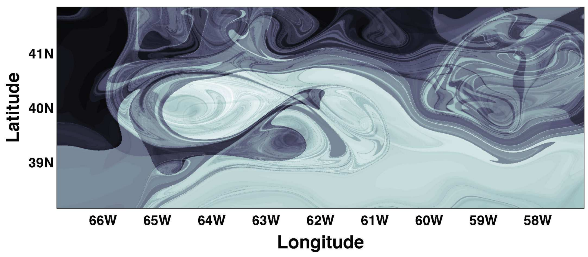

FIGURE 1. Computation of the continuous Lagrangian descriptor using AVISO data.

Keywords: Lagrangian descriptors, hyperbolic trajectories, invariant manifolds.

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Complexity of non linear robust design problems in control. Randomized Algorithms Approach

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Many non linear problems are very diffcult to solve if the number of variables becomes sufficiently large. Some of these problems are NP-hard type.

They can be any kind of problems: decision problems, search problems, or optimization problems. Recently, there have been interesting progresses on probabilistic analysis and design methods for systems and control, including uncertain and hybrid systems [**1, 2**]. This approach uses the theory of rare events and large deviation inequalities to bound the tail of the probability distribution. These inequalities are crucial in the area of Statistical Learning Theory [**2, 3**], and the use of this theory for feedback design of uncertain systems has been started in [**2**]. Recently, significant improvements regarding the sample complexity have been provided in [**4**]. For the special case of convex optimization problems, the scenario approach has been introduced in [**5**] for probabilistic controller design.

The importance of randomized algorithms is based on avoiding the complexity of nonconvex design problems. A set of random variables is independent and identically distributed (i.i.d.) if each random variable has the same probability distribution as the others and all are mutually independent. In this setting, one can draw N i.i.d. samples $\{w^{(1)},\dots,w^{(N)}\}$ from $\mathscr{W}$ according to probability $\Pr_{\mathscr{W}}$ and solve the sampled optimization problem.

Since obtaining a global solution to the previous problem is a difficult task in the general case, we analyze [**6**] the probabilistic properties of any suboptimal feasible solution. If one allows at most m violations of the N constraints.The idea of allowing some violations of the constraints is not new and can be found, for example, in the context of identification [**7**]. The randomized strategies corresponding to sampled optimization problem and allowing some violations of the constraints have been recently studied in [**4**]; see also [**1, 2**].

We remark that the probability of failure is slightly different from the probability of one-sided constrained failure introduced in [**4**].

Keywords: nonlinear, NP-Hard, control, randomized algorithms.

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Exponential stability for nonautonomous functional differential equations with state dependent delay. Applications to neural networks.

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The properties of stability of a compact semiflow $(\mathcal{K}, \Pi, \mathbb{R}^+)$ determined by a family of nonautonomous FDEs with state dependent delay taken values in $[0, r]$ are analyzed. The solutions of the variational equation through the orbits of $\mathcal{K}$ induce linear skew-product semiflows on the bundles $\mathcal{K} \times W^{1,\infty}([-r, 0], \mathbb{R}^n)$ and $\mathcal{K} \times C([-r, 0], \mathbb{R}^n)$. The coincidence of the upper-Lyapunov exponents for both semiflows is checked, and it is a fundamental tool to prove that the strictly negative character of this upper Lyapunov exponent is equivalent to the exponential stability of $\mathcal{K}$ in $\Omega \times W^{1,\infty}([-r, 0], \mathbb{R}^n)$ and also to the exponential stability of this minimal set when the supremum norm is taken in $W^{1,\infty}([-r, 0], \mathbb{R}^n)$. In particular, the existence of a uniformly exponentially stable solution of a uniformly almost periodic FDE ensures the existence of exponentially stable almost periodic solutions.

We apply the above results in order to analyze the dynamics of the solutions of a two-dimensional system of state-dependent delay equations which models the so-called delayed cellular neural networks. We find conditions guaranteing the existence of a global attractor which defines a copy of the base flow. In particular, this means that the properties of recurrence of the trajectories lying in this attractor are the same as those of the temporal coefficients of the model.

Keywords: nonautonomous FDEs, state-dependent delay, exponential stability, upper Lyapunov exponent, global attractor, neural networks.

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Pullback attractor for a non-classical and non-autonomous diffusion equation containing infinite delay

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In this paper we consider the following nonclassical and nonautonomous diffusion problem written in an abstract functional formulation:

$$(9)\quad \begin{cases} \dfrac{\partial u}{\partial t} - \gamma(t)\Delta \dfrac{\partial u}{\partial t} - \Delta u = g(u) + f(t, u_t) \\ \qquad\qquad \text{in } (\tau, +\infty)\times\Omega, \\ u = 0 \text{ on } (\tau, +\infty)\times\partial\Omega \\ u(t,x) = \phi(t-\tau, x),\ t \in (-\infty, \tau],\ x\in\Omega, \end{cases}$$

where $\tau \in \mathbb{R}$ is the initial time, $\Omega \subset \mathbb{R}^n$ is a smooth bounded domain, and the time-dependent delay term $f(t, u_t)$ represents, for instance, the influence of an external force with some kind of delay, memory or hereditary characteristic. Here, u_t denotes a segment of solution, in other words, given a function $u : (-\infty, +\infty)\times\Omega \to \mathbb{R}$, for each $t \in \mathbb{R}$ we can define the mapping $u_t : (-\infty, 0]\times\Omega \to \mathbb{R}$ by $u_t(\theta, x) = u(t+\theta, x)$, for $\theta \in (-\infty, 0]$, $x \in \Omega$.
In this way, this abstract formulation allows to consider several types of delay terms in a unified way. When $\gamma(t)$ is constant, this type of nonclassical parabolic equations has been very much studied and is often used to model physical phenomena, such as non-Newtonian flows, soil mechanics, heat conduction, etc (see, e.g., [1], [4], [5]). However, any physical model might experiment some kind of natural or artificial changes, therefore it needs consistence under perturbations (see, e.g., [3]). Moreover, in this paper we are interested in the case in which some kind of delay is taken into account in the forcing term. This is an important variant of the nondelay case because there are many situations in which the evolution of the model is determined not only by the present state of the system but by its past history (see, e.g., [2]).

Our aim in this work is the study of the existence and uniqueness of solutions for a non-classical and non-autonomous diffusion equation containing infinite delay terms. We also analyze the asymptotic behaviour of the system in the pullback sense and, under suitable additional conditions, we obtain global exponential decay of the solutions of the evolutionary problem to stationary solutions.

Keywords: nonautonomous diffusion problem, infinite delay, stationary solution, exponential decay, pullback attractor.

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Feedback stabilization fo a predator-prey model by using switched systems

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This paper studies the feedback stabilization of a predator–prey model. Concretely, the model presented by Krajewski and Viaro [**1**] is studied in order to obtain a new method of stabilizing unstable equilibrium points. The method is obtained by using the switched systems, that is, the results in [**2**] for second-order switched systems are applied to this model when two different controllers are considered. Consequently, it is proved the existence of regions in the plane where the feedback stabilization is possible although

In [**1**] Krajewski and Viaro employ feedback stabilization in a predator-prey model in order to choose an equilibrium point and establish the stability of this equilibrium point. Hence, they obtain the following system:

$$\begin{aligned}\dot{x}(t) &= \left(b\,x_e + \frac{c\,y_e}{1+\delta\,x_e} + \frac{h}{x_e}\right)x(t) \\ &\quad -b\,x^2(t) - c\,\frac{x(t)y(t)}{1+\delta\,x(t)} - h \\ \dot{y}(t) &= -\left(\frac{g\,x_e}{1+\delta\,x_e} - f\,y_e\right)y(t) - \\ &\quad f\,y^2(t) + g\,\frac{x(t)y(t)}{1+\delta\,x(t)}\end{aligned}$$

where (x_e, y_e) is the chosen equilibrium point, b, c, δ, g, f are positive constants, and h accounts for constant prey harvesting ($h > 0$) or feeding ($h < 0$). Therefore, depending on h, (x_e, y_e) is unstable or stable equilibrium point.

In this work, for the values of h when the equilibrium point is unstable, we design a strategy to stabilize this equilibrium point. In order to do that, we use switched systems. A switched system is a dynamical system that consists of a finite number of subsystems and a logical rule (called switching law) that orchestrates switching between these subsystems. That is, in the second-order case we can define a switched system as:

$$\begin{pmatrix}\dot{x}(t)\\ \dot{y}(t)\end{pmatrix} = \begin{pmatrix}F_{\sigma(t)}(x(t), y(t))\\ G_{\sigma(t)}(x(t), y(t))\end{pmatrix}$$

where (F_i, G_i), $i = 1,2$, is a vector field of class $\mathscr{C}^1$ in the open and connected set, D, $(x, y) \in \mathbb{R}^2$, and $\sigma : [0,\infty) \longrightarrow \{1,2\}$ is a piecewise constant function called *switching law* indicating the active subsystem at each instant. Hence, for the previous model and for different h_1 and h_2 we have a second-order switched system.

Now, by using the results about convergence of these switched systems in [**2**], it is possible to assure the convergence to (x_e, y_e) although for h_1 and h_2 the common equilibrium point is unstable for each subsystem (F_i, G_i). Hence, we can assure that there exist values of h_1 and h_2 such that for any initial condition in a connected set containing (x_e, y_e) the solution of the previous switched system converge to (x_e, y_e).

Keywords: stabilization, predator-prey model, switched systems.

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Following top-down and bottom-up approaches to discretize non-linear stochastic diffusion equations

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The description of transport processes in soft matter is usually described by non-linear partial differential equation (PDE). Stochastic versions of these PDE appear when Brownian motion, critical phenomena, transition events, and so on, are of interest. The numerical solution of such non-linear stochastic partial differential equations (SPDE) is delicate, because such equations have no mathematical sound basis. Typically, Gaussian noise is too violent in $D > 1$ to allow for a proper definition of a continuum limit.

In the present work we consider a set of *discrete* equations for the evolution of the stochastic fields [**1**]. The discrete equations are formulated from two different starting points, either a microscopic approach based on the theory of coarse-graining (named a bottom-up approach), or from a formal discretization of the continuum equations (named a top-down approach). Both approaches lead to the same discrete set of equations. From the assumptions made in the microscopic derivation we conclude that the continuum limit cannot be taken in the discrete equations. The methodology proposed for the introduction of thermal fluctuations in finite element methods is general and valid for both regular and irregular grids in arbitrary dimensions.

In this work we focus on simulations of the Gaussian and the Ginzburg-Landau free energy functional using both regular and irregular 1D grids. Convergence of numerical results (in 1D) is obtained for observables like the static and the dynamic structure factors as the resolution of the grid is increased.

Keywords: SPDE, SDE, finite element methods.

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Analysis of kinetic and macroscopic models of pursuit-evasion dynamics

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In this work we will analyse kinetic and macroscopic models intended to describe pursuit–evasion dynamics. We investigate well–posedness issues and the connection between the two modeling, based on asymptotic analysis. In particular, in dimension 2, we show that the macroscopic system has some regularizing effects: bounded solutions are produced, even when starting from integrable but possibly unbounded data. Our proof of the latter is based on De Giorgi's method.

Keywords: collective behavior, self–propelling particles, self–organization, kinetic models, hydrodynamic models, regularity of solutions, De Giorgi's method. sindexbehavior, collective, particles, self–propeling

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Boundary equilibrium bifurcations leading to limit cycles in piecewise linear systems

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Within the realm of nonlinear dynamical systems, the class of piecewise-linear differential systems (PWL systems, for short) is very important. First, PWL systems naturally appear in realistic nonlinear engineering models, as certain devices are accurately modeled by piecewise linear vector fields. In fact, this kind of models is frequent in applications from electronic engineering and nonlinear control systems, where piecewise linear models cannot be considered as idealized ones; they are used in mathematical biology as well, where they constitute approximate models.

Second, since non-smooth piecewise linear characteristics can be considered as the uniform limit of smooth nonlinearities, the global dynamics of smooth models has been sometimes approximated by piecewise linear models and viceversa. Note that, in practice, nonlinear characteristics use to have a saturated part, which is difficult to be approximated by polynomial functions. Therefore, this possibility of what we could call 'global linearization' by linear pieces emphasizes even more the importance of PWL systems, which so can be thought as the most natural extensions to linear systems in order to capture nonlinear phenomena.

In fact, it is a widely extended feeling among researchers in the field that the richness of dynamical behavior found in piecewise linear systems covers almost all the instances of dynamics found in general smooth nonlinear systems: limit cycles, homoclinic and heteroclinic orbits, strange attractors...

In this work, boundary equilibrium bifurcations in continuous planar piecewise linear systems with two and three zones are considered, with emphasis on the possible simultaneous appearance of limit cycles. Situations with two limit cycles surrounding the only equilibrium point are detected and rigorously shown for the first time in the family of systems under study.

The theoretical results are applied to the analysis of an electronic Wien bridge oscillator with biased polarization, characterizing the different parameter regions of oscillation.

Keywords: piecewise linear systems, limit cycles, bifurcations.

Acknowledgments

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Pseudo almost periodic solution for Nicholson's blowflies model with patch structure and linear harvesting terms

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In this paper, we study the existence end exponential convergence of positive pseudo almost periodic solutions for a class of Nicholson's blowfies model with patch structure and multiple harvesting terms. We use exponential dichotomy method and fixed point theorem to find existence of pseudo almost periodic solutions.Also under appropriate conditions, we establish some criteria to ensure that the solutions of this system converge locally exponentially to a positive almost periodic solution.

Keywords: pseudo almost periodic solution, Nicholson's blowfies model, exponential convergence.

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Part 2

BIOLOGY, FLUIDS, COSMOLOGY AND THE ENVIRONMENT

Arctic circulation from a Lagrangian perspective

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In the context of rapid increase of temperature in the polar regions, inducing a dramatic ice melting, Lagrangian transport in the Arctic Ocean becomes an area of key interest. There exists evidence of changes in the circulation of water masses at different depths such as a notable decrease in intensity of the so-called Beaufort Gyre and the discharge of a great amount of fresh water into the North-West Atlantic Ocean. These facts may have a direct impact on the global circulation of water and heat.

The aim of this talk is to show the phase portrait of the sea currents throughout the Arctic Ocean, which allows us to study aspects of their dynamics. This is done by means of the numerical method known as Lagrangian descriptors [4], which computes and graphically displays flow structures over a given domain of the ocean, highlighting coherent jet circulation patterns in red color.

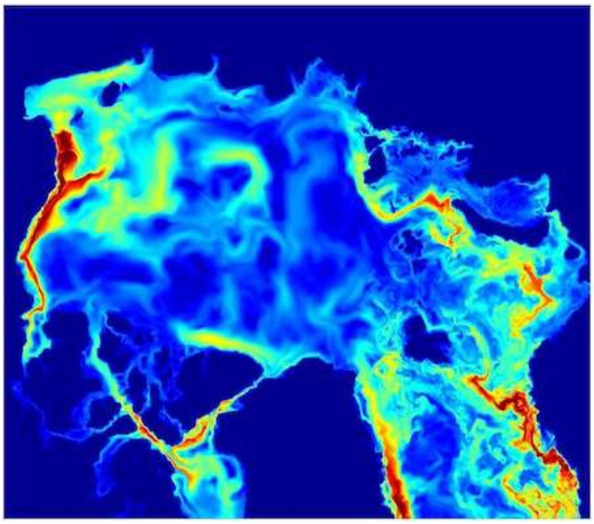

FIGURE 1. Lagrangian descriptor values over a grid of points located in the Arctic region and at a depth of 30 meters. This image corresponds to the 11th March 2013.

This method was originally based on the computation of the Euclidean arc length of trajectories of a dynamical system. In this work the considered dynamical system is a velocity field given by data sets collected through observations. Other quantities integrated along trajectories, such as the p-norm of the velocity field, also highlight remarkable features of the phase space such as distinguished trajectories and their stable and unstable manifolds [3].

The images provide us both a validation of already known and well reported flow structures [2], and also the discovery of singular features in the ocean which correspond to extreme weather events [1]. These observations quantify how much global climate warming has made impact to the general circulation of the Arctic Ocean.

Keywords: Lagrangian transport, phase space, Lagrangian descriptors.

Acknowledgments

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Protracted metronomic therapies to target low-grade glioma malignant transformation

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Grade II gliomas are slowly growing primary brain tumours that mostly affect young patients and become fatal only several years after diagnosis. Current clinical management includes surgery as first line treatment. Cytotoxic therapies such as radiotherapy or chemotherapy are used initially only for patients with bad prognosis. Therapies such as radiotherapy are administrated following the maximum dose in minimum time principle(Maximum Tolerated Dose, or MTD paradigm). This is basically the same schedule as for high grade brain tumours in spite of their growth being much faster. A series of previous studies has developed a model describing the basic features of grade II glioma progression and response to radiotherapy [**1**]. The model includes the time dynamics of two cellular compartments (active tumour cells and lethally damaged tumour cells). The fraction of tumour cells damaged by a radiation dose is estimated by the linear-quadratic model. The model describes most of the well-known clinical facts of grade II glioma response to radiotherapy [**2**]. Then maintaining one day (Monday to Friday) distance between doses, that if the toxicity is to be preserved, the most effective dose fractionation is that of the standard scheme [**3**]. However, the model predicts that there is a much more effective (protracted) fractionation scheme in which the doses of 1.8 Gy are spaced by a distance that can be estimated to be a fraction of the tumour potential doubling time and radiosensitivity, typically of the order of 1-2 months, the potential survival gain being of the order of years [**4**]. In this work, we have studied the optimal strategy when both the dose per fraction and the time spacing between fractions are left free under the restriction, same toxicity (biological effect on the healthy tissue) as the standard fractionation. Typically, the best scheme is a combination of protraction with metronomic therapies, lowering the dose per fractions to levels below 1 Gy [**5**]. Thus, the optimal scheme departs form what one would expect thinking in a “linear” way and is the result of the interplay of the nonlinearities and restrictions of the system.

Keywords: glioma, malignant transformation, radiotherapy, fractionation, protracted and metronomic therapy.

Acknowledgments

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Mathematical Modeling of the Emergence of Drug Resistance via Nonlinear and Nonlocal Exchange

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Multidrug resistance (MDR) in cancer, the prime cause of therapy failure [**1**], is strongly influenced by intratumour heterogeneity and by alterations induced on the microenvironment. Intratumour heterogeneity arises, to a large extent, from genetic mutations driving, via a Darwinian evolutionary analogue, the selection of tumour cells expressing phenotypes adapted to the tumour microenvironment. It has also been recognized that intratumour heterogeneity may also develop from extragenetic processes mediated by stochastic events or epigenetic mutations. An important exogenous source of alterations on the microenvironment originates from the action of chemotherapeutic agents targeting the malignant cells.

Mathematical models [**2**] may assist not only in identifying the relevant mechanisms that drive the emergence of drug resistance in specific tumour types but also in providing new perspectives on the risks of interventional chemotherapy on cancer patients by elucidating how the adaptation of extragenetically unstable cell populations exposed to antiproliferative drugs can be acted upon by selective forces, which eventually impel the outgrowth of tumour cell populations having both high proliferation and persistent drug resistance.

We have put forward a continuous nonlinear and nonlocal mathematical model built upon the kinetic theory of active particles [**3**]. The model considers the distribution functions $u_i(x,t):[x_{\min},x_{\max}]\times[0,T]\to\mathbb{R}^+$ depending on the level x of a cell membrane drug transporter for each $i=1,2$ cell subpopulation (i.e., $i=1$ sensitive and $i=2$ resistant phenotypes). The resulting system of hyperbolic integro-differential equations for the (x,t)-evolution of $u_i(x,t)$ incorporates the change in the x level by means of a resistance transfer mechanism (through an advection-like term), net proliferation (through a nonlocal kernel) and the effect of administration of chemotherapeutic agents. The model is solved via numerical simulations and describes the experimental results in various cancer cell lines (non-small cell lung carcinoma and glioma) where MDR is observed.

Keywords: biomathematics, hyperbolic integro-differential equations, mathematical models of multidrug resistance in cancer.

Acknowledgments

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Computation of capillary-gravity generalized solitary waves

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The goal of the present talk is two-fold. First, an efficient algorithm to compute capillary-gravity solitary waves of the irrotational incompressible Euler equations with free surface is introduced. Our research, [**2**], forms part of the study of generation and dynamics of capillary-gravity waves on the surface of incompressible fluids, a topic of relevant and permanent interest, [**3**]. Our starting mathematical model for a potential flow induced by a solitary wave involves an inviscid, homogeneous fluid in a horizontal channel of constant depth; the channel is modeled above by an impermeable free surface, where the pressure is equal to the surface tension, and bounded below by a fixed, impermeable horizontal seabed. The model assumes not negligible gravity and capillary forces. In Cartesian coordinates moving with the wave this is sketched in Figure 1, with x as the horizontal coordinate and y the upward vertical one, d stands for the depth of the channel, the bottom is at $y = -d$ and $y = \eta(x)$ is the free surface elevation from the mean water level at $y = 0$.

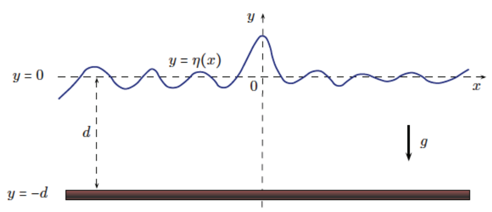

FIGURE 1. Sketch of the physical domain.

Conformal mapping techniques allow to transform the fluid domain and to reformulate the problem of steady capillary-gravity solitary waves in terms of a Babenko-type equation, [**1**]. The equation is discretized with a Fourier-type pseudospectral method and the resulting discrete system is solved with the Levenberg-Marquardt method, [**4, 5**]. The performance of the procedure is checked with several examples and allows to study the internal flow structure under a solitary wave. The success of the method in computing classical and generalized solitary waves (solitary waves with undamped oscillatory wings) will be illustrated. Of especial relevance is the generation of multi-pulse generalized solitary waves of elevation and depression which, to our knowledge, have never been computed in the context of capillary-gravity surface waves. The numerical experiments suggest the existence of an infinite number of generalized solitary waves for equal Froude and Bond numbers.

Keywords: solitary surface waves, capillary–gravity waves, Euler equations, generalized solitary waves.

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On stochastic second order Korteweg - de Vries type equations

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Nonlinear wave equations with soliton solutions play an important role in contemporary physics and mathematics. The simplest equation of that type is famous Korteweg-de Vries (KdV) equation. The KdV equation was originally derived for shallow water problem, that is, for gravitational waves on the surface of a shallow water. The set of Euler equations for irrotational motion of the fluid with proper boundary conditions is difficult to solve. However, in a limit of long small waves and shallow water it is possible to simplify Euler equations and obtain approximate nonlinear wave equations. The equation obtained in the first order approximation was obtained by Korteweg and de Vries in 1985 [**1**] and became a prototype of nonlinear wave equations. Recently more advanced second order approximation named *extended KdV* [**3**] or KdV2 [**4, 4**] attracted growing interest. In [**4**] we extended the derivation of the KdV2 equation for the case when the bottom of the fluid is not flat. In [**4**] we demonstrated in numerical simulations for stochastic KdV2 equations that both soliton and cnoidal solutions are very robust with respect to stochastic forces.

In the presentation, which is based on the paper [**5**], we give sufficient conditions for the existence and uniqueness of mild solution of a stochastic KdV2-type equation. The proof uses approach and estimates from [**6, 7, 8**]. Moreover, we supply conditions for exponential stability and stability in probability of the mild solution mentioned above. For obtaining stability results we used approach developed in [**9**].

Keywords: shallow water problem, KdV-type equation, stochastic KdV-type equation, mild solution, exponential stability, stability in probability.

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Control of bursting synchronization in Central Pattern Generators

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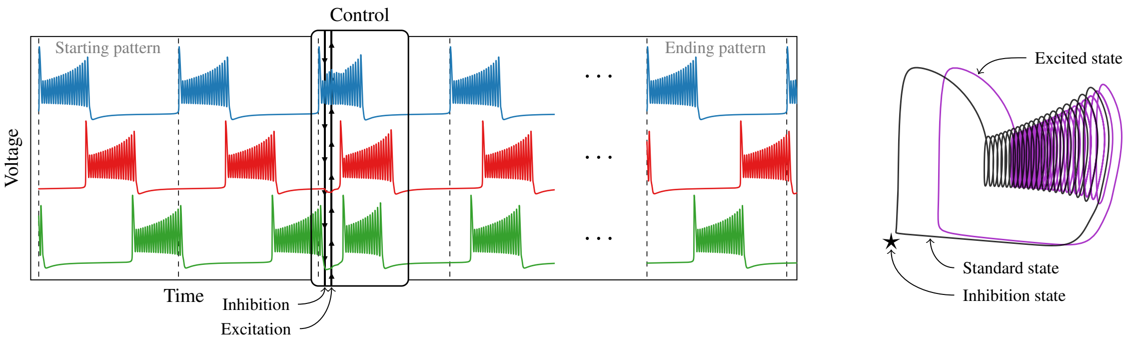

FIGURE 1. Control technique

The study of the synchronization patterns of small neuron networks (CPGs) that control several biological processes is an interesting growing discipline [**1, 4**]. Some synchronization patterns within CPGs are related to undesirable neurological diseases, and they are believed to play a crucial role in the emergence of pathological rhythmic brain activity in different diseases.

We show how, with a suitable combination of short and weak global stimuli (Figure 1-left), we can switch between different stable bursting patterns in CPGs [**3**]. We develop a systematic study based on the effect of pulses on a single neuron [**1, 2**], and select an inhibitory pulse and a excitatory one (Figure 1-right). Then we study the different effects of such pulses, varying lag between them, on the stability patterns the CPG eventually reaches. Moreover, we compare the technique on a completely symmetric CPG and on a slightly perturbed one (a much more realistic situation). The present approach shows that we can switch between stable patterns using global stimuli.

Keywords: control, CPG, synchronization pattern.

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Simulating Supermassive Black Holes in Coherent Nonlinear Systems

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We show how the interaction of super massive black holes (SMBH) and scalar field dark matter (ψDM) can be mimicked in systems of coherent waves. Our discussion is based on two basic assumptions, recently proposed: in first place that ψDM is mostly composed by ultralight axions forming a cosmic Bose-Einstein Condensate (BEC)[**4**] and secondly, that it is possible to induce “artificial gravity" in a coherent wave system, like those using lasers or matter waves. If both premises are considered, our numerical simulations indicate that it is possible to simulate nontrivial phenomena concerning SMBHs and ψDM in the framework of current experiments in photonics or ultracold atomic gases.

Our work is based on the fact that the mathematical descriptions are identical in different types of coherent nonlinear systems, the time evolution being described by the well-known Schrödinger-Poisson equation[**6**] which, in its adimensional form, reads:

$$i\frac{\partial\psi}{\partial t}+\nabla^2\psi+(V+\Phi)\psi=0$$

where $\nabla^2\Phi = |\psi|^2$. The wave-function ψ can correspond to the distribution of dark matter[**2**], laser amplitude[**5**] or the order parameter in BEC systems[**5**], respectively. The external potential $V(\mathbf{x})$ is any spatial distribution that can affect the dynamics of the nonlinear wave. In astrophysics, some of the most important objects to be considered are SMBHs, which can be modeled approximately by a gaussian function with $1/e$ width of the range of the Schwarzschild radius[**6**]. In photonic experiments, Φ is mimicked by using a nonlinear glass with thermo-optic effect. In a dipolar BEC, “artificial gravity" can be induced with an adequate laser setup and the external potential can be achieved by means of a tightly focused light beam[**7**].

The previous argument can be therefore used to propose experiments in photonics and cold atoms to simulate interactions of SMBHs and dark matter that can pave the way to a better understanding of fundamental questions like the or so-called M-sigma relation which connects the mass of a SMBH with the surrounding velocity dispersion.

Keywords: solitons, BEC, dark matter, nonlinear optics.

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Brain tumors: Textural heterogeneity as predictor of survival in Glioblastoma

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Glioblastoma (GBM) is the most frequent malignant brain tumor in adults and the most lethal type, with a median survival of 14.6 months. Preoperative magnetic resonance imaging is routinely used for diagnosis, therapy planning and follow-up. The advantage of imaging techniques is their non-invasive nature and the fact that the whole tumor is taken into account, whereas cellular diagnosis techniques (based on information from biopsies) are invasive and limited to a discrete set of tumor cells.

One of the most important characteristics of GBM is its marked intratumoral heterogeneity, which confers an evolutionary advantage in the face of fluctuations imposed by chemotherapy or radiotherapy. Due to this fact, the study of tumor image heterogeneity has attracted the interest of the scientific community [**1**].

Our purpose in this work is to measure the spatial complexity (heterogeneity) of specific types of MRIs (postcontrast T1 MRIs) in a data set of 79 GBM patients from 3 Spanish hospitals to check its possible influence on survival. We have computed the run-length matrix (RLM) and the co-ocurrence matrix (CM) derived measures in 3D providing regional and local heterogeneity information in our patient dataset [**2**]. We have analyzed their robustness [**3**] and a new familiy of heterogeneity measures computed on MRIs: the spatial p-energy and the global p-energy.

Kaplan-Meyer survival analysis showed that 4 of the 11 RLM features and 4 of the 5 CM features considered were robust predictors of survival. The median survival differences in the most significant cases were of over 6 months.

The main conclusion is that tumor complexity measures obtained from standard MRI images provide relevant information on survival.

Keywords: glioblastoma, texture, heterogeneity, robustness.

Acknowledgments

This research has been supported by Ministerio de Economí○a y Competitividad/FEDER, Spain [grant MTM2015-71200-R], Junta de Comunidades de Castilla-La Mancha, Spain [grant PEII-2014-031-P] and James S. Mc. Donnell Foundation (USA) 21st Century Science Initiative in Mathematical and Complex Systems Approaches for Brain Cancer (Special Initiative CollaborativeĂŞ Planning Grant 220020420 and Collaborative award 220020450).

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Dynamical mechanism of antifreeze proteins to prevent ice growth

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The fascinating ability of algae, insects and fishes to survive at temperatures below normal freezing is realized by antifreeze proteins (AFPs). Antifreeze proteins (AFPs) are surface-active molecules and interact with the diffusive water/ice interface preventing a complete solidification.

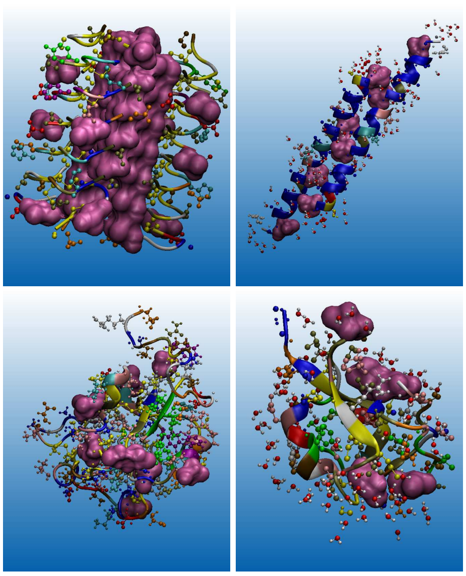

FIGURE 1. Four different classes of AFP structures from psodeupleuronectes americanus (1WFB), hemitripterus americanus (2AFP), macrozoarces americanus (1MSI) and tenebrio molitor (1EZG). Crystallographic data are from the RCSB Protein Data Bank.

A new dynamical mechanism is proposed how these proteins inhibit the freezing of water. We apply a Ginzburg-Landau type approach to describe the phase separation in the two-component system (ice, AFP). The free energy density involves two fields: one for the ice phase with low AFP concentration, and one for the liquid water with high AFP concentration. The time evolution of the ice reveals microstructures as a result of phase separation in the presence of AFPs. We observe a faster clustering of pre-ice structure connected with a locking of grain size by the action of AFP which is an essentially dynamical process. The adsorption of additional water molecules are inhibited and the further growth of ice grains are stopped. The interfacial energy between ice and water is lowered by which the AFPs allow only smaller critical ice nucleus to be formed. Analogously to hysteresis in magnetic materials we observe a thermodynamic hysteresis leading to a nonlinear density dependence of the freezing point depression [**1**] in agreement with the experiments [**2**].

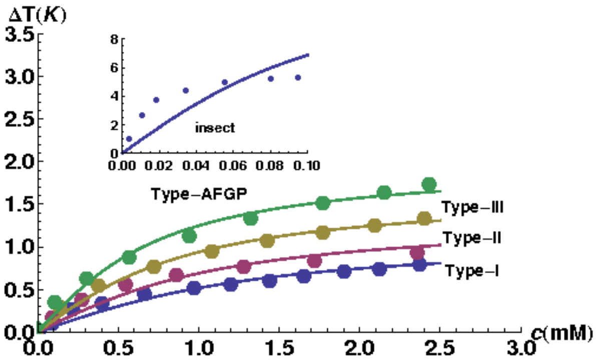

FIGURE 2. The freezing temperature depression of four different classes of AFP structures and insects with the fits compared to the experimental data (points) of [**2**].

Keywords: freezing suppression, antifreeze protein.

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Formation of brine channels in sea-ice as habitat for micro-algae

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Brine entrapment between growing ice platelets in sea ice is an important habitat for a variety of CO_2 - binding microalgaea and therefore crucial in polar ecosystems. We microscopically describe the structure formation of ice platelets and develop a phase field model for pattern formation during solidification of the two-dimensional interstitial liquid by two coupled order parameters, the tetrahedricity as structure of ice and the salinity. These parameters describing the velocity of the freezing process and the velocity of structure formation, determine the phase diagram, the super-cooling and super-heating region, and the specific heat respectively.

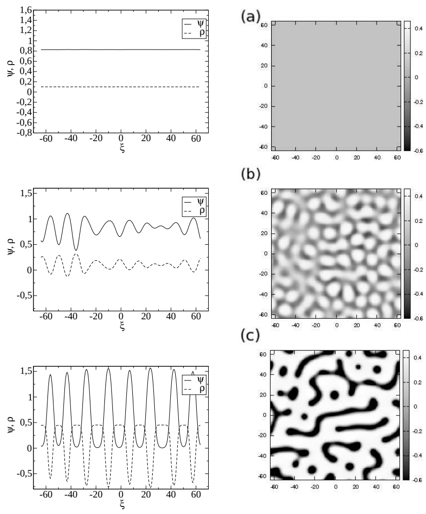

FIGURE 1. Time evolution of the order parameter ψ and salinity ρ as deviation from mean salinity for 1D (left) and the salinity for 2D (right) versus spatial coordinates for $\tau = 10, 150, 500$ (from above to below) with the initial random distribution $\psi(\tau = 0) = 0.9$ and $\rho(\tau = 0) = 0.1 \pm 0.001 N(0,1)$. The parameters are $\alpha_3 = 0.9$, $\alpha_1 = 0.1$, and $D = 0.5$.

We use the model to calculate the short-time frozen microstructures and compare the morphological structure with the vertical brine pore space obtained from Xray computed tomography [**4**].

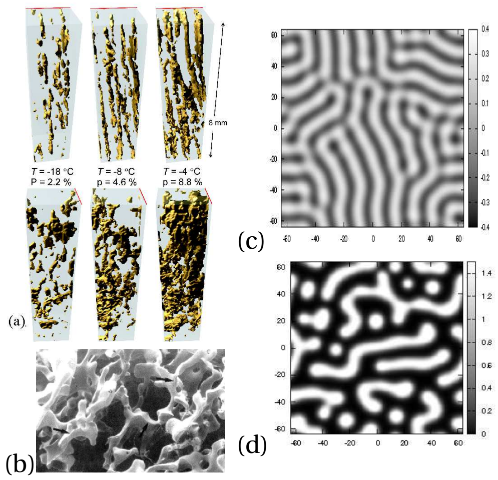

FIGURE 2. (a) Imaging brine pore space with X-ray computed tomography (image from [**1**]). The upper images shows the view approximately along the brine layers. The view across the brine layers is shown in the bottom images. (b) Scanning electron microscopy image of a cast of brine channels [**2**], (c) Turing structure after long time [**3**], (c) long-time phase field structure from figure 1.

Keywords: brine channel distribution, sea-ice, freezing point suppression, phase field, pattern formation.

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Nonlinear Dark Matter Waves

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We present here a plausible explanation to the first evidence of dark matter (DM) non-gravitational self-interaction that has been reported for the Abell 3827 cluster ($z \approx 0.1$), where a displacement of the luminous mass with respect to the maximum density of its DM halo has been observed, for some of the merging galaxies [**1**]. In our model, we consider that most of DM consists of Bose-Einstein condensed ultralight axions [**2**] that can form robust coherent solitons [**3**], which display interference properties that may be the cause of this kind of offsets [**4**].

This destructive interference behavior between robust wave lumps is well known in nonlinear waves and soliton systems [**5**], where the mathematical description of the phenomena is similar to the theory of coherent DM waves, as time evolution of non-relativistic coherent dark matter which only self-interacts through Newtonian gravity is governed by the well-known Schrödinger-Newton equation [**6**]:

$$i\hbar\partial_t\psi(t,\mathbf{x}) = -\frac{\hbar^2}{2m_a}\nabla^2\psi(t,\mathbf{x}) + \\ -G\, m_a^2\psi(t,\mathbf{x})\int\frac{|\psi(t,\mathbf{x}')|^2}{|\mathbf{x}'-\mathbf{x}|}d^3\mathbf{x}', \tag{10}$$

being ψ the wave-function of the dark matter distribution, t and $\mathbf{x}$ the time and position, G the gravitational constant, and m_a the mass of the ultralight axion.

Our numerical simulations show that it is plausible that interference between dark waves can have observational consequences for galactic mergers and, in particular, it can explain the significant results of [**1**]. Even if present data do not allow for a detailed modeling, we have calculated simplified situations, showing that qualitative features can be reproduced in a rather robust way and that they are reminiscent of soliton repulsion in nonlinear optics and atomic matter waves. In fact, refined control of trapped atoms and optical media, including the introduction of gravity-like interactions, might allow for laboratory analogue simulators of galactic-scale phenomena.

Keywords: solitons, dark matter, ultralight axions.

Acknowledgments

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On the role of Oscillations and Phases in Neural Communication

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Background oscillations, reflecting the excitability of neurons, are ubiquitous in the brain. Some studies have conjectured that when spikes sent by one population reach the other population in the peaks of excitability, then information transmission between two oscillating neuronal groups is more effected.

In this context, phase relationship between oscillating neuronal populations may have implications in neuronal communication between brain areas. To study this relationship, we will consider a population rate model and perturb it with a time-dependent input, and analyse the states achieved depending on the frequency and amplitude of the perturbation.

To perform this study, we consider the stroboscopic map, and perform a bifurcation analysis as a function of perturbation parameters (amplitude and frequency). We observe the existence of bistable solutions for some regions of the parameter space, suggesting that, for a given input, populations may operate in different regimes.

Keywords: oscillations, phase-locking, bistability.

Acknowledgments

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Reduced Basis method for a bifurcation in a Rayleigh-Bénard convection problem at low aspect ratio

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The reduced basis approximation is a discretization method that can be implemented for solving of parameter-dependent problems $\mathscr{P}(\phi(\mu),\mu) = 0$ with parameter μ in cases of many queries. This method consists of approximating the solution $\phi(\mu)$ of $\mathscr{P}(\phi(\mu),\mu) = 0$ by a linear combination of *appropriate* preliminary computed solutions $\phi(\mu_i)$ with $i = 1, 2, ..., N$ such that μ_i are parameters chosen by an iterative procedure using the *kolmogorov n-width* measures [**2, 4**].

In [**1**], the reduced basis method is applied to a two dimensional incompressible Navier-Stokes equations with constant viscosity and the Boussinesq approximation coupled with a heat equation that depends on the Rayleigh number, $\mathscr{P}\left(\phi(R), R\right) = \vec{0}$. The classical approximation scheme used here to solve the stationary problem with the corresponding boundary conditions for different values of the Rayleigh number R is a Legendre spectral collocation method.

Rayleigh-Bénard convection problem displays multiple steady solutions and bifurcations by varying the Rayleigh number, therefore the eigenvalue problem of the corresponding linear stability analysis has to be implemented. A linear stability analysis of these solutions is performed in [**3**] by Legendre spectral collocation method.

In this work the eigenvalue problem of the corresponding linear stability analysis is solved with the reduced basis method. It is considered the aspect ration $\Gamma = 3.495$ and R varies in $[1,000; 3,000]$ where different stable and unstable bifurcartion branches appear [**1, 3**]. We apply the reduced basis method within this framework to compute whole bifurcation diagram with the stable and unstable solutions corresponding to many values of R in an interval of the considered bifurcation parameter. Also, nine branches of solutions stable and unstable are obtained with this method and different basis sets are considered in each branch. The reduced basis method permits to obtain the bifurcation diagrams with much lower computational cost.

The problem is numerically solved by the Galerkin variational formulation using the Legendre Gauss-Lobatto quadrature formulas together with the reduced basis $\{\phi(R_i), i = 1, 2, ..., N\}$ such that $\phi(R) \sim \sum_{i=1}^{N} \lambda_i \phi(R_i)$.

Keywords: reduced basis approximation, bifurcations, Rayleigh-Bénard, Kolmogorov width, flow problem, model reduction.

Acknowledgments

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Statistical physics of active ionic channels

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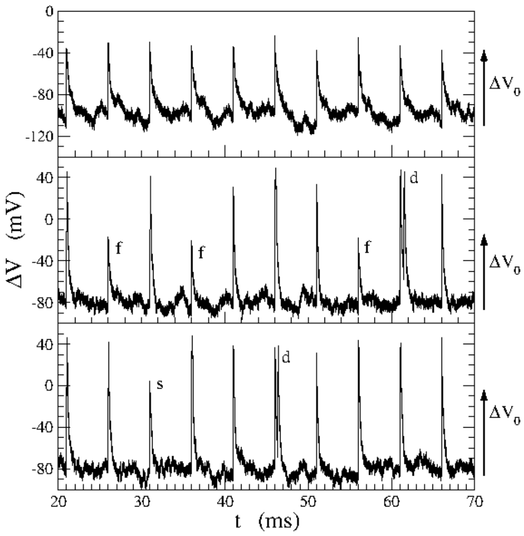
Experiments on single channels have contributed to a large extent to our current view on the function of membrane ionic channels. Such experiments evidence the presence of strong fluctuations both in the open-close gating dynamics and in the measured ionic fluxes. Here we study a semi-microscopic approach for the stochastic dynamics of voltage gated channels, in which the interaction between ions and the mechanical structure of the channel is represented by the dynamics of a minimum set of degrees of freedom, specifically ion positions and channel gate states. The dynamics is obtained by applying very general principles of statistical physics, which leads to the formulation of Langevin equations for these variables. Parameters of the model are obtained from biological experimental data on energies and time scales. The voltage sensitivity of the gate is due to the interaction of the "gating charge" located at the gate and the potential membrane, which controls also the balance between the open and close steady states.

Numerical simulations of the Langevin equations for ions and channel gates are implemented and the results are compared with experimental available data and analytical studies. We show that this approach explains qualitatively experimental results for the dynamics of Na and K channels, and also provides new results. In the figure we show that the excitable properties of the membrane are present in a single voltage-gated Na channel in the presence of K leak [**1**]. As a reference we see at the top of the figure the membrane potential when 10 small periodic depolarizing perturbations of order +70 mV are applied on the membrane with K channels only. In the middle and bottom parts of the figure we see the response of the membrane with both Na and K channels when pulses of +70 mV (middle) and +80 mV (bottom) are applied. The intensity of the response is twice larger in the true excitable events.

In addition, and as a consequence of the mechanical consistency of the formulation, it is predicted that ionic concentration should have an observable effect on the gating characteristics of the channels. Namely concentration favors the open probability in activating variables, and the opposite effect in inactivating variables.[**2**] This new prediction calls for further experimental research.



Keywords: stochastic modeling of ion channels, excitable channels, action potential dynamics.

Acknowledgments

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Takens-Bogdanov bifurcations and resonances of periodic orbits in the Lorenz system

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We study Takens–Bogdanov bifurcations of equilibria and periodic orbits in the classical Lorenz system, allowing the parameters to take any real value. First, by computing the corresponding normal form we determine where the Takens–Bogdanov bifurcation of equilibria is non-degenerate, namely of homoclinic or of heteroclinic type. The transition between these two types occurs by means of a triple-zero singularity. Moreover, we demonstrate that a degenerate homoclinic-type Takens–Bogdanov bifurcation of infinite codimension occurs. Secondly, taking advantage of the above analytical results, we carry out a numerical study of the Lorenz system. In this way, we find several kinds of degenerate homoclinic and heteroclinic connections as well as Takens–Bogdanov bifurcations of periodic orbits. The existence of these codimension-two degeneracies, that organize the symmetry-breaking, period-doubling, saddle-node and torus bifurcations undergone by the corresponding periodic orbits, guarantees in some cases the presence of Shilnikov chaos. We also show the existence of a codimension-three homoclinic connection that together with the triple-zero degeneracy act as main organizing centers in the parameter space of the Lorenz system.

We remark that the heteroclinic case of the Takens–Bogdanov bifurcation in the Lorenz system was found in the literature, in a region with negative parameters, in the study of a thermosolutal convection model and in the analysis of traveling-wave solutions of the Maxwell-Bloch equations. In this zone of the parameter space, we perform a detailed numerical study of the resonances of periodic orbits. The combination of numerical continuation methods and Poincaré sections of the flow provides important information of how the resonances appear and evolve giving rise to a very rich dynamical and bifurcation scenario.

Keywords: Lorenz system, Takens-Bogdanov bifurcation, resonances.

Acknowledgments

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Adiabatic invariants of second order Korteweg - de Vries type equation

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It is a well known fact that the Korteweg – de Vries equation (KdV) posesses an infinite number of invariants [**1**]. The lowest three invariants are related to conservation laws of mass (volume), momentum and energy of the fluid. However, the relation of KdV invariants to energy is a delicate matter because, as pointed in [**2**], energy is expressed by KdV invariants only when the system is considered in the reference frame moving with a natural velocity equal $\sqrt{gh}$, where h is a (constant) water depth.

When the Euler equations for shallow water are taken to the next order, beyond KdV, the only exact invariant which is left is the mass (volume). The KdV2 equation, second order in expansion parameters, was first derived by Marchant and Smyth [**3**] and named *extended KdV*. The KdV2 equation has the following form

$$\begin{aligned} &\eta_t+\eta_x+\frac{3}{2}\alpha\,\eta\eta_x+\frac{1}{6}\beta\,\eta_{3x}-\frac{3}{8}\alpha^2\eta^2\eta_x \\ &+\alpha\beta\left(\frac{23}{24}\eta_x\eta_{2x}+\frac{5}{12}\eta\eta_{3x}\right)+\frac{19}{360}\beta^2\eta_{5x} \;=\; 0, \end{aligned} \tag{11}$$

where small parameters α and β are ratios of the wave amplitude a, constant water depth d and average wavelangth l

$$\alpha=\frac{a}{h}, \qquad \beta=\left(\frac{h}{l}\right)^2. \tag{12}$$

In (11), $\eta(x,t)$ stands for a wave profile and low indexes denote partial derivatives (e.g. $\eta_{3x}=\frac{\partial^3\eta}{\partial x^3}$ and so on). Neglecting terms second order in α,β in (11) one arrives to KdV equation. The lowest invariants of KdV equation are:

$$I_1=\int_{-\infty}^{\infty}\eta\,dx, \qquad I_2=\int_{-\infty}^{\infty}\eta^2\,dx \tag{13}$$

$$\text{and} \qquad I_3=\int_{-\infty}^{\infty}\left(\eta^3-\frac{\beta}{3\alpha}\eta_x^2\right)dx. \tag{14}$$

In [**5**] we found two approximate invariants of equation (11) playing the same role like I_2 (13) for KdV and one approximate invariant corresponding to I_3 (14). They are approximate (adiabatic) in this sense that terms violating the constant values are at least one order higher in small paremeters than the constant ones.

Our method of construction of adiabatic invariants is so general that in principle allows to construct infinitely many of them, not only the lowest ones.

Keywords: shallow water waves, nonlinear equations, invariants of KdV2 equation, adiabatic invariants.

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Investigating Hilbert frequency dynamics and synchronisation in climate data

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A recent study demonstrated that, in a class of networks of oscillators, the optimal network reconstruction from dynamics is obtained when the similarity analysis is performed over time series obtained by Hilbert transform.[**1**] In spite of the fact that this transform has been widely used to analyse output signals of many complex systems, it has not yet been employed to construct climate networks. For these reasons, in this work we analyse large climate datasets of SAT (Surface Air Temperature) using Hilbert transform to compute frequency time series, with the goal of inferring new information about underlying climate interactions and dynamics – for example, signatures of frequency synchronisation.

We work on daily SAT time series, from year 1979 to 2015, in 16380 grid points over the Earth surface.[**2**] From each SAT time series we calculate the anomaly time series and also, by using the Hilbert transform, we calculate the frequency time series. By plotting the map of the average frequency in every grid point, we extract relevant information about SAT dynamics in different regions of the world.

Then, we calculate autocorrelations of frequency and anomaly series. With these results we plot autocorrelation maps, that allow to uncover geographical regions with different memory properties. In a second step, to find correlations between sites, we compute the zero-lag cross correlations (CC). According to statistical considerations, we put a threshold on the CC matrices (both from SAT anomalies and from Hilbert frequencies) and we build two undirected networks from the two derived series. Then, we analyse network topology, finding which nodes are most connected and exploring their long-range connections. A comparison between the two networks shows which new information can give us the approach based on frequency analysis.

Ultimately, our results suggest that, in fact, Hilbert transform and frequency series are a valid tool to build a climate network and give additional information about correlations, teleconnections and patterns of similar dynamical behaviour. As an example of our results, we report in Figure 1 the map of average frequency. It can be seen that most of the extra-tropics have an average frequency which corresponds very well to the expected value, i.e. the inverse of the period of the annual solar cycle (12 time steps, giving an angular frequency of $2\pi/12 \approx 0.52$ rad/month). Additionally, there are key tropical areas that diverge from this value: a wide zone in the Pacific Ocean with characteristic dynamics due to El Niño and another one in the Indian Ocean with different dynamics due to monsoons.

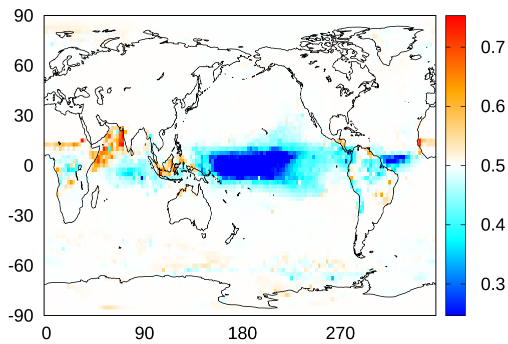

FIGURE 1. Map of time-average Hilbert frequency.

Keywords: climate network, synchronisation, Hilbert transform.

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Part 3

OPTICS AND BOSE-EINSTEIN CONDENSATES

Nonlinear Raman scattering techniques

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Conventional Raman spectroscopy (spontaneous Raman scattering) is a linear optical technique where one photon creates an optical phonon and a scattered photon of lower or higher frequency/energy (that of the incoming photon plus or minus the phonon energy) is emitted. In a solid, this is done through virtual electronic states. After the scattering, the electronic states return to the fundamental or ground state. Instead of the typical cross section, a Raman scattering event is measured through the scattering efficiency (cross section per unit volume), given by

$$\frac{dS}{d\Omega} = \frac{\omega_s^3 n_s^3 n_l}{h^2 c^4 \omega_l} \sum_{I,F} |W_{IF}|^2 (N_0 + 1) \tag{15}$$

where I and F are the initial and final states of the process, ω_l (ω_s) is the laser (scattered) frequency, n_l (n_s) the refractive index, W_{IF} the transition probability and N_0 the phonon population. On the other hand,

$$W_{IF} = \sum_{\mu,\nu} \frac{\langle I|H_{eR}|\mu\rangle\langle\mu|H_{ep}|\nu\rangle\langle\nu|H_{eR}|F\rangle}{(E_\mu - \hbar\omega_l)(E_\nu - \hbar\omega_s)} \tag{16}$$

where the sum is over the electronic intermediate states μ and ν, and H_{eR} and H_{ep} are the electron-radiation and electron-phonon interaction Hamiltonians. This expression comes from the Fermi Golden rule in second order perturbation theory. Equation (1) corresponds to the Stokes shift (phonon emission), the expression for the anti-Stokes shift has a factor of N_0 instead and it is only important at high enough temperatures.

There are also non linear Raman spectroscopy techniques where two or more photons are involved. One example is a technique called hyper-Raman, where two photons promotes an electron from the valence to the conduction band (used in dipole-forbidden transitions for instance) and, after the electron-phonon scattering process, a scattered photon is emitted. We will discuss in this talk the quantum mechanical expressions and Feynman diagrams, not only of hyper-Raman scattering, but also on other non-linear techniques like Coherent Anti-stokes Raman Spectroscopy. The advantages and limitations of the techniques will be explained and some examples will be given.

Keywords: nonlinear Raman spectroscopy, virtual electronic states, Feynman diagrams.

Vortex Rings in Bose-Einstein Condensates

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We review recent results for the emergence, existence, dynamics and interactions of vortex rings in Bose-Einstein condensates (BECs). Under appropriate conditions, a BEC can be accurately described by the Gross-Pitaevskii equation (GPE) which is a variant of the celebrated defocusing nonlinear Schrödinger equation incorporating external potentials. Using the GPE model, we focus our attention on the two opposite regimes of low and high atomic density limits in the BEC as well as in the intermediate transition between these two limits.

In the low density limit, corresponding to the linear limit, we study the emergence of single and multiple vortex rings emanating from planar 3D dark solitons through bifurcations. It is found that these bifurcations crucially depend on the aspect ratio (anisotropy) of the confining trapping potential. We characterize such bifurcations quantitatively using a Galerkin-type approach, and find good qualitative and quantitative agreement with our Bogoliubov-de Gennes (BdG) numerical analysis. Under appropriate conditions for the trapping strengths, we find that vortex rings might be stabilized for large enough atomic densities (large chemical potentials).

On the other hand, in the large density limit (the so-called Thomas-Fermi limit), the vortex rings acquire stability and behave like robust coherent structures. We study different single and multi-vortex-ring configurations together with their (normal) modes of vibration. Exotic structures such as Hopfions, the one-component counterpart to Skyrmions, are also constructed and tested for stability. Finally, we discuss some interactions dynamics between vortex rings such as periodic leapfrogging of co-axial vortex rings and the scattering behavior for co-planar collisions between vortex rings.

Keywords: Bose-Einstein condensates, Gross-Pitaevskii equation, nonlinear Schrödinger equation, vortex rings.

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Solitary waves in the NonLinear Dirac Equation

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In the last decades, the Nonlinear Schrödinger Equation has been the most ubiquitous model for nonlinear waves in optics, atomic physics, fluid mechanics, condensed matter and mathematical physics. However, its relativistic analogue, the Nonlinear Dirac Equation, has been forgotten despite its appearance almost 80 years ago in the context of high-energy physics.

This trend is starting to change because the Nonlinear Dirac Equation is emerging in physical systems of considerable interest, which includes Bose-Einstein condensates in honeycomb lattices, atomically thin 2d Dirac materials such as graphene, silicene or germanene, or honeycomb photorefractive lattices (the so-called photonic graphene).

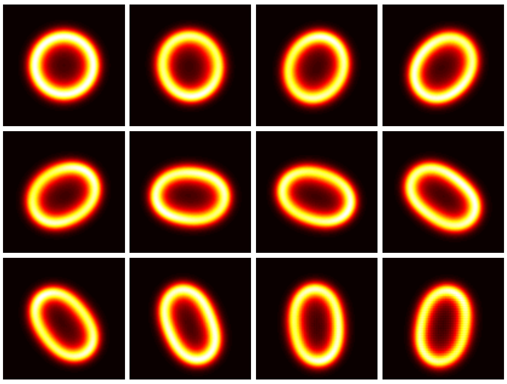

FIGURE 1. Evolution of an unstable soliton in (2+1)D settings.

One of the main interesting features of the Nonlinear Dirac Equation is that it permits the existence of soliton solutions. Mathematical analysis has demonstrated the stability of such solitons in (1+1)D settings. Solitons in (2+2)D systems have prone to oscillatory instabilities for low frequency, leading to deformation and rotation of the soliton (see Fig. 1); vortex solitons are unstable for every frequency and oscillatory instabilities lead to vortex breaking (see Fig. 2).

The aim of the present talk is to present the main models of the Nonlinear Dirac Equation and the most recent results regarding the stability of solitons and vortices in one-, two- and three dimensional settings, as those shown in the paragraph above, and comparing them with the main features of solitons in the relativistic (Nonlinear Schrödinger) limit.

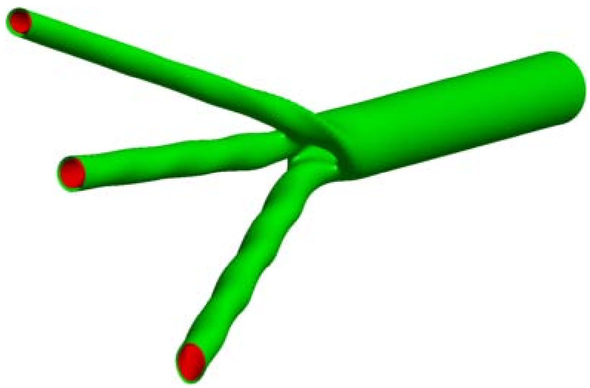

FIGURE 2. Isosurface for the time evolution of the density of an unstable vortex in (2+1)D settings.

Keywords: nonlinear Dirac equation, solitons, vortices.

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Analysis of the soliton solutions in a parity-time-symmetric triple-core waveguide

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In this contribution we present our study of a system of three two-dimensional (2D) nonlinear Schrödinger equations (NLSEs) coupled by linear terms and with the cubic(focusing)-quintic(defocusing) nonlinearity [**1**]. In particular, we discuss two versions of the system: a conservative and a parity-time($\mathcal{PT}$)-symmetric one. These models describe triple-core nonlinear optical waveguides, with balanced gain/losses in the ($\mathcal{PT}$)-symmetric case. The system of equations takes the following form:

$$i\frac{\partial\psi_1}{\partial z}+\nabla^2\psi_1+(|\psi_1|^2-|\psi_1|^4)\psi_1+\alpha\psi_2+\beta\psi_3 = i\gamma\psi_1,$$

$$i\frac{\partial\psi_2}{\partial z}+\nabla^2\psi_2+(|\psi_2|^2-|\psi_2|^4)\psi_2+\alpha\psi_1+\alpha\psi_3 = 0,$$

$$i\frac{\partial\psi_3}{\partial z}+\nabla^2\psi_3+(|\psi_3|^2-|\psi_3|^4)\psi_3+\alpha\psi_2+\beta\psi_1 = -i\gamma\psi_3,$$

where $\psi_{1,2,3}$ are the dimensionless amplitudes of the electric field in the three cores, z is the propagation distance, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the 2D Laplace operator in the transverse plane x and y, $\alpha>0$ and $\beta>0$ are the coupling coefficients and γ is the gain/loss parameter. For $\gamma=0$ the system becomes conservative, as no gain and losses are present. The case $\gamma>0$ enables the $\mathcal{PT}$ symmetry, where the first equation describes gain, the third equation describes a lossy waveguide, and the second equation remains neutral. We start studying the conservative model, where we obtain five different families of soliton solutions and discuss their stability using a linear stability analysis [**2**] and performing direct numerical simulations of the evolutional system of equations. Subsequently, we discuss the $\mathcal{PT}$-symmetric configuration and report the different stable solitons of the system (see Figure 1 for an example of a stable solution). Roughly speaking, we conclude that stable solitons can be found as long as γ does not exceed the $\mathcal{PT}$-symmetry breaking threshold (reality of the spectrum of the underlying linear system). Additionally, we also present different analogies and differences between our model and the cubic-quintic 2D couplers previously studied [**3**]. To conclude, we briefly investigate interactions and collisions between the conservative and $\mathcal{PT}$-symmetric solitons.

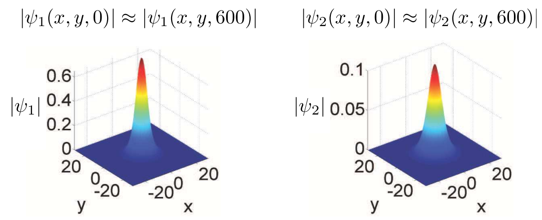

FIGURE 1. Example of long distance evolution of two components ($|\psi_1|$=$|\psi_3|$) of a stable 2D $\mathcal{PT}$-symmetric soliton.

Keywords: solitons, triple-core waveguide, parity-time symmetry, cubic-quintic nonlinearity.

Acknowledgments

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Creation of stable three-dimensional solitons and vortices: New perspectives

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Self-trapping of two- and three-dimensional (2D and 3D) localized modes in nonlinear dispersive/diffractive media, which are usually categorized as solitons, is a topic of great interest to many areas of physics [**1**]-[**3**]. This theme of theoretical and experimental studies is highly relevant to nonlinear optics and the dynamics of matter waves in Bose-Einstein condensates (BECs). Other realizations of multidimensional solitons are known in ferromagnets, superconductors and semiconductors, nuclear physics, and the classical-field theory.

Unlike 1D solitons, which are usually stable objects [**1**], stability is a major issue for their 2D and 3D counterparts. The common cubic self-focusing nonlinearity, which readily creates 2D and 3D solitons solutions, simultaneously gives rise to the critical wave collapse in 2D and supercritical collapse in 3D, that completely destabilizes the solitons. In particular, the first example of solitons which was introduced in nonlinear optics, *viz.*, the *Townes solitons*, i.e., 2D self-trapped modes supported by the cubic self-focusing [**4**], are unstable to the critical collapse, therefore they have not been observed in experiments. Multidimensional solitons with embedded vorticity, alias 2D vortex rings and 3D vortex tori, are vulnerable to a still stronger splitting instability initiated by azimuthal perturbations.

Thus, the stabilization of multidimensional fundamental and vortex solitons is an issue of great significance. The talk presents a short review of recently elaborated methods which predict stable solitons in unexpected settings. One scheme makes use of *self-defocusing* nonlinearity with the local strength growing, as a function of distance r from the center, at any rate faster than r^3, which may be realized in BEC. In addition to 3D solitons and vortex tori, it gives rise to more sophisticated modes, such as *hopfions*, i.e., vortex rings with internal twist, which carry two topological charges. The stability of these states is secured by the repulsive sign of the nonlinearity.

Completely novel results were reported in very recent works addressing two-component self-attractive BEC with spin-orbit coupling (SOC), in 2D [**6**] and 3D [**7**] settings. In 2D, the SOC breaks the scale invariance, which underlies the critical collapse, and creates a missing ground state (GS), in the form of *semi-vortices* (SVs) or *mixed modes* (MMs). The SVs are complexes built of fundamental and vortex solitons in two components of the binary system, while MMs mix vorticities 0 and ± 1 in the two components. In the 3D setting, the supercritical collapse cannot be suppressed, hence a true GS cannot be created. Nevertheless, robust 3D SVs and MMs have been constructed in the form of *metastable solitons*, which, in particular, are stable against small perturbations.

Keywords: Bose-Einstein condensates, spin-orbit coupling, semi-vortex.

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Kink–Antikink Collisions in the Kryuchkov–Kukhar' Equation

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Solitary electromagnetic waves can propagate in a graphene superlattice, a sheet of graphene deposited on a superlattice, several periodically alternating layers of SiO_2 and h-BN, under THz radiation [**1**]. The nonlinear d'Alembert equation for the transverse component of the potential vector for the electromagnetic wave field results in a generalization of sine-Gordon equation (sGeq) as derived by Kryuchkov and Kukhar' [**2**]. Taking the plane xz for the graphene sheet, the dimensionless value of the A_z component of the potential vector for the electromagnetic field, α, follows the graphene superlattice equation (GSLeq):

$$\frac{\partial^2\alpha}{\partial t^2} - c^2\frac{\partial^2\alpha}{\partial x^2} + \frac{\omega_0^2\, b^2 \sin\alpha}{\sqrt{1+b^2(1-\cos\alpha)}} = 0,$$

where ω_0^2 is the normalized frequency, $b \le 1$ is a geometrical parameter of the superlattice, c is light speed, x is position and t is time.

The GSLeq has a solitary wave solution of the form $u(x,t) = \alpha(\xi)$, where $\xi = (x - vt)/\gamma$, with $\gamma = \sqrt{1 - v^2/c^2}$. It corresponds to a one-dimensional topological solitary wave between consecutive multiples of 2π and propagating at speed $v < c$. It is referred to as a kink (antikink) when it is monotonically increasing (decreasing). Note that the GSLeq is non-integrable, but reduces to the sGeq for $b \to 0$. The goal of this work is the interactions of kinks and antikinks comparison in both equations by using numerical methods.

The Strauss–Vázquez finite difference scheme [**3**] for the GSLeq is given by

$$\frac{u_m^{n+1} - 2u_m^n + u_m^{n-1}}{\Delta t^2} - \frac{u_{m+1}^n - 2u_m^n + u_{m-1}^n}{\Delta x^2} + \frac{G(u_m^{n+1}) - G(u_m^{n-1})}{u_m^{n+1} - u_m^{n-1}} = 0,$$

where u_m^n approximates to $\alpha(m\Delta x, n\Delta t)$, Δx is the grid size, Δt is the time step, and $G'(u) = V(u)$ is the nonlinear potential in the GSLeq. This numerical method is second-order accurate in both space and time, and nonlinearly stable since it exactly conserves a discrete energy [**3**].

An extensive set of numerical results for the kink–antikink interaction have been obtained with $c = 1$ and $\omega_0^2\, b^2 = 1$, for several values of $b \in (0,1]$ and $v \in (0,1)$. For small b, the interaction is apparently elastic, without noticeable radiation, being very similar to that expected for the sGeq. For large b, the inelasticity of the interaction results in the emission of wave packets of radiation. No critical velocity similar to that of the ϕ^4 Klein–Gordon equation is observed in the inelastic interaction. Even though it is observed the annihilation of the kink-antikink pair. The whole set of results seems to be that the GSLeq behaves as a nearly integrable perturbation of the sGeq. The application of integrable perturbation methods to the GSLeq with the sGeq as leading-order approximation will be considered in further research.

Keywords: solitary waves, generalized sGeq, finite difference method.

Acknowledgments

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Solitons and vortices in Bose-Einstein condensates with finite-range interaction

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We study a dilute and ultracold Bose gas of interacting atoms by using an effective quantum field theory which takes account finite-range effects of the inter-atomic potential [**1, 2**]. Within the formalism of functional integration at finite temperature, we derive one-loop analytical results which depend on both scattering length and effective range of the interaction. At zero temperature, we obtain a modified Gross-Pitaevskii equation, which accounts for the energy dependence of the two-body scattering amplitude on the basis of the effective-range expansion [**3, 4, 5, 6, 7**]. We discuss the effect of the effective range on quasi-1D bright and dark solitons [**8**] and also on quantized vortices.

Keywords: effective quantum field theory, Bose-Einstein condensates, BEC solitons, quantized vortices.

Acknowledgments

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Part 4

CRYSTALS, METAMATERIALS AND OTHER CONDENSED MATTER

Multiple lattice kinks in a cation lattice

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One peculiarity of the mineral mica muscovite is the presence of dark tracks of magnetite in the cation layer, some due to swift particles and some along the lattice directions due to some kind of lattice excitation called *quodons*. See Ref. [**1**] for a recent review. Recently a model with realistic interactions for the cation layer of the silicate muscovite mica has been developed [**2, 3**], one of their findings was the existence of a supersonic lattice kink also called a *crowdion*. This lattice kink have a constant velocity and a constant energy of 26 eV. This energy was of particular interest because it is smaller that the recoil energy of the beta decay of ^{40}K, which is the probably source of *quodons*, but also larger than the energy to eject an atom from the lattice, which is about 8 eV. as demonstrated experimentally [**4**]. It was observed that only positive particles were able to leave a dark track in the mineral, which led to the conclusion that most of quodons also have a positive charge [**5**]. This property connected with lattice kinks, because they include the transport of an ion of K^+ and therefore of a positive charge, which is produced by the emission of an electron during beta decay.

The lattice kinks have an structure of a double kink or if described in the distances between particles of a double soliton. This is a particular case of bound solitons who have been described and analyzed in previous publications [**6**]. In this article we analyze which other multi-kinks can appear in the realistic model for the cation layer of mica muscovite, we analyze their energies and their possible role in the production of tracks in muscovite mica. We also study which is the role of the three potentials involved: repulsive electrostatic potential, short-range nuclear repulsion and on-site potential produced by the rest of the lattice in the appearance of the different lattice-kinks.

Keywords: mica muscovite, quodons, crowdions, lattice kinks, bound solitons, charge transfer.

Acknowledgments

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Discrete breathers in crystals: energy localization and transport

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Discrete breather (DB) is spatially localized vibrational mode in defect-free nonlinear lattice. Frequency of DB must lie outside the spectrum of small-amplitude traveling waves. Do not resonating with traveling waves and do not losing energy to their excitation, theoretically DB can maintain its vibrational energy forever, in the absence of thermal vibrations and other perturbations. Crystals are nonlinear discrete systems and they also support DB. Experimental studies of DB run into considerable technical difficulties, and the main tool of their study is by far the atomistic computer simulations. Having gained confidence in the existence of DB in crystals, we still poorly understand their role in solid state physics. This presentation covers issues specific to the physics of real crystals, which were not considered in the classical works on DB. Focus is placed on the energy transport through crystal lattice assisted by moving DB and by energy exchange between DB. Besides, the following topics are discussed: examples of DB in crystals; effect of lattice dimension on DB properties; interaction of DB with each other and with lattice defects; DB at crystal surface; effect of elastic strain of crystal lattice on DB; first-principles simulations. Solution of these problems will bring us closer to understanding the role of DB in solid state physics.

Keywords: nonlinear dynamics, discrete breathers, energy transport.

Acknowledgments

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Heterogeneous catalysis driven by localized anharmonic vibrations

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Catalysis is at the heart of almost every chemical or nuclear transformation process, and a detailed understanding of the active species and their related reaction mechanism is of great interest. An important parameter of the reaction kinetics is the activation energy, i.e. the energy required to overcome the reaction barrier. The lower is the activation energy, the faster the reaction rate, and so a catalyst may be thought to reduce somehow the activation energy. Dubinko et al [1, 2] have shown that in a crystalline matrix, the activation energy may be reduced at some sites due to a special class of localized anharmonic vibrations (LAVs) of atoms, known also as discrete breathers or intrinsic localized modes arising in regular crystals. LAV can be excited thermally or by irradiation, resulting in a drastic acceleration of chemical reaction rates driven by thermally-activated 'jumps' over the reaction barrier due to the time-periodic modulation of the barrier height in the LAV vicinity. At sufficiently low temperatures, the reaction rate is controlled by quantum oscillations rather than by thermal fluctuations. We demonstrate a strong increase of the Kramers rate of escape out of a parabolic potential well with the time-periodic eigenfrequency, when the modulation frequency exceeds the eigenfrequency by a factor of ∼ 2 (parametric regime). Such regimes can be realized in the vicinity of LAVs [3]. We solve the Schrödinger equation for a nonstationary harmonic oscillator, show that the localization probability distribution $|\psi|^2$ broadens with time t induced by the parametric modulation (see figure) and present an expression for the exponential increase of the energy of zero-point oscillations (ZPO) [4]:

$$E_{ZPO} = \frac{\hbar\omega_0}{2}\cosh\left(\frac{g\omega_0 t}{2}\right),$$

where ω_0 is the harmonic eigenfrequency, $\hbar$ is the Plank constant and g is the modulation amplitude. The Kramers rate (which is an archetype model for chemical reactions since 1940) modified with account of zero-point energy is given by [5]

$$R_k(T) \approx \frac{\omega_0}{2\pi}\exp\left(-\frac{\Delta U}{E_{ZPO}\coth(E_{ZPO}/k_B T)}\right),$$

where ΔU is the reaction barrier, k_B the Boltzmann constant and T is temperature. We may conclude that an account of the LAV induced increase of ZPO energy in the Kramers escape rate shows that LAVs can be strong catalysts at both high and low temperatures. This reasoning opens a way of engineering the active environment based on atomistic modeling of LAV excitation dynamics in solids.

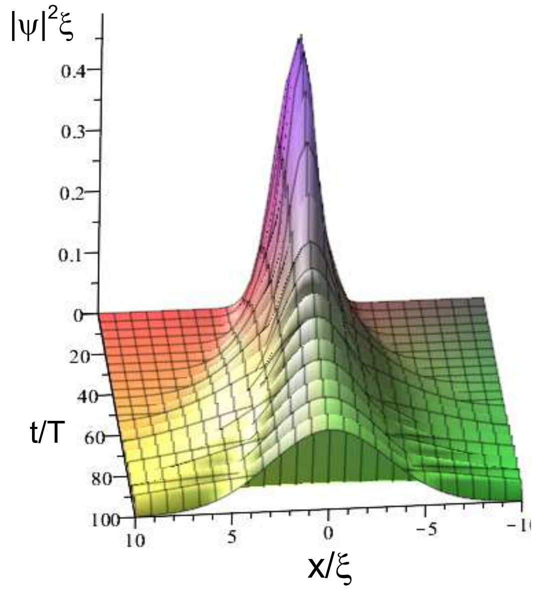

FIGURE 1. Broadening of the probability distribution $|\psi|^2$ with time t induced by parametric modulation. The parameter $\xi = \sqrt{\hbar/2m\omega_0}$ is the initial ZPO amplitude for the mas m.

Keywords: localized anharmonic vibrations, zero-point energy, tunnel effect, catalysis.

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A class of nonlinear complex elastic media in the vicinity of an equilibrium state behaving as acoustic metamaterials

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There are a lot of works on electromagnetic [**1**] and acoustic metamaterials [**2**], which demonstrate anomalous wave properties that can be used for cloaking, lensing, signal absorption. Most of them, with a few exceptions as [**3**] are experimental or use microstructural approach. We want to understand what a metamaterial could be from the point of view of continuum mechanics.

We consider complex elastic media, whose particles (point bodies) are not point masses but have a complex structure. Each point body may contain point masses, infinitesimal rigid bodies, and other degrees of freedom, subjected to holonomic and ideal constraints (Fig. 1), described by a vector of generalised co-ordinates $\mathbf{q}$ of any dimension. Such an elastic medium obeys Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} + \nabla \cdot \frac{\partial L}{\partial \nabla \mathbf{q}} = \mathbf{0}, \tag{17}$$

where $L = K - U$ is the Lagrange function, K and U are the densities of kinetic and strain energy, ∇ is the nabla operator, and t is time.

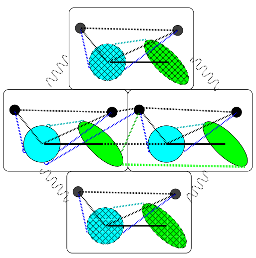

FIGURE 1. Example of a complex medium.

Consider a special case when the medium is described by a "special" vectorial generalised co-ordinate $\mathbf{q}_0$ such that L does not depend on $\nabla \mathbf{q}_0$, but depends on $\mathbf{q}_0$, and by a "bearing" vectorial generalised co-ordinate $\mathbf{q}_1$, i.e. $U = U(\mathbf{q}_0, \mathbf{q}_1, \nabla \mathbf{q}_1)$. Then the Lagrange equation with respect to $\mathbf{q}_1$ is similar to (17), and the equation with respect to $\mathbf{q}_0$ looks the same as for a discrete system:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}_0} - \frac{\partial L}{\partial \mathbf{q}_0} = \mathbf{0}. \tag{18}$$

We consider a nonlinear energy U of general kind. Suppose that the medium is in the vicinity of a certain equilibrium. We obtain the equations of motion for small deviations near it. They appear to be the Lagrange equations for a linear dispersive medium containing effective elastic moduli, depending on the equilibrium state. Requiring some symmetry conditions, we obtain that the medium is either a single negative acoustic metamaterial, i.e. has forbidden bands, where plane waves in the bulk do not propagate, or (and) a double negative acoustic metamaterial, i.e. has a frequency domain where the frequency of the plane wave decays while the wave number increases (the energy flux is opposed to the direction of the wave propagation), see Fig. 2.

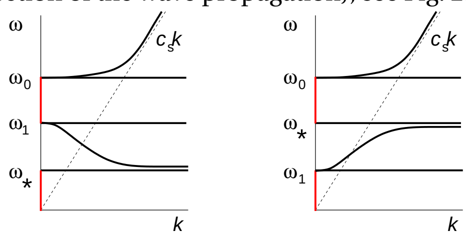

FIGURE 2. Typical dispersion curves

Changing the nonlinear strain state, we change the effective elastic parameters. If we succeed to construct the material with an appropriate strain energy, in this way we may control the properties of the acoustic metamaterials, changing their kind and frequency domains with interesting wave propagation properties. Some examples are given.
This approach could be applied also to electromagnetic materials with the same Lagrangian.

Keywords: acoustic metamaterials, nonlinear elasticity, complex media.

Acknowledgments

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Spatially localized modes in anharmonic lattices without gaps in phonon spectrum

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Spatially localized modes, called as intrinsic localized modes (ILMs) or discrete breathers may exist in crystals with the gaps in the phonon spectrum. ILMs in these crystals split down from the optical bands due to the softening of the pair potentials with the increase of the amplitude of the mode. Recently we have found that in some metals and covalent crystals ILMs may also exist with the frequencies above the top of the phonon spectrum [**1, 2**]. Examples are given by metallic Ni, Nd, Fe, Cu and by semiconductors Ge and diamond. In the metals mentioned the required hardening of the potentials comes from the Friedel oscillations of the electronic density reducing the odd anharmonicity at the intermediate distances. In covalent crystals the hardening comes from the strong orientation dependence of the covalent interactions. In the metals these excitations are highly mobile: they may propagate along the crystallographic directions transferring energy of $\gtrsim 1$ eV over large distances [**2**].

Recently a new type of ILMs in the lattices without gaps in the phonon spectrum was predicted [**3**] – self-localized transverse anharmonic vibrations in chains and planes (graphene). It was found that due to the lack of odd anharmonicities for displacements in transverse direction, there may exist ILMs with the frequencies above the spectrum of the corresponding phonons. The properties of these novel nonlinear vibrational excitations will be discussed. We will show that although the frequencies of the modes are in resonance with longitudinal (chain) or in-plane (graphene) phonons they can decay only due to a weak anharmonic process. In terms of quantum theory this process corresponds to the up-conversion: two quanta of the ILM merge together producing one quantum of the longitudinal (in-plain) phonon. We will show that the rate of this process may be very small. E.g. in the chain, according to our analytical and numerical calculations the lifetime of a transverse ILM may exceed 10^{10} periods. We call these vibrations as transverse intrinsic localized modes. The modes are mobile.

Keywords: intrinsic localized modes (ILMs), discrete breather, nonlinear dynamics.

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Quasiperiodic Intermittency in a Surface Reaction Model

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One of the most studied heterogeneous catalytic reactions is the oxidation of CO by O_2 over metal catalysts such a Pt, Pd or Ir. This reaction exhibits typical phenomena of nonlinear dynamics, kinetic phase transitions, bi-stability, hysteresis, and a rich variety of oscillatory behaviors which vary from periodic and quasiperiodic to aperiodic and chaotic. A simple and fruitful model to study the catalyzed reaction of CO is the well-known Ziff, Gulari and Barshad (ZGB) model [**1**] which considers that before the reaction can take place both CO and O have to be absorb from the gas phase on the surface. The simplicity of the ZGB model allows the inclusion of new physical phenomena to make the model more realistic.

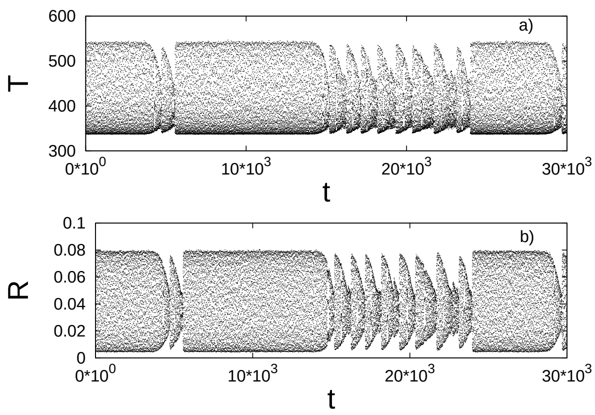

FIGURE 1. Time series of the temperature T and the production rate of CO_2 (R) for the parameters $\gamma = 0.30$, $p_1 = 0.012$, $p_2 = 0.01$. Windows of quasiperiodic oscillations are interrupted by burts in T and R.

In this work we study the effects of the temperature, desorption and diffusion on the reaction of CO and O over a catalytic surface. The simulations are made with a cellular automaton (CA) originally proposed in [**2**] that takes into account the temperature of the surface through a thermal relaxation parameter γ. We use an extended ZGB model in which only the desorption and diffusion of CO molecules have been considered whereas the O atoms are supposed to be immobile. Both processes are probabilistic with probabilities p_1 and p_2 respectively. The coupling among the termal effects, desorption and diffusion shows many different regimes that go from a poisoned state with O or CO to quasiperidic and chaotic oscillations. But the most interesting and striking behavior is found in the range of parameters ($0.009 \le p_1 \le 0.015$) and ($0.30 \le \gamma < 0.35$). A typical run is shown in Figure 1. In this kind of behavior a torus is destabilized giving rise to an intermitency between a laminar phase (which is of a quasiperiodic nature) and chaotic bursts whose abundance is larger as the thermal parameter γ increases. This behavior does not correspond to any of the classical intermittencies (I, II, III and on-off intermittency) and as far as we know the only physical system in which a similar phenomena has been reported is in a Zeeman Laser [**3**].

Keywords: surface reaction, quasi-periodic, intermittency.

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Discrete breathers in metals and ordered alloys

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Nonlinear, spatially localized vibrational modes called discrete breathers (DB) in defect free lattices recently have attracted considerable attention of researchers in many areas of modern physics. The existence of DB in crystal lattices is provided by the anharmonicity of the interatomic forces, leading to a dependence of vibration frequencies of atoms on their amplitudes. It has been shown that DB can exist in various types of materials including ionic and covalent crystals, carbon materials of different dimensionality, metals and alloys. Investigation of localised vibrations in metallic materials is a relevant direction of scientific studies due to possible contribution of DB to evolution of structure and properties of those materials.

DB can demonstrate soft or hard nonlinearity. In the former (later) case DB frequency decreases (increases) with increasing amplitude. Decrease of the soft nonlinearity DB frequency with growing amplitude can result in entering the phonon spectrum gap, if it exists. Considering metallic materials it is easy to excite gap DB in ordered alloys with a large difference in the atomic mass of the components, ensuring the existence of a wide gap in the phonon spectrum, e.g., in Pt_3Al [**1**]. These works were based on the Morse pair interatomic potentials. It should be mentioned that in alloys with large atomic masses difference it is possible to excite both gap DB and DB with frequency above the phonon spectrum [**2**].In case of Pt_3Al the gap DB is localized predominantly on one atom of aluminum and is immobile. In contrast, the DB with frequency above the phonon spectrum is localized on four to five atoms of aluminum belonging to a close-packed row and can move along the close-packed row. In case of mobile DB there exist a probability of their collisions with each other and with immobile gap DB [**2**]. Interactions of several DB can produce even higher spatial localization of energy and contribute to the decrease the potential barrier of defect migration.

Beginning with the work [**3**], where for the first time the mobile DB have been successfully excited in fcc Ni and bcc Nb, DB have been actively studied in pure metals. DB in all pure metals have the same structure. The atoms located in a close-packed row, oscillate in anti-phase with the nearest neighbours at a frequency above the phonon spectrum. The frequency increases with increasing amplitude of DB. It should be also mentioned that all the investigated objects should be related to the concept of quasibreathers rather than strictly time-periodic DB and can be characterised by slow energy radiation. Recent achievements in investigations of DB in metals and alloys, methods of DB excitations, their properties and possible contribution of localized vibrations to evolution of structure will be discussed.

Keywords: nonlinear dynamics, discrete breathers, metals, alloys.

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Ultradiscrete supersonic electron polarons in nonlinear molecular chains with realistic interatomic potentials and electron-phonon interaction

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In this Presentation we revisit the problem of the trapping of an excess electron and its transfer by lattice supersonic kink (supersonic acoustic soliton) in a one-dimensional molecular chain. For the first time, this problem was independently analytically studied in works [1,2]. Since that time, the problem was studied numerically in several papers, see, e.g., [3,4]. But in our opinion the detailed mechanism of the trapping is not yet clarified. Namely, since the first papers [1,2] it is claimed in most of the works on this topic that the electron is captured by the potential well produced by supersonic kink. On one hand, it is known that supersonic kink in the lattice with realistic interatomic potential (such as Lennard-Jones, Morse or the Coulomb potential) produces local compression of the lattice, see, e.g., [5]. On the other hand, lattice compression enhances electron Fermi energy and therefore the local compression should produce for the electron a local potential hill, instead of potential well, through the deformation potential of the corresponding sign. In this Presentation we show that only the electron on the top of its tight-binding band, where it possesses negative effective mass, can be trapped by supersonic kink in a molecular chain with realistic interatomic potentials and electron-phonon interaction. We also show that the localization length of the electron wave function is larger than lattice period in the case of adiabatic electron dynamics and decreases with the speed of the ultradiscrete supersonic kink with the (approximate) sinusoidal envelope with the "magic" wave number, which was revealed for ultradiscrete supersonic kinks in lattices with different interatomic potentials with hardening anharmonicity [5,6]. Electron (or exciton) can also be localized by the discrete breather (intrinsic localized mode) in the lattice with a realistic combined symmetric and asymmetric anharmonic potential. In the case of stationary or slowly moving discrete breather, there is a kink-like distribution of static or quasi-static lattice displacements, caused by the asymmetry of the interparticle potential. In the lattice with realistic interatomic potentials, these displacements cause local lattice stretching and the formation of local potential well for the electron (or exciton), produced by the discrete breather. This in turn results in the trapping of slowly-moving electron (or exciton) with its localization below the lower edge of the conduction (or exciton) band.

Keywords: supersonic kink, trapped electron, electron-phonon interaction.

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Second harmonic generation in a chain of magnetic pendulums

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A chain of magnetic pendulums has been demonstrated to be a good analogue model of a crystal lattice [**1**]. We consider an infinite chain of identical magnets with mass m aligned along the x-axis. The equation of motion for every magnet in such chain, considering a magnetic dipole-dipole interaction only between two nearest neighbors , takes the form:

$$m\ddot{u}_n = -\frac{g}{L}u_n + \frac{\epsilon}{(a-u_{n+1}+u_n)^4} - \frac{\epsilon}{(a-u_n+u_{n-1})^4}$$

where u_n represent the displacement of magnet n^{th} measured with respect to its equilibrium position, ϵ is a coupling constant, a is the distance between the center of magnets (lattice constant) and g is the gravitational acceleration. In particular, for small displacements, this equation reduces to the well-known α-FPU equation (quadratic approximation). Then, it is expected to observe the same kind of phenomena that appears in other systems like acoustic layered media or granular chains [**2**] . This discrete medium is called a superlattice and presents dispersion at frequencies close to the cutoff.

In this work, we studied numerically, analytically and experimentally the nonlinear dynamics of this system, in particular, harmonic generation and the balance between nonlinearity and dispersion. In the experimental setup, each magnet is attached to a T-shaped rod with length L and the system presents a very low damping. Three regimes have been explored exciting the chain, forcing the first pendulum with an harmonic force of long wavelength with respect to a and with such amplitude that the dispersion relation still remains valid. We have studied harmonic generation when; (a) the frequencies of the $First$ harmonic (driven force) and the $second$ harmonic generated by the nonlinearity lies into the non-dispersive part of the dispersion relation (almost linear $k(\omega)$). In this case the second harmonic starts to increase while first harmonic decreases. (b) The driven frequency is in the non-dispersive part of the dispersion relation but the second harmonic is evanescent. The amplitude of the second harmonic reaches a constant value along the chain. (c) The frequency of the second harmonic lies in the dispersive part of the dispersion relation. Harmonic components travel with different velocities and beating appears. Analytical results and numerical simulations are in good agreement with the experimental results.

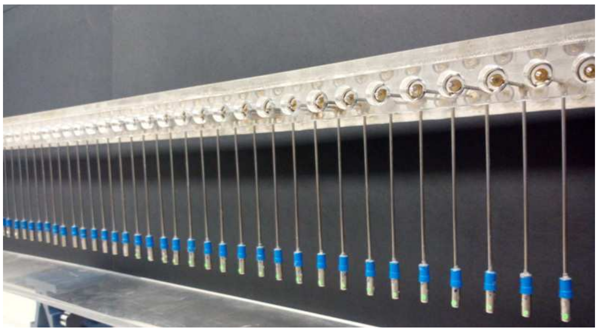

FIGURE 1. Experimental setup

Keywords: FPU, magnetic pendulums, dipole-dipole potential, Nonlinear lattice, Harmonic generation.

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Dynamics of homogeneous and inhomogeneous nonlinear lattices formed by repelling magnets

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Nonlinear lattices, represented by nonlinear mass-spring systems, exhibit a number of fascinating physical phenomena, as chaos, bifurcations, nonlinear resonances, or the formation of solitary waves and breathers. Understanding and exploiting nonlinearity in these kinds of systems has led to the design of materials or devices with unprecedented properties, including mechanical diodes and logic gates.

In this work we investigate experimentally and theoretically the nonlinear dynamic response of a one–dimensional lattice composed of repelling magnets. We consider a homogeneous chain as well as a chain containing mass defects. The chains are built experimentally using neodymium magnets placed on an air-bearing table, and the dynamics of the system is measured using Digital Image Correlation. The system is modelled as a Fermi-Pasta-Ulam lattice with nearest neighbour interactions.

In the case of a homogeneous chain, we demonstrate the formation of solitary waves with profile and propagation speed depending on the excitation amplitude [**1**]. More specifically, the system belongs to the kind of nonlinear lattices studied in [**2, 3**] and exhibits a sech^2 profile in the low energy regime and atomic scale localization in the high energy regime.

In the case of an inhomogeneous chain, we demonstrate the existence of localized vibrational modes (breathers) induced by mass defects. The transfer of energy between different frequencies through the excitation of these modes is also discussed.

In addition to their use as a toy model for the study of fundamental nonlinear dynamical systems, such systems could find potential applications in energy mitigation, localization and harvesting, or in the design of acoustic lenses capable to emit very narrow pulses. Moreover, the similarities of the magnetic potential with the potentials governing atomic lattices may suggest the use of this system as experimentally accessible platforms for a better understanding of physical phenomena arising in atomic chains.

Keywords: nonlinear lattice, solitons, breathers.

Acknowledgments

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The Geometry of Transition State Theory

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Transition State Theory (TST) plays a central role in chemical reactions as it provides a simple answer to two of the most important tasks in Chemistry, namely: which is the reaction rate and which is the reaction mechanism responsible of that reaction.

In this contribution, we will revisit some of the last advances on TST based on the identification of the geometrical structures (the invariant manifolds) that determine the rate constant in systems that interact strongly with their environments [**1, 2**].

Keywords: transition state, rate theory, invariant manifold.

Acknowledgments

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Effect of cohesion on sound propagation in disordered powder packings

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In this job we present different models (Hertz, DMT, JKR) for the elastic deformation of spheres in a load carrying contact [**1**]. The transmission of sound between particles without adhesion can be described by a Hertz contact ($\sigma \propto \varepsilon^{3/2}$) if we consider the stress σ and consequently the strain ε as the superposition of a static contribution σ_0, ε_0 and a dynamic (acoustic) contribution $\tilde{\sigma}_0$, $\tilde{\varepsilon}$ with $|\tilde{\varepsilon}_0| \ll |\varepsilon_0|$ ($\varepsilon_0 < 0$) [**2**]. Different experiments have shown that models based on Hertz contact are valid when adhesive forces between particles are negligible compared with the contact loads arising from the compacting pressure [**3**]. However, if adhesive forces are relevant, the contact between spheres is best described by the DMT or the JKR model that may yield a different dependence of the sound speed with pressure.

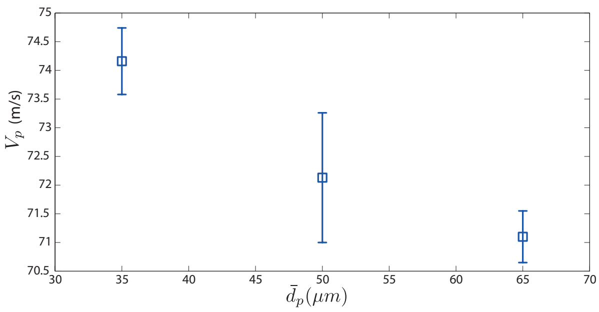

FIGURE 1. Longitudinal wave velocity as a function of the average particle size $\bar{d}_p$ in magnetite particle samples.

Experimental results have been shown that sound velocity increases with increasing cohesion in the particles with a certain size ($d_p < 110\mu m$) [**4**]. As shown in Fig 1 , we can see the longitudinal wave velocity in magnetite particle samples to observe this behaviour before cited. Another recent study has also shown that polydispersity plays a important role in the wave velocity of the transmitted longitudinal wave [**5**]. It is important to say that previous experiments [**4, 5**] have been done at very low consolidation because the pressure is only due to gravity. In our experiment, we have applied an external pressure to a powder to evaluate the importance of cohesive forces and the network of contacts when a powder is loosely packed.

Keywords: ultrasonics, wave propagation, cohesive granular media.

Acknowledgements

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Transport properties of quodons

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Studies of fossil tracks of charged high energy particles and lattice quodons in muscovite crystals have given evidence for the transport over macroscopic distances of electronic charge at near-sonic speed in an insulator at high temperature [**1**]. The speed of transport is several orders of magnitude greater than the classical speed observed in metals.

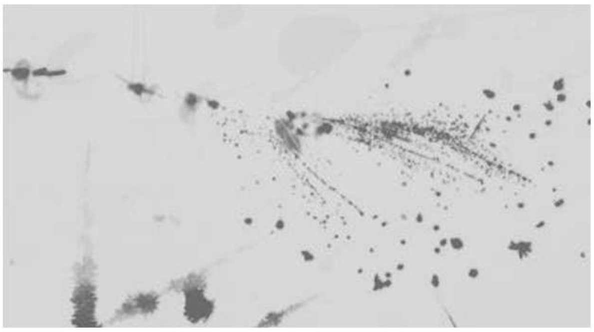

FIGURE 1. Photomicrograph of the fossil record of a fracture of a crystal of muscovite. The macro-size fracture enters at top left and ends near the image centre, where it creates many micro-fractures and dislocations that extend to the right. The severely distorted lattice in the vicinity of the fractures inhibits the recording process decorating the tracks of quodons, which are unable to propagate through fracture regions. The fading of quodon tracks entering at bottom left is visible. The width of the image is 30 mm.

The results of computer studies are compared with experiments on the transport of energy by quodons [**2**]. This is extended to include secondary quodons created by scattering at dislocations of primary quodons. The fossil record of crystal damage arising from fractures is contrasted with that of quodon tracks. The theory and design of proposed experiments to study the transport properties of quodons is described. In particular, the possibility of observing the motion of atoms within a quodon envelope by means of pulsed coherent X-rays is considered. Other experiments include the detection of the transport of charge by quodons and the transport of heat energy at sonic speed. The inability at present to model bare or charged quodons in muscovite is inhibiting their possible relevance to cuprate superconductors, which have a layered structure similar to that of muscovite, and to the transport of electrical energy at high temperatures [**3**]. The relationship between quodons and solectrons is explored [**4**]. The first order criteria for lattice excitations to achieve nuclear fusion is derived. The aim of this paper is to bridge the gaps between theoretical and computational studies of quodons, experimental studies of energy transport, the generation of quodons and practical applications.

Keywords: quodons, charge-trapping, superconductors.

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Acoustic gap solitons in layered media

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Wave propagation is common to many fields of physics such as mechanics, optics or acoustics. In the case of acoustics, high intensity waves present specific features such as distortion, shock formation or harmonic generation. These phenomena have been studied since the second half of 18th century [**1**]. Common acoustic media are characterized by a quadratic nonlinearity and very weak dispersion. During last decades a class of artificial materials, the so-called sonic crystals, have received increasing interest, owing to their ability of manipulating sound wave propagation. They consist of a periodic arrangement of scatterers embedded in a host medium. Due to periodicity, sonic crystals present *band gaps*, that is, ranges of frequency where waves are not allowed to propagate. At the edges of the bands near the band gaps, waves propagate but experience high dispersion. There is a recent interest in studying the propagation of intense waves through periodic acoustic media [**2**]. The interplay between nonlinearity and dispersion leads to several phenomena including selective harmonic generation and propagation of solitary waves. In fact, in the band gap, waves are not allowed to propagate in linear regime, but under certain conditions, solitary waves can propagate without changing its shape. These waves have been called *gap solitons*. In the case of quadratic media, such solutions take the name of simultons, since localization affects both to the fundamental wave and its second harmonic. In this work, we study a 1D periodic system, formed by the periodic repetition of a set of two layers of different fluid materials, producing a 1D periodic modulation of the physical properties.

The propagation of acoustic waves in the multilayered media is described by a second-order nonlinear wave equation, also known as the lossless-Westervelt equation [**1**],

$$\frac{\partial^2 p}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\frac{\beta}{\rho c^4}\frac{\partial^2 p^2}{\partial t^2}, \tag{19}$$

where p is the acoustic pressure field, c is the space-dependent sound velocity, ρ is the fluid density and β the nonlinearity parameter. From (19), we have derived a set of coupled-mode equations for the envelopes of the acoustic pressure profiles, analogous to the ones that appear in the optical case [**3**].

In this work, we demonstrate numerically the existence of acoustic gap solitons (simultons) in a 1D layered acoustic media in the frame of the coupled-mode theory, and discuss the physical parameters corresponding to a realistic acoustic medium where such solitary waves could be observed experimentally.

Keywords: acoustics, phononics, harmonic generation, gap solitons.

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Peculiarity of propagating self-sustained annealing of radiation-induced interstitial loops

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It is well-known [**1**] that the degradation of materials under irradiation reduces the operating life of nuclear facilities. Radiation affects materials in many ways, but the main reason for their degradation is radiation-induced defects, which cause changes in material microstructure and properties. The simplest and most often used way of material recovery from radiation damage is thermal annealing. The irradiated material is a metastable system with non-linear feedbacks, which can give rise to different scenarios of annealing and one of them is an auto-wave propagation [**2**].

A theoretical approach to propagating self-sustained annealing of radiation induced defects as a result of thermal-concentration instability is studied. Defects that are considered in the model are the interstitial loops with given distribution of their sizes. The loop, which consists of n interstitial atoms (n-loop), can release one interstitial atom at any given moment in time. As a result, the number of n-loops decreases by one, while the number of $(n-1)$-loops and of interstitial atoms increase by 1 correspondingly. A crystal with defects has extra non-thermal energy which is approximately equal to the energy of defect formation. In the annealing area this energy transforms into heat (thermal energy) and the local temperature of material in this area grows. This leads to the acceleration of annealing. Therefore the system of non-linear equations for loop distribution and temperature profile of the material are considered. This system of non-linear equations resembles models encountered in the physics of combustion [**3**].

Simulation of the auto-wave of annealing (evolution of defect concentration and of material temperature) has been performed. The front shape and the speed of the auto-wave have been obtained. It was observed that annealing occurs in a narrow region of time and space. Simulations have revealed another peculiarity of auto-wave annealing, namely different regimes of propagation. In some cases, depending on the parameters of the model, the speed of the auto-wave oscillates near its constant mean value and the temperature in the front oscillates in a complex way. In the other cases the speed of propagation is constant and fronts of temperature and concentration resemble sigmoid functions.

Keywords: material, radiation-induced defects, annealing, non-linear feedback, auto-wave.

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Embedded solitons in the asymmetric array of Josephson junctions

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Embedded solitons (after [**1**] are solitary waves that are *embedded* into the continuous spectrum of the underlying system. In other words, there exists a resonance between some internal frequency of the solitary wave and the linear waves. Embedded solitons appear in many nonlinear systems, both continuous and discrete.

Here we investigate topological soliton (fluxon or josephson vortex) dynamics in the dc-biased array of asymmetric three-junction superconducting quantum interference devices (SQUIDs). This array is described by the discrete double sine-Gordon equation. It appears that this equation possesses a finite set of velocities (called *sliding velocities*) at which the fluxon propagates with the constant shape and without radiation [**2**]. Thus, the fluxon has the properties of an embedded soliton, although the resonance condition with the linear spectrum $\omega(q)$

$$\omega(q) - sq = 0,$$

is always fulfilled for any fluxon velocity s for at least one (or more) value of the wavenumber q.

The signatures of these sliding velocities appear on the respective current-voltage characteristics of the array as inaccessible voltage intervals (gaps). The critical depinning current has a clear minimum as a function of the asymmetry parameter (the ratio of the critical currents of the left and right junctions of the SQUID), which coincides with the minimum of the Peierls-Nabarro potential.

Keywords: solitons, kinks, Josephson junctions.

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POSTERS

On Discontinuous Piecewise Linear Models for Memristor Oscillators

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Leon Chua in 1971 [**3**] postulated the theoretical existence of a fundamental two-terminal passive device called memristor, a contraction for memory resistor, for which a nonlinear relashionship links charge and flux. In 2008 [**4**] a team of scientists of Hewlett-Packard Company announced the fabrication of the first memristor. Itoh and Chua [**5**] defined the basic equations for a third-order canonical memristor oscillator, obtained by replacing Chua's diode with a memristor, this system is three-dimensional, five parameter piecewise-linear system of ordinary differential equations (20) (for details, see [**5**] section 3.2).

$$\begin{aligned} \dot{x} &= \alpha\big(-W(z)x + y\big), \\ \dot{y} &= -\gamma x + \beta y, \\ \dot{z} &= x, \end{aligned} \tag{20}$$

where $W(z) = \dfrac{dq(z)}{dz}$

$$q(z) = \begin{cases} b(z-1)+a, & \text{if} \quad z > 1, \\ az, & \text{if} \quad |z| \leq 1, \\ b(z+1)-a, & \text{if} \quad z < -1, \end{cases}$$

and $\alpha, \beta, \gamma, a, b > 0$. For this model, which exhibits a continuum of equilibria, in [**1**] and [**2**] authors detect numerically the existence of oscillations when $b < \beta/\alpha < a$. In fact, they conjecture under certain hypotheses the existence of a topological sphere foliated by such periodic orbits, as the system determines a discontinuous vector field on each invariant surface, the existence of oscillation was only numerically confirmed.
In this work, we clarify the reasons for the seemingly strange behavior for the third-order memristor models worked by Itoh and Chua in [**5**], by putting in evidence the role of initial conditions, what justifies the infinite number of periodic orbits exhibited by these models. This will be evident once we show the existence of a conserved quantity, to be determined by such initial conditions.

Keywords: memristor oscillator, limit cycles, piecewise linear systems.

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Using the small alignment index chaos indicator to characterize the phase space of LiNC-LiCN molecular system

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The structure of the social networks in which individuals are embedded influences their political choices and therefore their voting behavior. Nowadays, social media represent a new channel for individuals to communicate, what together with the availability of the data, makes it possible to analyze the online social network resulting from political conversations. Here, by taking advantage of the recently developed techniques to analyze complex systems, we map the communication patterns resulting from Spanish political conversations. We identify the different existing communities, building networks of communities, and finding that users cluster themselves in politically homogeneous networks. We found that while most of the collective attention was monopolized by politicians, traditional media accounts were still the preferred sources from which to propagate information.

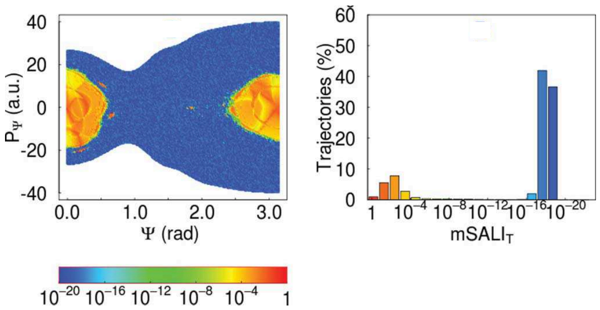

FIGURE 1. SALI Map for LiNC/LiCN at 4000 cm^{-1}. The corresponding histogram of $mSALI(T)$ ($T = 2.5 \cdot 10^5$ a.u.) are plotted in the right column.

In two dimensional (2D) systems SALI maps are computed as 2D phase space representations, where the SALI asymptotic values are represented in color scale. We show here how these maps provide full information on the dynamical phase space structure of the LiNC/LiCN system, even quantifying numerically the volume of the different zones of chaos and regularity as a function of the molecule excitation energy.

Keywords: molecular nonlinear dynamics, Hamiltonian system, chaos indicators.

Acknowledgments

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Mapping the online communication patterns of political conversations

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The structure of the social networks in which individuals are embedded influences their political choices and therefore their voting behavior. Nowadays, social media represent a new channel for individuals to communicate, what together with the availability of the data, makes it possible to analyze the online social network resulting from political conversations.

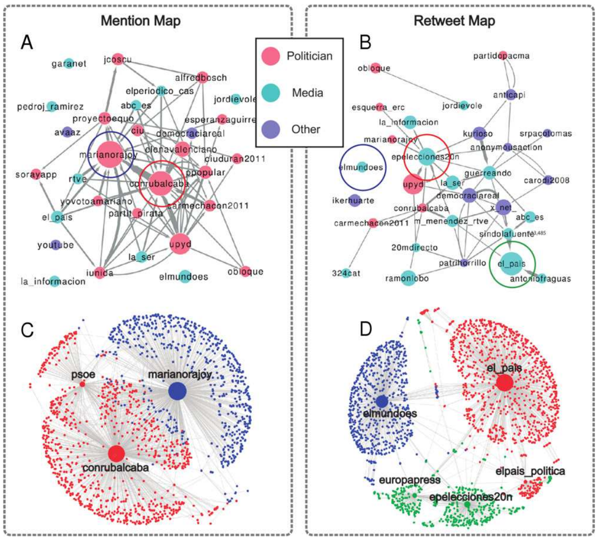

FIGURE 1. Map of the 20N mention and retweet c-networks (networks of communities), showing the 30 most important communities. The size represent their pagerank centrality. We only show the links representing at least 0.005 % of the total inter-module flow. A detail of the inside structure of some important communities (marked with circles) is displayed in (C) for the mention c-network and (D) for the retweet c-network. Nodes size represent their centrality and their color indicate the community to which they belong.

Here, by taking advantage of the recently developed techniques to analyze complex systems, we map the communication patterns resulting from Spanish political conversations about the 20th of November 2011 Spanish general elections. We downloaded all the messages that included the keyword 20N posted in a three week period including the official electoral campaign and voting day. This dataset has already been used to show that the activity taking place on Twitter was correlated to the election outcomes; and to characterize politicians behavior, observing a lack of debate among political parties.

We identify the different existing communities, building networks of communities, and finding that users cluster themselves in politically homogeneous networks. We found that while most of the collective attention was monopolized by politicians, traditional media accounts were still the preferred sources from which to propagate information.

Keywords: Social network analysis, political conversations, Twitter, political elections, online communication patterns.

Acknowledgments

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Impulse-induced optimum signal amplification in scale-free networks

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Optimizing information transmission across a network is an essential task for controlling and manipulating generic information-processing systems. Here, we show how topological amplification effects in scale-free networks of signaling devices are optimally enhanced when the *impulse* transmitted by periodic external signals (time integral over two consecutive zeros) is maximum. This is demonstrated theoretically by means of a star-like network of overdamped bistable systems subjected to *generic* zero-mean periodic signals, and confirmed numerically by simulations of scale-free networks of such systems. Our results show that the enhancer effect of increasing values of the signal's impulse is due to a correlative increase of the energy transmitted by the periodic signals, while it is found to be resonant-like with respect to the topology-induced amplification mechanism.

Keywords: complex networks, signal amplification.

Acknowledgments

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On the TS-Bifurcation in $\mathbb{R}^3$

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For a discontinuous piecewise linear dynamical system (DPWL) in $\mathbb{R}^3$, whose state space is divided in two open regions R^- and R^+ by a smooth switching manifold Σ, the vector field determines generically two lines of quadratic tangency, one for each side of the Σ. When these two lines have transversal intersection, such a point is called a two-fold singularity. In the case that both tangencies are of invisible type, the two-fold point is known as *Teixeira singularity* (TS-point), [**1, 2**].

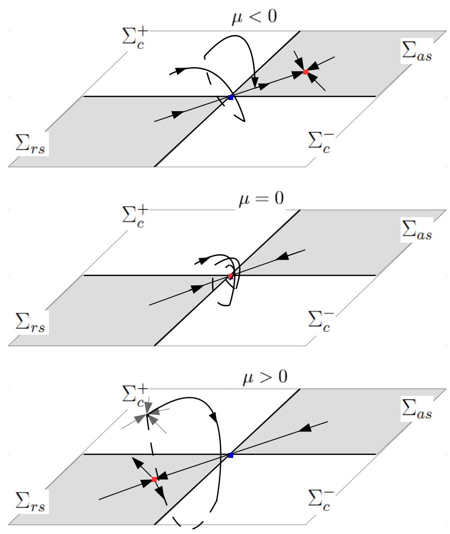

FIGURE 1. An illustrative example of the TS-bifurcation.

The Teixeira singularity can undergo an interesting bifurcation involving both a pseudo-equilibrium point and a crossing limit cycle (CLC). Here we analyse such a *compound* bifurcation, which occurs when a pseudo-equilibrium point crosses the TS-point, passing from the attractive sliding region Σ_{as} to the repulsive sliding region Σ_{rs} (or vice versa) and, simultaneously, a CLC arises, [**3**]. Such bifurcation will be named in this work as *TS-bifurcation*.

Figure 1 illustrates the birth of a CLC, after the TS-bifurcation. For $\mu < 0$ the pseudo-equilibrium (red point) it's at Σ_{as} and has stable node dynamics; when $\mu = 0$ the pseudo-equilibrium collides with the TS-point (blue point); for $\mu > 0$ the pseudo-equilibrium it's at Σ_{rs} and has saddle dynamics. The CLC arises to $\mu > 0$ and is stable. This is only a particular case, once that the TS-bifurcation exhibits ten possible different dynamic scenarios, [**3, 4**].

The dynamics around the Teixeira singularity in the regions of sliding (Σ_{as} and Σ_{rs}) and crossing ($\Sigma_c^{\pm}$) is analysed for a generic DPWL dynamical system in $\mathbb{R}^3$, obtaining a canonical form and giving conditions for the occurrence of the TS-bifurcation. Some examples are given in order to illustrate the proposed algorithm of analysis, [**4**].

Keywords: TS-bifurcation, Teixeira singularity, Pseudo-equilibrium, Crossing limit cycle.

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Analysis of coherent cavitation in the liquid of light

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In this contribution we discuss one of the properties of the so-called liquid light, system which was reported theoretically for the first time in [**1**] and experimentally in [**2**]. In particular, we analyze numerically the phenomenon of coherent cavitation (i.e. the formation of cavities inside a liquid) in this model, studying the cubic (focusing)-quintic (defocusing) nonlinear Schrödinger equation (CQNLSE) in two transverse dimensions (2D) [**3**]. This equation reads:

$$i\partial_z\psi = -\nabla_\perp^2\psi - (|\psi|^2 - |\psi|^4)\psi,$$

where ψ is the dimensionless amplitude of the electromagnetic field, z is the propagation distance, ${\nabla_\perp}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the 2D transverse Laplacian operator and $(|\psi|^2 - |\psi|^4)\psi$ denotes the nonlinear (cubic-quintic) term. We start calculating the family of stationary travelling wave solutions of the CQNLSE, including rarefaction pulses and vortex-antivortex pairs (bubbles without and with vorticity respectively), in a background of critical amplitude. This computation is developed adapting the methods displayed in [**4**]. Subsequently, we study collisions between two solitons (i.e. droplets of liquid light) of significantly different sizes. Specifically, we consider one big quiescent soliton and a small soliton which is launched with an initial velocity to the previous one. We observe different scenarios in the evolution, which strongly depends on the imprinted initial velocity and the relative phase of the solitons. For velocities below a certain limit value, the most probable outcome is that the small soliton bounces back. Otherwise, a rarefaction pulse appears unless the relative phase is around an integer multiple of 2π at the collision. This rarefaction pulse or "dark void" can be generated in such a way that it exits the big soliton retransformed in a "bright soliton", performing the phenomenon of coherent cavitation (see Figure [1]). To conclude, we demonstrate that these rarefaction pulses in motion correspond to the previous ones of the family of stationary solutions.

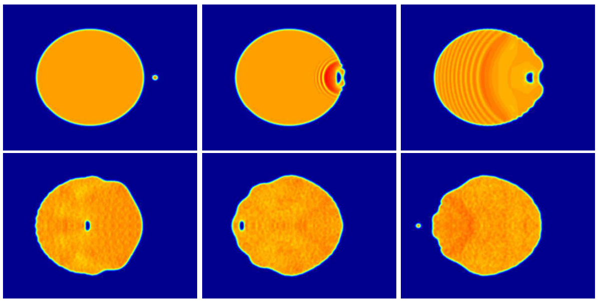

FIGURE 1. Different snapshots of the process of coherent cavitation in the liquid of light.

Keywords: coherent cavitation, liquid light, solitons, rarefaction pulses, bright-dark-bright reconversion.

Acknowledgments

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Simulation of Antiphase Dynamics in Lasers with Cellular Automata. A Work in Progress

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Cellular automata (CA) are a class of spatially and temporally discrete mathematical systems characterized by local interactions and synchronous dynamical evolution. They have the ability to generate complex behavior from sets of components that follow simple rules. xA wide variety of physical systems such as magnetization in solids, reaction diffusion processes, fluid dynamics, growth phenomena,... have been modeled by CA. In [**1**] a CA was proposed to simulate laser dynamics which was able to capture the essential features and phenomenology encountered in lasers such as relaxation oscillations, spiking behavior and pattern formation. The model can also simulate the physics of pulsed pumped lasers [**2**].

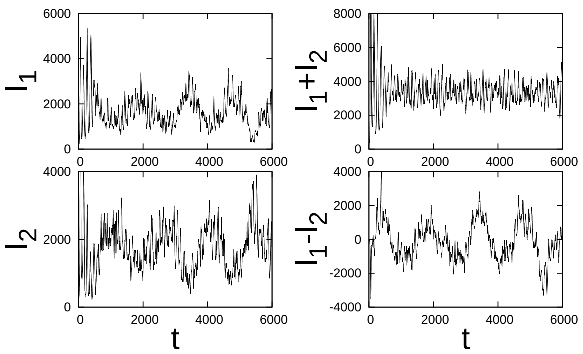

FIGURE 1. To model antiphase dynamics in lasers with a CA two laser subsystems are considered. Time series of the intensity of both populations, the total intensity and the difference of intensities. The pumping probability is $p = 0.01$ and the coupling parameter $\beta = 0.8$.

In this work we extend the original CA to study antiphase oscillations in lasers that was experimentally observed in a class B Nd:YAG laser [**3**]. The CA rule of evolution is composed of four different processes representing the pumping of electrons, the stimulated emission, and the decaying of electrons and photons. We also consider the laser to be composed of two subsystems associated with two orthogonal polarization eigenstates. Therefore, each one is described by its intensity $I_{1,2}$ and population inversion $D_{1,2}$. The two subsystems are coupled by cross-saturation phenomena: the intensity of one polarization is amplified by the population inversion associated to the other population and the stimulated emission in one polarization saturates the population inversion of the other polarization. Theoretically it has been shown that the total intensity $I_1 + I_2$ and the total population inversion $D_1 + D_2$ present relaxation oscillations with a frequency f_R while the difference $I_1 - I_2$ and $D_1 - D_2$ exhibit slow relaxation oscillations whose frequency is $f_L = \frac{1-\beta}{1+\beta} f_R$. Preliminary results of the CA simulations are shown in Figure 1. The classical point of view to study the laser dynamics are the Maxwell-Bloch equations however the CA models are complementary tools that represent an advantage in cases in which the system of differential equations have convergence problems.

Keywords: cellular automata, lasers, antiphase dynamics.

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Nonlinear Mathieu equation in particle accelerator physics

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In particle accelerator physics the so called *smooth approximation* [**1**] allows studying the particle's radial motions using equation (1),

$$\frac{d^2y}{ds^2}+[\mu_0^2+\alpha\sin(2\pi s)]\;y=0, \tag{21}$$

with $s = z/L$ the distance along the beam propagation axis normalized to the accelerator lattice length L, y the particle radial position, μ_0 the particle unperturbed phase advance per focusing period and the bracket second term a harmonic perturbation. This is a "classical" Mathieu equation traditionally written (2),

$$\frac{d^2y}{dx^2}+[p-2q\cos(2x)]\;y=0. \tag{22}$$

In the longitudinal phase plane, the acceleration generating the longitudinal focusing is done by radio-frequency cavities. The perturbed synchrotron motion [**2**] must be studied using the nonlinear Mathieu equation (3),

$$\frac{d^2y}{dx^2}+[p-2q\cos(2x)]\sin(y)=0. \tag{23}$$

This is the equation of motion of the pendulum with a periodically vibrating point of suspension in the vertical direction, a subject of theoretical, numerical and experimental studies for over a century ([**3**], [**4**]).

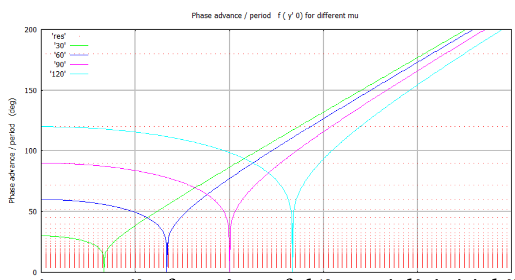

FIGURE 1. μ as a function of the particle initial position ($y0 = 0$, $y'0 = 0$ to 2.5)

Figure 1 shows the evolution of μ, the unperturbed phase advances per focusing period, as a function of the particle initial position ($y0$=0, $y'0$) for $\mu_0 = 30$, 60, 90 and 120°. It allows to predict the range and position of the resonances (red dots) in the oscillation and rotation phase-space areas.

Figures 2 shows the phase-space portraits for two perturbation levels when the phase advance per focusing period is $\mu(0) = 65°$.

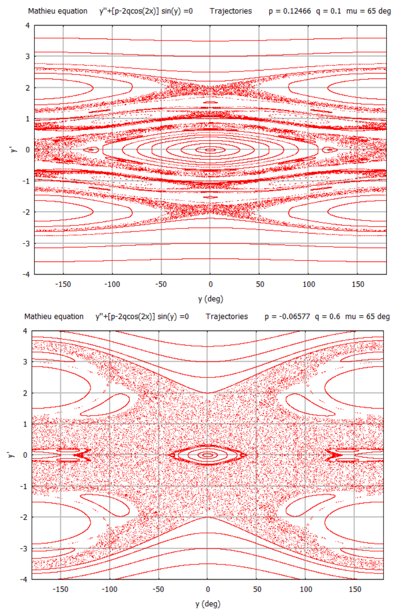

FIGURE 2. $\mu(0)$= 65° phase-space portraits, top: q = 0.1, bottom: $q = 0.6$

Keywords: particle accelerator physics, nonlinear Mathieu equation.

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Fractional diffusion equations modeling chemotaxis

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In this work, published in [**1**], we are interested in the microscopic description of fractional diffusion chemotactic models. We will use the kinetic framework of collisional equations having a heavy–tailed distribution as equilibrium state (see [**2**]) and take an adequate hydrodynamic scaling to deduce the fractional Keller–Segel system for the cell dynamics. In addition, we use this frame to deduce some models for chemotaxis with fractional diffusion including biological effects and non standard drift terms.

Keywords: fractional diffusion, chemotaxis, asymptotic limits.

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Stable nonlinear vortices in self-focusing Kerr media with nonlinear absorption

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We have studied the stability properties of nonlinear Bessel vortices in self-focusing Kerr media with nonlinear absorption. Nonlinear Bessel vortices are propagation-invariant solutions of the nonlinear Schrödinger equation with cubic nonlinearity and nonlinear absorption. We have found that these vortices can be stable against perturbations, including the azimuthal perturbations that usually break the cylindrical symmetry of standard vortex solitons in self-focusing Kerr media. This property is seen to arise from the stabilizing effect of nonlinear absorption.

Our model is the nonlinear Schrödinger equation

$$\partial_z A = i\Delta_\perp A + i\alpha|A|^2 A - |A|^{2M-2} M \quad (24)$$

where $\Delta_\perp = \partial_x^2 + \partial_y^2$, describing, for instance, light beam propagation in a self-focusing ($\alpha > 0$) Kerr medium with M-photon absorption.

Localized solutions with z-independent intensity profile $|A|^2$ of the form

$$A = a(r)\exp[i\phi(r)]\exp(-iz)\exp(is\varphi),$$

where (r,φ,z) are cylindrical coordinates and $s = 0, \pm1, \pm2, \ldots$ is the topological charge, do exist, and are nonlinear Bessel vortices, also called nonlinear unbalanced Bessel beams [**1, 2**]. These beams carry a vortex of the type $a(r)\exp[i\phi(r)] \simeq b J_s(r)$ about the origin $r = 0$, where b is a constant. Figure 1(a) shows an example of radial intensity profile compared to that of the Bessel profile $b^2 J_s^2(r)$.

A linearized stability analysis of these beams reveals that they may be stable against all type of small perturbations, including the most dangerous azimuthal perturbations usually leading to azimuthal breaking of vortex solitons in self-focusing Kerr media. Figure 1(b) illustrates that the growth rate of unstable radial ($m = 0$) and azimuthal modes ($m > 0$) vanish at positive values of the Kerr nonlinearity coefficient α.

This result has important implications in actual applications of nonlinear light beams, as laser material processing for waveguide writing or micromachining with vortex light beams [**2, 3**].

Keywords: spatial solitons, optical vortices, nonlinear optics.

Acknowledgments

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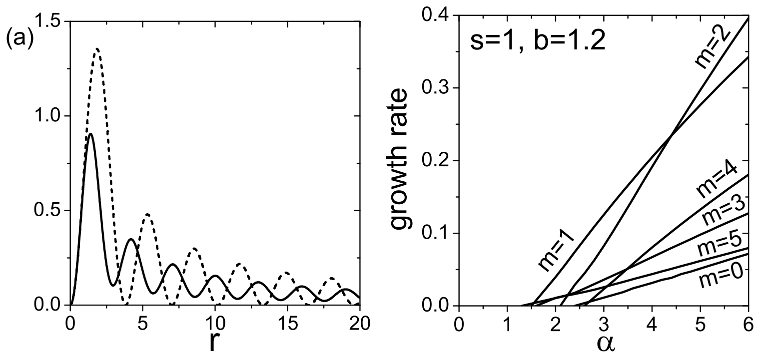

FIGURE 1. (a) Radial intensity profile of the nonlinear Bessel vortex beam with $M = 4$, $s = 1$, $b = 2$ and $\alpha = 1$ (solid curve) compared to the radial profile $b^2 J_s^2(\rho)$ of the linear Bessel beam with the same vortex core (dashed curve). (b) Growth rates of unstable azimuthal modes m of nonlinear Bessel vortices with $M = 4$, $s = 1$, $b = 1.2$ and different values of the Kerr nonlinearity α.

Two-component vortex solitons in photonic crystal fibres

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Photonic crystal fibers (FCF) have been very successful devices since their proposal because of their interesting properties, not present in conventional fibres[**1**]. They are summarized in the strong monomode character, the posibility of designing the minimum dispersion point and the fact that it is very easy to actuate on their birefringence properties. They are constituted by an array of holey cylinders running parallel to the fiber axis forming a periodic structure on the transversal plane with a central defect, which can be a lack of a hole (solid-core fibers) or a geometrically different one (hollow-core fibres). Solid-core PCFs are specially interesting for observing and exploiting nonlinear effects as they allow a high energy confinement inside the core and consequently a high energy density even for relatively small optical powers.

In the past it was demonstrated the existence of fundamental solitons and vortices, as well as vector (two-component) fundamental solitons which are stable under particular conditions[**2**]. In this work I demonstrate the existence of two-component incoherently coupled solitons showing angular momentum in at least one of the components. A calculation of the stationary states for different values of the coupling coefficient and the propagation constants shows that there exist combined systems of a fundamental and a vortex mode [see Fig. 1-(A,B)] as well as dual-vortex modes [Fig. 1-(C–E)]. They bifurcate from the single-component states[**3**] and exist for a particular range of propagation constant values. Vortex components can be found with a doughnut shape as well as a tripole shape since such discrete states partially have the symmetry of the PCF network. Tripole shapes may also be of two different types, one with the lobes facing the network inter-hole spaces (see for example panel B) and another one with lobes facing the holes (see panel D). In case of double-vortex states I also found double-tripoles with coincident lobes in both components (panel D) and with non-coincident ones (not shown). The latter ones only exist for low enough coupling coefficients. At high powers, the symmetry of the network can be partially broken and quadrupole modes appear (panel E).

Stability properties were studied numerically with the result that generally two component states are more stable than their single component counterparts. Also, for double-vortex states stability is higher for oposite vorticity between components (the so-called hidden vorticity states) than for same vorticity components. Unstable states develop the azimuthal instability decaying into fundamental solitons moving inside the PCF core, and may suffer further collapse for those less stable cases.

Keywords: vortices, photonic crystal fibres, vector solitons.

Acknowledgments

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Self-accelerating solution of NLS with parabolic potential

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We find self-accelerating solution of nonlinear Schrödinger equation (NLS) with a time-dependent parabolic potential, which models harmonically trapped Bose-Einstein condensate. We derive a formula for self-acceleration and show that the initial form of the wave function and an initial phase contribute the self-acceleration.

In 1979, Berry and Balazs theoretically showed that the Schrodinger equation describing a free particle admits a non-trivial Airy wave packet solution [**1**]. This free particle wave packet is unique in the sense that it accelerates. Furthermore, the Airy wave packet doesn't spread out as it accelerates. The Airy wave packet is also called self-accelerating wave packet since it accelerates in the absence of an external potential. The accelerating behavior is not consistent with the Ehrenfest theorem, which describes the motion of the center of mass of the wave packet. The reason of this inconsistency is the non-integrability of the Airy function. The self-accelerating Airy wave packet was also experimentally realized within the context of optics three decades after its theoretical prediction [**2, 3**]. In the present study, we consider the 1-D NLS equation for a BEC with time dependent nonlinear interaction strength in a time dependent harmonic trap

$$i\hbar\frac{\partial\psi}{\partial t}=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{m\omega^2(t)}{2}x^2+g(t)|\psi|^2\right)\psi \tag{25}$$

where $\omega(t)$ and $g(t)$ are the time dependent angular frequency and nonlinear interaction strength, respectively. We assume that the nonlinear interaction strength is positive for all time. In the non-interacting limit, the system obeys the Ehrenfest's theorem provided that the wave function is square integrable.

To get self-accelerating wave packet solution, let us first rewrite the GP equation in an accelerating frame, $x'=\dfrac{x-x_c(t)}{L(t)}$ where the time dependent function $x_c(t)$ describes translation and $L(t)$ is a time dependent dimensionless scale factor to be determined later. More precisely, we will see that the width of the wave packet changes according to $L(t)$. Initially, we take $L(0)=1$. Under this coordinate transformation, the time derivative operator transforms as $\partial_t \to \partial_t-(\dot{L}\ x'+\dot{x}_c)/L\ \partial_{x'}$, where dot denotes time derivation.

In the accelerating frame, we will seek the solution of the form $\psi(x',t)=\dfrac{1}{\sqrt{L}}\, e^{i\Lambda(x',t)}\,\psi(x')$, where the position dependent phase reads $\Lambda(x',t)=\dfrac{m}{\hbar}\left(\alpha x'+\dfrac{\beta}{2}x'^2+S\right)$ and the time dependent functions are given by $\alpha(t)=L\dot{x}_c$, $\beta(t)=L\dot{L}$ and $\dot{S}(t)=\dfrac{1}{2}\dot{x}_c{}^2-\dfrac{\omega^2}{2}x_c^2$. Substitute these transformations into the GP equation and assume that the following equations are satisfied $\ddot{L}+\omega^2(t)\,L=0$ and $\ddot{x}_c+\omega^2(t)\,x_c=\dfrac{a_0}{L^3}$ where $a_0>0$ is an arbitrary constant in units of acceleration that can be experimentally manipulated and L is the wave packet width.

Suppose the nonlinear interaction changes according to $g(t)=g_0/L(t)$, where $g_0>0$ is a constant. Then the time dependent GP equation is transformed to the time independent second Painleve equation. One can find a non-integrable stationary solution of this equation. By transforming backwards, the wave packet in the lab. frame can be obtained.

Keywords: self-acceleration, Airy waves.

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SUBJECT INDEX

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SCHEDULE

Conference Schedule

Oral communications (including questions): 25 minutes (20+5)

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9.30-10.00 Presentation	10.00-11.00 Plenary session. García Ojalvo	10.00-11.00 Plenary session. Desroches	10.30-11.30. Plenary session. Velarde (II)
10.00-11.00 Antonio Castellanos in memoriam. His life and work	11.00-11.30 Coffee break	11.00-11.30 Coffee break	11.30-12.00 Coffee break
11.30- 12.30 Plenary session. Velarde (I). In Spanish. (Science for the lay audience)	11.30-12.45 Scientific Sessions. S2.1 Archilla/Kosevich/ Russell	11.30-12.45 Scientific Sessions. S6.1 Lopesino/Balibrea-Iniesta/J.-Morales	12.00-13.15 Scientific Session. S10.1 Márquez-Durán/Urrutia/(***)
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	17.30-19.00 Scientific Session. S5.2 F.-García/Vela/ Pérez	18.00-19.15 Scientific Session. S9.2 Zolotaryuk/Moleron/Mehrem	
	20.00 Panoramic tour		
		21.00 Conference dinner	

Plenary sessions

1. Manuel G. Velarde (I). "Del surfeo en los ríos y en el mar al surfeo electrónico".
2. Francisco R. Villatoro. "Las ondas gravitacionales como ondas no lineales".
3. A. J. Sievers. "Shepherding intrinsic localized modes in microscopic and macroscopic nonlinear lattices".
4. Frank Jülicher. "Droplet formation in living cells".
5. Jordi Garcia-Ojalvo. "Dynamical regulation in living systems".
6. Mathieu Desroches. "Simplifying canard theory with piecewise-linear systems Applications to neuronal dynamics".
7. Eduardo Liz. "Complexity in discrete-time population models: other bifurcation diagrams are possible".
8. Leonor Cruzeiro. "The folding of a small protein".
9. Manuel G. Velarde (II). "From macrosurf (hydrodynamics) to nanosurf (electron transfer in crystals): a common line of nonlinear thinking with useful consequences".

Oral communications (including questions): 25 minutes (20+5)

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1. A. Henares-Molina, A. Martínez-González, S. Benzekry, VM Pérez-García. "Protracted metronomic therapies to target low-grade glioma malignant transformation".
2. D. Molina J. Pérez-Beteta, A. Martínez-González, E. Arana, L.A. Pérez-Romasanta, and V.M. Pérez-García. "Brain tumors: Textural heterogeneity as predictor of survival in Glioblastoma".
3. Gabriel F. Calvo, Arturo Álvarez-Arenas, Juan Belmonte-Beitia, Víctor M. Pérez-García. "Mathematical Modeling of the Emergence of Drug Resistance via Nonlinear and Nonlocal Exchange".

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1. Algaba A., Fuentes N. García C.. "Normal forms for a class of tridimensional vector fields with free-divergence in its first component".
2. A. Algaba, I. Checa, C. García, J. Giné. "Analytic integrability of some degenerate centers".

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1. Juan F.R. Archilla, Yaroslav O. Zolotaryuk, Yuriy A. Kosevich and Víctor J. Sánchez-Morcillo "Multiple lattice kinks in a cation lattice".
2. Yuriy A. Kosevich. "Ultradiscrete supersonic electron polarons in nonlinear molecular chains with realistic interatomic potentials and electron-phonon interaction".
3. F. Michael Russell. "Transport properties of quodons".

Session S2.2. Lecture room 2

1. A. Algaba, M.C. Domínguez-Moreno, E. Gamero, M. Merino, A.J. Rodríguez-Luis. "Takens-Bogdanov bifurcations and resonances of periodic orbits in the Lorenz system".
2. A. Algaba, C. Domínguez, M. Merino. "Analysis of the Hopf-zero bifurcation and their degenerations in a quasi-Lorenz system".
3. Juan Vicente Gutiérrez-Santacreu. "Potential singularities for the Navier-Stokes equations".

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1. Elena A. Korznikova, Sergey V. Dmitriev. "Discrete breathers in metals and ordered alloys".
2. Sergey V. Dmitriev. "Discrete breathers in crystals: energy localization and transport".
3. V. Hizhnyakov, M. Klopov, A. Shelkan. "Spatially localized modes in anharmonic lattices without gaps in phonon spectrum".

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1. L. Ramírez-Piscina, J.M. Sancho. "Statistical physics of active ionic channels".
2. Álvaro Lozano, Marcos Rodríguez, Roberto Barrio, Sergio Serrano, Andrey Shilnikov. "Control of bursting synchronization in Central Pattern Generators".
3. Roberto Barrio, M. Angeles Martínez, Sergio Serrano, Daniel Wilczak. "When chaos meets hyperchaos: a Computer-assisted proof".

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2. H. Michinel, A. Paredes. "Simulating Supermassive Black Holes in Coherent Nonlinear Systems".
3. A. Paredes, H. Michinel. "Nonlinear Dark Matter Waves".

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2. Francisco Pla, Yvon Maday, Henar Herrero. "Reduced Basis method for a bifurcation in a Rayleigh-Bénard convection problem at low aspect ratio".
3. Ramazan Yazgan, Cemil Tunc. "Pseudo almost periodic solution for Nicholson's blowflies model with patch structure and linear harvesting terms".

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2. R. Carretero-González, Wenlong Wang, R.M. Caplan, J.D.Talley, P.G. Kevrekidis, R.N. Bisset, C. Ticknor, D.J. Frantzeskakis, and L.A. Collins. "Vortex Rings in Bose-Einstein Condensates".
3. Jesús Cuevas-Maraver. "Solitons in the Nonlinear Dirac Equation".

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2. Enrique Ponce, Javier Ros, Elísabet Vela. “Boundary equilibrium bifurcations leading to limit cycles in piecewise linear systems”.
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2. Francisco Balibrea-Iniesta, Vícctor J. García-Garrido, Ana M. Mancho, Stephen Wiggins. “Arctic circulation from a Lagrangian perspective”.
3. F. Jiménez-Morales, M.C. Lemos. “Quasiperiodic Intermittency in a Surface Reaction Model”

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2. F. Balibrea, M.V. Caballero. “On autonomous and non-autonomous discrete versions of the Goodwin's model”.
3. Anna Karczewska. “On stochastic second order Korteweg - de Vries type equations”.

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2. F. Martin-Vergara, F. Rus, and F. R. Villatoro. “Kink--Antikink Collisions in the Kryuchkov--Kukhar' Equation”.
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3. A. Luque, R.Oulad Ben Zarouala, M.J. Ávila, M.E. Peralta. “Complexity of non linear robust design problems in control. Randomized Algorithms Approach”.

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2. D. Clamond, D. Dutykh, A. Durán. “Computation of capillary-gravity generalized solitary waves”.
3. Andrés Cantarero. “Nonlinear Raman scattering techniques”

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2. K. Morawetz, B. Kutschan, S. Thoms. “Formation of brine channels in sea-ice as habitat for micro-algae”.
3. Dario A. Zappalà, Giulio Tirabassi, Cristina Masoller. “Investigating Hilbert frequency dynamics and synchronisation in climate data”.

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2. Ismael Maroto, Carmen Núñez, Rafael Obaya. “Exponential stability for nonautonomous functional differential equations with state dependent delay. Applications to neural networks”
3. Piotr Rozmej, Anna Karczewska, Eryk Infeld. “Adiabatic invariants of second order Korteweg - de Vries type equation”.

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2. Miguel Molerón, Marc Serra-García, André Foehr, C. Chong, C. Daraio. “Dynamics of homogeneous and inhomogeneous nonlinear lattices formed by repelling magnets”.
3. A. Mehrem, N. Jiménez, L. J. Salmerón-Contreras, X. García-Andrés, R. Picó, L. M. García-Raffi, V. J. Sánchez-Morcillo. “Second harmonic generation in a chain of magnetic pendulums”.

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3. Vladimir I. Dubinko, Denis V. Laptev. “Heterogeneous catalysis driven by localized anharmonic vibrations”.