DECISION MAKING WIH DEMSPTER-SHAFER BELIEF STRUCTURE USING GENERALIZED AGGREGATION OPERATORS

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RESUMEN

Se presenta un método general para la toma de decisiones con la estructura de credibilidad de Dempster-Shafer mediante el uso de operadores de agregación generalizados. La principal ventaja de este nuevo modelo es la posibilidad de obtener una formulación mucho más completa en el marco de Dempster-Shafer porque se obtienen una amplia gama de casos particulares a través de utilizar medias generalizadas, cuasi-aritméticas y operadores OWA (medias ponderadas ordenadas). Por eso, podemos representar el problema mediante el uso de las medias aritméticas clásicas pero también mediante el uso de otro tipo de medias como las geométricas o las cuadráticas. Se estudian diferentes propiedades y casos particulares de estos nuevos operadores. Finalmente, se presenta un ejemplo ilustrativo sobre selección de estrategias.

Palabras clave: Toma de decisiones; Teoría de la evidencia de Dempster-Shafer; Operador OWA; Incertidumbre; Operadores de agregación generalizados.

ABSTRACT

We present a general method for decision making with Dempster-Shafer belief structure based on generalized aggregation operators. The main advantage of this approach is that it gives a more complete formulation of the D-S framework because it is able to provide a wide range of aggregation operators by using generalized means, quasi-arithmetic means and ordered weighted averaging (OWA) operators. Thus, we are able to formulate the D-S approach by using the usual arithmetic means but also using other types of means such as geometric or quadratic means. We study different properties and particular cases based on the generalized OWA operator. We further generalize this approach by using the Quasi-OWA operator. The paper ends with an illustrative example of the new approach in a decision making problem about selection of strategies.

Keywords: Decision making; Dempster-Shafer theory of evidence; OWA operator; Uncertainty; Generalized aggregation operators.

1. INTRODUCTION

The Dempster-Shafer (D-S) theory of evidence (Dempster, 1967; Shafer, 1976) provides a unifying framework for representing the uncertainty because it includes the cases of risk and ignorance as particular cases. Since its appearance, it has been studied by a lot of authors (Engemann et al., 1996; Merigó and Casanovas, 2007; 2008a; 2008b; Merigó et al., 2007; Srivastava and Mock, 2002; Yager, 1992; Yager et al., 1994; Yager and Liu, 2008). In decision making problems, it is very useful because it is able to include probabilistic information and situations of complete uncertainty in the same formulation. In (Yager, 1992), Yager developed an approach that used the ordered weighted averaging (OWA) operator in decision making with D-S framework. Thus, he provided a model that included a wide range of aggregation operators between the maximum and the minimum. This approach has also been studied by other authors such as (Casanovas and Merigó, 2007; Engemann et al., 1996; Merigó and Casanovas, 2007; 2008a; 2008b; Merigó et al., 2007).

The OWA operator (Yager, 1988) is an aggregation operator that provides a parameterized family of aggregation operators that includes the maximum, the minimum and the average, among others. Since its appearance, it has received increasing attention by a lot of authors and it has been applied in a lot of problems such as (Beliakov et al., 2007; Calvo et al., 2002; Fodor et al., 1995; Herrera et al., 2003; Karayiannis, 2000; Merigó, 2008; Merigó and Gil-Lafuente, 2009; Xu, 2006; Xu and Da, 2003; Yager, 1993; 1996; 2003; 2004; 2007; Yager and Filev, 1994; Yager and Kacprzyk, 1997).

Among the different extensions of the OWA operator, in this paper we will focus on the generalized OWA (GOWA) operator (Karayiannis, 2000; Yager, 2004; Merigó and Gil-Lafuente, 2009). It is a generalization of the OWA operator by using generalized means. Thus, it includes a lot of particular cases such as the generalized mean (GM), the average, the OWA operator, the ordered weighted geometric (OWG) (Herrera et al., 2003), and others. The GOWA operator can be further generalized by using quasi-arithmetic means. The result is the Quasi-OWA operator (Fodor et al., 1995). For further reading on the GOWA and the Quasi-OWA, refer, e.g., to (Beliakov, 2007; Fodor et al., 1995; Calvo et al., 2002; Merigó, 2008).

The aim of this paper is to present a new decision making process with D-S theory based on generalized aggregation operators. Thus, we are able to provide a more general formulation that includes a lot of different situations by using different particular cases of the GOWA operator. For example, it is possible to use the traditional OWA operator in the D-S framework (Yager, 1992), the OWG operator (Merigó and Casanovas, 2008) and other situations. We study some of the main properties of this approach and we find a new aggregation operator: the belief structure - GOWA (BS-GOWA) operator. It is an aggregation operator that aggregates the information found in the belief structure by using the GOWA operator.

We study the aggregation process with the GOWA operator and we find a wide range of particular cases. We see that this approach is very useful in a lot of situations according to the particular case of GOWA operator used. The reason for doing so is that we want to show that the decision maker has a wide range of alternatives to follow in the decision process according to his interests. Among others, we consider the olympic-GOWA, the centered-GOWA, the GM, and the S-GOWA operator.

We further generalize this approach by using quasi-arithmetic means. As a result we get the BS-Quasi-OWA operator. The main advantage of this model is that we get a more general formulation of the D-S framework that includes the BS-GOWA operator as a particular case.

This paper is organized as follows. In Section 2, we briefly review some basic concepts about the D-S theory, the GOWA and the Quasi-OWA operator. In Section 3, we present the new approach about using the GOWA operator in decision making with D-S belief structure. Section 4 presents a more general formulation by using quasi-arithmetic means. In Section 5 we end the paper summarizing the main results.

2. PRELIMINARIES

In this Section, we briefly review some basic concepts about the D-S theory of evidence, the GOWA and the Quasi-OWA operator.

2.1. DEMPSTER-SHAFER THEORY OF EVIDENCE

The D-S theory of evidence was suggested by (Dempster, 1967; Shafer, 1976). Since then, a lot of new developments have been developed about it such as (Engemann et al., 1996; Merigó and Casanovas, 2007; 2008a; 2008b; Merigó et al., 2007; Reformat and Yager, 2008; Srivastava and Mock, 2002; Yager, 1992; 2004; Yager et al., 1994; Yager and Liu, 2008). This type of formulation provides a unifying framework for representing uncertainty as it can include the cases of risk and ignorance as special situations of this framework. Apart from these traditional cases, the D-S framework allows to represent various other forms of information that a decision maker may have about the states of nature. It can be defined as follows.

Definition 1. A D-S belief structure defined on a space X consists of a collection of n nonnull subsets of X, B_j for $j = 1,\ldots,n$, called focal elements and a mapping m, called the basic probability assignment, defined as, $m: 2^X \to [0, 1]$ such that:

- $m(B_j) \in [0, 1]$.
- $m(A) = 0$, $\forall A \neq B_j$.
- $\sum_{j=1}^{n} m(B_j) = 1$.

The cases of risk and ignorance are included as special cases of belief structure in the D-S framework. For the case of risk, a belief structure is called Bayesian belief structure, if it consists of n focal elements such that $B_j = \{x_j\}$, where each focal element is a singleton. Then, we can see that we are in a situation of decision making under risk environment as $m(B_j) = P_j = \text{Prob}\{x_j\}$.

For the case of ignorance, the belief structure consists in only one focal element B, where $m(B)$ essentially is the decision making under ignorance environment as this focal element comprises all the states of nature. Thus, $m(B) = 1$. Other cases of belief structures are studied in (Shafer, 1976).

2.2. GENERALIZED OWA AND QUASI-OWA OPERATOR

The generalized OWA (GOWA) operator was introduced in (Karayiannis, 2000; Yager, 2004; Merigó and Gil-Lafuente, 2009). It generalizes a wide range of aggregation operators that includes the OWA operator with its particular cases, and a lot of other cases. It can be defined as follows.

Definition 2. A GOWA operator of dimension n is a mapping GOWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, and such that:

$$\text{GOWA}(a_1, a_2, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda} \right)^{1/\lambda} \tag{1}$$

where b_j is the jth largest of the a_i, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

The GOWA operator can be further generalized by using quasi-arithmetic means. The result is the Quasi-OWA operator (Fodor et al., 1995). It can be defined as follows.

Definition 3. A Quasi-OWA operator of dimension n is a mapping QOWA: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n such that the sum of the weights is 1 and $w_j \in [0, 1]$, according to the following formula:

$$\text{QOWA}(a_1, a_2, \ldots, a_n) = g^{-1} \left(\sum_{j=1}^{n} w_j g(b_j) \right) \tag{2}$$

where b_j is the jth largest of the a_i, and g is a strictly continuous monotonic function.

3. DECISION MAKING WITH D-S THEORY USING GENERALIZED AGGREGATION OPERATORS

In this Section, we present the new approach about using generalized aggregation operators in decision making with D-S belief structure. First, we describe the decision making process. Second, we analyze the aggregation process. And third, we study some families of BS-GOWA operators.

3.1. DECISION MAKING APPROACH

The problem of decision making with D-S theory of evidence has been studied by different authors such as (Engemann et al., 1996; Merigó and Casanovas, 2007; 2008a; 2008b; Merigó et al., 2007; Yager, 1992; 2004; Yager et al., 1994; Yager and Liu, 2008). In (Yager, 1992), Yager suggested the use of the OWA operator in D-S framework. Then, he provided a more complete aggregation model that was able to consider a wide range of aggregation operators.

In this paper, we suggest a new method that uses the GOWA operator. The reason for using this aggregation operator is that we get a more complete formulation of the decision process because it generalizes a wide range of operators such as the OWA operator, the OWG operator, the WGM, and others. Therefore, by using this approach the decision maker gets a more complete view of the decision problem because he can consider a wide range of scenarios and select the one that is in accordance with his interests. The decision process can be summarized as follows.

Assume we have a decision problem in which we have a collection of alternatives $\{A_1, \ldots, A_q\}$ with states of nature $\{S_1, \ldots, S_n\}$. a_{ik} is the payoff to the decision maker if he selects alternative A_i and the state of nature is S_k. The knowledge of the state of nature is captured in terms of a belief structure m with focal elements $B_1, \ldots, B_r$ and associated with each of these focal elements is a weight $m(B_k)$. The objective of the problem is to select the alternative which best satisfies the payoff to the decision maker. In order to do so, we should follow the following steps:

Step1: Calculate the payoff matrix.

Step2: Determine the belief function m about the states of nature and the decision makers degree of optimism α.

Step3: Calculate the collection of weights, w, to be used in the GOWA aggregation for each different cardinality of focal elements.

Step4: Determine the payoff collection, M_{ik}, if we select alternative A_i and the focal element B_k occurs, for all the values of i and k. Hence $M_{ik} = \{a_{ik} \mid S_k \in B_k\}$.

Step5: Calculate the aggregated payoff, V_{ik} = GOWA(M_{ik}), using Eq. (1), for all the values of i and k. Note that it is possible to use for each focal element a different type of GOWA operator. That is, for each focal element we can use a different weighting vector W.

Step6: For each alternative, calculate the generalized expected value, C_i, where:

$$C_i = \sum_{k=1}^{r} V_{ik}\, m(B_k) \quad (3)$$

Step7: Select the alternative with the largest C_i as the optimal. Note that it is possible to establish an order of the results obtained.

Remark 1. Note that sometimes it is better to use the AGOWA operator instead of the GOWA (or DGOWA) operator. The main reason for doing so is because we have to distinguish between situations where the highest value is the best result and situations where the lowest value is the best result.

3.2. BELIEF STRUCTURES WITH THE GOWA OPERATOR

Analyzing the aggregation in Steps 6 and 7 of Section 3.1., it is possible to formulate in one equation the whole aggregation process. Then, the result obtained is that the focal weights are aggregating the results obtained by using the GOWA operator. We will call this process the belief structure - GOWA (BS-GOWA) aggregation. It can be defined as follows.

Definition 4. A BS-GOWA operator is defined by

$$C_i = \left(\sum_{k=1}^{r} \sum_{j_k=1}^{q_k} m(B_k) w_{j_k} b_{j_k}^{\lambda} \right)^{1/\lambda} \tag{4}$$

where w_{j_k} is the weighting vector of the kth focal element such that $\sum_{j=1}^{n} w_{j_k} = 1$ and $w_{j_k} \in [0, 1]$, b_{j_k} is the j_kth largest of the a_{i_k}, $m(B_k)$ is the basic probability assignment, and λ is a parameter such that $\lambda \in (-\infty, \infty)$.

Note that q_k refers to the cardinality of each focal element and r is the total number of focal elements. The BS-GOWA operator is monotonic, commutative, bounded and idempotent.

3.3. FAMILIES OF BS-GOWA OPERATORS

Different types of GOWA operators may be used in the aggregation of the BS-GOWA operator. Mainly, we can distinguish between those families found in the weighting vector W and those found in the parameter λ.

Remark 2. By looking to the parameter λ, we find the following particular cases:

- The OWA operator if $\lambda = 1$ (arithmetic).
- The OWG operator if λ approaches to 0 (geometric).
- The OWQA operator if $\lambda = 2$ (quadratic).
- The OWHA operator if $\lambda = -1$ (harmonic).
- Etc.

Remark 3. If we analyse the weighting vector W, then, we find the following cases:

- The maximum ($w_1 = 1$ and $w_j = 0$, for all $j \neq 1$).
- The minimum ($w_n = 1$ and $w_j = 0$, for all $j \neq n$).
- The GM ($w_j = 1/n$, for all a_i).
- The WGM (the ordered position of i is the same as the ordered position of j).

The generalized Hurwicz criteria ($w_1 = \alpha$, $w_n = 1 - \alpha$ and $w_j = 0$, for all $j \neq 1, n$).

- The step-GOWA ($w_k = 1$ and $w_j = 0$, for all $j \neq k$).
- The olympic-GOWA operator ($w_1 = w_n = 0$, and $w_j = 1/(n - 2)$ for all others).
- The general olympic-GOWA operator ($w_j = 0$ for $j = 1, 2, \ldots, k, n, n - 1, \ldots, n - k + 1$; and for all others $w_{j^*} = 1/(n - 2k)$, where $k < n/2$).
- The S-GOWA ($w_1 = (1/n)(1 - (\alpha + \beta)) + \alpha$, $w_n = (1/n)(1 - (\alpha + \beta)) + \beta$, and $w_j = (1/n)(1 - (\alpha + \beta))$ for $j = 2$ to $n - 1$ where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$).
- The centered-GOWA (if it is symmetric, strongly decaying from the center to the maximum and the minimum, and inclusive).
- Etc.

Remark 4. Using a similar methodology, we could develop a lot of other families of GOWA weights in a similar way as it has been developed in a lot of studies for the OWA operator such as (Beliakov et al., 2007; Calvo et al., 2002; Karayiannis, 2000; Merigó, 2008; Merigó and Gil-Lafuente, 2009; Xu and Da, 2003; Yager, 1993; 1996; 2003; 2007; Yager and Filev, 1994).

Remark 5. Note that it is possible to use different families of GOWA operators for each focal element of the decision problem. If we strictly use only one case in the whole aggregation, then, we could refer to it as the BS-centered-GOWA, BS-olympic-GOWA, BS-S-GOWA, BS-GM, BS-WGM, etc.

4. QUASI-OWA OPERATORS IN D-S FRAMEWORK

Although we have already considered a wide range of aggregation operators that can be used in the D-S framework, it is interesting to present a more general formulation that includes more types of aggregation operators by using quasi-arithmetic means. The main advantage of using this approach is that it generalizes the GOWA and it includes a lot of other particular cases. Therefore, the decision maker gets a more complete view of the decision problem because he is able to consider a lot of different situations and select the one that is in accordance with his interests. By using this approach in decision making with D-S belief structure, we are able to develop a unifying framework that provides a general formulation with probabilities and different types of OWAs.

In order to use this type of aggregation operator in D-S framework, we should make the following changes to the decision process explained in the previous section for the GOWA operator.

In Step3, when calculating the collection of weights, W, we have to consider that we are using the Quasi-OWA operator in the aggregation for each different cardinality of focal elements.

In Step5, when calculating the aggregated payoff, we should use V_{ik} = QOWA(M_{ik}), using Eq. (2) for all the values of i and k.

In this case, we could also formulate in one equation the whole aggregation process as follows. We will call it the BS-Quasi-OWA operator.

Definition 5. A BS-Quasi-OWA operator is defined by

$$C_i = g^{-1}\left(\sum_{k=1}^{r}\sum_{j_k=1}^{q_k} m(B_k) w_{j_k} g(b_{j_k})\right) \tag{5}$$

where w_{j_k} is the weighting vector of the kth focal element such that $\sum_{j=1}^{n} w_{j_k} = 1$ and $w_{j_k} \in [0, 1]$, b_{j_k} is the j_kth largest of the a_{i_k}, $m(B_k)$ is the basic probability assignment, and g is a strictly continuous monotonic function.

Remark 6: Note that the BS-Quasi-OWA operator has similar properties than the BS-GOWA operator.

5. APPLICATION IN STRATEGIC DECISION MAKING

In the following, we are going to develop an application of the new approach in a decision making problem. We will develop an application in the selection of strategies. Note that other decision making applications could be developed such as the selection of financial products, human resources, assets, etc.

Assume a company that wants to invest some money in a market and they consider 5 general strategies to follow.

- A_1 = Invest in the American market.
- A_2 = Invest in the European market.
- A_3 = Invest in the Asian market.
- A_4 = Do not develop any investment.

In order to evaluate these strategies, the company uses a group of experts. They consider that the key factor is the economic situation of the company for the next year. After careful analysis, the experts have considered five possible situations that could happen in the future: S_1 = Very good, S_2 = Good, S_3 = Normal, S_4 = Bad, S_5 = Very bad.

Depending on the uncertain situations that could happen in the future, the experts establish the payoff matrix. The results are shown in Table 1.

Table 1: Payoff matrix

	S_1	S_2	S_3	S_4	S_5
A_1	60	70	30	40	60
A_2	80	40	90	20	40
A_3	30	50	80	90	20
A_4	40	30	80	70	40

After careful analysis of the information, the experts have obtained some probabilistic information about which state of nature will happen in the future. This information is represented by the following belief structure about the states of nature.

Focal element

$B_1 = \{S_2, S_3, S_4\} = 0.3$

$B_2 = \{S_1, S_2, S_3\} = 0.3$

$B_3 = \{S_1, S_2, S_3, S_4\} = 0.4$

The experts establish the following weighting vectors for the aggregation operators to be used in the problem. In this example, we will use the arithmetic mean (AM), the weighted average (WA), the OWA operator, the quadratic mean (QA) and the OWQA operator. With this information, we can obtain the aggregated payoffs. The results are shown in Table 3.

Weighting vector

$W_3 = (0.3, 0.3, 0.4)$

$W_4 = (0.2, 0.2, 0.3, 0.3)$

$W_5 = (0.1,0.2, 0.2, 0.2, 0.3)$

Table 2: Aggregated payoffs

	AM	WA	OWA	QA	OWQA
V_{11}	46.6	46	45	49.66	48.06
V_{12}	63.3	63	63	63.5	63.16
V_{13}	50	47	47	52.44	49.49
V_{21}	50	47	47	58.02	55.4
V_{22}	53.3	52	52	56.56	55.13
V_{23}	57.5	57	52	64.22	59.16
V_{31}	73.3	75	71	75.27	73.14
V_{32}	33.3	32	32	35.59	34.35
V_{33}	62.5	67	58	66.89	62.61
V_{41}	60	61	57	63.77	61.23
V_{42}	36.6	37	36	36.96	36.33
V_{43}	55	59	51	58.73	54.86

Once we have the aggregated results, we have to calculate the generalized expected value. The results are shown in Table 4.

Table 3: Generalized expected value

	AM	WA	OWA	QA	OWQA
A_1	53	51.5	51.2	54.92	53.16
A_2	54	52.5	50.5	60.06	56.82
A_3	57	58.9	54.1	60.01	57.29
A_4	51	53	48.3	53.71	51.21

As we can see, depending on the aggregation operator used, the results and decisions may be different. A further interesting issue is to establish an ordering of the financial strategies. Note that this is very useful when the decision maker wants to consider more than one alternative. The results are shown in Table 5.

Table 4: Ordering of the strategies

	Ordering		Ordering
AM	$A_3 \geq A_2 \geq A_1 \geq A_4$	QA	$A_2 \geq A_3 \geq A_1 \geq A_4$
WA	$A_3 \geq A_4 \geq A_2 \geq A_1$	OWQA	$A_3 \geq A_2 \geq A_1 \geq A_4$
OWA	$A_3 \geq A_1 \geq A_2 \geq A_4$		

As we can see, the ordering of the strategies may be different depending on the aggregation operator used. In this example, we see that in most of the cases, A_3 seems to be the optimal choice. However, it is not clear which is the second best and so on.

6. CONCLUSIONS

We have presented a new approach for decision making with D-S belief structure by using generalized aggregation operators. The main advantage of this method is that it provides a general formulation that includes a lot of situations by using different particular cases of the GOWA operator. Therefore, we have been able to provide a more complete model for the decision maker because he is able to consider more scenarios and select the one that it is in accordance with his interests. We have studied some of the main properties of this approach and we have analyzed a wide range of particular cases of the GOWA operator to be used in the D-S framework. Although today it seems more useful to use the traditional arithmetic aggregation operators such as the OWA (a particular case of the GOWA), it is interesting to see the general formulation given in this paper as a general model of the decision process with D-S.

We have further generalized this approach by using quasi-arithmetic means. Thus, we have obtained a more general formulation that includes the BS-GOWA operator as a particular case and a lot of other situations.

In future research, we expect to develop further extensions of this approach by using other extensions of the GOWA operator. For example, we will consider the use of order-inducing variables and uncertain information (interval numbers, fuzzy numbers, linguistic variables, etc.) in the decision process. We will also consider the applicability of this approach in a real decision making problem such as in investment selection, product management, strategic decision making, etc.

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