Regression Modeling for Digital Test of $\Sigma\Delta$ Modulators

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***Abstract*—The cost of Analogue and Mixed-Signal circuit testing is an important bottleneck in the industry, due to time-consuming verification of specifications that require state-of-the-art Automatic Test Equipment. In this paper, we apply the concept of Alternate Test to achieve digital testing of $\Sigma\Delta$ converters. By training an ensemble of regression models that maps simple digital defect-oriented signatures onto Signal to Noise and Distortion Ratio (SNDR), an average error of 1.7% is achieved. Beyond the inference of functional metrics, we show that the approach can provide interesting diagnosis information.**

I. Introduction

Model-based testing is an old research path that has primarily been developed to reduce the number of tests in the test plan. The principal idea was that many specifications were likely to be highly correlated. A trivial example of this would be the Integral Non-Linearity (INL) and Total Harmonic Distortion (THD) in a data converter, which are both measurements of linearity. A more subtle thinking in terms of degradation mechanisms may lead to the same conclusion: conceptually different specifications may be affected by the same defects or parametric variations and are thus likely to be correlated.

An early approach has consisted in building a linear model to either infer the complete set of specifications from a reduced set of measurements [1]–[3] or relax the precision requirements on individual measurements [4], using correlations as a sort of averaging to reduce total noise. Such linear models work reasonably well in a wide variety of situations because the circuits are usually designed with some guardbands and the parametric variations are expected to induce only small deviations around the "operating point". The linear model can thus be understood as a first order Taylor development. While a sensitivity analysis would retrieve the influence of degradation mechanisms on each specification, blind modeling approaches only extract the consequences of these common causes in the form of correlations. Instead of performing expansive specification measurements, it has also been proposed to use simple, cost-effective measurements to infer the circuit performance [5]. In an attempt to refine model-based approach and to extend the validity range beyond the linear limits, other statistical models have been proposed. Neural Networks have been extensively used not only for regression [6] but also for classification [7], [8]. Nearest-neighbors approaches have also been reported. In [9], for instance, the technique is used to estimate the joint probability of the parameters and measurement. Finally, many papers rely on multivariate adaptive splines (MARS) [10], which are particularly well suited for building models in high dimensional spaces but with local correlations between few variables. Combining non-linear advanced statistical models and simple measurements forms the basement of the Alternate Test concept that has gained much attention in the past few years [11]–[13].

In this paper, we will apply this concept to the test of $\Sigma\Delta$ converters. This is a challenging task since the behavior of $\Sigma\Delta$ modulator is in essence highly non-linear. For this purpose, we use a set of digital signatures that have been proposed in [14]–[17], as detailed in Section 2. These digital tests have been designed to capture the non-idealities of the modulator building blocks and thus lend themselves well to Alternate Test. In order to build the regression model, we will make use of a statistical tool called mixed ensemble learning [18]. This in an advanced tool in the sense that it trains several models over the available data and combines the outcome to give composite model whose generalization error can be demonstrated to be lower than the individual models. This tool will be introduced in Section 3. Finally Section 4 discusses the results obtained for a 2-1 cascaded modulator.

II. Digital tests of $\Sigma\Delta$ modulators

A. Test generation

The generation and validation of simple digital test for the detection of important parametric variations in $\Sigma\Delta$ modulators have been described in detail in previous articles [14]–[16]. Here, we will only briefly review the main concepts and describe the high-level simulations that we have carried out for model-based testing purpose.

Figure 1 shows a possible implementation of a 2-1 cascaded modulator. The first stage is a simple structure, with two delaying integrators of gain 0.5 and unit feedback coefficient. The second stage makes use of the same integrator and is fed directly with the second integrator output. Hence, the reconstruction filter $R(z)$ must be,

$$R(z) = z^{-1}Y_1 + \left(z^{-1}Y_1 - 4Y_2\right) \times \left(1 - z^{-1}\right)^2 \qquad (1)$$

where Y_1 and Y_2 are the first and second stage output bit-streams, respectively.

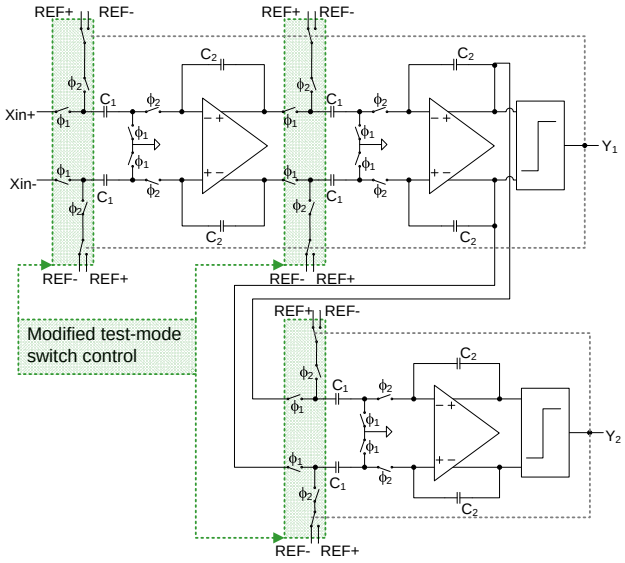

Fig. 1. Schematic of the 2-1 $\Sigma\Delta$ modulator used for digital tests. The input switches and feedback DAC of each integrator admit a modified control in test mode.

In order to generate the different digital signatures, the modulator must implement some Design-for-Test (DfT) modifications. Namely, the control of the input switches and the feedback DAC for each integrator are modified during test modes. These modifications only require additional digital gates and do not alter the design of the analog signal path. The main idea is that the nominal input of the integrators can be disconnected while the feedback DAC can be re-used during the sampling phase to input a digital test sequence of our choice. Using test sequences of different mean values, we can generate signatures that are sensitive to the pole error and output range of the different integrators.

Another important modification is that the master clock that drives the non-overlapping phase generator can be controlled and correlated to the test sequence. The clock period for a digital one input (alternatively a digital zero) can be made last twice as long as a clock period for a digital zero input (alternatively a digital one). In this way, the integrator has twice as much time to properly settle for a one input as for a zero. This simple correlation makes the settling error appear as a DC component in the modulator output.

The feedback DAC is also modified to include the possibility to input the common-mode voltage instead of the voltage references. The differential DAC in test mode is thus three-valued (-1, 1 and 0, referred to Full-Scale). This modification brings the opportunity to generate a signature that is mainly sensitive to the non-linear part of the settling error [19].

All these modifications are discussed in detail in [16]. For all the proposed tests, the effects of non-idealities appear in the DC component of the signatures. The initial proposals focused on BIST implementations and proposed to use simple counters to extract this DC component. However, it has been shown in [17] that using higher order filtering is a better approach to reduce signature uncertainty.

For our regression purpose, we perform a total of 27 tests, some of which in parallel:

- Two leakage tests with a digital sequence of mean value $2/3$ (referred to full-scale) and its opposite, for each integrator.
- Two leakage tests with a sequence of mean value $1/3$ and its opposite, only for the first integrator.
- Two settling tests using a pseudorandom sequence and doubling the clock period for the 1 and then for the -1 input samples. These tests are performed at nominal frequency, for each integrator, and repeated at 120% of the nominal frequency.
- Two settling tests using a random sequence of 1 and 0 (using the DAC modification that allows to send the common-mode voltage), for each integrator. These tests do not require clock modification.
- A noise test for the overall modulator, with a sequence of zero mean value. Here we measure the mean value and standard deviation of the converter output, after the decimation filter (a 4^{th} order Comb filter).

For each acquisition, the DC component at the stage outputs (either Y_1 of Y_2) are retrieved by a 3^{rd} order Comb filter with an OSR of 6000. We thus have to simulate at least 18001 points for each acquisition to account for filter settling time. The last test of the list uses the decimation filter of the converter, with an OSR of 64. Simulating 18000 samples thus provides 276 samples to calculate the standard deviation and the mean value. From all these acquisitions, we generate a total of 18 different signatures that are, in principle, sensitive to the offset, leakage and settling of the three integrators, to the output range of the first integrator and to the excess noise of the overall converter.

In previous works, we have derived analytical closed-form expressions that relate these signatures to some important behavioral parameters of $\Sigma\Delta$ modulators. This is important because we know that there is a relation between signatures and performance degradation mechanisms. However, in this paper we want to build a blind regression model without assuming any analytical first-order relationship.

B. High-level simulation

Machine-learning approaches require a training set of devices that exercise the parameter space in a wide range. The model generalization error grows significantly if only few samples are available. This is a general rule that find its roots in the well-known "Curse of Dimensionality", though it affects different models in greater or lesser extent.

In order to verify that the digital tests have the potential to correctly predict performance, we must try to avoid model learning limitations and thus use a large training set. Ideally, we could perform Monte-Carlo simulations on the electrical schematic or even the extracted layout of the circuit. However, this would be extremely time-consuming. In order to get more flexibility, high-level simulations using Matlab have been preferred.

It has been demonstrated in various papers that most relevant effects can be modeled in event-driven simulators and give accurate results, in accordance to electrical simulations [20]. All our digital tests rely on the processing of a digital sequence – sent by the re-used feedback DAC – by a $\Sigma\Delta$ modulator. The behavioral model is thus adequate for both the nominal operating mode and the digital tests.

We thus built a high-level model of our 2-1 modulator considering the following effects:

- Amplifier offset, finite DC gain, static non-linearity, Slew-Rate and Gain-Bandwidth product
- Integrator coefficient error and noise (both kT/C and amplifier noise)
- Comparator offset and hysteresis

For three integrators and two comparators, this leads to a total of 28 parameters. We thus performed a Monte-Carlo simulation of 1200 runs, sampling the 28-dimensions parameter space using Latin Hyper-Square method. For each run we perform the set of tests proposed in previous subsection. In addition, we also perform a classical functional test. The input signal is a half-scale sine-wave. The output of the decimation filter (with $OSR = 64$) is processed to compute the SNDR from the FFT over 512 samples. The objective is thus to map the 18-dimension signature space to the SNDR.

III. Ensemble Learning

Machine-learning, regression modeling, function approximation, data mining, all this terminology belongs to the vast mathematical field of statistics. Researchers have been struggling to develop the best modeling approach from more than a hundred years. Unfortunately, the idea of best model is always relative to the application and nobody has come out with the definitive approach. Some models perform better on low-dimension spaces, other require few training samples, etc. Actually, the task of model selection has already been investigated (see Chapter 7 in [21]), and a number of criteria have been developed to assess model quality, usually in terms of expected prediction error. Anyhow, managing these concepts is not an easy task to the profane. As a matter of fact most papers that apply machine-learning algorithms to circuit testing usually do not justify the choice of their statistical tool.

In this context, the concept of ensemble learning is very appealing because it builds a mosaic model from a collection of statistical tools. It implements a routine that trains different models using cross-validation principles to deduce the expected prediction error. The final model is a weighted average of a subset of all the trained models, being the weights a function of the calculated prediction error.

We use a Matlab toolbox developed by Wichard and Merkwirth [18], which itself uses elements of [22]. Let us briefly present the different models that can be trained by the toolbox.

A. Linear and Polynomial models

The most straightforward model is the linear one. As explained in the introduction, the linear model expresses the measurement vector – in our case the SNDR of the $\Sigma\Delta$ converter – as a linear combination of the input vectors – in our case the digital signatures. The regression is performed in the least-square sense. The validity range of linear models is usually limited as they are not able to capture non-linear interactions. The results are usually highly biased but present rather low variability, and the generalization error is usually quite close to the training error. Conceptually, the bias is potentially high because the model imposes a strong constraint on the relationship between the measurement vector and the input vectors (i.e. linearity). On the other hand, this strong constraint also ensures that no unreasonable outlier will be produced by the model. In particular, if the model is exercised out of the training range, it may still lead to reasonable asymptotical results, which is not the case for more flexible models.

A simple modification of linear model is the polynomial model. Here the number of input vectors is extended by adding the product of two or more input vectors. Taking into account these new vectors, a linear regression is then performed. Obviously, this approach is better suited for input spaces of low dimensions. Otherwise, even considering low order polynomials terms would lead to a large number of input vectors. The toolbox limits the polynomial degree to the lowest order that generates a number of variables higher than a given threshold. In our case, we have 18 variables and the polynomial is limited to the 3^{rd} degree. A forward stepwise algorithm then searches all the variables among the available ones that improve fitting and add them to the pool of useful variables on a step by step basis. Then, a backward stepwise algorithm tries to remove one variable at a time to avoid redundancy and overfitting.

B. Nearest-neighbors

In a sense, Nearest-Neighbors approaches are the complete opposite to linear modeling. In this approach, no implicit structure is supposed in the data and the model prediction is more an interpolation of the training data than the result of a given function. The prediction for a given input is simply equal to the value of the nearest neighbor in the training set. As a result, Nearest neighbors approaches must store the complete training set in order to make further predictions. The result is a model with low bias but high variability. This means that it adapts very well to local fluctuations but on the other hand is very sensitive to the non-ideality of the available data like noise, outliers or simply density. Indeed, it appears obvious that the larger the training set – which supposedly samples the input space adequately – the better the results. This is particularly troublesome in high-dimensional spaces. Refinements of the method usually rely on defining a larger neighborhood and considering some kind of weighted average (accordingly to a given kernel, for instance Gaussian or Epanechnikovs kernel). It is also possible to adapt the size of the neighborhood to the local sampling density to mitigate the effects of noise and outliers in regions with sparse data. The toolbox uses a genetic algorithm to select the optimal methodology, varying the number of nearest neighbors and

also the metric used for distance calculation and for local weighting.

C. Neural Networks

The field of function approximation through Neural Networks is very extended and many different networks topologies have been investigated. The toolbox implements two families of Neural Networks. The first family is that of the well-known Perceptron Networks, which involve only feedforward layers of neurons. The multilayer perceptrons can be trained with either 1^{st} order of 2^{nd} order gradient decent. The second family is that of Radial Basis Function Networks. While Perceptron networks use linear or sigmoid activation functions for regression, RBF networks use radial activation functions that are parameterized at least by their centers and radii. Several training algorithm exist that give different results. The best known training method is the Expectation Maximization (EM) algorithm which optimizes an initial guess. A simplified version of it is the David MacKay (DM) algorithm, which is faster but has no formal proof of convergence. And finally the combination of Regression Trees with RBF network [23] has given good results. A regression tree is grown to split the input space in smaller hyperrectangles which will then determine the center and size of the RBFs. The size of the tree is limited by a parameter that defines the minimum number of samples in a split. Furthermore, the complexity of the model is then reduced by pruning which is done by forward selection and can be combined with backward elimination. Finally, Perceptron Radial Basis Function Nets (PRBFN) [24] are, as its name indicates, a neural network with hidden layers composed of a mixture of perceptron and RBF units. The motivation of this approach is that it has been shown that a function can be decomposed into exclusive radial and projection parts. The RBF units thus tend to capture the radial components while the ridge (perceptron) units try to reproduce the projection part.

D. Multivariate Adaptive Regression Splines

The MARS approach proposed by Friedman [10] has been used successfully in a large number of papers in the past few years [11]–[13]. As its name indicates, it is based on common regression splines. For high-dimensional data, only low-order splines are considered (typically lower than three)in order to limit the complexity of the model. The Adaptive Multivariate part of the name comes from the recursive partitioning of the input space, in a way very similar to the regression tree used for RBF networks previous subsection. This kind of model presents the advantage of being continuous with continuous derivatives, a constraint which may be a desirable feature and which contributes to reduce the variability of the model prediction. Moreover, MARS models consider additive contributions of either local or global interactions but always between few variables at a time. This structure makes easier to extract diagnosis information from the model.

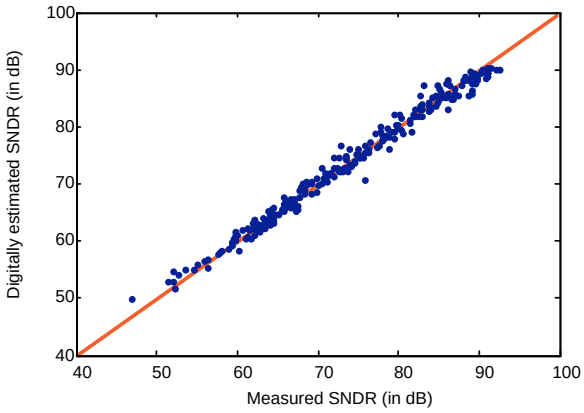

Fig. 2. Scatter plot of the SNDR estimated by the ensemble model on the base of the digital signatures, versus the measured SNDR.

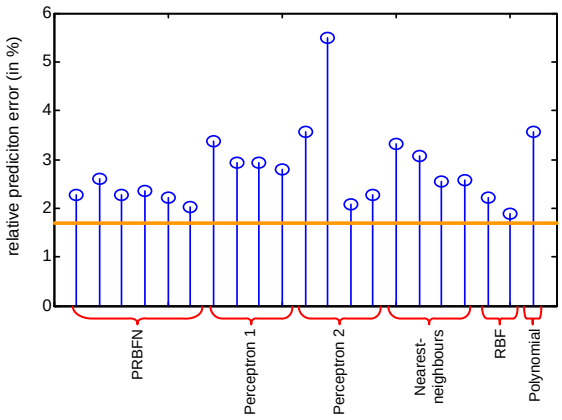

Fig. 3. Relative prediction error for the individual models that from the ensemble. The horizontal line corresponds to the ensemble prediction error.

IV. Result Exploration and Discussion

A. Fitting the SNDR

In order to guarantee that the generalization error for models obtained through the described method is fully reliable, we preferred to discard the estimate provided by the crossvalidation training routine of the ensemble. Instead, we set apart 200 modulators, randomly chosen from the 1200 simulated and defined then as the test set. Such test set is never used for training purpose in any of the models that compose the ensemble. Then we trained an ensemble model containing all the class of models described in Section 3, considering 10-fold crossvalidation.

Once the model has been trained, the SNDR is estimated for the test set. Figure 2 shows a scatter plot of the obtained results. It can clearly be seen that the model has actually managed to extract the relationship between the different signatures and the SNDR. As a matter of fact, the standard deviation of the prediction error is as low as 1.7%, that is $1.27dB$.

Taking a look at the model construction, we can see that it is composed of:

- 6 PBRFN models

- 4 perceptrons trained with 1^{st} order gradient decent
- 4 perceptrons trained with 2^{nd} order gradient decent
- 4 nearest-neighbors models, two obtained with Euclidean distance, and the other two with Manhattan distance.
- 2 RBF networks using Cauchy distribution, trained by regression trees with forward selection
- 1 polynomial model

Figure 3 shows a stem plot of the individual errors of the constituting models. The horizontal line represents the ensemble prediction error and it can be verified that it is slightly better than the best of the individual models. It is interesting to notice that none of the MARS models have been retained by the optimization algorithm of the ensemble selection. On the other hand, one polynomial model has been retained. This is unexpected since it means that the data present a quite strong global structure. One of the benefits of MARS models is that they are able to capture both local and global interactions. It seems reasonable to think that hybrid ensemble models may reach the same benefit by combining inherently local models (like nearest-neighbors) and global models (like polynomial). Considering individual models, the MARS approach may be a good performer but in the ensemble it does not bring anything new.

B. The relation between test and diagnosis

In the field of test, everybody is familiar with the functional and defect-oriented philosophies. One of the advantages of functional test is that it directly measures the specifications of the circuit and thus provides a great confidence in the quality of the circuit. However, this confidence is reasonable at the instant of test. Indeed, a circuit with an important defect may fulfill all the specifications. For instance, an unexpected strong deviation can occur in a block that was designed with a very large guard-band. The circuit performance is not affected and the functional test is thus unable to detect the defect. It could be argued that such a defect is irrelevant, but who can ensure that it would not represent a reliability issue? What is the evolution of this defect with external conditions? Alternate-test approaches that rely on several dedicated tests to capture degradation mechanisms should detect more reliability defects. However, they are partially submitted to the same issue as functional test. Indeed, in order to limit overfitting – a common issue that increases the variability of the model and the overall generalization error – many statistical tools involve some form of complexity penalty which tend to set to zero the contributors with little influence. This happens with tree pruning in RBF networks and MARS approach, with the ridge regression in polynomial fitting, etc. This last case is interesting because it is quite easy to analyze. By looking at the details of the polynomial model fitted to our data, we can see that it makes use of only 13 of the 18 signatures. Actually, the discarded signatures are those which are, in first order, sensitive to the offset in the three integrators, to the leakage of the third integrator and one of the settling signatures of the third integrator. This is not at all surprising, since integrator offset have little impact, if any, on the performance of the modulator.

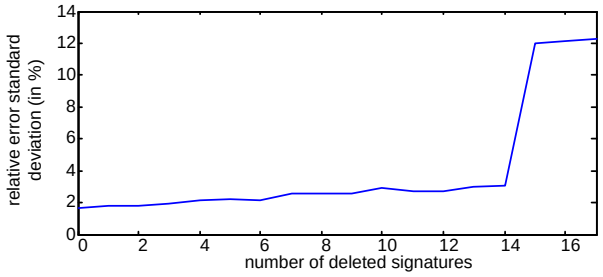

Fig. 4. Model prediction error versus the number signatures removed from the training data.

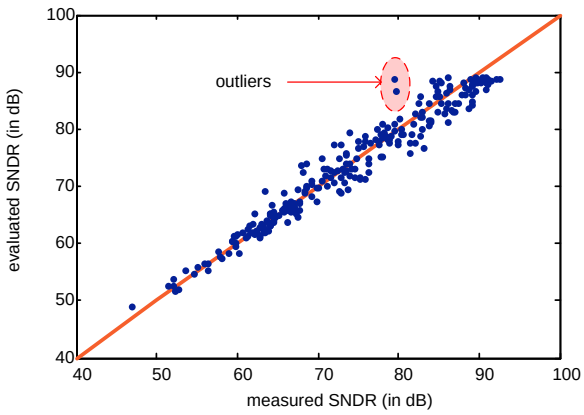

Fig. 5. Scatter plot of the SNDR estimated by the ensemble model with only the four most relevant signatures, versus the measured SNDR.

Similarly, the nonidealities of the third integrator are much less relevant than that of the first two integrators. Being located further in the $\Sigma\Delta$ loop, the errors are partially shaped to the high-frequencies and subsequently filtered out. Though the performance of the modulator can still be accurately predicted, the polynomial model have completely removed some signatures related to the behavior of the third integrator and may thus not be able to capture some defects occurring in it. To further illustrate this effect, we have ordered the 18 signatures from the less to the most relevant using a quite naive criterion relying on Singular Value Decomposition that still gives interesting results. Using the ordered list, we train the ensemble model iteratively removing one variable from the training set, from the supposedly less relevant to the most.

Figure 4 shows the prediction error of the model (calculated on the independent test set), versus the number of removed variables. It appears clearly that as many as 14 variables can be removed from the set with a small impact on the prediction error, which slowly grows from 1.7% to 3.1%. If one more variable is removed, the prediction error jumps to 12%. The remaining question is: With only four signatures, how many possible reliability defects may pass through the test?

Figure 5 shows a scatter plot of the fitted data for the model that contains only the four most relevant signatures. This plot can be compared directly with Fig. 2, and apart from a slightly larger dispersion, we can notice two more relevant outliers.

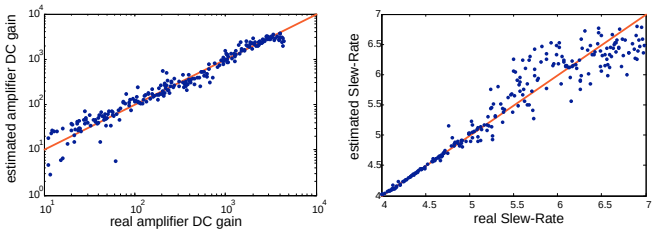

Fig. 6. Scatter plot of the predicted DC gain and Slew-rate of the 3^{rd} amplifier.

Going to the source of the MonteCarlo data, we can verify that these outliers correspond to defects in the 3^{rd} integrator. They do not affect the performance in a large extent but this small, statistically irrelevant deviation was captured by the full model and not by the reduced model.

Up to this point, following the Alternate test concept we have built a model regression mapping the digital signatures to the SNDR of the converter. However, in order to show the diagnosis potential of our defect-oriented signatures, we have also performed regression from the training data to the behavioral parameters of the MonteCarlo data. Notice that this is possible here because we are performing a highlevel MonteCarlo simulation and we thus have access to the true underlying parameters. This may also be done through electrical simulation but not on a set of real devices because we do not have a way to independently measure behavioural parameters like amplifier DC gain, etc. As can be seen in the example of Fig. 6, our digital signatures do sense parametric variations in the 3^{rd} amplifier that a functional test hardly notice. Obviously, the precision of the measurement is not very high but the diagnosis information is still very valuable, as even simple fault location is an interesting feature.

V. Conclusions

In this paper, the Alternate Test approach has been applied to $\Sigma\Delta$ moddulators. It has been shown that simple digital tests that had been primarily developed for Go/No-Go BIST solutions can provide sufficient information to predict the SNDR with good accuracy. It has been shown that the high performance statistical tool called ensemble learning performs better than the most used MARS tool, at least for this application. Furthermore, the analysis of the data points out a relevant aspect of the Alternate Test approach: its relation with defectoriented approaches and diagnosis. If some care is taken in the model construction, considering defect–oriented signatures and training the model over a wide variation range, Alternate Test approach may add significant diagnosis capabilities to accurate performance prediction.

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