On the Rees algebra of a bipartite graph

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Abstract

Let G be a bipartite graph and let I be its edge ideal. First it is shown that if A is the incidence matrix of G, then adding a row of 1's to A preserves total unimodularity. Then as a consequence it is shown that the toric ideal of the Rees algebra $R(I)$ of I is generated by square-free binomials. As another consequence we give a simple proof of the fact that $R(I)$ is normal. We prove that the facets of the Rees cone are in one to one correspondence with the minimal covers of the graph. Thus as a byproduct we obtain a method to compute the minimal covers using linear programming. We are able to prove that the a-invariant of $R(I)$ as a standard graded algebra over a field K is equal to $-(\beta_0 + 1)$, where β_0 is the independence number of G.

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