

# Physically-insightful equivalent circuit models for electromagnetic periodic structures

F Mesa<sup>1</sup>, R Rodríguez-Berral<sup>1</sup>, F Medina<sup>2</sup>

<sup>1</sup> Dpt. Física Aplicada 1, Universidad de Sevilla, Sevilla, Spain

<sup>2</sup> Dpt. Electrónica & Electromagnetismo, Universidad de Sevilla, Sevilla, Spain

E-mail: mesa@us.es, rrberral@us.es, medina@us.es

## Abstract.

In this presentation it will be discussed how to obtain analytical or quasi-analytical equivalent circuits to deal with periodic structures such as frequency selective surfaces and/or metasurfaces. Both the topology and the values of the involved elements of these circuits are obtained from a basic rationale to solve the corresponding integral equation. This procedure, besides providing a very efficient analysis/design tool, allows for a good physical insight into the operating mechanisms of the structure in contrast with the almost blind numerical scheme of commercial simulators.

## 1. Introduction

The study of periodic structures in Microwave Engineering has been a topic of intense research from early [1–3]. More recently the interest on this topic was boosted by the discovery of the so-called extraordinary optical transmission [4,5] and the further emergence of metasurfaces and related structures [6,7]. The study of these periodic or quasi-periodic structures finds a relevant precedent in the Frequency Selective Surfaces (FSSs), widely used to control the amplitude, phase and polarization of electromagnetic waves in antenna and scattering applications [8]. As is well known, a metasurface can be regarded as a special case of a FSS in which the electrical size of the constituent unit cell is small in comparison with the wavelength. In this case, the possibility of carrying out an “homogenization” procedure of the surface properties [9] conveniently allows for the treatment of the periodic surface as a sort of 2-D material [10]. Although metasurfaces and FSSs can be studied following different approaches, currently it is quite common to resort to electromagnetic commercial simulators, which, despite their high demands of computational resources, are now available in most research laboratories. Clearly, these commercial simulators have greatly benefited from the intense research carried out in the past decades on efficient numerical methods to deal with scattering problems [11].

## 2. Equivalent Circuit Approach

Another type of analysis of the periodic structures has been the Equivalent Circuit Approach (ECA), which aims at finding an equivalent network made up of transmission lines and lumped passive elements [12,13]. This method has its roots in studies made in the fifties of the last century [14,15]. Three different approaches can be distinguished in this line of action: i) obtaining the topology and values of the equivalent network from the *a priori* knowledge of



the frequency response of the periodic structure [16]; ii) a general multi-mode analysis involving a “black-box” admittance matrix whose elements are derived from a rigorous numerical or semi-analytical analysis [17–19]; and iii) making use of some simplifying assumption (for instance, operation regime in the long-wavelength limit) that allows for a quasi-static analytical modeling of the structure’s performance [9]. A clear drawback of the first family of methods is that the full dynamical behavior of the structure has to be known *a priori* in order to obtain a reduced-order model that works appropriately in the given frequency range of interest. Also, the network topology as well as the number and values of the involved elements are obtained from a purely numerical strategy. This general-purpose procedure provides little physical insight into the electromagnetic behavior of the structure, and the topology and elements of the equivalent network may vary depending on the frequency range of operation and/or the desired accuracy of the circuit model. The second family of methods is also a general tool that does not provide an explicit topology of the network, except in some limiting cases where it can be simplified. The third type of methods does provide a convenient physical insight but its range of application comes completely determined by the validity of the simplifying assumptions. Although there are numerous interesting situations covered by this last analytical approach, it fails for many other practical cases of interest. It is then the purpose of this talk to present an analytical approach that can be applied in a wide frequency range of operation but that preserves the advantages of the analytical derivation of an explicit topology of the ECA as well as closed-form expressions for the involved lumped elements and transmission lines. Similar relevant attempts in this direction have been reported in the past [20] and some examples of more recent effort of the authors of this presentation to extend the validity and applicability of this type of ECA can be found in [21, 22].

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