Extending Qualitative Spatial Theories with Emergent Spatial Concepts

An Automated Reasoning Approach

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Abstract. Qualitative Spatial Reasoning is an exciting research field of the Knowledge Representation and Reasoning paradigm whose application often requires the extension, refinement or combination of existent theories (as well as the associated calculus). This paper addresses the issue of the sound spatial interpretation of formal extensions of such theories; particularly the interpretation of the extension and the desired representational features. The paper shows how to interpret certain kinds of extensions of Region Connection Calculus (RCC) theory. We also show how to rebuild the qualitative calculus of these extensions.

1 Introduction

One of the main challenges in Qualitative Spatial Reasoning (QSR) is the need to combine or extend the existing theories to include new aspects in the same formalism [18,12]. In order to face the problem, several features and viewpoints must be considered.

The focus here is the logical aspect of the challenge, particularly the relationship among models of initial theories and that of the new ones. A key aspect to consider in Artificial Intelligence in general is the feasibility/complexity of the reasoning process, by providing, for example, a qualitative calculus. This approach contrasts with the qualitative and nature inspired one [14]. Several of the purely logical features could be solved if a sound methodology is adopted, for example the *definitional methodology* for building formal ontologies [4]. It can be too rigid because of strong requirements such as logical categoricity. In contrast with this framework, in QSR the (characterization of) the class from *intended models* is more important than the general class of models. In fact, a sound interpretation of the revised ontology/theory for preserving those models is a key step especially if previous definitions have to be changed. For example, any extension by definition of a new concept/relationship should be supported by a

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good theory about its relationship with the original theory, as well as by a nice way of expanding a representative class of models of the source theory to the new one.

From this point of view, the use of automated reasoning systems can ensure the correctness of the results as well as that it has not been used spatial intuitions which are not formalized in the theory (c.f. [21]). It is very important both the soundness of the associated calculus and the use of a spatial theory as basis for building and reasoning with ontologies [17].

The aim of the paper is to show how the assistance of automated reasoning systems (ARS) can help to classify, interpret and compute abstract extensions of QSR theories, required to accommodate new concepts and insights which Knowledge Engineering problems induces. The use of ARS provides an formal framework where contrast hypothesis, specifications and axioms.

Specifically, the case of the extension of RCC theory [12,18] by insertion of an undefined relationship is analyzed. In a broad scope, the aim is to describe how rudiments of First Order Model Theory (and computational logic) can be used for increasing the knowledge on generic extensions of the QSR theory: On the one hand, by providing a formal support to the reasoning both from lattice of spatial relationships and transition tables. On the other hand, since the computing of the extensions is assisted by automated reasoning, it provides information to the designer which comes from the logical entailment. In this way the designer only has to re-interpret if necessary, elements from the older theory in order to satisfy those information requirements. This task, non algorithmic in essence, is the responsibility of experts in the domain represented by the ontology. In fact, such re-interpretation can force us to reconsider the initial ontological commitments. This paper addresses these issues.

The rest of the paper proceeds as follows. Next section motivates the need of qualitative reasoning on abstract extensions of standard theories. Section 3 introduces basic features of lattice categorical extensions, a formal notion for extending theories and it recalls a result on extensions of RCC. Sections 4,5,6 represent the main contributions of the paper. In Section 4 the interpretation of the extensions by means topological pulsation is described. Section 5 shows how the transition table for the extensions from interpretation can be rebuilt. Section 6 shows other interpretation framework (egg-yolk approach). Conclusions and new insights are summarized in Section 7.

2 Interpretation of Generic Extensions

The paper addresses in first place the problem of obtaining a sound interpretation (by providing a spatial meaning) of the extensions obtained by means automated reasoning; and, in the second one, it studies how from that interpretation, other tools for QSR (as transition tables) can be deduced. Formally:

Definition 1. *Let Ω be a topological space and T be a mereotopological theory. An* **interpretation** *on Ω is an interpretation of the language of T whose universe is Ω. T is* **interpretable** *on Ω if there exists an interpretation on Ω which is model of T.*

Roughly speaking, an interpretation is a (logical) interpretation which interprets spatial entities as open sets in the space, and relations as spatial relations, often on spatial regular regions. If an abstract extension of a standard QSR theory is obtained, it

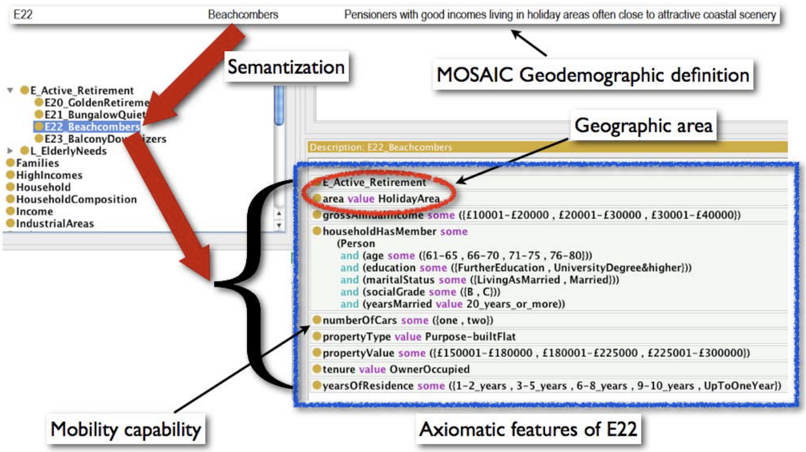

Fig. 1. Semantic approach to a geodemographic class [11]

is necessary to extend the standard interpretation by interpreting the new concepts or relationships as spatial regions and relations respectively.

2.1 A Motivating Example

The needs of generic extensions of (classic) qualitative reasoning theories comes from the analysis of spatial relationships partially defined by different specifications. For example, in [11] authors show how to build a (semantic web) ontology from a state-of-art geodemographic system. Such kind of systems are composed by high-level specifications of spatio-temporal and geodemographic features. Geodemographic classes extracted from the system are underspecified by the formalization of a number of geographic, demographic and sociological restrictions that really do not define the intent of geodemographic specialist (see Fig. 1). Therefore, when automated reasoning work on specifications poor results are obtained: formal class can not soundly interpreted as geodemographic expert desires, which really represents a vague region contained in the intersection of a number of anonymous classes.

The refinement of geodemographic ontologies can not be sufficient if the system can not reason with rough, generic spatial relations which provides a basic spatial calculus. The selection of QSR for refining ontologies was showed in a range of papers [6,9,2,3] in which are presented both the foundational issues as well as their applications. The paper [3] describes an intelligent interface (called Paella), based on qualitative spatial reasoning which is designed to (spatially) reason with ontology classes (see [2] for an application). The refining cycle to be applied (once extended standard qualitative reasoning to work with the new kind of spatial entities) is represented in Fig. 2. It can be considered other possibility consisting on the refinement of the definition by means of the combined use of two or more classifier systems (and the sound topology) [24]. However the qualitative nature of ontological definitions discourages this approach. Standard mereotopological interpretation leads an abstract spatial configuration which

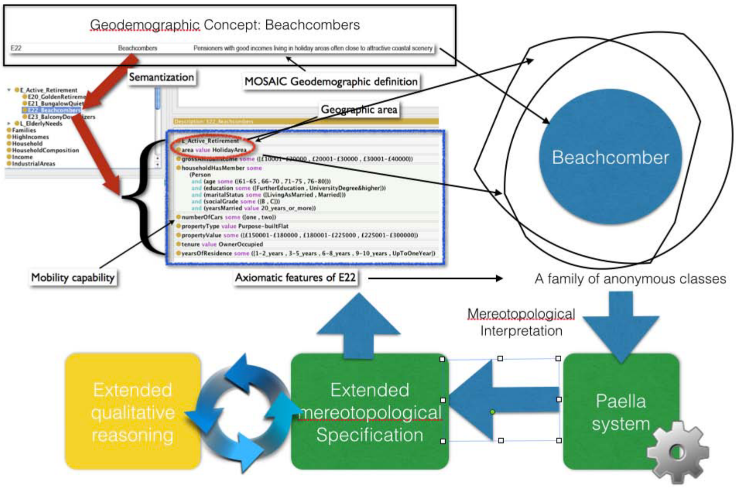

Fig. 2. Augmenting reasoning cycle with extended spatial reasoning [3]

has to be used by purely formal methods (because the spatial intuition may fail on these new relationships). Therefore, it needs a new formal framework where the reasoning is founded with strong logical theories on spatial reasoning, which must be topologically interpretable in turn.

2.2 The Mereotopological Theory RCC

RCC theory [12], a mereotopological approach to QSR, describes topological features of the spatial relations. It has been useful in several fields of Artificial Intelligence such as Geographic Information Systems (GIS) and Spatial Databases (see e.g. [16]) It allows us both to reason on spatial regions and to interchange knowledge between ontologies and their spatial models. We consider a ground relation, the connection between two regions, which enjoys the reflexive and symmetrical properties. The meaning of connection is: the topological closures of two connected regions intersect. The set of axioms expressing the properties and definitions of the remaining relations (Fig. 3 (left) conforms the set of axioms of RCC (see [12]).

On one hand, the set of the eight binary relations depicted in Fig. 3 is denoted by RCC8. These relations are jointly exhaustive and pairwise disjoint (JEPD) and RCC8 is regarded a calculus for Constraints Satisfaction Problems (CSP) (see e.g. [22]). On the other hand, there is another interesting calculus, $RCC5 = \{DR, PO, PP, PPi, EQ\}$. The difference between them is that while the former allows us to enrich the representation of knowledge by using frontiers of the regions, the latter do not. This fact will be discussed next. Although it has been empirically established [19] that RCC8 is more suitable than RCC5 for the representation of topological relations discriminated by humans, both of them are used here: RCC5 is appropriate for solving CSPs associate to

$DC(x,y) \leftrightarrow \neg C(x,y)$	(x is disconnected from y)
$P(x,y) \leftrightarrow \forall z[C(z,x) \rightarrow C(z,y)]$	(x is part of y)
$PP(x,y) \leftrightarrow P(x,y) \wedge \neg P(y,x)$	(x is proper part of y)
$EQ(x,y) \leftrightarrow P(x,y) \wedge P(y,x)$	(x is identical with y)
$O(x,y) \leftrightarrow \exists z[P(z,x) \wedge P(z,y)]$	(x overlaps y)
$DR(x,y) \leftrightarrow \neg O(x,y)$	(x is discrete from y)
$PO(x,y) \leftrightarrow O(x,y) \wedge \neg P(x,y) \wedge \neg P(y,x)$	(x partially overlaps y)
$EC(x,y) \leftrightarrow C(x,y) \wedge \neg O(x,y)$	(x is externally connected to y)
$TPP(x,y) \leftrightarrow PP(x,y) \wedge \exists z[EC(z,x) \wedge EC(z,y)]$	(x is a tangential prop. part of y)
$NTPP(x,y) \leftrightarrow PP(x,y) \wedge \neg\exists z[EC(z,x) \wedge EC(z,y)]$	(x is a non-tang. prop. part of y)

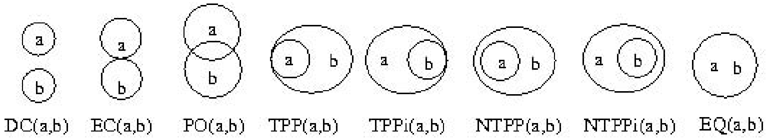

Fig. 3. Axioms of RCC (top) and RCC8 spatial relations (bottom)

a mereotopological representation and RCC8 is useful to design a rich translation of a spatial representation to the ontology code.

Models of RCC have been deeply studied from different viewpoints [20,22]. The study of the lattice of spatial relationships of extensions of RCC was made in [10]. The last work raise several questions about the relation between the original theory and its extensions. This can be studied from the QSR paradigm, or from the logical consequences of the extension of the theory (see e.g. [15] for the combination of RCC and reasoning about qualitative size and [9] for the same problem).

3 Background: Lattice-Categorical Extensions

An essential requirement to a qualitative theory should be that if it is possible to entail the basic relationships among the concepts considered. For example, RCC entails both the relationship between the spatial defined relations (which has lattice structure) and the transition calculus [21]. Likewise the extension should satisfy the same requirement. Inspired by foundational questions on the Semantic Web [1], in [10] a formal definition of *robust ontology* is proposed, called *lattice categorical extension* [8] used for computing a range of RCC-extensions used in the paper.

A *lattice categorical* theory is the one that proves the lattice structure of its basic relations. Formally, given a fixed language, let $\mathcal{C} = \{C_1, \ldots, C_n\}$ be a (finite) set of concept symbols, let T be a theory. Given M a model of T, $M \models T$, we consider the structure $L(M, \mathcal{C})$, in the language $L_{\mathcal{C}} = \{\top, \bot, \leq\} \cup \{c_1, \ldots, c_n\}$, whose universe are the interpretations in M of the concepts (interpreting c_i as C_i^M), $\top$ is M, $\bot$ is $\emptyset$ and $\leq$ is the subset relation. We assume that $L(M, \mathcal{C})$ is requested to have a lattice structure for every theory we consider.

The relationship between $L(M, \mathcal{C})$ and the model M itself is based on that the lattice L can be characterized by a finite set of equations E_L, plus a set of formulas $\Theta_{\mathcal{C}}$

categorizing the lattice under completion, that is, $\Theta_{\mathcal{C}}$ includes the domain closure axiom, the unique names axioms and, additionally, the axioms of lattice theory.

Definition 2. *Let E be a $L_{\mathcal{C}}$-theory. We say that E is a* **lattice skeleton** *(l.s.) for a theory T if E verifies that*

- *There is $M \models T$ such that $L(M, \mathcal{C}) \models E \cup \Theta_{\mathcal{C}}$, and*
- *$E \cup \Theta_{\mathcal{C}}$ has an unique model (modulo isomorphism).*

Every consistent theory has a lattice skeleton [10]. The existence of non equivalent l.s. makes it difficult to reason with the relations, while the existence of only one would make it easy due to the relationship among the relations is the same in any model of T.

Definition 3. *T is called a* **lattice categorical (l.c.) theory** *if every pair of lattice skeletons for T are equivalent modulo $\Theta_{\mathcal{C}}$.*

Note also that every consistent theory T has an extension T' which is lattice categorical: it suffices to consider a model $M \models T$, and then to find a set E of equations such that $\Theta_{\mathcal{C}} \cup E$ has $L(M, \mathcal{C})$ as only model.

A method -assisted by ATP an MF- for obtaining the skeleton is described [10].

Finally, we can give a formalization of *robust ontological extension*, based in the categorical extension of the ontology:

Definition 4. *Given two pairs $(T_1, E_1), (T_2, E_2)$ we will say that (T_2, E_2) is a* **lattice categorical extension** *of (T_1, E_1) with respect to the sets of concepts $\mathcal{C}_1$ and $\mathcal{C}_2$ respectively, if $\mathcal{C}_1 \subseteq \mathcal{C}_2$ and $L(T_2, \mathcal{C}_2)$ is an E_1-conservative extension of $L(T_1, \mathcal{C}_1)$.*

The most important feature of l.c. theories is that this allows use only the lattice relationships for reasoning with the relations. Lattice categoricity has been used for extending ontologies by decision of the user [10], motivated by data and designed by the user [8], data-driven [7] and ontology merging [9].

In [10] l.c. extensions of RCC for supporting undefinition are computed: those that insert the undefinition into RCC8 calculus, so obtaining a new JEPD set. There exist other kind of extensions designed for other uses. See [8] for details.

Theorem 1. *[10] There are only eight l.c. extensions of the lattice of RCC by insertion of a new relation D such that $RCC8 \cup \{D\}$ is a JEPD set.*

The analysis of the extensions (fig. 4) suggests us that the new relations represent *undefinition up to a degree*.

4 Interpreting with Pulsation/Contraction

The above result is an example of a purely logical result obtained by automated reasoning. As it is commented the method ensures the correctness of the result. It is necessary to complete the study by interpreting (if possible) the new elements (and the reinterpreting the older ones). This way it qualifies the designer to use it as QSR theory. In order to obtain specific interpretations, it need to work with concrete spaces. In this section we illustrate this idea by using $\mathcal{R}(\Omega)$ as the set of regular sets of the topological space Ω.

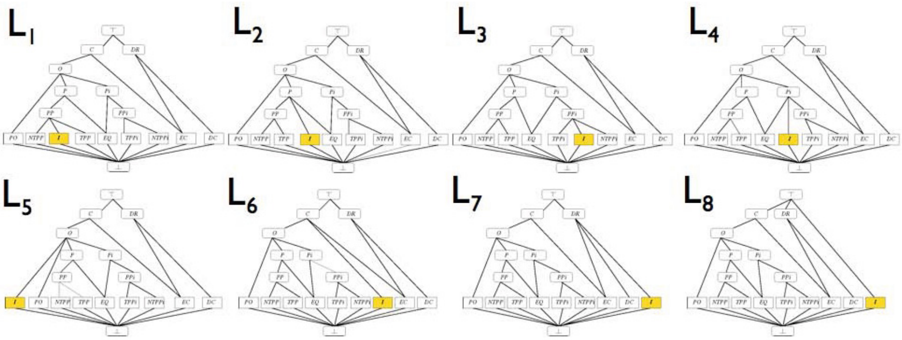

Fig. 4. The eight lattice describing the l.c. extensions of RCC by a undefinition relation

Definition 5. *A* **pulsation** *on a topological space* $\Omega = (\mathcal{X}, \mathcal{T})$ *is a map* $\sigma : \mathcal{R}(\Omega) \longmapsto \mathcal{R}(\Omega)$ *such that the closure of* $\sigma(X)$ *contains that of* X*;* $\overline{X} \subset \overline{\sigma(X)}$*. The pair* (Ω, σ) *where* Ω *is nontrivial, connected and regular is a* ***topological space with pulsation****.*

The interpretation on these spaces is based on considering the pairs $(x, \sigma(x))$.

Theorem 2. *Seven of the eight extensions from theorem 1 are interpretable in topological spaces with pulsation.*

Proof. We denote by R^{Ω} ($R \in RCC$) the natural interpretation of R in the topological space Ω. For the sake of simplicity, we make use of the following conventions,

$$R^{\sigma}(a, b) := R(\sigma(a), \sigma(b)) \text{ and } \bigvee RCC8^{\sigma} := \bigcup_{R \in RCC8} R^{\sigma}$$

Let (Ω, σ) be a topological space with pulsation σ. Ω_k is defined like the structure on the language of $RCC + \{I_k\}$ where $k \in \{1, 2, 3, 4, 5, 7, 8\}$, and for every $R \in \mathcal{R}_{\mathcal{RCC}}$, the interpretation of R in Ω_k is obtained by combination of regions. It only shows two of such interpretations. The others explanations are similar.

L_1 : $R^{\Omega_1} = R^{\Omega}$ if $R \in \mathcal{R}_{RCC} \setminus \{NTPP, TPP\}$, $\quad TPP^{\Omega_1} = TPP^{\Omega} \cap TPP^{\sigma}$,
$NTPP^{\Omega_1} = NTPP^{\Omega} \cap NTPP^{\sigma}$ and
${I_1}^{\Omega_1} = (TPP^{\Omega} \cap (\bigvee RCC8^{\sigma} \setminus \{TPP^{\sigma}\})) \cup (NTPP^{\Omega} \cap (\bigvee RCC8^{\sigma} \setminus \{NTPP^{\sigma}\}))$

L_3 : $R^{\Omega_3} = R^{\Omega}$ if $R \in \mathcal{R}_{RCC} \setminus \{TPPi, NTPPi\}$, $TPPi^{\Omega_3} = TPPi^{\Omega} \cap TPPi^{\sigma}$,
$NTPPi^{\Omega_3} = NTPPi^{\Omega} \cap NTPPi^{\sigma}$ and
${I_3}^{\Omega_3} = (TPPi^{\Omega} \cap (\bigvee RCC8^{\sigma} \setminus \{TPPi^{\sigma}\})) \cup (NTPPi^{\Omega} \cap (\bigvee RCC8^{\sigma} \setminus \{NTPPi^{\sigma}\}))$

The relation I_6 does not have interpretation on pulsation. It has to use *contraction*.

Definition 6. *A* **contraction** *in a topological space* $\Omega = (\mathcal{X}, \mathcal{T})$ *is a map* $\sigma : \mathcal{R}(\Omega) \longmapsto \mathcal{R}(\Omega)$ *such that* $\overline{(}\xi(A)) \subset \overline{A}$ *for each* A *with nonempty inner. The pair* (Ω, ξ) *where* Ω *is nontrivial, connected is called a* ***topological space with contraction****.*

Theorem 3. I_6 *is interpretable in a topological space with contraction*

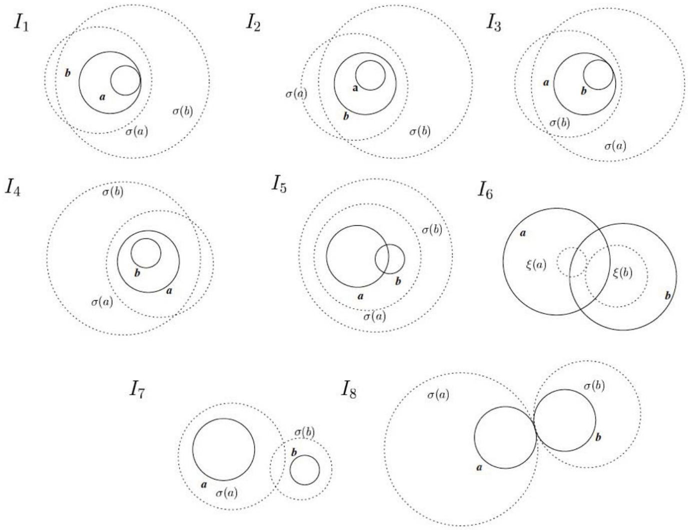

Fig. 5. Interpretation of the relations by undefintion (I_6 by contraction.The rest by pulsation)

Proof. Given (Ω, ξ) define Ω_6 the structure of the language $RCC + \{I_6\}$ as follows:

- $R^{\Omega_6} = R^{\Omega}$ if $R \in \{C, DR, EC, DC\}$
- $R^{\Omega_6} = R^{\Omega} \cap O^{\xi}$ if $R \in \mathcal{R}_{RCC} \setminus \{C, DR, EC, DC\}$
- ${I_6}^{\Omega_6} = O^{\Omega} \cap DR^{\xi}$

Fig. 5 summarizes the interpretations. In fact, it verifies:

Theorem 4. *The set of interpretations Ω_k, $k \in \{1, 2, \ldots, 8\}$ defined above entails the lattice structure L_k depicted in the Fig. 4.*

It suffices to check exhaustively the properties of the reticle according to the corresponding interpretation. The details of such long and tedious process are omitted.

Corollary 1. *The set $RCC8 + \{I_k\}$ is a JEPD set under the interpretation Ω_k, for $k = \{1, 2, \ldots, 8\}$.*

The interpretations correspond, in essence, to a skeleton of every possible extension of RCC. The skeleton (the set of lattice equations characterizing the lattice) can be obtained by using a model finder (MACE4 in our case), but the calculus is out of the scope of this paper.

5 Building the New Transition Tables

One of the advantages of interpreting l.c. extensions is that it allows to build a transition table for the new theory, particularly in the case of the news JEPDs. As for RCC8, it is

Table 1. Composition table for the extension corresponding to L_1

R2(b, c) / R1(a, b)	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ	I_1
DC	RCC8[I_1]	DC, EC, PO, TPP, NTPP, I_1	DC, EC, PO, TPP, NTPP, I_1	DC, EC, PO, TPP, NTPP, I_1	DC, EC, PO, TPP, NTPP, I_1	DC	DC	DC	DC, EC, PO, TPP, NTPP, I_1
EC	DC, EC, PO, TPPi, NTPPi	DC, EC, PO, TPP, TPPi, EQ, I_1	DC, EC, PO, TPP, NTPP, I_1	EC, PO, TPP NTPP, I_1	PO, TPP NTPP, I_1	DC, EC	DC	EC	EC, PO, TPP NTPP, I_1
PO	DC, EC, PO, TPPi, NTPPi	DC, EC, PO, TPPi NTPPi	RCC8[I_1]	PO, TPP NTPP, I_1	PO, TPP NTPP, I_1	DC, EC, PO, TPPi NTPPi	DC, EC, PO, TPPi NTPPi	PO	PO, TPP NTPP, I_1
TPP	DC	DC, EC	DC, EC, PO, TPP NTPP, I_1	TPP NTPP, I_1	NTPP	DC, EC, PO, TPP TPPi, EQ I_1	DC, EC, PO, TPPi NTPPi	TPP	TPP NTPP, I_1
NTPP	DC	DC	DC, EC, PO, TPP NTPP, I_1	NTPP	NTPP	DC, EC, PO, TPP NTPP, I_1	RCC8[I_1]	NTPP	NTPP, I_1
TPPi	DC, EC, PO, TPPi, NTPPi	EC, PO, TPPi, NTPPi	PO, TPPi, NTPPi	PO, EQ TPP, TPPi	PO, TPP NTPP I_1	TPPi NTPPi	NTPPi	TPPi	PO, EQ TPP, TPPi, NTPP, I_1
NTPPi	DC, EC, PO, TPPi, NTPPi	PO, TPPi, NTPPi	PO, TPPi, NTPPi	PO, TPPi, NTPPi	PO, TPPi TPP, NTPP NTPPi, EQ I_1	NTPPi	NTPPi	NTPPi	PO, TPPi TPP, NTPP NTPPi, EQ I_1
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ	I_1
I_1	DC	DC, EC	DC, EC, PO, TPP NTPP, I_1	TPP NTPP, I_1	NTPP, I_1	DC, EC, PO, TPP EQ NTPP, I_1	RCC8[I_1]	I_1	TPP, NTPP, I_1

possible to prove the transition table for the new JEPD sets $RCC8+I_k, k = \{1, \dots, 8\}$. To illustrate the method, the table for $RCC8 + \{I_1\}$ is computed.

Table 1 shows the transition table for $RCC8 + \{I_1\}$.

The part of the table that corresponds(fits) to the composition of relations $R_1^{\Omega}, R_2^{\Omega}$ where $R_1, R_2 \in RCC8$, coincides with the table we obtain for $RCC8$, except:

- If from the composition of two relations R_1, R_2 in $RCC8$ is obtained TPP or $NTPP$ (or both of them),then it will appear TPP or $NTPP$ (or both of them), besides the relation I_1.
- As a consequence of that, if in the composition table of $RCC8$ the result of composing two relations is $RCC8$, then, the result is the set $RCC8 + \{I_1\}$, which we have denoted as $RCC8[I_1]$.

In table 2 it shows an example of calculus.

6 Interpretation in the "egg-yolk" Approach

In this section another interpretation, in the egg-yolk paradigm [13] is studied. This is naturally related with the pulsation one. A complete picture of the relationship between undefinition relations is given (as well as with RCC5) instead of a separate interpretation for each one. In egg-yolk paradigm, regions (which we call *e-y regions*) have undetermined boundaries (a 'vague region'), and they are represented by a pair of concentric regions with determinate boundaries ('crisp regions'), which provide limits (not necessarily the tightest limits possible) on the range of indeterminacy. In this paradigm

Table 2. Computing $I_1 \circ I_1 \equiv NTPP \vee TPP \vee I_1$ under interpretation with pulsation

$$I_1^{\Omega_1}(a,b) \wedge I_1^{\Omega_1}(b,c) =$$
$$= ((TPP^{\Omega}(a,b) \cap (\bigvee RCC8^{\sigma} \setminus \{TPP^{\sigma}\})(a,b)) \cup (NTPP^{\Omega}(a,b) \cap (\bigvee RCC8^{\sigma} \setminus \{NTPP^{\sigma}\})(a,b)))$$
$$\cap ((TPP^{\Omega}(b,c) \cap (\bigvee RCC8^{\sigma} \setminus \{TPP^{\sigma}\})(b,c)) \cup (NTPP^{\Omega}(b,c) \cap (\bigvee RCC8^{\sigma} \setminus \{NTPP^{\sigma}\})(b,c))) =$$
$$= (NTPP^{\Omega}(a,c) \cap (\bigvee RCC8^{\sigma}(a,c))) \cup (TPP^{\Omega}(a,c) \cap (\bigvee RCC8^{\sigma}(a,c))) =$$
$$= (NTPP^{\Omega}(a,c) \cap (NTPP^{\sigma}(a,b) \cup (\bigvee RCC8^{\sigma} \setminus \{NTPP^{\sigma}\})(a,c))) \cup$$
$$\cup (TPP^{\Omega}(a,c) \cap (TPP^{\sigma}(a,b) \cup (\bigvee RCC8^{\sigma} \setminus \{TPP^{\sigma}\})(a,c))) =$$
$$= (NTPP^{\Omega}(a,c) \cap NTPP^{\sigma}(a,b)) \cup (NTPP^{\Omega}(a,c) \cap (\bigvee RCC8^{\sigma} \setminus \{NTPP^{\sigma}\})(a,c)) \cup$$
$$\cup (TPP^{\Omega}(a,c) \cap TPP^{\sigma}(a,b)) \cup (TPP^{\Omega}(a,c) \cap (\bigvee RCC8^{\sigma} \setminus \{TPP^{\sigma}\})(a,c)) =$$
$$= NTPP^{\Omega_1}(a,c) \cup (NTPP^{\Omega}(a,c) \cap (\bigvee RCC8^{\sigma} \setminus \{NTPP^{\sigma}\})(a,c)) \cup$$
$$\cup TPP^{\Omega_1}(a,c) \cup (TPP^{\Omega}(a,c) \cap (\bigvee RCC8^{\sigma} \setminus \{TPP^{\sigma}\})(a,c)) =$$
$$= NTPP^{\Omega_1}(a,c) \vee TPP^{\Omega_1}(a,c) \vee I_1^{\Omega_1}(a,c)$$

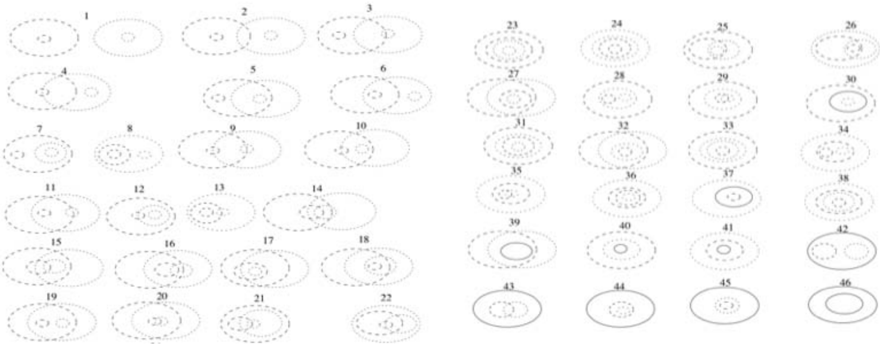

Fig. 6. Egg-yolk relations [13]

RCC5 is used instead of RCC8 by evident reasons. In Fig. 6 a complete description of e-y relations is shown.

Given two e-y regions $A = (a, \overline{a})$, $B = (b, \overline{b})$ and $R \in RCC5$ define $\overline{R}$ by: $(A, B) \in \overline{R} \iff (\bar{a}, \bar{b}) \in R$. Thus the interpretation of $\{I_1, \ldots, I_5, I_7, I_8\} \cup RCC5$ is

- $\overline{DR} = \{1\}$ and $\overline{EQ} = \{42, 43, 44, 45, 46\}$
- $\overline{PP} = \{(A, B) : PP(\overline{a}, \overline{b})\}$ which agree with I_1 (acording to lattice L_1). Thus, $I_1 = \overline{PP} = \{8, 13, 22, 24, 26, 34, 35, 36, 37, 38, 41\}$.
- $I_2 = \overline{PP} \cup \overline{EQ} = \{8, 13, 22, 24, 26, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46\}$,
- By symmetry, $I_3 = \overline{PPi} = \{7, 12, 21, 23, 25, 28, 29, 30, 31, 33, 40\}$ and $I_4 = \overline{PPi} \cup \overline{EQ} = \{7, 12, 21, 23, 25, 28, 29, 30, 31, 33, 40, 42, 43, 44, 45, 46\}$.
- $\overline{PO} = \{2, 3, 4, 5, 6, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 27, 32, 39\}$, thus
- $I_5 = I_2 \cup I_4 \cup \overline{PO} = \{2, \ldots, 46\}$
- $I_7 = \overline{DR} = \{1\}$.
- $I_8 = \{1, 2, \ldots, 46\} = \bigcup_{k \in \{1,2,3,4,5,7\}} I_k = I_5 \cup I_7$

Theorem 5. *$RCC5 \cup \{I_1, \ldots, I_5, I_7, I_8\}$ have the lattice structure depicted in 7 in the egg-yolk interpretation.*

Therefore, the set $\{I_1, \overline{EQ}, I_3, \overline{PO}, I_7\}$ is JEPD, and also $\{I_2, I_4, \overline{PO}, I_7\}$ y $\{I_5, I_7\}$. Likewise it is possible to build transition tables for these calculus.

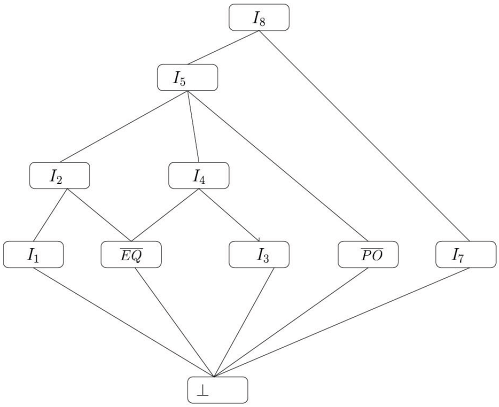

Fig. 7. Lattice of egg-yolk interpretations of new relationships

7 Conclusions and Future Work

The paper is a first step towards the classification/interpretation of generic kind of extensions of RCC as well as the computing of their transition tables. They represent the logical side of the extensions of RCC used in [6], and it is the basis of the associated tool [2,3]. The method for obtaining all the extensions was shown in [10]. The main contribution (sections 4,5 and 6) rest unpublished by now.

Due the lack of space, we do not present related interpretation based on rough sets [23]. This approach is based on totally disconnected topological spaces as models. Thus it starts from a different class of intended models. Also, the approach described in [5] can be used as a basis for a interpretation. The future work is to implement the abstract interpretations as an extended feature of Paella system, in order to specify ontologies which vaguely represent specialized concepts (as for example in the geodemography field).

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