TOUGHNESS DETERMINATION IN COMPOSITE MULTIMATERIAL CLOSED CORNERS

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1 Abstract

In the present work, a general procedure for the experimental evaluation of the generalized fracture toughness in multimaterial corners is defined. The proposed method is suitable for closed corners (all material wedges being bonded) having two singular terms in the asymptotic stress representation at the corner tip. For a particular corner configuration, the method finds the load configuration at which one of the singular terms vanishes, thus the main stress contribution being controlled by the other non-vanishing singular term. The experimental test, until failure, using the previously defined load configuration allows the generalized fracture toughness associated to each singular term to be evaluated.

The whole procedure has been applied to a bimaterial CFRP-Adhesive bimaterial corner and the generalized fracture toughness values have been obtained. The testing of mixed modes has permitted a failure envelope based on the generalized fracture toughness values at the corner tip to be defined. Previously published results, with different geometries, but involving the same corner, have shown that the failure envelope can predict accurately the failure initiation at these corners.

2 Introduction

The stress and displacement fields in the neighbourhood of linear elastic anisotropic multimaterial corners, assuming 2D elastic state and considering a polar coordinate system (r,θ) centred at the corner tip, can be represented by an asymptotic series expansion, with variable separation, see Wieghardt (1907), Williams (1952), Dempsey and Sinclair (1981), Ting (1997) and Barroso *et al.* (2003).

Under some simplifying assumptions (e.g. neglecting the possible existence of logarithmic terms) the series expansion for displacements and stresses at a corner tip can be written in the following form:

$$u_\alpha(r,\theta) \cong \sum_{k=1}^{n} K_k r^{\lambda_k} g_\alpha^k(\theta), \quad \sigma_{\alpha\beta}(r,\theta) \cong \sum_{k=1}^{n} K_k r^{\lambda_k - 1} f_{\alpha\beta}^k(\theta) \quad (\alpha = r,\theta) \qquad (1)$$

where K_k (k=1,...,n) are the Generalized Stress Intensity Factors (GSIFs), λ_k (k=1,...,n) are the characteristic exponents ($0<\lambda_k<1$, $1-\lambda_k$ being the order of stress singularity) and $g_\alpha^k(\theta)$ and $f_{\alpha\beta}^k(\theta)$ (k=1,...,n) are the angular shape functions for displacements ($g_r^k(\theta)$, $g_\theta^k(\theta)$) and stresses ($f_{rr}^k(\theta)$, $f_{\theta\theta}^k(\theta)$, $f_{r\theta}^k(\theta)$), respectively. The angular shape functions $g_\alpha^k(\theta)$ and $f_{\alpha\beta}^k(\theta)$ have been normalized in the present work according to Pageau *et al.* (1996).

Unlike the well defined test standards for the experimental determination of fracture toughness values for cracks in homogeneous isotropic materials (K_{IC}, and K_{IIC} respectively for the symmetrical and unsymmetrical cases), the lack of symmetries in the stress fields in general configurations of anisotropic multimaterial corners, makes difficult to develop a general procedure for the generalized-fracture-toughness determination in corners of this kind.

In this work, the evaluation of λ_k, $g_{\alpha}^{k}(\theta)$ and $f_{\alpha\beta}^{k}(\theta)$ is based on a general analytical procedure proposed in Barroso *et al*. (2003) which applies for linear elastic generalized plane strain states, without any limitation in the number and nature of linear elastic materials. The evaluation of K_k is based on a numerical procedure (Barroso *et al* 2011), which has proved to be accurate in most difficult cases with multiple singularities.

The aim of the present work is to propose a general procedure for the generalized fracture toughness determination in 2D multimaterial anisotropic closed corners having two singular terms. With two singular terms, the evaluation of generalized fracture toughness K_{kC} (k=1,2) is based on the possibility of isolating each singular term with a particular external load distribution.

The procedure presented here is only valid for closed corners (with all materials wedges perfectly bonded, without any external boundaries, sometimes referred to as cross-points) and is based on a novel modified configuration of the Brazilian test geometry (introduced almost simultaneously by Carneiro, 1943, and Akazawa, 1943). The multimaterial corner tip is placed at the centre of the disk and the disk is loaded in compression in the diametric direction at any generic point along the external perimeter, the procedure being obviously only valid for closed corners.

For a practical illustration of the procedure in the field of composite materials, it has been applied to a particular CFRP-epoxy bimaterial closed corner. The experimental results and failure envelope, based of critical values of the GSIFs, are also presented. Previously published results of different geometries having, locally, the same corner configuration have shown to agree with the predictions of the failure envelope.

3. Description of the test procedure

Figure 1a shows an example of an adhesively bonded joint between a composite laminate and an aluminium plate containing three different multimaterial closed corners. In particular, Figure 1b shows a 90º wedge of a unidirectional carbon fibre layer, with the fibre in the *x* direction, bonded to a 270º wedge of adhesive. The bimaterial corner configuration, shown in Figure 1b, will be the one chosen for the application of the numerical procedure and experimental testing.

In particular, the procedure consists in the application of a compressive loading to a specimen like the one shown in Figure 1b, at any generic position of the external perimeter, and the evaluation of the corresponding generalized fracture toughness values of the two singular modes (critical values of GSIFs K_1 and K_2) for each loading angle α, which is schematically depicted in Figure 1c.

The experimental testing until failure at loading angles α_1 (where K_1=0) and α_2 (where K_2=0) allow the evaluation of the critical values of K_2 and K_1, respectively, which will be defined in what follows as K_{2C} (for the test at α_1) and K_{1C} (for the test at α_2). The evaluation of such values is obtained by substituting the experimental failure load in the linear elastic simulation of the Brazilian disk (by means of a FEM model).

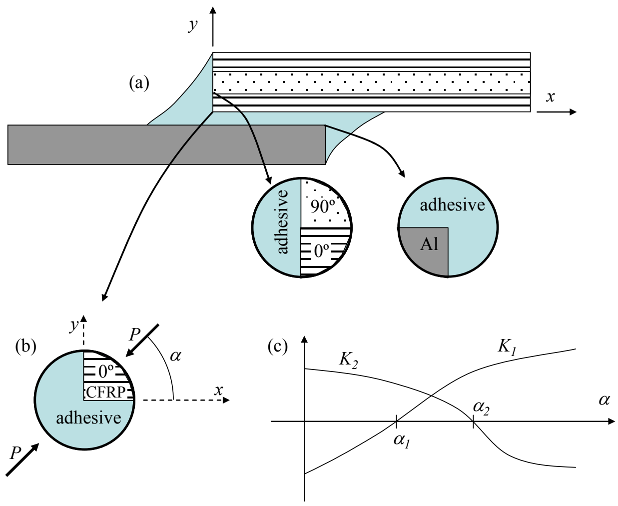

Fig. 1. Schematic representation of the procedure for isolating the singular terms.

4. Application to a real bimaterial corner

The previously introduced procedures will be applied for the bi-material corner shown in Figure 1b. The calculation of the orders of stress singularities and angular shape functions in (1) can be obtained by the well-known techniques, see for example Ting (1997), Barroso *et al*. (2003), Hwu *et al*. (2003) and Yin (2003) among others. In particular, the procedure by Barroso *et al*. (2003) has been used in the present work.

The mechanical properties of the carbon/epoxy unidirectional lamina (AS4/8552), modelled as an orthotropic material, are: E_{11}=141.3 GPa, E_{22}=E_{33}=9.58 GPa, G_{12}=G_{13}=5.0 GPa, G_{23}=3.5 GPa, ν_{12}=ν_{13}=0.3, ν_{23}=0.32, whereas the isotropic elastic properties of the epoxy adhesive (FM-73M.06) are E=3.0 GPa and ν=0.35.

The values of the three smallest non-trivial characteristic exponents are (λ_1=0.763236, λ_2=0.889389, λ_3=1.106980). The angular shape functions can also be computed from the semianalytic expressions given by Barroso *et al.* (2003).

The GSIFs have been standardized following the procedure by Pageau *et al*. (1996) giving rise to expressions of the angular shape functions which fulfil the condition $f_{\theta\theta}^{k}\Big|_{\theta=0^{\circ}} = (2\pi)^{\lambda_k - 1}$ (k=1,2,3), in order to have $\sigma_{\theta\theta}\Big|_{\theta=0^{\circ}} = \sum_{k=1}^{3} \frac{K_k}{(2\pi r)^{1-\lambda_k}}$.

The GSIFs K_k (k=1,2,3) for the particular corner under study have been extracted from FEM results using the postprocessing procedure by Barroso *et al.* (2011). In this procedure, the determination of K_k is based on the minimization of the sum of quadratic differences between the analytical series expansion (1) and numerically computed displacements at common bonded interfaces (θ=0º, 90º) by a simple and robust least squares technique.

Any other available technique could be used for evaluating the GSIFs. The normalized values, according to Pageau *et al*. (1996), of the GSIFs associated to the two singular terms (K_1, K_2) for 0º<α<180º are shown in Figure 2, where it can be observed that $\alpha \approx$ 13º and $\alpha \approx$ 60º are the test configurations at which K_2 and K_1 respectively vanish.

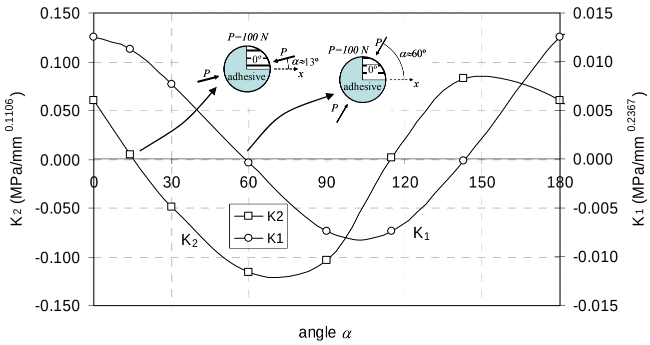

Fig. 2. Standardized values of K_1 and K_2 vs the loading compression orientation α.

5. Experimental results

The samples were prepared for the application of the compression load which should be distributed along a small flat area at both sides, as usually recommended in the standard Brazilian test to avoid premature failure (Wang and Xing, 1999). The samples were tested in compression. Values of K_k (k=1,2,3) were evaluated numerically (by FEM) for a compression value of P=100 N, a radius R=1 mm and thickness t=1 mm. The experimental results have to be properly scaled with the data from the real tested specimens by means of the following expression:

$$K_{kC} = \frac{F_u}{t \cdot R \cdot P} K_k^{numerical\,(P=100N)} \quad (2)$$

where F_u is the failure load (in Newtons), t is the thickness (in mm), R is the radius (in mm) and P=100 N is the factor of the applied load in the numerical FEM model. Using equation (2) the generalized fracture toughness values K_{1C} and K_{2C} can be calculated. Due to the fact that K_{1C} and K_{2C} have different units, it is convenient to divide the critical values of K_k for other load orientations by the mean value of K_{kC} in order to eliminate these units. In this sense, Figure 3, and Table 1, shows the dimensionless values of critical values of K_k obtained in all experiments. This representation allows a failure envelope based on the generalized fracture toughness values of the corner to be proposed. The failure envelope has been defined using the mean values of critical GSIFs K_k for each loading angle, and linear interpolation between loading angles.

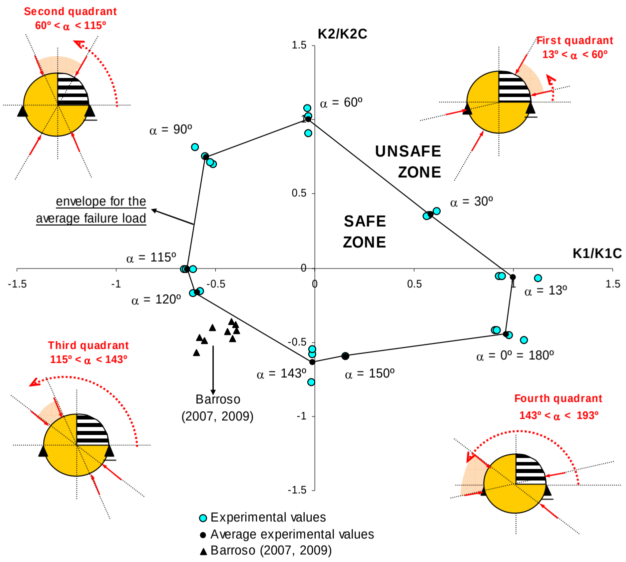

Fig. 3. Failure envelope based on the generalized fracture toughness values.

Load angle / sample		$\frac{K_1}{K_{1C}}$	$\frac{K_2}{K_{2C}}$
0º = 180º	a	1.05	-0.49
	b	0.91	-0.42
	c	0.98	-0.45
	d	0.92	-0.42
13º (K_2≈0)	a	0.93	-0.06
	b	1.13	-0.07
	c	0.94	-0.06
30º	a	0.62	0.38
	b	0.58	0.35
	c	0.56	0.35
60º (K_1≈0)	a	-0.03	0.91
	b	-0.03	1.02
	c	-0.03	1.07
90º	a	-0.51	0.69
	b	-0.55	0.75
	c	-0.60	0.81
	d	-0.53	0.71
115º (K_2≈0)	a	-0.61	-0.01
	b	-0.66	-0.01
	c	-0.65	-0.01
120º	a	-0.61	-0.17
	b	-0.58	-0.16
143º (K_1≈0)	a	-0.01	-0.58
	b	-0.01	-0.77
	c	-0.01	-0.55
150º	a	0.15	-0.59

Table 1. Results of K_1/K_{1C} and K_2/K_{2C}.

Additionally, experimental results by Barroso (2007, 2009), testing adhesively bonded double-lap joints with the same local geometry at the end of the overlap zone, in the joint of the unidirectional laminate corner with the adhesive spew fillet, have been included in Figure 3. However, although the results fall outside the failure envelope obtained in the present work, they are very close to it. This result is quite significant as the samples tested in Barroso (2007, 2009), although having the same local corner configuration, are completely different in size, geometry and manufacturing process. Pictures of the tested samples and their corresponding failure schemes,, for the load orientations (α=0º, 13º, 30º, 60º and 90º) which include pure mode loadings (α=13º and α=60º) are shown in Figure 4.

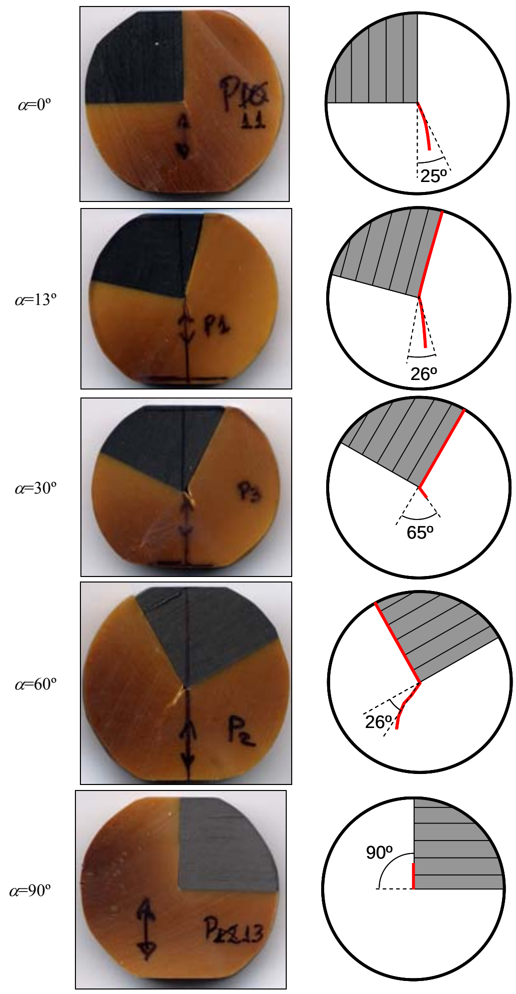

Fig. 4. Some tested samples and failure schemes (α=0º, 13º, 30º, 60º and 90º).

Conclusions

In the present work an experimental test procedure has been developed for the generalized-fracture-toughness determination in multimaterial closed corners. The procedure is based on a novel modified configuration of the Brazilian test applied to the corner geometry. The procedure is especially suitable for non-symmetric multimaterial corners involving isotropic and non-isotropic materials and having two stress singularities.

The method is able to isolate any of the singular modes, which is not possible with standard tests procedures defined for homogeneous isotropic materials, due to the lack of general symmetries of the local stress states at these corners. The method has been applied to a particular bimaterial corner typically appearing in adhesive joints involving composites, and generalized fracture toughness values K_{1C} and K_{2C} have been obtained.

With the generalized fracture toughness values, a failure envelope based on the critical GSIF values has been defined, which can be used as a failure criterion in joints of this type. Previous experimental results already published, involving the same local corner configuration but completely different global geometry, have shown to be in a good agreement with the proposed failure envelope.

Acknowledgements

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