A Middle Income Trap in a Small Open Economy: Modelling the Argentinean Case

Abstract
This paper presents a model of the conditions that may lead a small open economy towards a middle income trap. This situation has shown to be pervasive in Latin America. As Argentina is a salient instance of this phenomenon, we develop a stylized model of its economy at the first decades of the XXth century. It consists of a general equilibrium model of an open emerging economy, which is a price-taking primary goods exporter. A growth process is triggered by an increase of commodity prices, due to an upward jump of the world demand of these goods. The economy goes through several phases of growth, starting from a subsistence stage. Once decreasing returns set in, the economy reaches a steady state. Only a sustained high demand of its export products allows the economy to thrive. Otherwise, the economy gets entrapped in a middle income level.

Keywords: middle income trap, stages of development, small open economy, steady state, external conditions.

JEL Classification: O10, O11.

1 Introduction

Following the 2007/2008 world crisis, which mainly affected the developed economies, it seemed that developing countries were the only ones able to help the world to avoid a downfall towards a long-run recession. While China was the most conspicuous economy pushing the global economy, other countries contributed as well. The countries in the BRICS group, in particular, seemed to be steered towards the status of advanced economies (Sharma, 2014). By 2015 these expectations have largely faded, mostly due to the fall in the world prices of commodities, the main production of developing nations (Qureshi et al., 2015). So, paralleling the discussion on the "secular stagnation" of advanced economies, the awareness on the existence of a *middle income trap* for developing nations spurred a literature to which this paper intends to be a contribution (Griffith, 2011; Eichengreen et. al, 2012; Agénor and Canuto, 2015).

Our take is based on the realization that this is by no means a new phenomenon. Latin America has been prone to generate false hopes of rapid development followed by stops and setbacks (Paus, 2012; King and Ramlogan-Dobson, 2015). Argentina, for instance, is an early example of this.[1] This economy does in fact show all the features that may serve for an explanation of the existence of middle income traps. Instead of presenting a case study, we proceed by developing a model capturing those aspects. The main idea is that the particular features of the growth process of a late-coming small open economy like Argentina's may help to explain how a middle income trap is reached. More precisely, we intend to show how the absence of increasing returns in the production of industrial goods and services leads to the stagnation of small agricultural goods producer economies. Unlike the usual approach in growth theory, we develop a general equilibrium model of a stylized version of the Argentinean economy, undergoing different stages of growth. We assume that foreign trade is balanced, and the economy is responsive to the changes in the external economic environment. In this economy, the initial stage consists of an agricultural sector, which uses land as its main production factor. At this stage there exists only a domestic market for these products. The proceeds are barely enough to satisfy current consumption, so that the bequest to the next generation consists only of land.

In a second stage the economy is able to export its agricultural goods. Since this economy is assumed to be small, the local producers are price takers

[1] A famous -but perhaps apocryphal- remark of Nobel Prize winner Simon Kuznets is: "There are four kinds of countries in the world: developed countries, undeveloped countries, Japan and Argentina." *The Economist* (2014).

in the world market. The rise of an external demand increases the prices of the local production, and then the income level.[2] This induces a change in the consumption pattern from agricultural to industrial goods and services. Since industrial products are tradable, they can be imported. Domestic production of industrial goods and services requires the use of physical and human capital, and thus the higher yields of agricultural production are devoted to the accumulation of both types of capital. Physical capital is assumed tradable, so that its price is taken from the world market. On the other hand, human capital is generated from an income tax applied to finance a public education system. The complementarity of physical and human capital requires critical levels of both to start the production process. Thus, in a further stage, once enough capital has been accumulated, the economy is able to produce industrial goods and services, and to reach a middle income steady state level. If there is no upgrade in the technology, in presence of decreasing returns, the economy will not grow further than that steady state level.

That is, the country gets trapped at a middle income level which, furthermore, is highly dependent on the international prices of agricultural products. Only a permanent increase of those prices can ensure an upgrade towards a higher income level. This result could illustrate the fact that many countries that reached a middle income level before the Great Depression.[3] A policy recommendation that arises from this analysis is to act on the features of the economy that lead to a middle steady-state result. That is, on the existence of decreasing returns in the production function. Structural reforms leading to continuous increases in productivity can achieve this goal and allow the escape from the middle income trap. This is consistent with recent analyses based on empirical evidence (Agénor and Canuto, 2015; Qureshi et. al, 2015).

The next section presents a review of the relevant literature, on which we base our main modeling choices. Section 3 presents the basic setting of the model. First the supply and demand functions of each sector are derived, and then we obtain the conditions for a general equilibrium. Up from the basic model, section 4 characterizes the different phases of the growth process: from the economy of subsistence to the transitional stage, and finally to the modern phase, in which we obtain the steady state of the economy. Finally, section 5 concludes.

[2]The importance of the existence of a successful "client" nation, like the UK was for Argentina is crucial for the ensuing growth process (Osborn and Vehbi, 2015).

[3]In fact several of these countries remained stagnated until the surge in commodity prices of the 2000s and were affected by the recent downturn of these prices.

2 Literature Review

In the last decades growth theory tried to explain the sustained growth of per capita income. Most of the literature emphasized the role of technical progress. While originally it was assumed to be exogenous, later approaches focused on endogenous growth processes. Within this branch of the literature, two approaches can be distinguished. On the one hand, the AK model is based on the crucial assumption of perfect competition, postulating the existence of externalities and scale effects as responsible of the sustained growth of the economy (see Romer (1986) and Lucas (1988, 1990)). On the other hand, the Research and Development (R & D) models assume imperfect competition; they explain growth in the long-run as a consequence of investment devoted to the creation of new processes and products, as well as from their spillovers (see Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), among others).

Recent theoretical developments analyzed how growth processes go through different stages. In this sense, Galor and Moav (2002, 2006) exhibit how a growth process may give way to higher stages of development. In particular, they explain phenomena like the demographic transition and the demise of the class structure in advanced economies. The main feature of their models is that, at some point in the evolution of an economy, external shocks may induce a new phase in the dynamics, leading towards a new equilibrium. More precisely, Galor and Moav (2002) show how growth facilitates the rapid reproduction of individuals that carry less children and invest more in education. This leads to the avoidance of a Malthusian growth trap and the enhancement of the environment for technological progress. In this vein, Galor and Moav (2006) show how the creation of a public education system in an economy with a class structure (capitalists and workers) leads to the creation of human capital, which generates the conditions for further growth in an economy where the class distinctions fade away.

All these theoretical developments are intended to explain the stylized facts of growth in advanced economies. Moreover, Galiani et al. (2008) present an extension of this approach, in order to model the development of land rich economies as triggered by the emergence of large-scale education systems. The idea is that the elite of landowners accepts collectively to be taxed in order to finance the education of credit-constrained workers. The rationale of this behavior is that a wide education system induces the accumulation of human capital able to sustain the human-capital-intensive services demanded by high-income groups.

Similarly, small open economies, specialized in the production of agricultural goods, tend to follow growth paths that differ from those of developed

economies due to their exposure to foreign markets. In particular, they can reach a higher stage of development as long as external conditions are favorable enough. In this sense, some successful cases faced favorable foreign conditions that lead to a long period of high growth, associated with physical and human capital accumulation and later with the development of domestic production of services and an incipient industry. Argentina illustrates well this process, particularly from 1870 until the midst of the XXth century. Furthermore, this case is interesting also because it exhibits the stagnation associated to a reversion of terms of trade, without having generated endogenous mechanisms to ensure an autonomous continuation of the growth process. But, as a recent study of the case of Latin America presented by Rodrigues (2010) in the framework of a dynamic Heckscher-Ohlin model has shown, even though Latin America deepened its capital between the 1950s and the 1980s, the adoption of import substitution policies created a low productivity trap and put the region on a long-run stagnation path. As Taylor (2014) asserts, the import substitution industrialization of Latin American countries increased the gap in industrial development with respect to the developed countries. In fact, investment in capital goods has barely increased even during the recent surge in commodity prices which triggered a process of fast growth in the region. For example, for the Argentinian case Santarcángelo et al. (2011) show that this growth did not lead to a structural change. All these arguments seem to indicate that the technology in the industrial and services sectors in Latin America lacks the increasing returns proper of advanced economies, and their growth has been mostly due to the improvement of the international prices of agricultural goods.

3 The Basic Setting

3.1 The Demand Side

The economy has a constant number of adult agents, n. Each agent i born in $t-1$ lives for two periods. At t he is an adult, has just one child, and decides the level of consumption and bequest at this period. The log-linear utility function is:

$$\begin{aligned} U_i(c_t^i, b_t^i) &= (1-\beta)[\alpha_1 \log({}^A c_t^i) + \\ &\quad \alpha_2 \log({}^I c_t^i + {}^I \bar{c}^i) + \alpha_3 \log({}^S c_t^i + {}^S \bar{c}^i)] + \beta \log(b_t^i + \bar{b}^i) \end{aligned}$$

where ${}^l c_t^i$ for $l = A, I, S$ is the consumption of either agricultural products, industrial goods or services. The constants ${}^l \bar{c}^i$, for $l = I, S$, and $\bar{b}^i$ are

assumed positive, with ${}^I\bar{c}^i <^S \bar{c}^i$.[4] For simplicity, we assume that the effect of the consumption of each type of good on the utility is the same, i.e. the parameters are such that $\alpha_1 = \alpha_2 = \alpha_3$, while $\alpha_1 + \alpha_2 + \alpha_3 = 1$. In turn, industrial goods are available in the world markets while services are non-tradeable. The parameter that indicates the savings (and bequeathing) propensity is $\beta \in [0, 1]$. Thus, an individual's satisfaction depends on how much he consumes as well as how much he bequests to his (only) child. While the satisfaction of consuming any of the three kinds of goods has the same weight, there is a difference between consuming and bequeathing.

The bequest b_t^i consists of the value of land (≥ 0) as well as of physical and human capital. To distinguish father and son, we indicate i's child with $i+$ and so the bequest is:

$$b_t^i = p_{t+1}^k k_{t+1}^{i+} + p_{t+1}^H h_{t+1}^{i+} + p_{t+1}^K K_{t+1}^{i+} \tag{1}$$

where k_{t+1}^{i+}, h_{t+1}^{i+} and K_{t+1}^{i+} are the bequests in land, human and physical capital, respectively.

Human capital is assumed to arise solely from education:

$$h_{t+1}^{i+} = (e_t^{i+})^\gamma \tag{2}$$

where e_t^{i+} is the amount of education received by i's child, and $\gamma < 1$; that is, the production of human capital has decreasing returns.

The income at t arises from the benefits that agent i accrues from participating in production units, the production of agricultural products and the sale of land and human capital:

$$Y_t^i \equiv \sum_j \theta_{ij} B_t^j + p_t^A A(k_t^i)^\alpha + p_t^k k_t^i + p_t^H h_t^i \tag{3a}$$

where B_t^j is the benefit of a firm j that produces industrial goods and services, and in which i has a participation $\theta_{ij} \leq 1$. $p_t^A A(k_t^i)^\alpha$ is the agricultural production, which uses land as input at an intensity $A > 1$, while $\alpha \in (0, 1)$ indicates the existence of decreasing returns to the scale of land. k_t^i and h_t^i are, respectively, the amounts of land and human capital sold. Finally, the weights in the production of industrial goods and services satisfy $\sum_j \theta_{ij} = 1$.

On the other hand, i's income at t is devoted to consume, to finance the child's education, to purchase land and to accumulate physical capital:

$$Y_t^i = p_t^k k_{t+1}^{i+} + p_t^K K_{t+1}^{i+} + p_t^E e_t^{i+} + \bar{Y}_t^i \tag{3b}$$

[4]Actually, the arguments X are assumed to be such that $\log(X) > 0$. This is ensured if $X > 1$. To simplify we assume that the variables we use are $\bar{X} = X + 1$.

where k_{t+1}^{i+} and K_{t+1}^{i+} are the amounts of land and physical capital bought as bequest for $i+$, and $\bar{Y}_t^i$ is the composite consumption index, i.e. $\bar{Y}_t^i = p_t^{A\ A}c_t^i + p_t^{I\ I}c_t^i + p_t^{S\ S}c_t^i$. We assume here that in a given period the land-owner sells *both* the land and its production. The income generated allows him to buy back land as well as physical capital in the next period. In turn, e_t^{i+} constitutes the amount education required for generating the human capital for the next generation, so that

$$b_t^i = p_{t+1}^k k_{t+1}^{i+} + p_{t+1}^K K_{t+1}^{i+} + p_{t+1}^H (e_t^{i+})^\gamma \qquad (1')$$

Let us note that it seems, at face value, that each father *sells* land in order to later *buy* land for his child. But this covers also the case in which the father just leaves the land to his son. It would be different if transaction costs were at play, but here we assume that these transactions are frictionless.

Therefore, agent i's goal is to choose the optimal values of c_t^l for $l = A, I, S$, k_{t+1}^{i+}, K_{t+1}^{i+} and e_t^{i+} such that maximizing his utility under the aforementioned constraints. The solutions depend on both income and prices. The conditions for non-zero solutions are the following $((7a), (7b), (7c)$ and $(7d)$ in the Appendix):

$$\begin{aligned}
c_t^A > 0 &\Leftrightarrow Y_t^{i\prime} + p_t^{I\ I}\bar{c}^i + p_t^{S\ S}\bar{c}^i + \bar{b}^i > 0 \\
c_t^I > 0 &\Leftrightarrow (1-\beta)\alpha_2[Y_t^{i\prime} + p_t^{I\ I}\bar{c}^i + p_t^{S\ S}\bar{c}^i + \bar{b}^i] > p_t^{I\ I}\bar{c}^i \\
c_t^S > 0 &\Leftrightarrow (1-\beta)\alpha_3[Y_t^{i\prime} + p_t^{I\ I}\bar{c}^i + p_t^{S\ S}\bar{c}^i + \bar{b}^i] > p_t^{S\ S}\bar{c}^i \\
b_t > 0 &\Leftrightarrow Y_t^{i\prime} + p_t^{I\ I}\bar{c}^i + p_t^{S\ S}\bar{c}^i > \frac{1-\beta}{\beta}\bar{b}^i
\end{aligned}$$

These inequalities show the existence of thresholds of consumption and bequest at different levels of income. It follows that at low levels of income, only agricultural goods will be consumed, while no bequest will be left. On the other hand, a necessary condition for the consumption of industrial goods and services is the consumption of agricultural products.

3.2 The Supply Side

The benefits of the agricultural sector is (Expression $(8a)$ in the Appendix):

$$B_t^A \equiv p_t^A q_t^A - p_t^k \frac{(q_t^A)^{\frac{1}{\alpha}}}{A^{\frac{1}{\alpha}}}$$

while each firm, j, producing either industrial goods or services in the economy, has the following profit function ($(8b)$ in the Appendix):

$$B_t^j = p_t^I q_t^j + p_t^S s_t^j - (p_t^K K_t^j + p_t^H h_t^j)$$

where K_t^j and h_t^j are the inputs in the production of industrial goods and services, q_t^j and s_t^j. The transformation functions are $s_t^j = f_j(K_t^j, h_t^j)$ and $q_t^j = g_j(K_t^j, h_t^j)$ which we assume concave and with constants returns to scale, allowing their aggregation (see Subsection 6.2 in the Appendix).

These conditions reflect the assumption that this economy is not subject to technical change (either exogenous or endogenous). We also assume that the production factors have perfect mobility between both sectors. That is, each factor is allocated as to reach the same marginal productivity in both sectors.[5]

3.3 The Equilibrium

The equilibrium in each market is reached through the interplay of supply and demand. The equilibrium price of land (the production factor) equals the value of its marginal product and thus we have (Expression $(9a)$ in the Appendix):

$$p_t^k = p_t^A(\alpha A \mathcal{K}^{\alpha-1})$$

The equilibrium in the agricultural goods, in the case in which the production is consumed domestically, yields:[6]

$$p_t^A = [\alpha_1(1-\beta)\hat{Y}_t^i]^{1-\alpha} A^{-1} \left(\frac{1}{\alpha}\right)^{\alpha} (p_t^k)^{\alpha}.$$

Since physical capital as well as the agricultural and industrial goods are tradable, their prices are taken from the world market. On the contrary, given that services are not tradable, their prices are determined in the domestic market ($(10d)$ in the Appendix):

$$p_t^S = \frac{\alpha_3(1-\beta)[Y_t\prime + p_t^I \ {}^I\bar{c} - \bar{b}]}{f(K_t^S, h_t^S) + {}^S\bar{c}[1-\alpha_2(1-\beta)]}$$

[5]This is a stylization of the fact that the size of the industry with respect to the size of the services sector remained relatively constant in the case of Argentina at least until the 1940s, in a time of free market policies, suggesting the absence of extra advantages in either one of the activities (Llach and Gerchunoff, 1998).

[6]Here $\hat{Y}_t^i = Y_t^i\prime + p_t^I \ {}^I\bar{c}^i + p_t^S \ {}^S\bar{c}^i - \bar{b}^i$ while $Y_t^i\prime$ is given by $(5b)$ in the Appendix.

while the equilibrium price of human capital is $p_t^H = (p_t^I + p_t^S)\frac{\partial f}{\partial h}(K_t^S, h_t^S)$. The amount of human capital produced will be $\bar{h}_t$ and the corresponding level of education, $\bar{e}_{t-1} = (\bar{h}_t)^{\frac{1}{\gamma}}$, is such that ((10$e$) in the Appendix):

$$p_{t-1}^E = p_t^H (\bar{h}_t)^{\frac{\gamma-1}{\gamma}}$$

In what follows we will analyze the dynamic transition among different stages of growth at changes in the external conditions faced by the economy. Figure 1 shows how Y^i depends log-linearly on p^A,[7] indicating the thresholds of income (Y_I and Y_S represent the thresholds that allow consumption of manufactures and services, respectively, derived from (5b), (7b) and (7c)), and p_O^A, the critical international price of agricultural goods that triggers the growth process:

[INSERT FIGURE 1 HERE]

Figure 1: The relation between income and the international prices of agricultural goods.

In particular we are interested in determining the steady state income at different prices of the agricultural goods. In order to do that we will keep at minimum the number of equilibrium conditions explicitly stated, with the proviso that *all* the economy is in equilibrium.

4 Stages of Growth

4.1 The Subsistence Economy

In this stage agents produce agricultural goods, which are only sold in the domestic market at the international price p^A. We assume that for all this stage $p_t^A = p^A$, for $t = 1, \ldots T_1$. Furthermore, there is no trade other than in agricultural products and land, and income arises from the sale of these items, for $t = 1, \ldots, T_1$ (Expression (11) in the Appendix):

$$Y_t^i \equiv p^A A(k_t^i)^\alpha + p_t^k k_t^i$$

The bequest will be ((11a) in the Appendix):

[7] Taking the equilibrium condition (9c) in the Appendix, a straightforward partial derivation yields $\frac{\partial Y^i}{\partial p^A} = \gamma(\frac{Y^i}{p^A})^\alpha$, where γ is assumed constant. Integrating we obtain that $\log Y^i = \frac{\log \gamma}{1-\alpha} + \log p^A$. We denote lY^i and lp^A to $\log Y^i$ and $\log p^A$, respectively.

$$b_t = \beta[p^A A(k_t^i)^\alpha + p_t^k k_t^i] - (1+\beta)\bar{b}^i$$

which in turn equals $p_{t+1}^k k_{t+1}^{i+}$, since the agent can leave only land to his child. That is, in this stage the older generation can only bequest land net of depreciation.

4.2 The Transition

In this stage the economy begins at $T_1 + 1$. Here the economy satisfies a growing external demand of its agricultural products, which increases the international prices. Consequently, the price of land also increases. The new prices for this phase, at $t = T_1 + 1$ are $p_t^A = p_O^A$ and $p_t^k = p_O^k$. At these new prices the income increases to a new level Y_O^i. With this higher income the owners of land are able to consume also industrial goods. On the other hand, assuming trade equilibrium,[8] all the excess in income, due to exports, must be devoted to import from foreign markets. Tradeable goods like industrial products, physical capital and education services can be imported. Notice that, since we do not assume the existence of foreign credit and the only way pay for imports is with the proceeds of exportation, no trade unbalances can be sustained in the model. Furthermore, assuming that the prices do not decrease from p_O^A the economy cannot revert to the subsistence stage, since the income will be enough to surpass the thresholds described in expressions ($7b$) and ($7c$) in the Appendix.

Physical and human capital become valuable for the production of industrial goods and services. But, since both types of capital are necessary to start production and human capital has to be produced locally, the acquisition of capital goods has to be delayed until enough h is produced. That is, only when h_t surpasses a critical level h^*, the economy will be able to produce both industrial goods and services. Only then capital goods will be imported. If $h_t < h^*$ the demand for industrial goods is fulfilled with imports.

On the other hand, since $h(\cdot)$ has decreasing returns to scale, to provide h^* a massive education system is needed. Besides, at this point no market exist for education in the economy. Consequently, in order to accumulate enough human capital, a public education system is created. At an early

[8] In the case of Argentina, in the 1871-1940 period the yearly average trade surplus was 50.8 million dollars (in current values). From those 70 years, only 27 witnessed negative values. In particular, the years from 1891 up to the 1930 were salient, with significant trade surpluses in proportion to the GNP, even considering that great influxes of foreign investment (railroads, subway lines, power stations, etc.) in those years (Ferreres, 2010).

stage educational services are imported[9], and they are purchased by the proceeds of an income tax.[10]

So, the bequest becomes

$$b^i_{T_1+1} = p^k_O k^{i+}_{T_1+2} + p^K_{T_1+2} K^{i+}_{T_1+2} + p^H_{T_1+2} h^{i+}_{T_1+2}$$

$\tau_{T_1+1} Y^i_O < Y^i_O$ is the income tax, that is devoted to the purchase of education, thus $\sum_{i=1}^n \tau_{T_1+1} Y^i_O = p^E_{T_1+1} e$, being e the aggregate level of education. From then on, the tax rate $\tau_t < 1$ will be such that $\sum_{i=1}^n \tau_t Y^i_t = p^E_t e$. The effects of this tax are twofold. On one hand the domestic consumption of agricultural and industrial goods as well as bequests would be lower than without the tax. On the other, human capital for the future generation increases and thus lowers its cost and that of services[11]

The amount of education e is chosen as to ensure the attainment of the critical level of human capital $h^* < h(e)$. It is assumed that the public system provides an uniform education to all the single children of the n households in the economy, i.e. $e^i_{T_1+1} = \frac{e}{n}$. Therefore $h^{i+}_{T_1+2} = h(e^i_{T_1+1})$ for each i. Furthermore, we assume that $h(e)$ is ready to go to the market in a single period. That is, that at $T_1 + 1$ all the human capital required is available in the economy.[12]

4.3 The Modern Phase and the Steady State

At $T_2 = T_1 + 2$ the economy is already able to import or to produce physical capital and start the domestic production of goods and services, due to the formation of human capital through education. Thus, both industrial goods and services will be consumed. Among the services that can be acquired are educational services that are now produced domestically. Nevertheless, assuming that trade equilibrium is kept in the long run, the reduction in purchase of foreign education services and industrial products is balanced by higher imports of goods. The full model, as presented in section 2, describes the situation in this stage. This yields a sustained growth path towards a steady state, as follows: the tax income increases, which means that higher

[9] An instance on how this may happen consider that in the 1870s Argentina hired American teachers, who started mentoring aspiring Argentinean teachers (Galiani et al., 2008).

[10] Following Galiani et al. (2008), the role of a government is highly stylized, acting as a frictionless mechanism that taxes the agents to purchase education.

[11] The welfare effects of this tax are analyzed in detail in Subsection 6.5 of the Appendix.

[12] Alternatively, we could think of m periods (school years) necessary for the generation of $h(e)$. In that case, the transition would be completed at $T_1 + m$.

amounts of education can be bought. Therefore, human capital increases as well as the production of industrial goods and services, and therefore income for $t > T_2$. As shown in the Appendix, we obtain that $Y^i_{t+1} = \beta Y^i_t + \bar{r}$, where $\bar{r} = Y^i_{T_2} - \beta Y^i_{T_1} > 0$, i.e. the change induced by changes in the agricultural products in global markets.

This process reaches a steady state (Expression $(11h)$ in the Appendix):

$$\bar{Y}^i_t = \frac{Y^i_{T_2} - \beta Y^i_{T_1}}{1 - \beta}$$

which grows with the magnitude of the initial jump up from the subsistence level, and decreases with the weight given in the preferences to consumption over bequest, $1 - \beta$. Figure 2 represents the evolution towards this steady state up from the upward jump in international prices of agricultural products (also shown here):

[INSERT FIGURE 2 HERE]

Figure 2: Evolution of Y^i in time.

The explanation for this behavior is rather intuitive: when land is fully used, and since the domestic industry is not able to affect the international price, income is devoted to consume more imported goods, while the bequest only impacts in the production of services. But then, the decreasing returns in their production set in, and the marginal productivity of physical and human capital decreases until the economy reaches its steady state. The level of income at the steady state depends on the consumption weight on the utility function and the size of the initial jump, which is determined by the world prices of agricultural goods.

5 Discussion and Conclusions

The model shows how a small agricultural goods producer open economy can go from a subsistence to a modern middle-income phase. The gap with advanced economies that are not subject to decreasing returns in their production functions may even grow, if the induced growth process in the latter countries does not increase their demand for agricultural products. This is so because once our economy reaches its steady state only continuous increments in the foreign demand of its production is able to increase its income level. That is, only a major change in the foreign conditions of the economy may lead the economy towards a new steady state. On the contrary, an unfavorable change in these conditions provokes a low steady state.

The model presented in this paper can be conceived as representation of many cases of economic development towards a middle income level from which it is hard to get out towards full development. Notice that the economy gets entrapped in a middle level of income even in the best international conditions (as illustrated in Figure 2). As indicated in the Introduction, Argentina provides our main illustration of this process. It is a small, price-taking economy whose exports have consistently been around the 0.4 or 0.5% of the world total. The fertility of the soil favored the subsistence of the original scarce population, based on the exploitation of large herds of cattle that required little human attention. The grasslands did most of the job: small teams of uneducated *gauchos* just captured and slaughter the animals needed to feed the small population. No education nor health system existed and the rule of law was notoriously absent. Warlords exerted property rights by sheer force, transmitting them to their progeny, which in turn had the same kind of life as their elders.[13]

But around the 1870s a technological revolution changed all this. Trans-oceanic steamboats with refrigerated cargo bays opened the production of the Pampas to the world. A rapid transition towards a modern economic setting ensued, in which the former warlords became landlords of enormous wealth. The per-head GDP jumped from a yearly mean of (in constant 1990 dollar values) U$S 1264 in 1880 to 3310 in 1930.[14] After the Depression and up to the 1960s (when, as a consequence, political and social turmoil started to affect further the economy), the per-capita income stalled at an average of U$S 4847. The growth rate accompanied this process, jumping from a dismal 0.52% in the 1810-1880 period, to an average of 2.62% from then on to 1930. This process slowed down afterwards, with a growth rate of 0.9% from 1930 to 1960. As emphasized in our model, this process paralleled the evolution of the terms of trade which *tripled* from the subsistence to the modern period, stalling in the Depression era and afterwards, when the prices of agricultural products were no longer as high as before 1930. During all this process, the Argentinean economy remained grounded on the production of agricultural products. Its industrial exports only became noticeable in its commercial balance after 1960, being their production until then only destined to the domestic market. On the contrary, industrial imports were highly important, consisting mainly of capital and intermediate goods which amounted to more than 36% of all imports. Finally, as it also derived in the model, human capital increased rapidly, from a literacy rate of 22% of the population in 1869 to a 62% in 1914 and 87% in 1947 (Llach and Gerchunoff,

[13]See Charles Darwin narrative of the life in the Pampas around 1830 (Darwin, 2014).

[14]All the data provided here are drawn from (Ferreres, 2010).

2004).

In summary, the case of Argentina could be fairly well described by the model, up to the long-term stagnation in an less amicable international environment, in which the prices of commodities are no longer favorable. Furthermore, even the downfall of the Argentinean economy seems to be also be understood in terms of this model. Since the international prices of agricultural goods decreased drastically, the income of the land-owners was also reduced. This led to lower levels of education and in turn of human capital generation. While after WWII international prices recovered, industrial goods became more valuable than agricultural ones. Therefore, an economy strongly based on the production of the latter and with decreased stocks of human capital was unable to cope with these changes. Nonetheless, wrong-headed import-substitution policies led the (incipient) industrial production towards the domestic market (Hirschman, 1968). Therefore, the Argentinean economy remained trapped in a middle income steady state and ended up betraying the expectations it had risen in the early XXth century.

Unless a process of technological development is started in the industrial and services sectors, avoiding decreasing returns, making the marginal value of human capital larger than its price,[15] the economy will end up be only affected by changes in the international prices of the agricultural goods.

More precisely, if the technologies in the industrial and services sectors, represented by the aggregate functions f and g had increasing returns, such that $\frac{\partial B_t}{\partial h} > 0$, the condition $(10b)$ would become $p_t^H < (p_t^S + p_t^I)\frac{\partial f}{\partial h}$. That is, the marginal value of human capital in the industrial sector (the same is true for services) would be larger than its price, inducing to the continued accumulation of this factor. In turn, if these differences were large enough would lead to $\frac{dY_{t+1}^i}{dY_t^i} > 1$, i.e. to the progressive increase in income over time.

These theoretical results represent the general case of emerging economies specialized in export of agricultural goods (i.e. commodities) in the early development stages, then in turn import industrial goods. Once the rest of the world begins to demand increasingly those commodities, they face better external conditions and are able to finance and develop the production of industrial goods and services domestically. The failure in improving the technology allows us to understand the pervasiveness of countries mired in middle income traps, particularly in Latin America. Since their industrial production has been historically oriented to their small domestic markets, capital accumulation has been low, which in turn affected investment in human capital. These features of the Latin American economies could explain

[15] An analogous argument, based on the condition $(10b\prime)$ and adapting $(11d)$ accordingly, can be made with respect to physical capital.

their vulnerability to external conditions, as well as the difficulties to develop a competitive and dynamic industrial sector.

The policy recommendations that ensue from this work are the following. Firstly, to promote the incorporation of technological change in the less competitive and inward-looking activities, in particular the industrial and low-grade services sectors. Secondly, to diversify the production of tradeable goods, instead of just an agricultural based basket of exports. Finally, and related to the previous recommendations, the generation of human capital devoted to R&D is crucial for the success of such proposals.

From a more theoretical point of view this paper can be extended in several directions, all aimed to show ways in which the middle income trap may be avoided. One possibility is to drop the assumption of trade equilibrium. If so, if the economy restricts consumption, importing less than the amount exported, the initial jump in prices may remain in time. A sustained favorable trade balance disequilibrium may push towards a higher income steady state. Another extension can be to assume non-decreasing returns, so that we can avoid the convergence to a steady state, and, instead, the economy can follow a more interesting sustained growth dynamic path. Both extensions could allow to explain how some small agricultural producers, for instance countries in South East Asia, overcame the middle income trap and started a growth process that converted them in developed countries.

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6 Appendix

6.1 Consumer behavior

The problem in Subsection 3.1 is the following:[16]

[16]From now on we omit the distinction between indexes i and $i+$, except when necessary for distinguishing father and son. This allows us to simplify the notation, changing from

$$\begin{aligned}\max U_i(c_t, b_t^i) &= (1-\beta)[\alpha_1 \log(c_t^A) + \alpha_2 \log(c_t^I +^I \bar{c}^i) + \alpha_3 \log(c_t^S +^S \bar{c}^i)] + \\ &\beta \log[p_{t+1}^k k_{t+1}^{i+} + p_{t+1}^K K_{t+1}^{i+} + p_{t+1}^H (e_t^{i+})^\gamma + \bar{b}^i]\end{aligned}$$

subject to

$$\begin{aligned}&\sum_j \theta_{ij} B_t^j + p_t^A A(k_t^i)^\alpha + p_t^k k_t^i + p_t^H h_t^i = p_t^k k_{t+1}^{i+} + \\ &+p_{t+1}^K K_{t+1}^{i+} + p_t^E e_t^{i+} + p_t^{A\ A} c_t^i + p_t^{I\ I} c_t^I + p_t^{S\ S} c_t^S \end{aligned} \tag{3c}$$

Or:

$$Y_t^i = \bar{Y}_t^i + b_t^i - p_{t+1}^H h_{t+1}^{i+} + p_t^E e_t^{i+} \tag{3d}$$

It follows that the demanded amounts of the consumption goods and the bequest obey to the following three equations derived from the first order conditions,

$$c_t^A : (1-\beta)\alpha_1 \frac{1}{c_t^A} = \lambda p_t^A \tag{4a}$$

$$c_t^I : (1-\beta)\alpha_2 \frac{1}{c_t^I +^I \bar{c}^i} = \lambda p_t^I \tag{4b}$$

$$c_t^S : (1-\beta)\alpha_3 \frac{1}{c_t^S +^I \bar{c}^i} = \lambda p_t^S \tag{4c}$$

$$b_t : \frac{\beta}{b_t + \bar{b}^i} = \lambda \tag{4d}$$

where λ is the Lagrange multiplier corresponding to the income constraint. Using these conditions we have that:

$$(1-\beta)\alpha_1 \frac{(b_t + \bar{b}^i)}{\beta} = p_t^A c_t^A \tag{4a'}$$

$$(1-\beta)\alpha_2 \frac{(b_t + \bar{b}^i)}{\beta} = p_t^I c_t^I + p_t^{I\ I} \bar{c}^i \tag{4b'}$$

$$(1-\beta)\alpha_3 \frac{(b_t + \bar{b}^i)}{\beta} = p_t^S c_t^S + p_t^{S\ S} \bar{c}^i \tag{4c'}$$

Adding these expressions we have:

[l] c_t^i to c_t^l.

$$\frac{(1-\beta)}{\beta}[\alpha_1+\alpha_2+\alpha_3](b_t+\bar{b}^i) \;=\; p_t^A c_t^A + p_t^I c_t^I + p_t^S c_t^S + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i$$

that is,

$$\frac{(1-\beta)}{\beta}(b_t+\bar{b}^i) \;=\; \bar{Y}_t + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i$$

from which it follows that:

$$\frac{1}{\beta}(b_t+\bar{b}^i) \;=\; \bar{Y}_t + b_t + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i \tag{5a}$$

If we recall that the amount of income plus the net benefits of education is

$$Y_t^{i\prime} = Y_t^i + p_{t+1}^H h_{t+1}^{i+} - p_t^E e_t^{i+} \tag{5b}$$

which from $(3d)$ yields:

$$Y_t^{i\prime} = p_t^A c_t^A + p_t^I c_t^I + p_t^S c_t^S + b_t \tag{5c}$$

In turn $(5a)$ is equivalent to:

$$b_t = \beta Y_t^{i\prime} + \beta(p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i) - \bar{b}^i \tag{5d}$$

On the other hand, replacing $(5a)$ in $(4a'), (4b')$ and $(4c')$:

$$c_t^A = \alpha_1 \frac{(1-\beta)}{p_t^A}[Y_t^{i\prime} + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i] \tag{6a}$$

$$c_t^I = \alpha_2 \frac{(1-\beta)}{p_t^I}[Y_t^{i\prime} + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i] -{}^I\ \bar{c}^i \tag{6b}$$

$$c_t^S = \alpha_3 \frac{(1-\beta)}{p_t^S}[Y_t^{i\prime} + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i] -{}^S\ \bar{c}^i \tag{6c}$$

These results show that the amounts consumed depend on both income and prices. The conditions for non-zero solutions are the following:

$$c_t^A > 0 \Leftrightarrow Y_t^{i\prime} + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i \;>\; 0 \tag{7a}$$

$$c_t^I > 0 \Leftrightarrow (1-\beta)\alpha_2[Y_t^{i\prime} + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i] > p_t^I\ {}^I\bar{c}^i \tag{7b}$$

$$c_t^S > 0 \Leftrightarrow (1-\beta)\alpha_3[Y_t^{i\prime} + p_t^I\ {}^I\bar{c}^i + p_t^S\ {}^S\bar{c}^i + \bar{b}^i] > p_t^S\ {}^S\bar{c}^i \tag{7c}$$

while, from $(5d)$, the condition for the existence of bequest is the following:

$$b_t > 0 \Leftrightarrow Y_t^i \prime + p_t^I \ {}^I\bar{c}^i + p_t^S \ {}^S\bar{c}^i > \frac{1-\beta}{\beta}\bar{b}^i \tag{7d}$$

The set of equations (7) shows the existence of thresholds of consumption and bequest at different levels of income. Comparing conditions (7a) and (7d), since $\frac{1+\beta}{\beta}\bar{b}^i \geq \bar{b}^i$, for low levels of income, only agricultural goods will be consumed, while no bequest will be left. From (7b) and (7c)) a necessary condition for the consumption of industrial goods and services is the consumption of agricultural products, because in this case $Y_t^i \prime + p_t^I \ {}^I\bar{c}^i + p_t^S \ {}^S\bar{c}^i - \bar{b}^i \geq 0$.

6.2 Supply side

Since the agricultural sector produces using just land, its benefits are:

$$B_t^A \equiv p_t^A q_t^A - p_t^k \frac{(q_t^A)^{\frac{1}{\alpha}}}{A^{\frac{1}{\alpha}}} \tag{8a}$$

where q_t^A is the amount of agricultural goods produced (recall that $q_t^A \equiv A(k_t)^\alpha$).

Each firm, j, producing either industrial goods or services in the economy, has the following profit function:

$$B_t^j = p_t^I q_t^j + p_t^S s_t^j - (p_t^K K_t^j + p_t^H h_t^j) \tag{8b}$$

where K_t^j and h_t^j are the inputs in the production of industrial goods and services, q_t^j and s_t^j. The transformation functions are $s_t^j = f_j(K_t^j, h_t^j)$ and $q_t^j = g_j(K_t^j, h_t^j)$ with $\frac{\partial f_j}{\partial K} > 0$, $\frac{\partial f_j}{\partial h} > 0$ while $\frac{\partial^2 f_j}{\partial K^2} < 0$and $\frac{\partial^2 f_j}{\partial h^2} < 0$. The same assumption of concavity in its arguments is assumed for g_j: $\frac{\partial g_j}{\partial K} > 0$, $\frac{\partial g_j}{\partial h} > 0$, $\frac{\partial^2 g_j}{\partial K^2} < 0$ and $\frac{\partial^2 g_j}{\partial h^2} < 0$. Furthermore, we assume that the production functions of industrial goods and services have constants returns to scale. This implies that we can postulate aggregation functions: $q_t = g(\sum_j K_t^j, \sum_j h_t^j) = \sum_j g_j(K_t^j, h_t^j)$ and $s_t = f(\sum_j K_t^j, \sum_j h_t^j) = \sum_j f_j(K_t^j, h_t^j)$.

6.3 Equilibrium

The results in Subsection 3.3 are obtained as follows. In equilibrium, $\sum_i k_t^i = n\mathcal{K}$. Therefore, since the equilibrium price of land (the production factor) equals the value of its marginal product, we have that:

$$p_t^k = p_t^A(\alpha A \mathcal{K}^{\alpha-1}) \tag{9a}$$

The equilibrium in the agricultural goods market arises from maximizing the profits function ($8a$). In the case in which the production is consumed domestically, its demand is, from ($6a$):

$$q_t^A = \alpha_1 \frac{(1-\beta)}{p_t^A} \hat{Y}_t^i \tag{9b}$$

where $\hat{Y}_t^i = Y_t^{i\prime} + p_t^I \ {}^I\bar{c}^i + p_t^S \ {}^S\bar{c}^i - \bar{b}^i$. From this equation and the first order condition from ($8a$) it follows that, in equilibrium:

$$p_t^A = [\alpha_1(1-\beta)\hat{Y}_t^i]^{1-\alpha} A^{-1} \left(\frac{1}{\alpha}\right)^{\alpha} (p_t^k)^{\alpha}. \tag{9c}$$

Finally, since physical capital as well as the agricultural and industrial goods are tradable, their prices are taken from the world market. On the contrary, given that services are not tradable, their prices are determined in the domestic market. The demand of human capital responds to the maximization of the aggregate benefits, derived from ($8b$). Given that g and f are, respectively, the aggregate production functions of industrial goods and services we have that:

$$\frac{\partial B_t}{\partial h} = p_t^I \frac{\partial g}{\partial h} + p_t^S \frac{\partial f}{\partial h} - p_t^H = 0.$$

Therefore:

$$\frac{p_t^I}{p_t^H} \frac{\partial g}{\partial h} + \frac{p_t^S}{p_t^H} \frac{\partial f}{\partial h} = 1 \tag{10a}$$

Equation ($10a$) yields the derived demand of human capital. Recall that we assumed perfect mobility of this production factor among sectors, $\frac{\partial g}{\partial h} = \frac{\partial f}{\partial h}$. Thus, from ($10a$):

$$(p_t^I + p_t^S)\frac{\partial g}{\partial h} = p_t^H = (p_t^S + p_t^I)\frac{\partial f}{\partial h} \tag{10b}$$

By the same token, the condition on physical capital yields:

$$(p_t^I + p_t^S)\frac{\partial g}{\partial K} = p_t^K = (p_t^S + p_t^I)\frac{\partial f}{\partial K} \tag{10b\prime}$$

The equilibrium between demand and supply of services is represented by the following condition, derived from ($6c$), ($10b$) and ($10b\prime$):

$$\alpha_3 \frac{(1-\beta)}{p_t^S}[Y_t\prime + p_t^I \ {}^I\bar{c} + p_t^S \ {}^S\bar{c} - \bar{b}] - {}^S\bar{c} \ =$$

$$= \ f\left(\left[\frac{\partial f}{\partial K}\right]^{-1}_{|K}\left(\frac{p_t^K}{p_t^S + p_t^I}\right), \left[\frac{\partial f}{\partial h}\right]^{-1}_{|h}\left(\frac{p_t^H}{p_t^S + p_t^I}\right)\right) \qquad (10c)$$

The left hand of this equation obtains as the aggregation over i of all the demands of services described by (6c). Notice that $[\frac{\partial f}{\partial h}]^{-1}_{|h}(\cdot)$ and $[\frac{\partial f}{\partial K}]^{-1}_{|K}(\cdot)$, the projections over the arguments of f (namely, K and h) are well defined, given the concavity of f in h and K and the optimality condition in equilibrium:

$$\frac{\frac{\partial f}{\partial h}}{\frac{\partial f}{\partial K}} \ = \ \frac{p^H}{p^K} \ = \frac{\frac{\partial g}{\partial h}}{\frac{\partial g}{\partial K}}.$$

Finally, if K_t^S and h_t^S are the optimal amounts of physical and human capital used in the production of services, from (10b) we have that:

$$p_t^S = \frac{\alpha_3(1-\beta)[Y_t\prime + p_t^I \ {}^I\bar{c} - \bar{b}]}{f(K_t^S, h_t^S) + {}^S\bar{c}[1 - \alpha_2(1-\beta)]} \qquad (10d)$$

while $p_t^H = (p_t^I + p_t^S)\frac{\partial f}{\partial h}(K_t^S, h_t^S)$.

Given that the only factor in the production of human capital is education, the price of the latter may reflect the future value of the former. On the other hand, investments in education will continue as long as benefits are positive. Therefore, in equilibrium benefits will be zero, and the amount of human capital produced at that point, $\bar{h}_t$ is such that the price of the corresponding level of education, $\bar{e}_{t-1} = (\bar{h}_t)^{\frac{1}{\gamma}}$, is such that $p_t^H \bar{h}_t = p_{t-1}^E \bar{e}_{t-1}$. Therefore:

$$p_{t-1}^E = p_t^H (\bar{h}_t)^{\frac{\gamma-1}{\gamma}} \qquad (10e)$$

6.4 Subsistence stage

In the conditions of Subsection 4.1 the income of agent i is:

$$Y_t^i \equiv p^A A(k_t^i)^\alpha + p_t^k k_t^i \qquad (11)$$

where p_t^k arises from (9a). From (6a), and the fact that there is no market for education, or any other good (which implies that the corresponding prices, p_t^I, p_t^S, p_t^E and p_t^H are 0), if condition (7a) is fulfilled:

$$p^A c_t^A = \alpha_1(1-\beta)[Y_t^i - \bar{b}^i]$$

since, in this case $Y_t^{i\prime} = Y_t^i$

There will exist a bequest only if $(7d)$ is fulfilled, that is, if

$$Y_t^i > \frac{1+\beta}{\beta}\bar{b}^i$$

There will be a bequest only if:

$$\beta\frac{p_t^A c_t^A}{\alpha_1(1-\beta)} > \bar{b}^i$$

On the other hand, if the agent owns land his income is described by (11), and from $(5a)$ the bequest will be:

$$b_t = \beta[p^A A(k_t^i)^\alpha + p_t^k k_t^i] - (1+\beta)\bar{b}^i \qquad (11a)$$

6.5 Transition

The welfare effects of the tax rate τ_t for $t > T_1$ can be evaluated as follows. First of all, notice that from $(5d)$ the bequest b^i will be vary in an amount $\beta\tau_t Y_t^i$ while from $(6a), (6b)$ and $(6c)$ for each type of good $j = A, I, S$ the variation will be $\alpha_j \frac{(1-\beta)}{p_t^j}\tau_t Y_t^i$. τ_t must be such that the remaining amounts still ensure the satisfaction of conditions $(7a)$ to $(7d)$. This, in turn depends on the new prices of the agricultural prices (internationally determined), but as indicated, we assume that their variation is large enough.

Given that the international demand is high, the produced amount is unaffected by the tax, while the price of land, $(9a)$ increases. Moreover, with a large enough factor A the increase is more than proportionally higher. Thus, the bequest is lower than in the absence of the tax, but compensated by the increase of prices. On the other hand, the price of industrial goods and that of physical goods is given by the world markets. Thus, the consumption of industrial goods is lower.

On the other hand the ways in which the prices of services and human capital change depend on the functional form of f. Consider the case of p_t^S: if p_t^S decreases, by $(10b\prime)$, since p_t^I and p_t^K remain constant, $\frac{\partial f}{\partial K}$ must increase. By the concavity of f, K_t must decrease, and thus also K_t^S, which in turn to a decrease in $f(K_t^S, h_t^S)$ (h_t^S remains fixed because of the delay in its production). Then in $(10d)$ the numerator must decrease less than the denominator. The contrary happens if p_t^S increases.

In turn, the price of human capital is directly affected by the price of services. If p_t^S decreases, we saw that $f(K_t^S, h_t^S)$ must also decrease. Then $\frac{\partial f}{\partial h}(K_t^S, h_t^S)$ increases. Then $p_t^H = (p_t^I + p_t^S)\frac{\partial f}{\partial h}(K_t^S, h_t^S)$ may or may not

decrease depending on the relative variations of p_t^S and $\frac{\partial f}{\partial h}(K_t^S, h_t^S)$. This, in turn affects the price of education, as indicated by $(10e)$.

Finally, since h_{t+1} is larger than without the tax, created in order to ensure that $h_{t+1} > h^*$, the local production of industrial goods and services is ensured, leading to a partial replacement of imports and increased benefits that in turn increase the income of agents.

6.6 Modern phase

This positive feedback process described in Subsection 4.3 can be seen recalling equations $(5b)$ and $(5d)$, which yield:

$$b_t = \beta[Y_t^i + p_{t+1}^H h_{t+1}^{i+} - p_t^E e_t^{i+}] + \beta(p_t^I \ {}^I\bar{c}^i + p_t^S \ {}^S\bar{c}^i - \bar{b}^i) - \bar{b}^i \tag{11b}$$

On the other hand, from (1) we have that:

$$p_{t+1}^H h_{t+1}^{i+} = b_t^i - \kappa^{i+} - p_{t+1}^K K_{t+1}^{i+} \tag{11c}$$

where κ^{i+} is the value of the largest extension of land that can be bought by i for his progeny.

Aggregate income at $t+1$ depends on the bequests left at t, so, from $(3a)$, $(8b)$ and $(11c)$:[17]

$$\begin{aligned} Y_{t+1}^i \equiv p_{t+1}^I g\left(K_{t+1}, \frac{b_t - \kappa - p_{t+1}^K K_{t+1}}{p_{t+1}^H}\right) \ + \\ p_{t+1}^S f\left(K_{t+1}, \frac{b_t - \kappa - p_{t+1}^K K_{t+1}}{p_{t+1}^H}\right) - p_{t+1}^H h_{t+1} - p_{t+1}^K K_{t+1} + \\ + p^A A(\mathcal{K})^\alpha + \ p^k \mathcal{K} \end{aligned} \tag{11d}$$

Replacing the aggregate bequests b_t according to $(11b)$ we have an expression of Y_{t+1} as a function of Y_t:

$$\begin{aligned} Y_{t+1}^i \equiv p_{t+1}^I g\left(K_{t+1}, \frac{\beta[Y_t + p_{t+1}^H h_{t+1} - p_t^E e_t] + \beta(p_t^I \ {}^I\bar{c} + p_t^S \ {}^S\bar{c} - \bar{b}) - \bar{b} - \kappa - p_{t+1}^K K_{t+1}}{p_{t+1}^H}\right) \ + \\ p_{t+1}^S f\left(K_{t+1}, \frac{\beta[Y_t + p_{t+1}^H h_{t+1} - p_t^E e_t] + \beta(p_t^I \ {}^I\bar{c} + p_t^S \ {}^S\bar{c} - \bar{b}) - \bar{b} - \kappa - p_{t+1}^K K_{t+1}}{p_{t+1}^H}\right) - \\ p_{t+1}^H h_{t+1}^j \ + \ p^A A(\mathcal{K})^\alpha \ + \ p^k \mathcal{K} \end{aligned} \tag{11e}$$

from which:

[17] κ is $p_t^k k_t^i$.

$$\frac{dY_{t+1}^i}{dY_t^i} = \frac{\beta}{p_{t+1}^H}[p_{t+1}^I + p_{t+1}^S]\frac{\partial f}{\partial h} \tag{11f}$$

Since from (10*c*) $p_{t+1}^H = (p^I + p_{t+1}^S)\frac{\partial f}{\partial h}$ in equilibrium, and if $p_{t+1}^H > 0$ it follows that:

$$\frac{dY_{t+1}^i}{dY_t^i} = \beta < 1 \tag{11g}$$

A straightforward integration yields that $Y_{t+1}^i = \beta Y_t^i + \bar{r}$, where $\bar{r}$ is the integration constant. By the initial conditions $\bar{r} = Y_{T_2}^i - \beta Y_{T_1}^i > 0$. Therefore, there exists a steady state:

$$\bar{Y}_t^i = \frac{Y_{T_2}^i - \beta Y_{T_1}^i}{1 - \beta} \tag{11h}$$

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