Damage localization and quantification in simply supported beams using static test data

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Abstract. A novel simple method using static test data for damage detection, localization, and quantification in beams is presented in this paper. The method is based on the change of the deflections of the beam between a reference and a damaged state. For simply supported beams with a single damage, the maximum value of the change of deflections indicates the location of damage. Once the damage is located, one could estimate the rotational stiffness at the damaged cross section by applying a superposition scheme to isolate the effect of damage and by using basic structural analysis equilibrium equations. Afterwards, damage extent is evaluated through an existing relation between rotational stiffness and damage severity. Several static tests of a simply supported steel beam with a point load at different locations were conducted to exam the performance of the strategy. The damage is artificially introduced as an opened crack located at the bottom of the beam. The deflections of the beam were measured by using a Digital Image Correlation system. The results show that the method can accurately detect and quantify the damage. The method is non-model based and can be easily conducted. No specific loading positions are required and damage identification objective can be achieved from just one single static test.

1. Introduction
The fundamental objective of damage detection is to identify the change of properies in a structure caused by damage, which includes natural frequencies, dampings, stiffness or flexibility matrix, mode shapes, and etc [1]. Many methodologies and techniques proposed by researchers are based on system identification or parameter estimation through the dynamic response of the structure. Various numerical methods are applied for damage detection and localization, such as the transimissibility function [2–4], the BAT algorithm [5], etc.

Some researchers also applied similar ideas using static response data of the structure. Caddemi and Morassi [6] identified a single crack in beams with different boundary conditions using the static displacements. Lee and Eun [7] presented a method for locating damage through the change of curvature of static deflections. Bakhtiari-Nejad, Rahai and Esfandiari [8] developed an algorithm based on the change of stored strain energy in the elements using static noisy data for damage detection. Seyedpoor and Yazdanpanah [9] also illustrated a method through the change of strain energy using static noisy data.

In this paper, a novel non-model based simple method for single crack damage detection and localization of beams is presented. Firstly, the theory of the method is presented to illustrate how the change of pre- and post-damaged static displacements of the beam under external
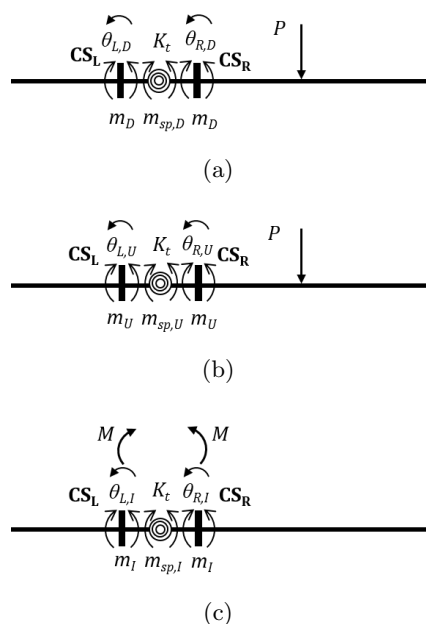
Figure 1. States of the superposition scheme: (a) Damaged State (D) , (b) reference or Undamaged State (U) and (c) Incremental State (I).

loads can be used for damage localization. From the information about the damage location, a damage quantification method based on structural analysis is introduced. Next, a series of experimental tests of a cracked simply supported beam were conducted to exam the performance of the method.

2. Theory background

It is known that the prescence of a crack will cause a reduction in the local stiffness at the cracked cross section. Hence a single damaged beam could be modeled as a rotational spring at the cracked location that connects two undamaged parts of the beam [10]. The problem of a cracked beam under some external forces (P) (*Damaged State, D*) can be decomposed into an *Undamaged State* (U) plus an *Incremental State* (I) (Fig.1). Thus, the deformation (U) and internal forces (f) can be written as

$$U_D = U_U + U_I \quad \text{and} \quad f_D = f_U + f_I \tag{1}$$

In figures Fig. 1 (a), (b) and (c), CS_L and CS_R are the left and right sides of the damage cross section respectively, K_t is the rotational stiffness of the spring that models the cracked cross section, m is the internal bending moment at CS_L and CS_R (they are equal to each other), m_{sp} is the internal torsional moment of the spring, θ_L and θ_R are the rotations at CS_L and CS_R, respectively, and the footnotes U, D, and I stand for the *Undamaged* (or reference), *Damaged*, and *Incremental States* respectively. In the *Undamaged State*, the rotations at CS_L and CS_R are set to be equal ($\theta_{L,U} = \theta_{R,U}$), which indicates the spring is not present in the undamaged beam. It is found that this superposition is valid when the applied moment (M) in the *Incremental State* is equal to the internal bending moment at damage location (m_U) in the *Undamaged State*.

The damage locations are revealed in the overall deformed shape of U_I since the external forces will introduce slope (rotation) discontinuities at damaged cross sections (Fig. 1 (c)). For a single cracked simply-supported beam, the shape of U_I is piecewise linear and it is independent from

the loading position. Its magnitude depends on the magnitude of external loads, the severity of damage and the relative position of the load and the damage.

A finite element model of a simply-supported Timoshenko beam with a 1200mm length (L) and a 100×20mm rectangle cross section was built in ANSYS (mesh size 120mm). A spring with a rotational stiffness (K_t) of $1.8e^5$N/m^2 was used to model the crack at $0.4L$ from the left end. A concentrated load, 1kN, was applied at $0.6L$ from the left end. The deflections of the beams for the *Damaged* and *Undamaged States* are shown in Fig 2 (a). The deflection under a self-equilibrated bending moment m_U corresponding to the *Incremental State* (U_I) and the difference between the displacements of the *Undamaged* and *Damaged States* (ΔU) are shown in Fig. 2 (b). It is shown that U_I is equal to ΔU. The discontinuity in the slope indicates the damage location precisely. The slight difference between ΔU and U_I is due to numerical errors.

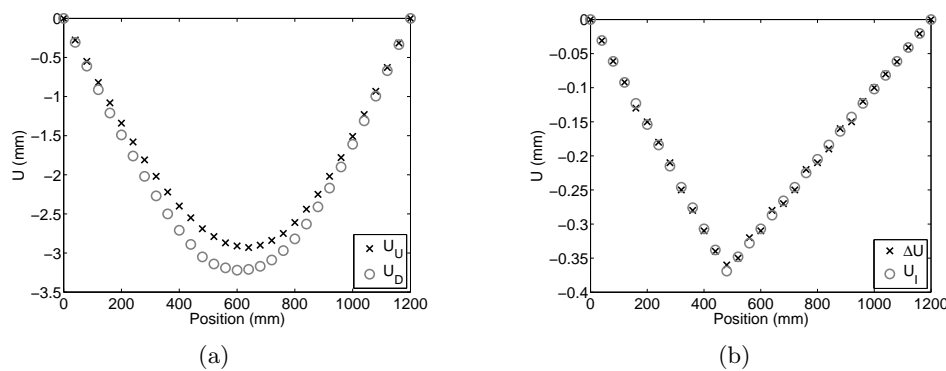(a)   (b)

Figure 2. Deflection results of the finite element model: (a) U_D and U_U of simply-supported beam; (b) ΔU and U_I of simply-supported beam

Once the damage is localized, the rotational stiffness of the cracked beam could be estimated from the *Incremental State* using the following expression:

$$K_t = \frac{m_{sp,I}}{(\theta_{L,I} - \theta_{R,I})} = \frac{m_{sp,I}}{\Delta\theta_I} \tag{2}$$

The rotation discontinuity ($\Delta\theta_I$) can be directly computed from ΔU (U_I) and the moment absorbed by the spring ($m_{sp,I}$) can be automatically calculated for a statically determinate beam since the reactions of the beam for the *Incremental State* are null and therefore $m_{sp,I}$ equals M (and m_U, as indicated previously). Once the rotational stiffness of the damaged cross-section is determined, the extent of damage can be estimated by comparing it with an existing correlation between damage size and rotational stiffness.

3. Experimental Test of A Simply-Supported Beam
3.1. Test Setup
An experimental test of a simply-supported steel beam was conducted to test the performance of the methodology. The dimension of the beam was $1200 \times 100 \times 20$mm. A notch was cut at the bottom of the beam at $0.35L$ (425mm) from the left end. The depth of the notch was set to be 7mm (35% of the beam height). A Digital Imagine Correlation(DIC) system (Fig. 3 (a), (b) and (c)) was used for measuring the deflection of the beam under loading. A total number of 241 measuring points (damage at the 86th) were marked along the beam with an equal spacing of 5mm. A concentrated force was applied on the beam vertically through hanging a 120kg mass on it. 21 tests were performed by putting the mass at 21 equally distributed positions along the beam. The scheme of the test is shown in Fig. 3 (d).
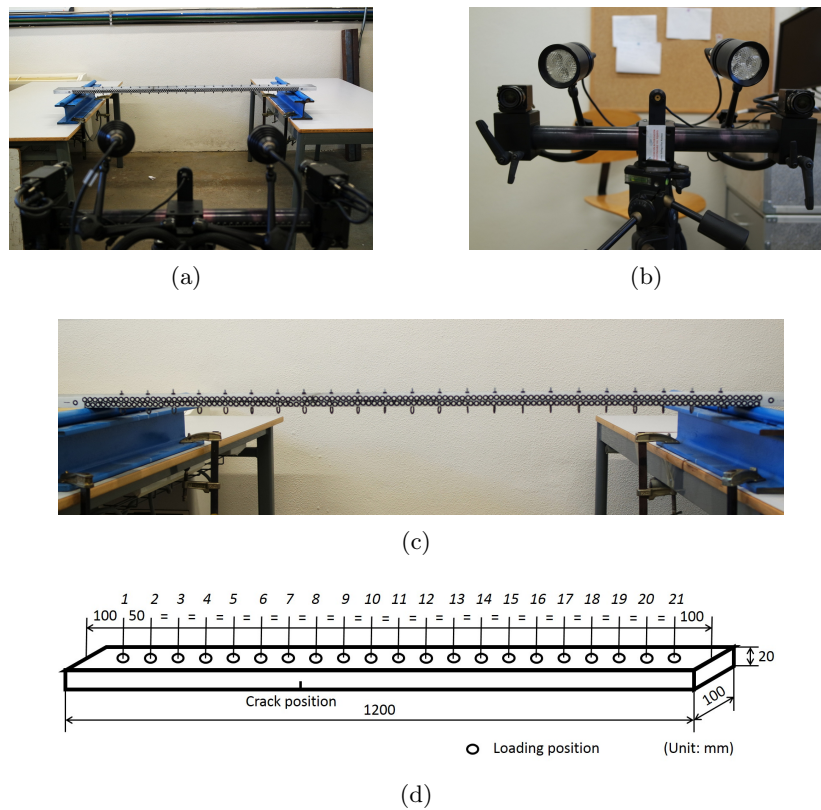
(a)

(b)

(c)

(d)

Figure 3. (a) Experimental setup, (b) DIC measuring system, (c) Tested beam and (d) Test scheme

3.2. Implementation of the methodology

Due to the effect of noise on the measured data, a trend estimate function named $l1$ Trending Filter is used to estimate the overall shape of ΔU. The $l1$ Trending Filter produces trend estimation that is piecewise linear through minimize the objective function in Eqn. (3), where λ is a nonnegative parameter. x_t is the estimated trend and y_t is the signal [11]. This trending filter automatically identifies the turning point along the piecewise shape data.

$$(1/2) \sum_{t=1}^{n} (y_t - x_t)^2 + \lambda \sum_{t=2}^{n-1} |x_{t-1} - 2x_t + x_{t+1}|^2 \tag{3}$$

The results of ΔU and the application of $l1$ Filter to ΔU (ΔU_{l1}) are displayed in Fig. 4 and 5. For all 21 loading positions, the shape of ΔU was estimated correctly. The effect of noise only takes a relative high influence when the loading positions are close to supports of the beam (at positions 1, 20 and 21). The position where the maximum value of ΔU_{l1} takes place is considered as the damage location. The predicted results are listed in table 1. All the predicted damage locations fall into a small range from the correct location. The furthest predition is at point 93 (for loading position 20), which is 35mm to the right of the real damage. Therefore, it is shown that the methodology can successfully localize the damage for this damage scenario.

For a notch type opened cracked on an elastic beam with rectangular cross-section, the equivalent rotational stiffness of the damaged cros-section (K_t) proposed by Rizos, Aspragathos, and Dimarogonas [12] (Eqn. (4) and (5)) is used in this paper for damage extent estimation:
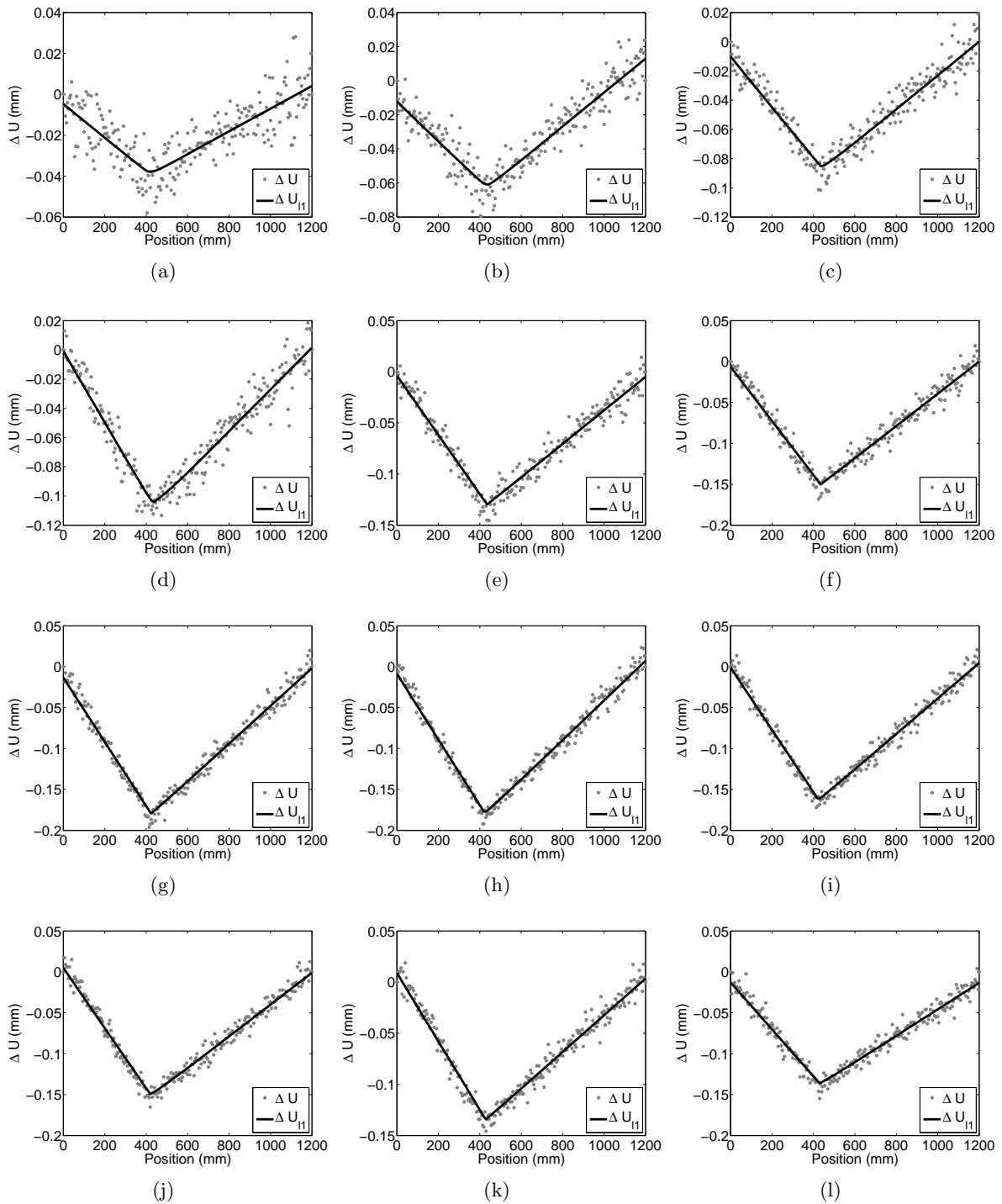
Figure 4. ΔU and ΔU_{l1} with loading at positions 1 (a) to 12 (l).
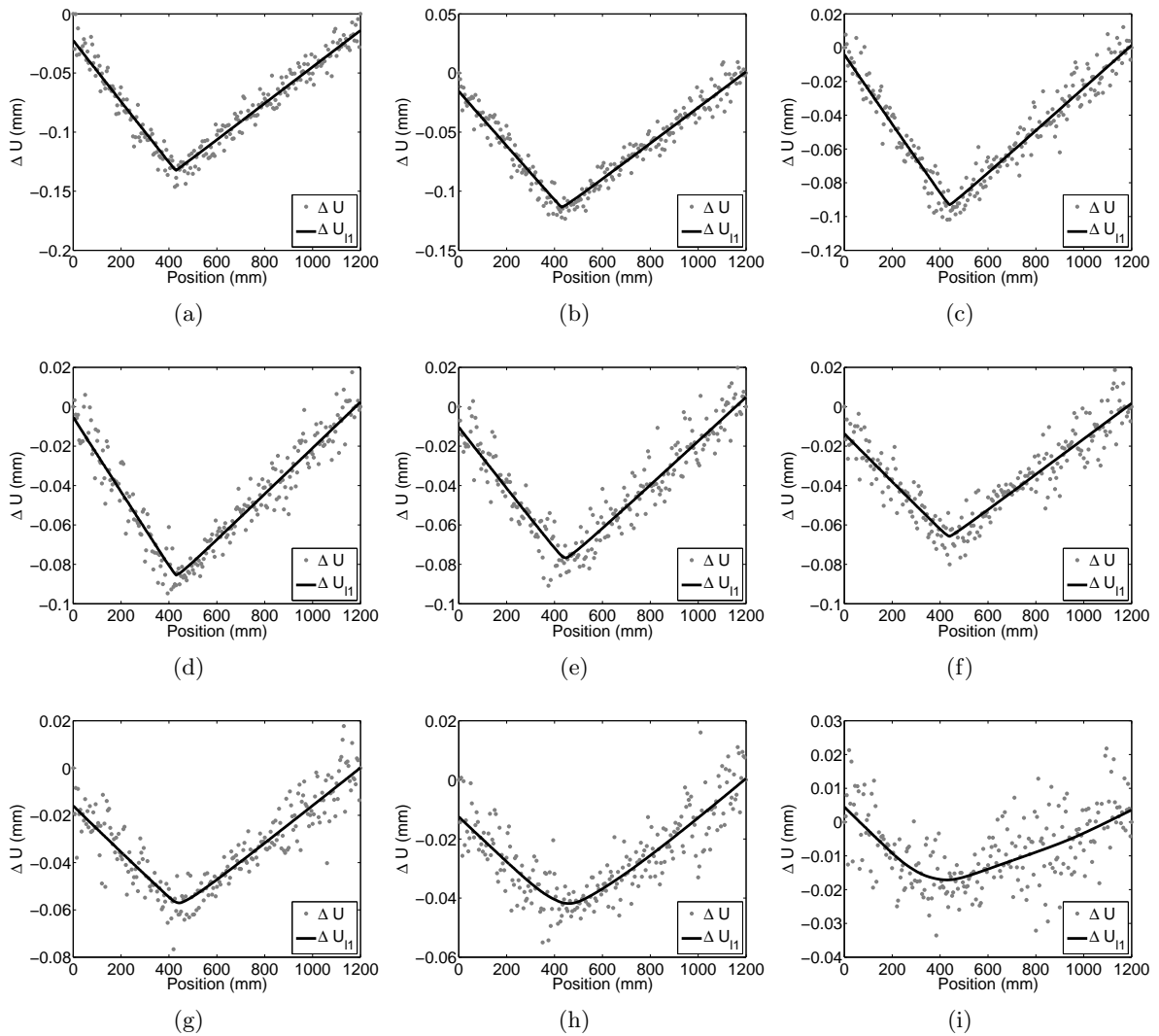
Figure 5. ΔU and ΔU_{l1} with loading at positions 13 (a) to 21 (i).

$$K_t = 1/c \qquad c = 6\pi h J(\xi)/EI \tag{4}$$

where h is the height of the beam, E the elastic modulus of the material of the beam, I the inertia of the cross-section and J is the following function of the ratio (ξ) between the notch depth and the height of the beam.

$$
\begin{aligned}
J(\xi) = {} & 1.86(\xi)^2 - 3.95(\xi)^3 + 16.375(\xi)^4 - 37.226(\xi)^5 \\
& + 76.81(\xi)^6 - 126.9(\xi)^7 + 172.5(\xi)^8 - 143.97(\xi)^9 + 66.56(\xi)^{10}
\end{aligned}
\tag{5}
$$

By using this empirical relationship, damage severity can be estimated from the experimentally identified rotational stiffness (K_t) in Eqn.(2). The rotation discontinuity $(\Delta\theta_I)$ can be evaluated using a piecewise linear regression function of U_I, whereas $m_{sp,I}$ (equivalent to m_U) can be easily computed from equilibrium equations of the undamaged beam. The estimated damage extent for each test are also listed in table 1. All of the estimated damage severities are bigger than the real value. This may indicate some discrepancy between the analytical model of

Table 1. Estimated damage location and severity and their errors

	Localization			Quantification		
Loading Position	Measuring Point	Location (mm)	Deviation (mm)	Severity (mm)	Error (mm)	Error (%)
1	85	415	5	7.5	0.5	7
2	89	440	15	7.7	0.7	10
3	89	440	15	7.7	0.7	10
4	88	435	10	7.9	0.9	13
5	88	435	10	7.8	0.8	11
6	88	435	10	7.8	0.8	11
7	86	425	0	7.8	0.8	11
8	86	425	0	7.8	0.8	11
9	87	430	5	7.8	0.8	11
10	85	415	5	7.9	0.9	13
11	87	430	5	7.8	0.8	11
12	87	430	5	7.7	0.7	10
13	87	430	5	7.7	0.7	10
14	87	430	5	7.8	0.8	11
15	89	440	15	7.7	0.7	10
16	87	430	5	8.0	1.0	14
17	91	450	25	7.8	0.8	11
18	89	440	15	7.8	0.8	11
19	90	445	20	7.9	0.9	13
20	93	460	35	7.7	0.7	10
21	87	430	5	7.7	0.7	10

the crack as a rotational spring and the actual behavior of the damaged cross section. However, for all the loading positions, the method provides predictions with high accuracy even for those with low signal to noise ratio.

4. Conclusion

A non-model based damage detection and localization methodology based on the static displacements is presented in this paper. No specific loading positions are needed for the experimental test and structural identification is not required. Experimental results of a simply-supported steel beam with a single crack are provided. The methodology successfully predicts the crack location with a very high accuracy for all loading positions. From the predicted damage locations, the damage extent can be estimated using an existing analytical correlations between damage extent and rotational stiffness of the damaged cross section. The method provides results with high accuracy as well. In summary, the paper proves the efficiency and simplicity of the method for practical purpose.

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