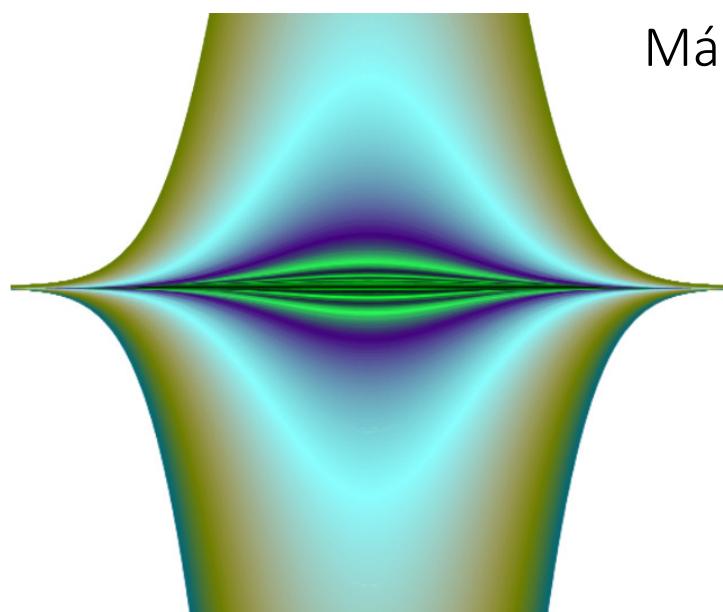


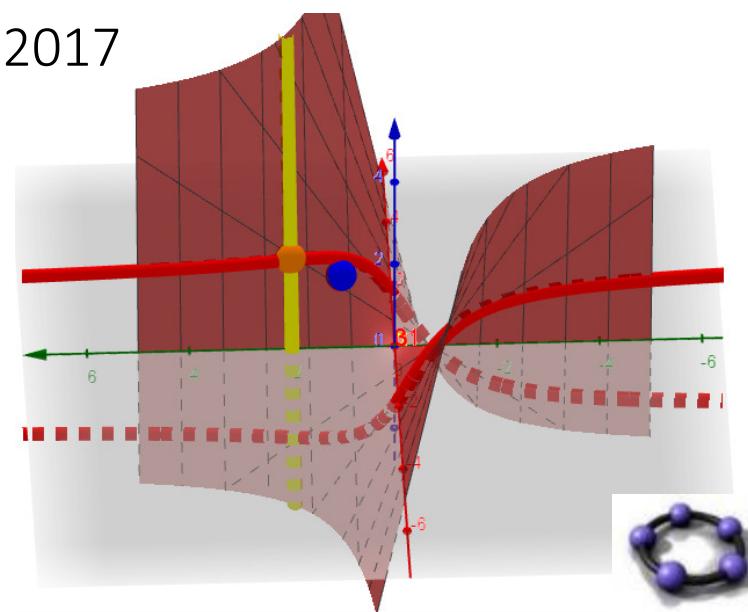
# GeoGebra, un primer paso para diseñar la Arquitectura Dinámica del Siglo XXI

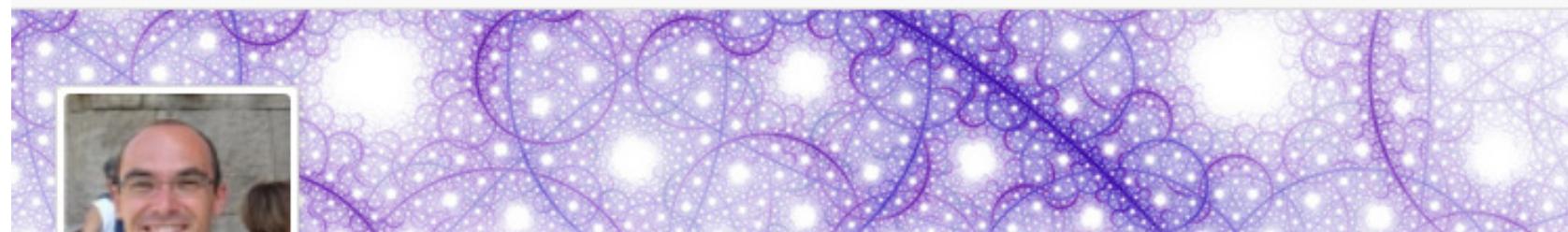
Raúl M. Falcón Ganfornina  
[rafalgan@us.es](mailto:rafalgan@us.es)

V Encuentro en Andalucía GeoGebra en el aula



Málaga, 22 de abril de 2017





Recursos

Seguidores

Raúl Manuel Falcón  
Ganfornina ⓘ

Buscar en los materiales de Raúl Manuel Fal... ▾

A-Z



Modelado 3D con GeoGeb...

Raúl Manuel Falcón Ganfornina  
29 de enero de 2017



Superficie tubular

Raúl Manuel Falcón Ganfornina  
29 de enero de 2017



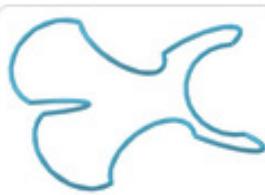
Superficie tubular

Raúl Manuel Falcón Ganfornina  
28 de enero de 2017



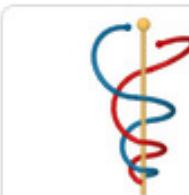
Superficie tubular

Raúl Manuel Falcón Ganfornina  
28 de enero de 2017



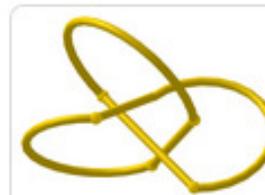
Superficie tubular

Raúl Manuel Falcón Ganfornina  
27 de enero de 2017



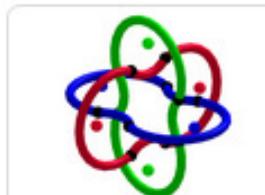
Superficie tubular

Raúl Manuel Falcón Ganfornina  
27 de enero de 2017



Superficie tubular

Raúl Manuel Falcón Ganfornina  
27 de enero de 2017



Superficie tubular

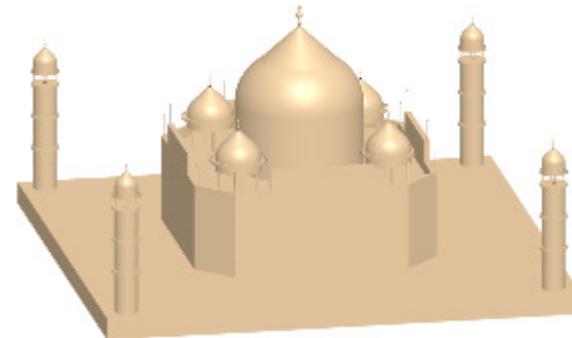
Raúl Manuel Falcón Ganfornina  
26 de enero de 2017



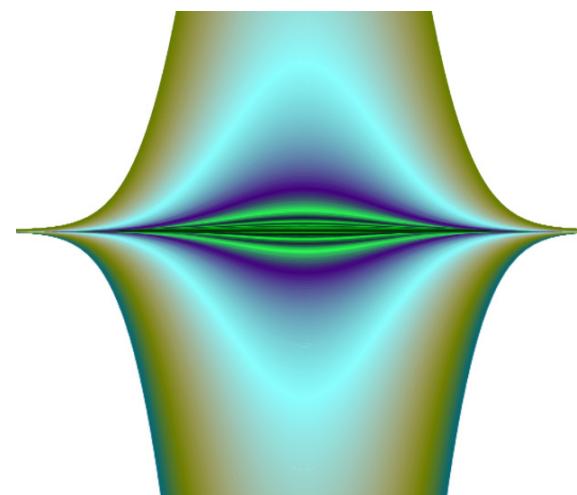
<https://www.geogebra.org/raúl+falcón>

# INDICE

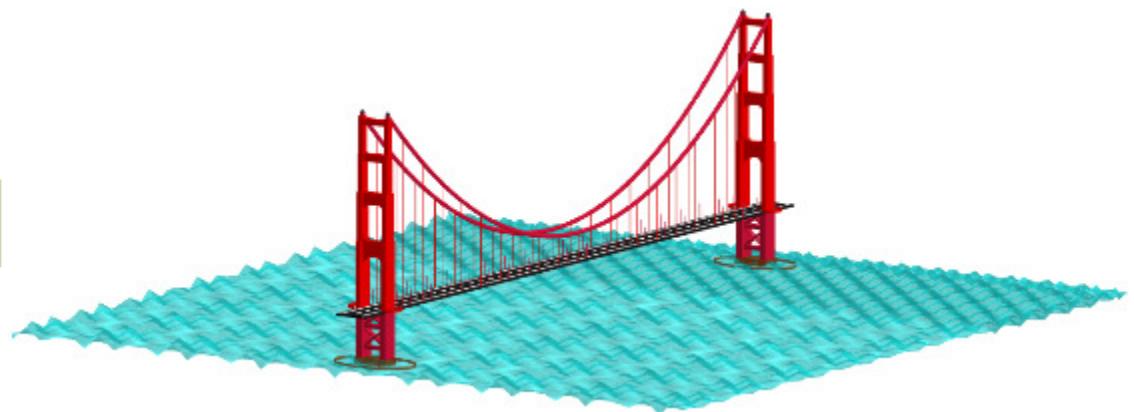
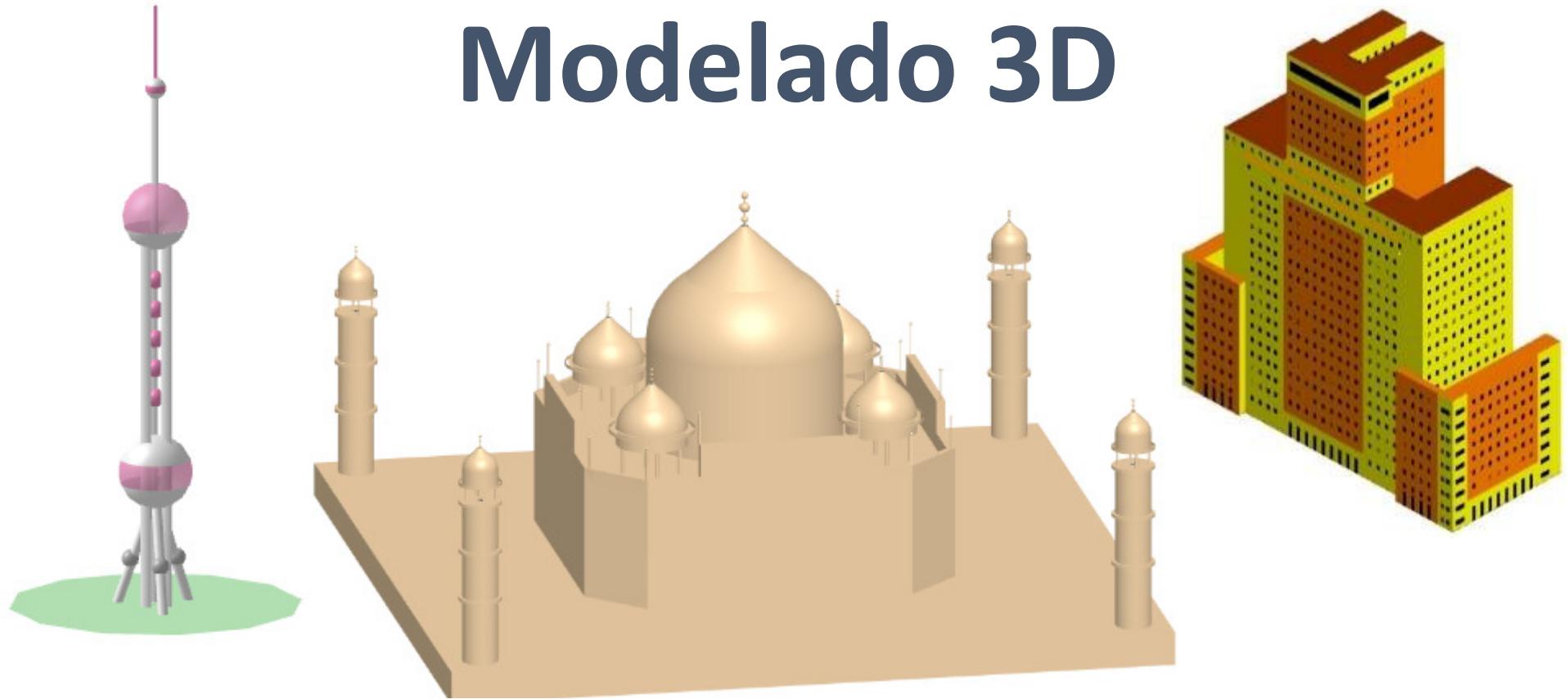
## Modelado 3D.



Diseños basados en EDO's y  
Geometría Diferencial.



# Modelado 3D



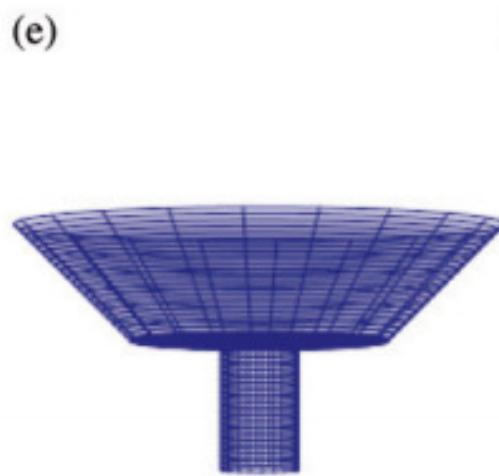
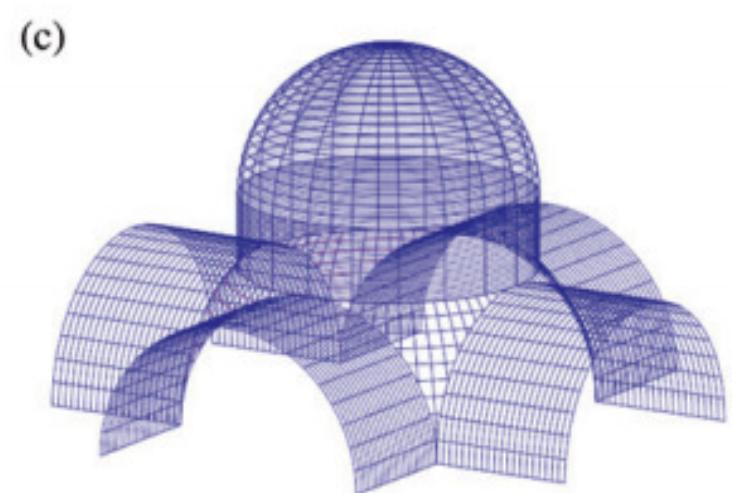
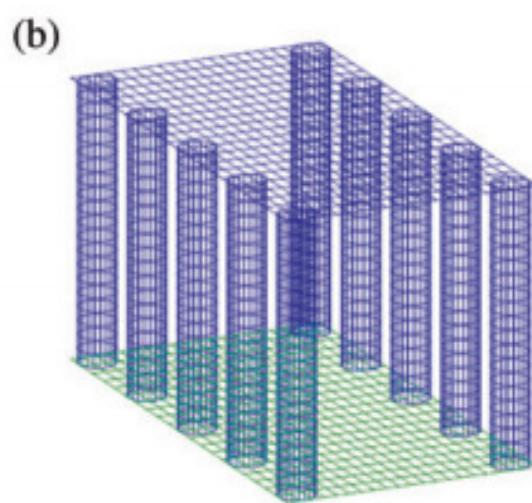
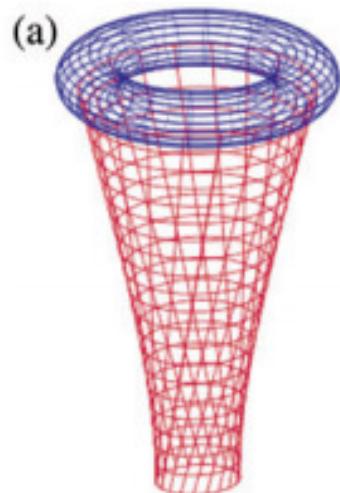
# Modelado 3D

← GeoGebra

7. Big Ben (Londres)  
8. Iglesia de San José (Monterrey)  
9. Museo Guggenheim (New York)  
10. Sinagoga (Florencia)  
11. Bauhaus (Dessau)  
12. Shah Faisal Masjid (Islamabad)  
13. Edificio CEOE (Zaragoza)  
14. Giralda (Sevilla)  
15. Torre del Oro (Sevilla)  
16. Biblioteca Felipe González Márquez (Sevilla)  
17. Torre Triana (Sevilla)  
18. Torre Mapfre (Sevilla)  
19. Torres Kio (Madrid)  
20. Hipódromo de la Zarzuela (Madrid)  
21. Museo de Arte (Sao Paulo)  
22. Institución educativa La Samaria (Pereira, Colombia)  
23. Oceanográfic (Valencia)  
24. Plaza de San Pedro (Vaticano)  
25. Taj Mahal (Agra)  
26. Burj Al Arab (Dubai)  
27. Third World Trade Center Memorial (New York)  
28. New Museum of Contemporary Arts (New York)  
29. Unilever Nederland BV (Rotterdam)  
30. Atomium (Bruselas)  
31. Cúpula de la Roca (Jerusalén)  
32. Eileen Gray's E-1027 (Roquabruna-Cap-Martin, Francia)  
33. Casa Levene (El Escorial)  
34. Casa Sert (Cambridge, Boston)  
35. Casa Gentil Wandel (Gent...)  
36. Casa Plaza de la Alianza (S...)  
37. Casa Plaza de la Alianza (S...)  
38. Casa Plaza de la Alianza (S...)  
39. Casa Plaza de la Alianza (S...)  
40. Casa Plaza de la Alianza  
41. Casa Plaza de la Alianza (S...)  
42. Casa Plaza de la Alianza (S...)  
43. Casa Plaza de la Alianza (S...)  
44. Casa Plaza de la Alianza (S...)  
45. Homer^3

The image shows a collection of 45 3D models of various famous landmarks, each accompanied by a small thumbnail image and a descriptive label. The landmarks are arranged in a grid format. The labels include:  
7. Big Ben (Londres)  
8. Iglesia de San José (Monterrey)  
9. Museo Guggenheim (New York)  
10. Sinagoga (Florencia)  
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40. Casa Plaza de la Alianza  
41. Casa Plaza de la Alianza (S...)  
42. Casa Plaza de la Alianza (S...)  
43. Casa Plaza de la Alianza (S...)  
44. Casa Plaza de la Alianza (S...)  
45. Homer^3

# Modelado 3D



# Modelado 3D

## Ángulos de Euler

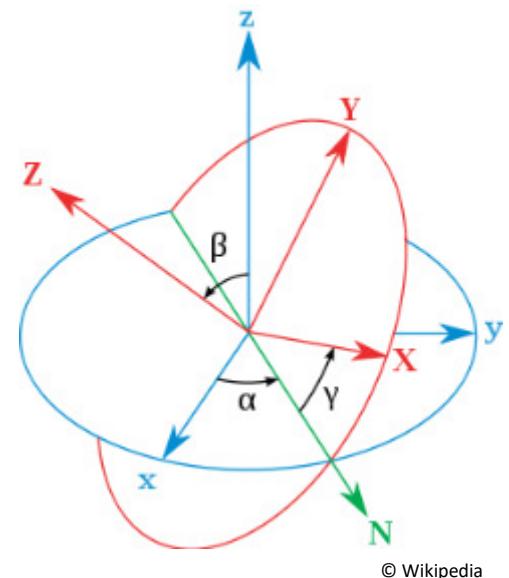
Proyección ortogonal 3D en 2D:

$$f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

$$\vec{x} = (x, y, z) \rightarrow f(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}))$$

$$f_1(\vec{x}) = x \sin(\beta) + y \cos(\beta)$$

$$f_2(\vec{x}) = -x \cos(\beta) \sin(\alpha) + y \sin(\beta) \sin(\alpha) + z \cos(\alpha)$$



# Modelado 3D



INSTITUTO DE MEDICINA LEGAL  
(Alejandro Zaera)  
Madrid (Campus de la Justicia)

SolidWorks interface showing the geometric parameters for the building's dome:

Parameter	Value	Parameter	Value
$u_0$	-2.4	$r_1$	12.8
$u_1$	0	$n$	15
$v_0$	11.2		
$v_1$	20		
		$u_0$	-0.47
		$u_1$	0.03
		$r_2$	37.2
		$n_2$	20
		$v_0$	2
		$v_1$	11.2

Checkboxes and labels:

- Torus
- MainCurves<sub>1</sub>  $\delta = 144^\circ$
- MainCurve<sub>2</sub>  $\zeta = 158^\circ$
- Sphere
- Axes  $\alpha = 169^\circ$ ,  $\beta = 317^\circ$ ,  $r = 5$

3D model diagram showing the geometric construction of the dome:

- A wireframe torus labeled "Torus" is shown.
- A wireframe sphere labeled "Sphere<sub>1</sub>" is positioned at the top of the torus.
- A wireframe sphere labeled "Sphere<sub>2</sub>" is positioned at the bottom of the torus.
- Coordinate axes are labeled A, B, C, and W.

Photograph of the completed 3D model of the dome, showing its metallic surface and spherical cap.

José Enrique Pozo Sierra  
Curso 2009-2010

# Modelado 3D

Move  
Drag or select objects (Esc)

3daxis

Free Objects

- $B = (-1.35, 1.55)$
- $C = (-0.45, 1.55)$
- $F = (-0.35, 1.53)$
- $G = (0.56, 1.53)$
- $J = (0.67, 1.52)$
- $K = (0.67, 2.43)$
- $N = (1.68, 2.44)$
- $O = (1.69, 1.53)$
- $c = \text{true}$
- $c_1(x) = \cos(x)$
- $c_2(x) = \sin(x)$
- $c_3(x) = 1$
- $d_1(x) = x$
- $d_2(x) = x$
- $d_3(x) = x$
- $e(x) = 1$
- $f_1(x) = \cos(x)$
- $f_2(x) = \sin(x)$
- $f_3(x) = 1$
- $g_1(x) = 1$
- $g_2(x) = 1$
- $g_3(x) = x$
- $n = 20$
- $r = 0.9$
- $r_1 = 0$
- $r_2 = 3.3$
- $u_0 = 0$
- $u_1 = 6.28$
- $u_2 = 0$
- $u_3 = 6.28$
- $v_0 = 0$
- $v_1 = 3.3$
- $v_2 = 3.3$
- $v_3 = 6.28$
- $w_0 = 0$

Orthographic projection      Cylinder      Cone      Circumference

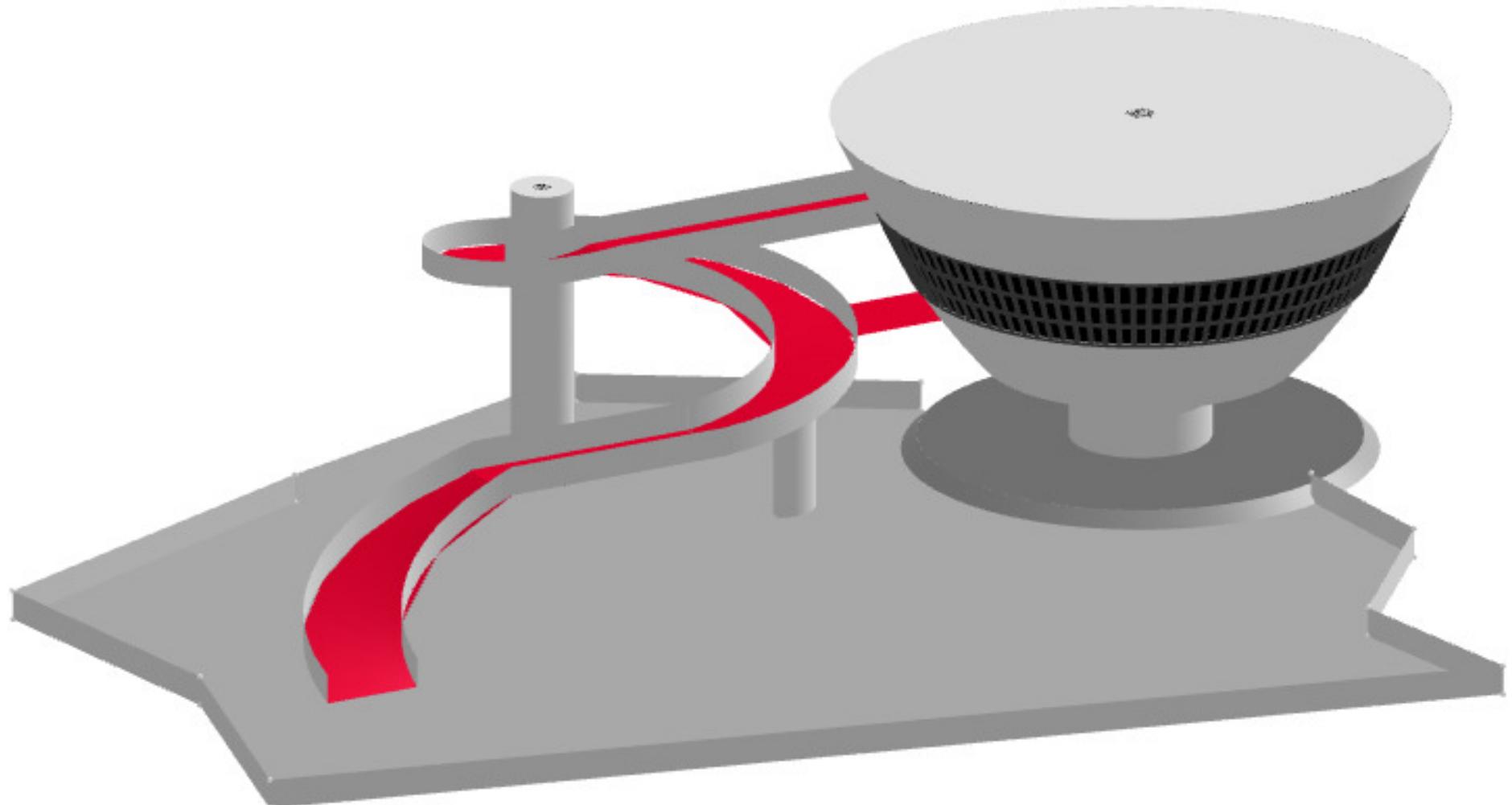
$\alpha = 4^\circ$        $\beta = 310^\circ$        $u_0 = 0$ ,  $u_1 = 6.28$        $u_2 = 0$ ,  $u_3 = 6.28$        $r_1 = 0$ ,  $r_2 = 3.3$

$r = 0.9$        $v_0 = 0$ ,  $v_1 = 3.3$        $v_2 = 3.3$ ,  $v_3 = 6.28$        $n = 20$

MUSEO DE ARTE CONTEMPORÁNEO (1996)  
(Óscar Niemeyer)  
Niteroi, Brasil

Miriam González Roca  
Curso 2009-2010

# Modelado 3D



**MUSEO DE ARTE CONTEMPORÁNEO (1996)**

(Óscar Niemeyer)

Niteroi, Brasil

**Manuel Delgado Liébana**

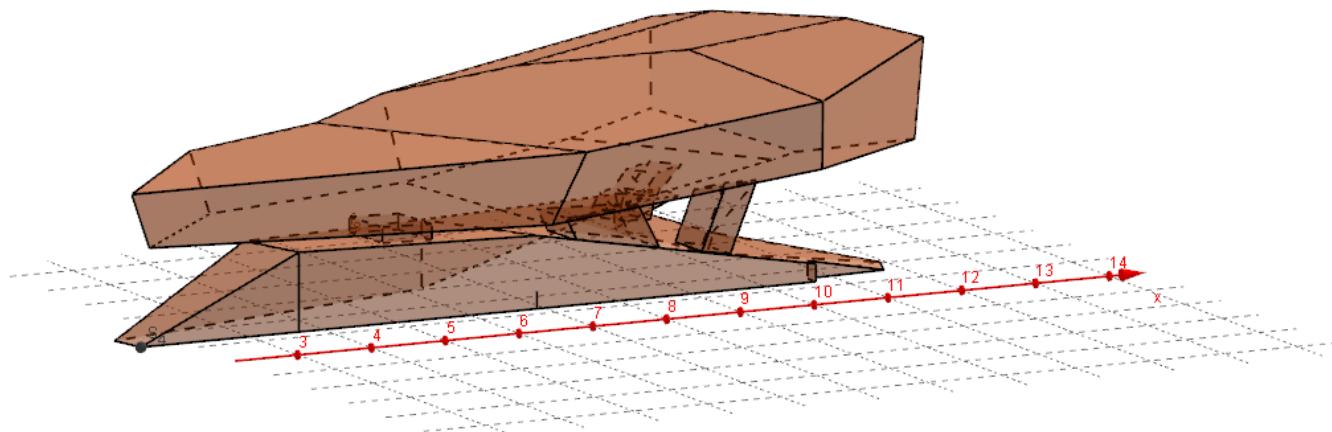
Curso 2015-2016

# Modelado 3D



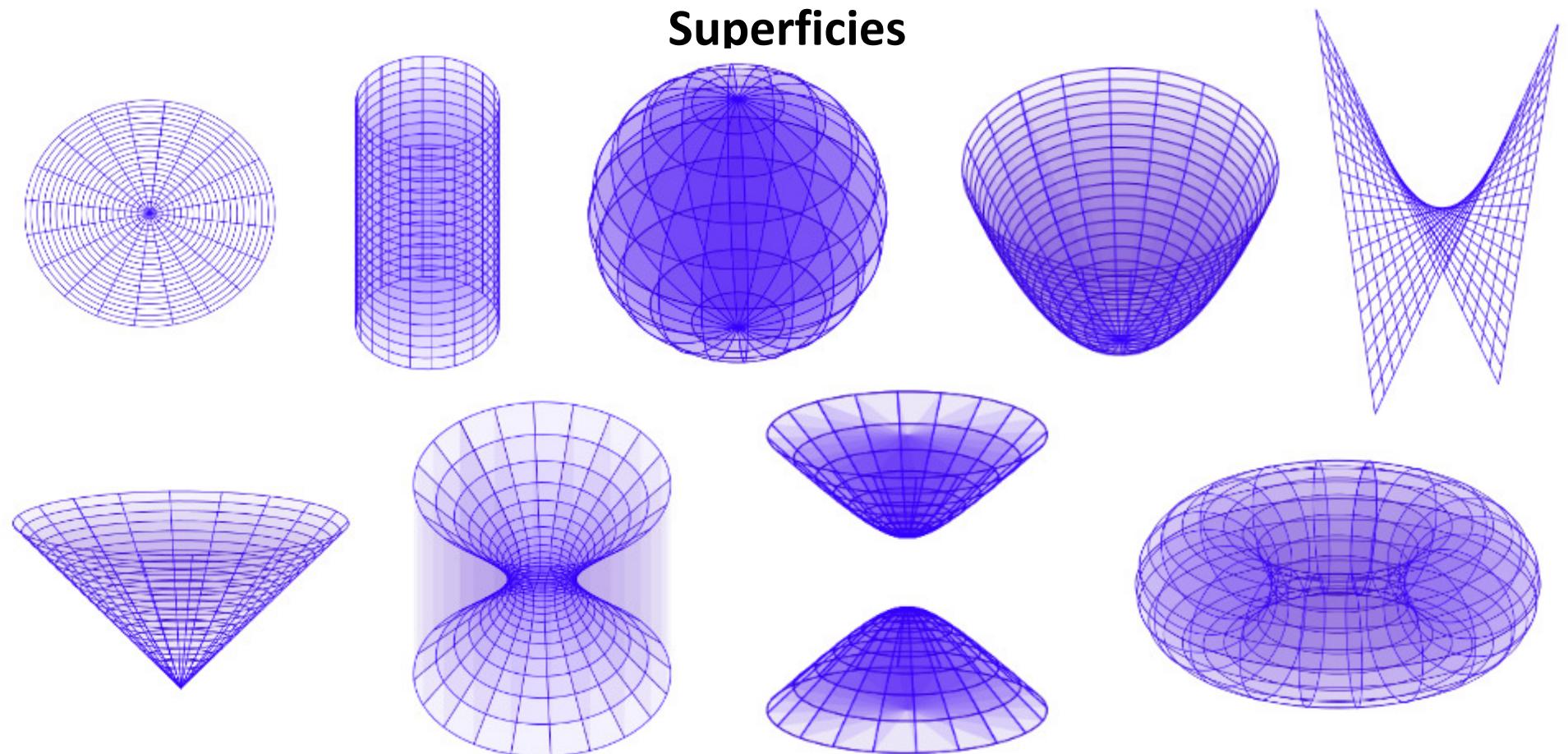
© Porsche

**MUSEO PORSCHE (2009)**  
(Delugan Meissl)  
Stuttgart



Carlos Palacios Gil  
Curso 2011-2012

# Modelado 3D



# Modelado 3D



**ALDAR HEADQUARTERS (2010)**

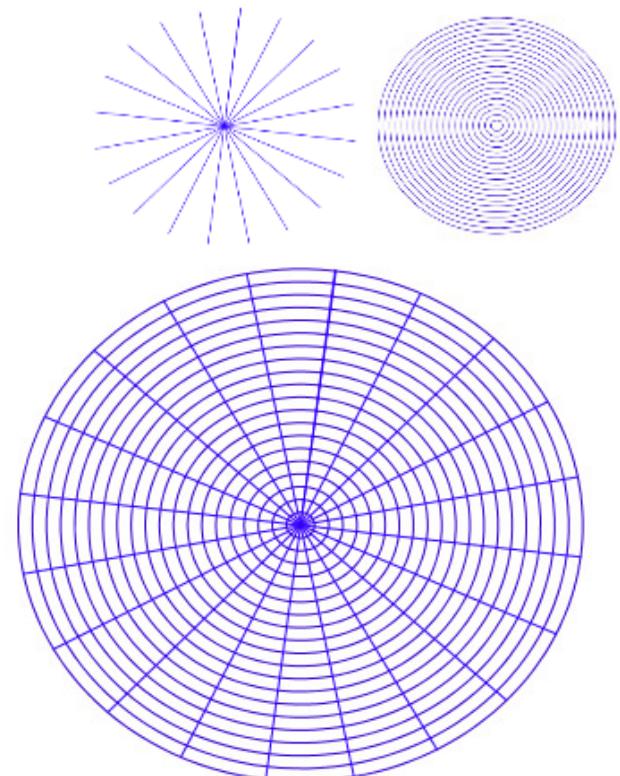
110 metros  
(MZ Architects)  
Abu Dhabi

## Disco

$$\begin{cases} x = u \cos(v), \\ y = u \sin(v), \\ z = 0. \end{cases}$$

$$u \in [0, a]$$

$$v \in [0, 2\pi]$$



# Modelado 3D



**WESTHAFEN TOWER (2004)**

109,9 metros

(Schneider & Schumacher)  
Frankfurt

© Wikipedia

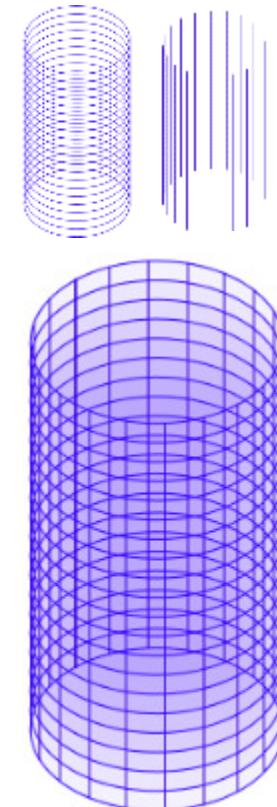
## Cilindro

$$\begin{cases} x = \cos(u), \\ y = \sin(u), \\ z = v. \end{cases}$$

$$u \in [0, 2\pi]$$

$$v \in [a, b]$$

$$x^2 + y^2 = 1$$



# Modelado 3D



© Wikipedia

**ATOMIUM (1958)**  
102 metros  
(André Waterkeyn)  
Bruselas

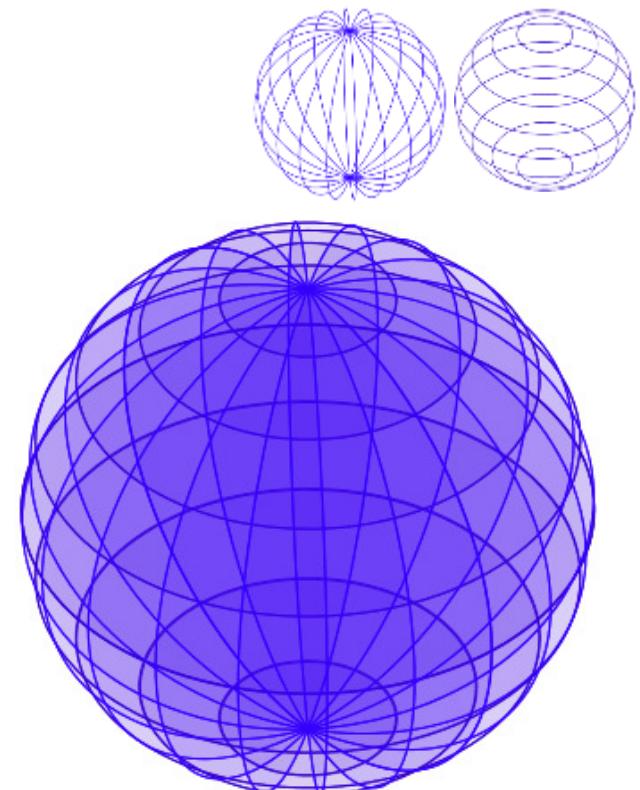
## Esfera

$$\begin{cases} x = \cos(u) \cos(v), \\ y = \cos(u) \sin(v), \\ z = \sin(u). \end{cases}$$

$$u \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$v \in [0, 2\pi]$$

$$x^2 + y^2 + z^2 = 1$$



# Modelado 3D

## Elipsoide



© Planetden

BEIJING NATIONAL GRAND THEATRE (2008)  
(Paul Andreu)  
Pekín

$$\begin{cases} x = R \cos(u) \cos(v), \\ y = r \cos(u) \sin(v), \\ z = \sin(u). \end{cases}$$

$$u \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$v \in [0, 2\pi]$$

$$x^2 + y^2 + z^2 = 1$$

# Modelado 3D



**THE GHERKIN (2003)**

102 metros

(Norma Foster)  
Londres

© Wikipedia

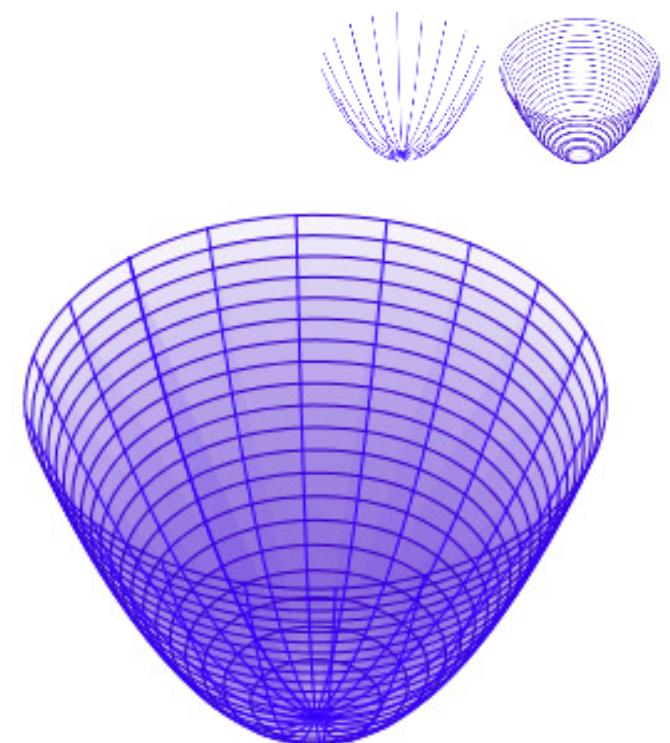
## Paraboloide elíptico

$$\begin{cases} x = \sqrt{u} \cos(v), \\ y = \sqrt{u} \sin(v), \\ z = u. \end{cases}$$

$$u \in [0, a]$$

$$v \in [0, 2\pi]$$

$$x^2 + y^2 = z$$



# Modelado 3D

## Paraboloide hiperbólico



© Elnuevodia

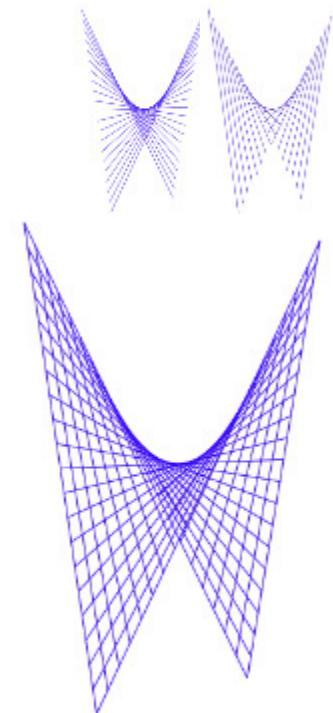
**PABELLÓN PHILIPS (1958)**  
(Le Corbusier e Iannis Xenakis)  
Bruselas

$$\begin{cases} x = u, \\ y = v, \\ z = u \cdot v. \end{cases}$$

$$u \in [a, b]$$

$$v \in [c, d]$$

$$x \cdot y = z$$



# Modelado 3D



© Wikipedia

**AUDITORIO DE TENERIFE (2003)**  
(Santiago Calatrava)  
Tenerife

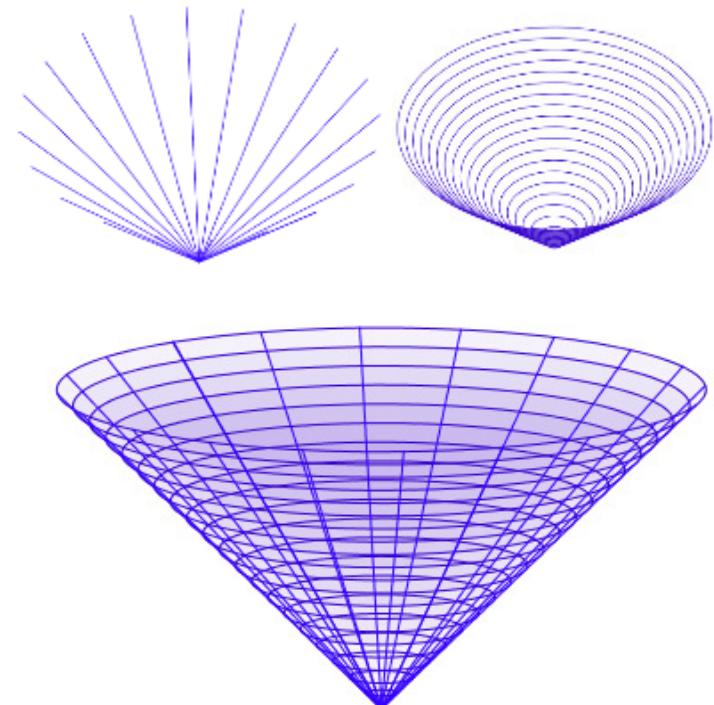
## Cono

$$\begin{cases} x = u \cos(v), \\ y = u \sin(v), \\ z = u. \end{cases}$$

$$u \in [0, a]$$

$$v \in [0, 2\pi]$$

$$x^2 + y^2 = z^2$$



# Modelado 3D

## Hiperboloide de una hoja



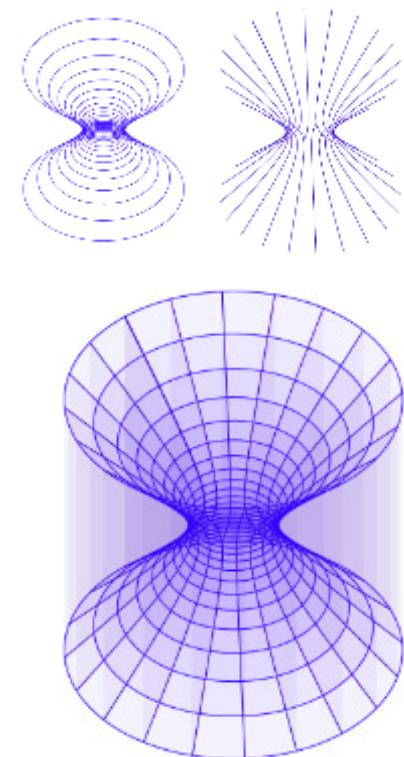
BOULEVARD EXPO SHANGHAI 2010  
Shanghai

$$\begin{cases} x = \cos(u) \cosh(v), \\ y = \cos(u) \sinh(v), \\ z = \sin(u). \end{cases}$$

$$u \in [0, 2\pi]$$

$$v \in [a, b]$$

$$x^2 - y^2 + z^2 = 1$$



# Modelado 3D

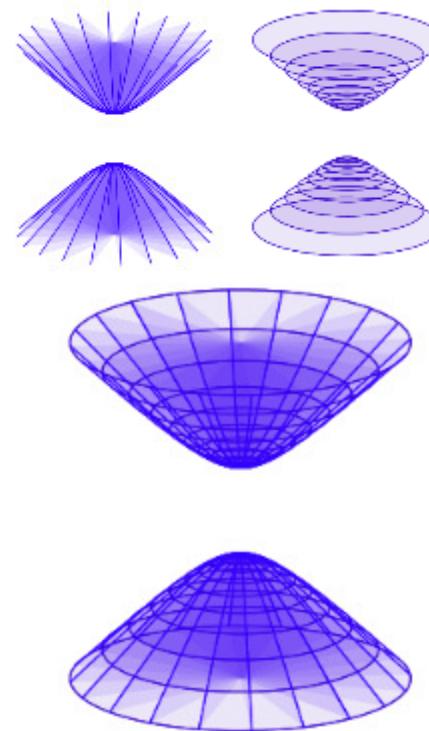
## Hiperboloide de dos hojas

$$\begin{cases} x = \sinh(u) \cos(v), \\ y = \sinh(u) \sin(v), \\ z = \cosh(u). \end{cases}$$

$$u \in [a, b]$$

$$v \in [0, 2\pi]$$

$$x^2 + y^2 - z^2 = -1$$



# Modelado 3D



© Wikipedia

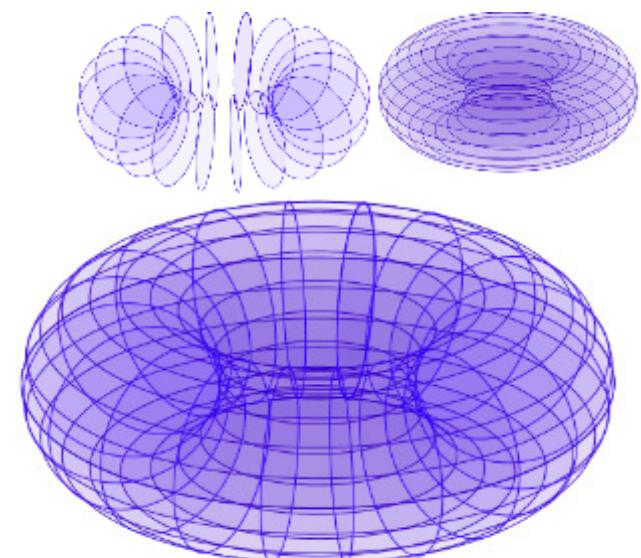
INSTITUTO DE MEDICINA LEGAL  
**(Alejandro Zaera)**  
Madrid (Campus de la Justicia)

## Toro

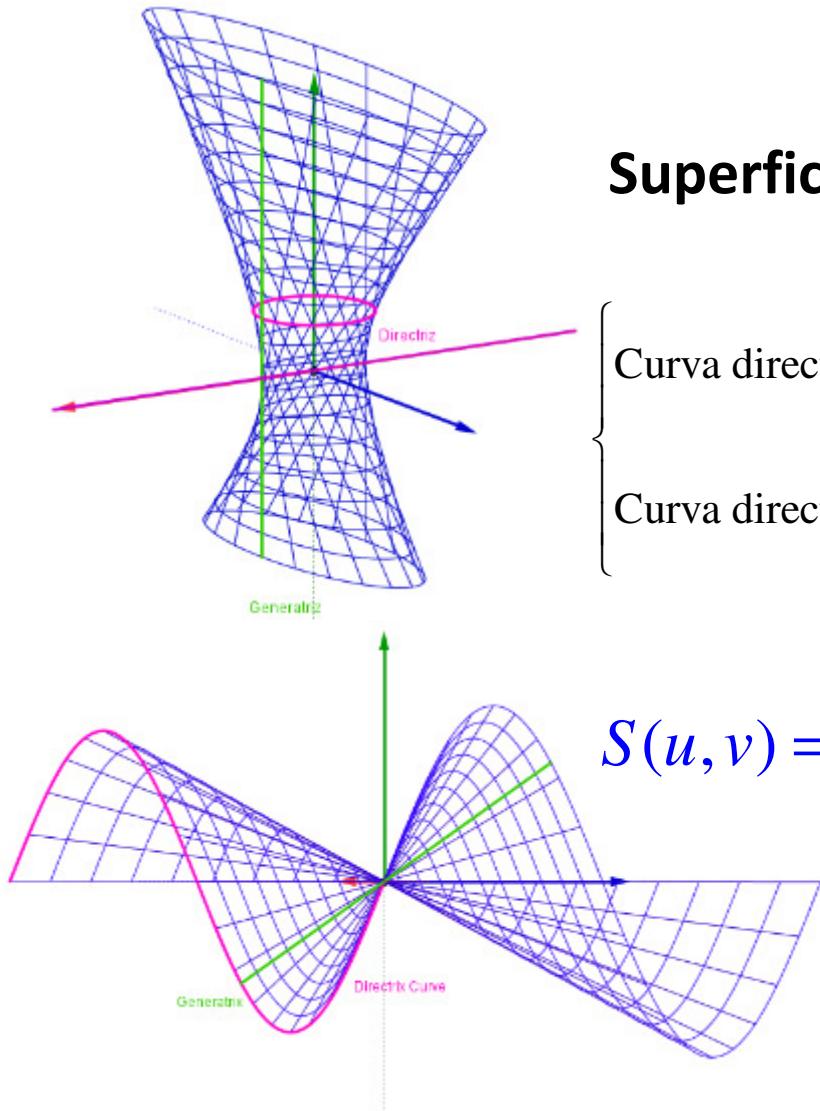
$$\begin{cases} x = (R + r \cos(u)) \cdot \cos(v), \\ y = (R + r \cos(u)) \cdot \sin(v), \\ z = r \sin(u). \end{cases}$$

$$u \in [0, 2\pi]$$

$$v \in [0, 2\pi]$$



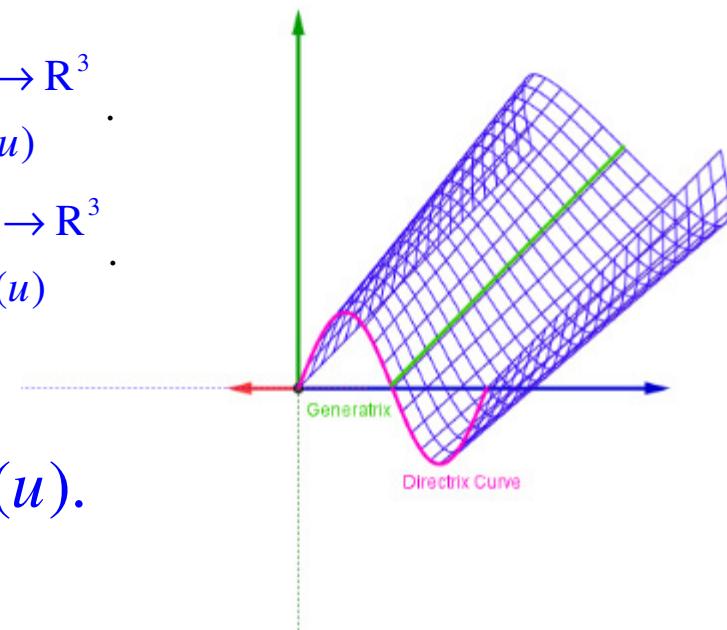
# Modelado 3D



## Superficies regladas

$$\begin{cases} \text{Curva directriz} \equiv \left\{ \begin{array}{l} C : D \subseteq \mathbb{R} \rightarrow \mathbb{R}^3 \\ u \mapsto C(u) \end{array} \right. \\ \text{Curva directora} \equiv \left\{ \begin{array}{l} \beta : D \subseteq \mathbb{R} \rightarrow \mathbb{R}^3 \\ u \mapsto \beta(u) \end{array} \right. \end{cases}.$$

$$S(u, v) = C(u) + v \cdot \beta(u).$$



# Modelado 3D

## Superficies regladas: Plano

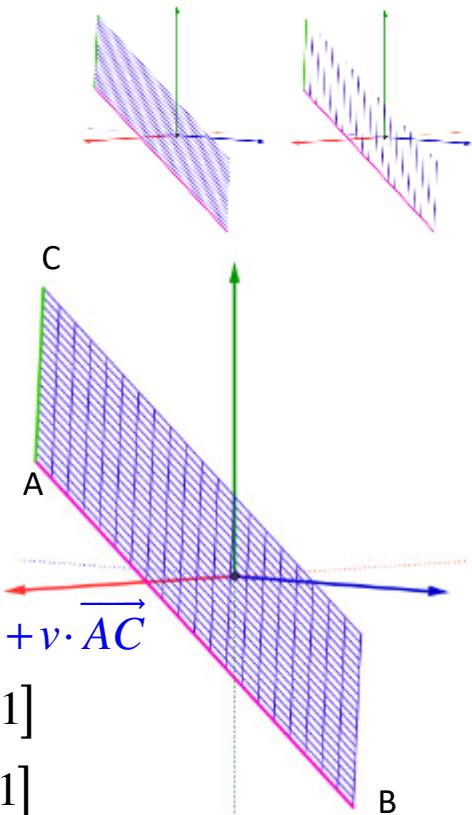


© Porshe

**MUSEO PORSCHE (2009)**  
(Delugan Meissl)  
Stuttgart

$$\begin{cases} C(u) = (a \cdot u + b, c \cdot u + d, e \cdot u + f), \\ \beta(u) = (g, h, i). \end{cases} \quad u \in [k, l] \quad u \in [0, 1]$$

$$v \in [0, 1]$$



# Modelado 3D

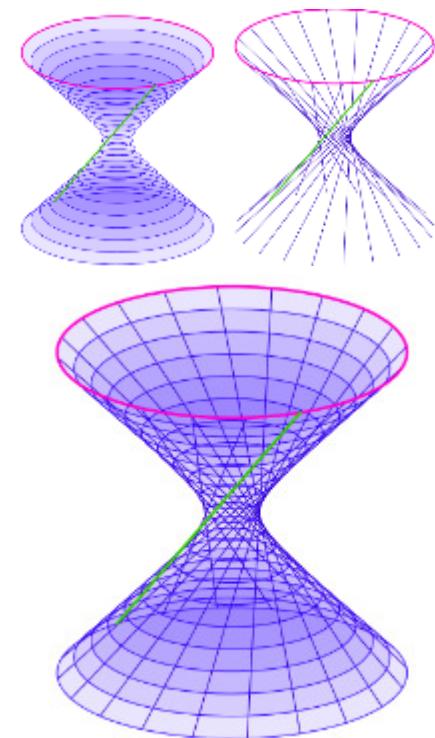
**Superficies regladas: Hiperboloide de una hoja**



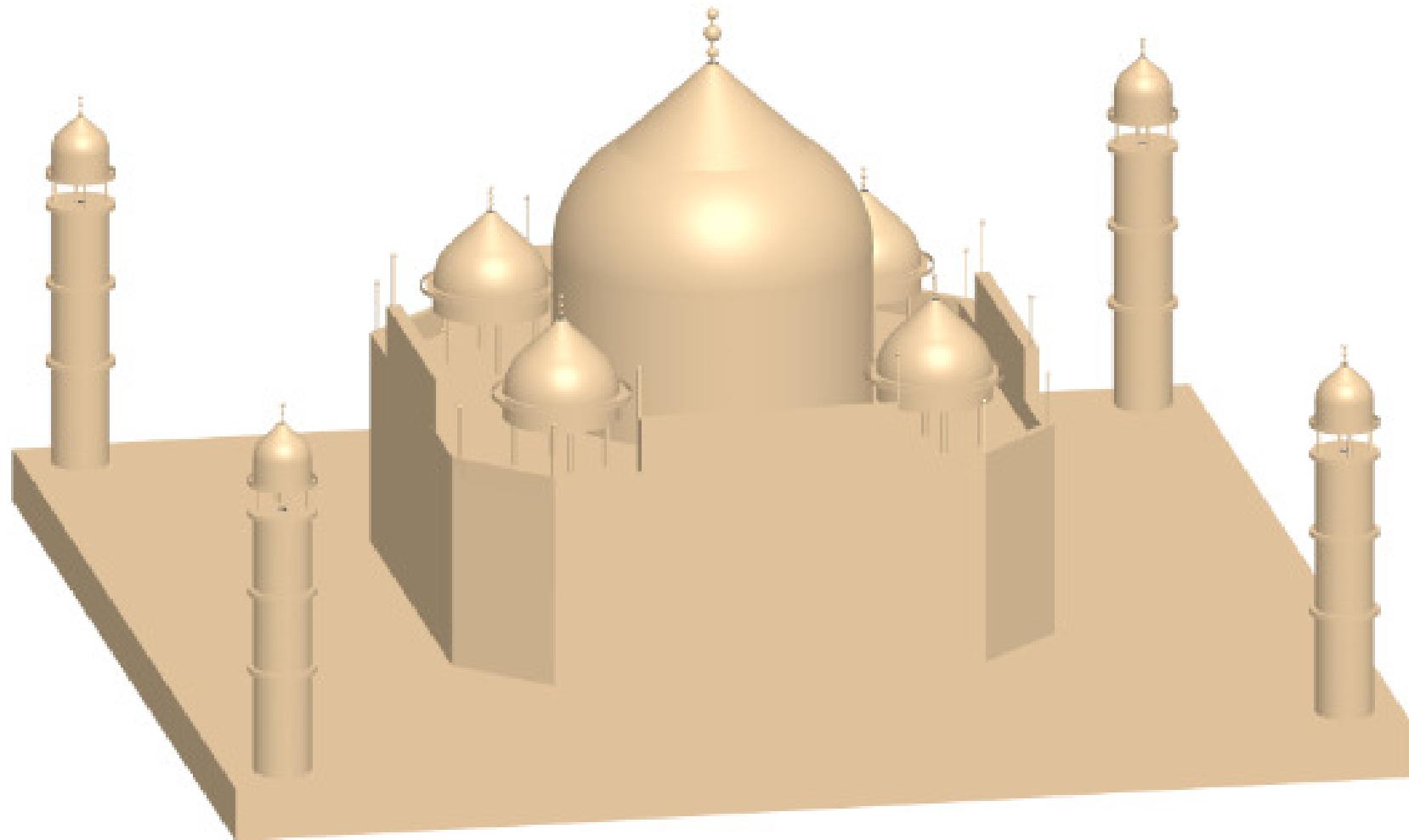
**CORPORATION STREET BRIDGE (1999)**  
(Hodder + Partners)  
Manchester

$$\begin{cases} C(u) = (\cos(u), \sin(u), 0), \\ \beta(u) = (-\sin(u), \cos(u), 1). \end{cases}$$

$$u \in [0, 2\pi]$$



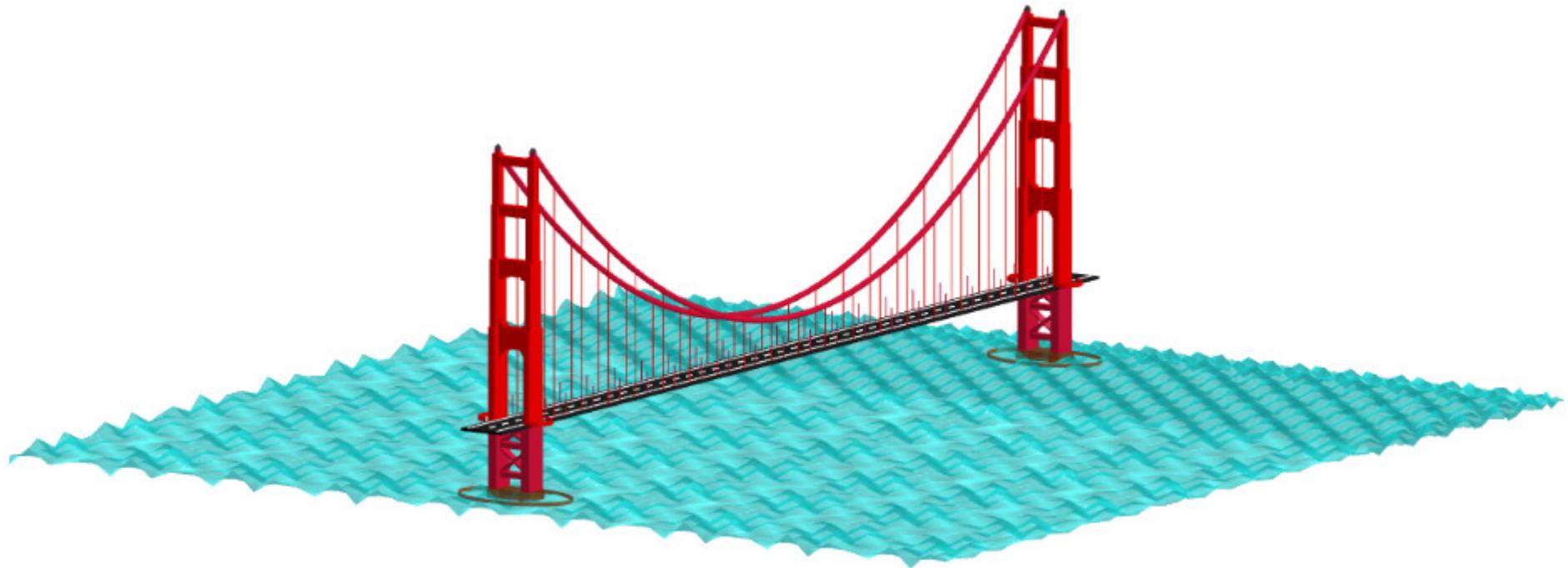
# Modelado 3D



**TAJ MAHAL (S. XVII)**  
Agra

**Sandra María Garzón Ruíz**  
Curso 2015-2016

# Modelado 3D



**GOLDEN GATE (1930's)**  
San Francisco

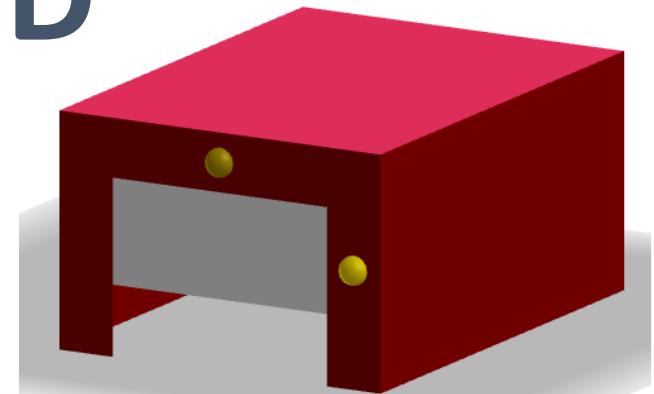
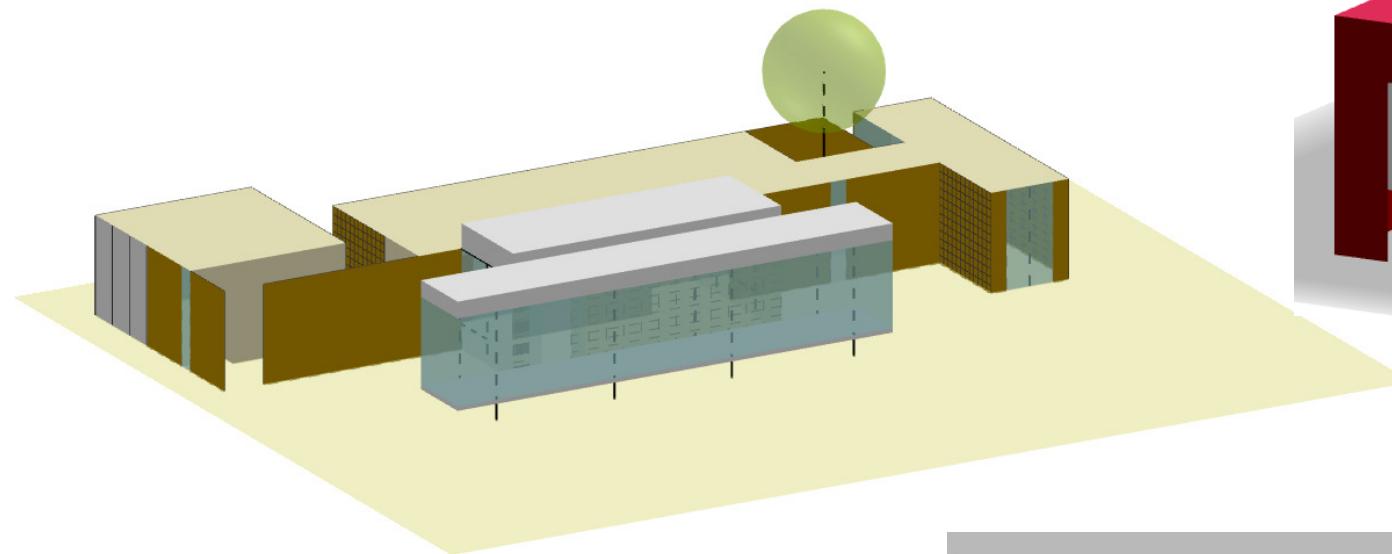
**Cristian Camilo Manrique Espinosa**  
Curso 2015-2016

# Modelado 3D

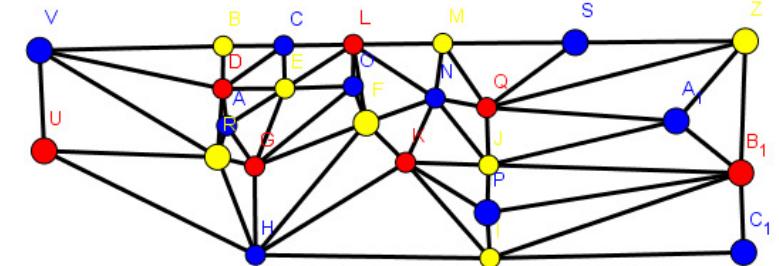
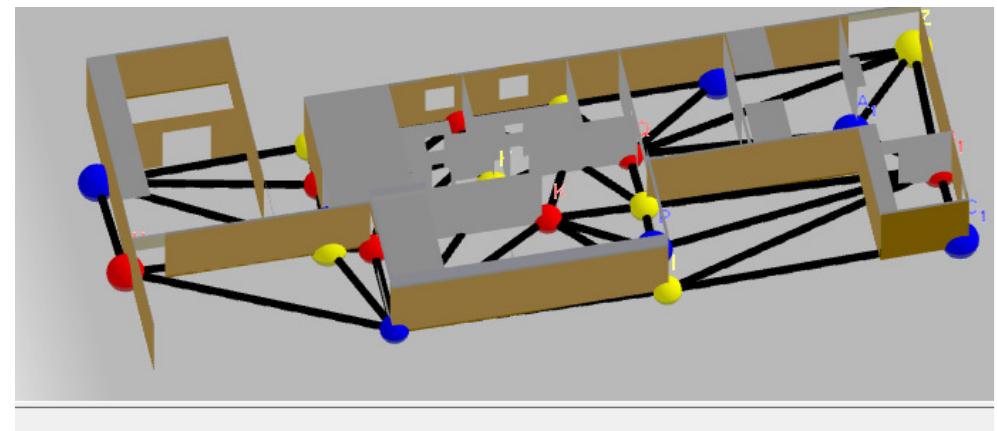


**Javier Caro Rueda**  
Curso 2015-2016

# Modelado 3D

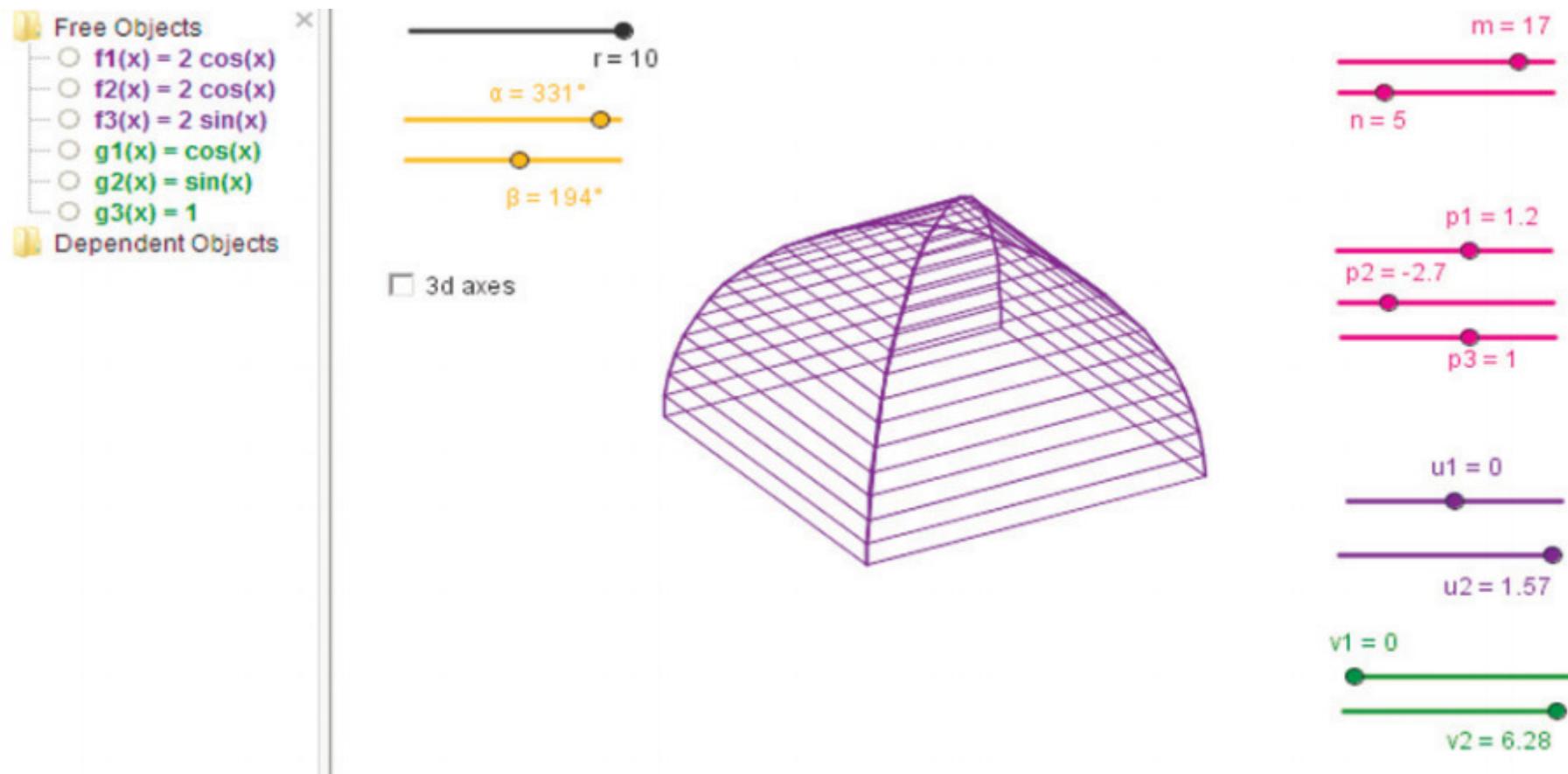


Marta Colón Paniagua  
Curso 2016-2017

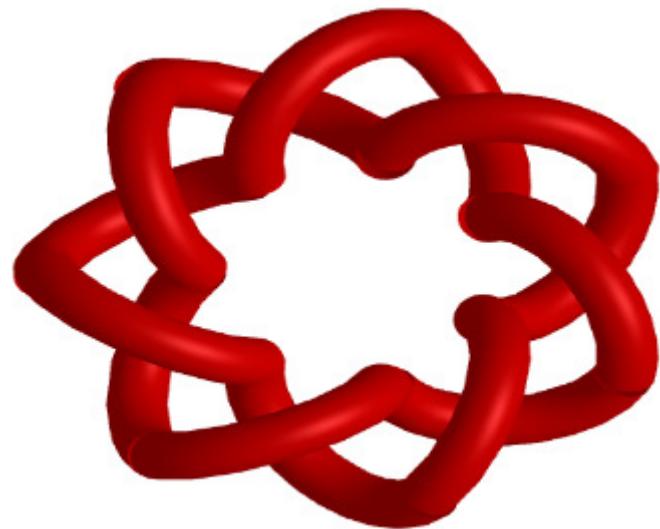
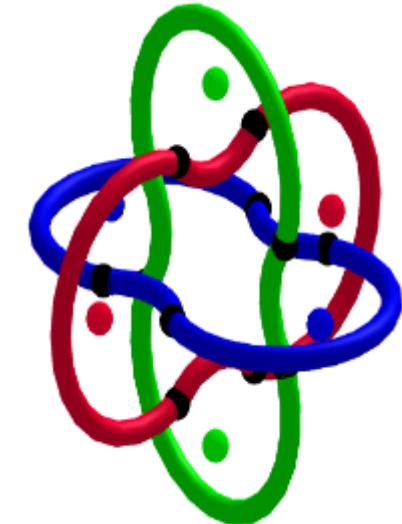
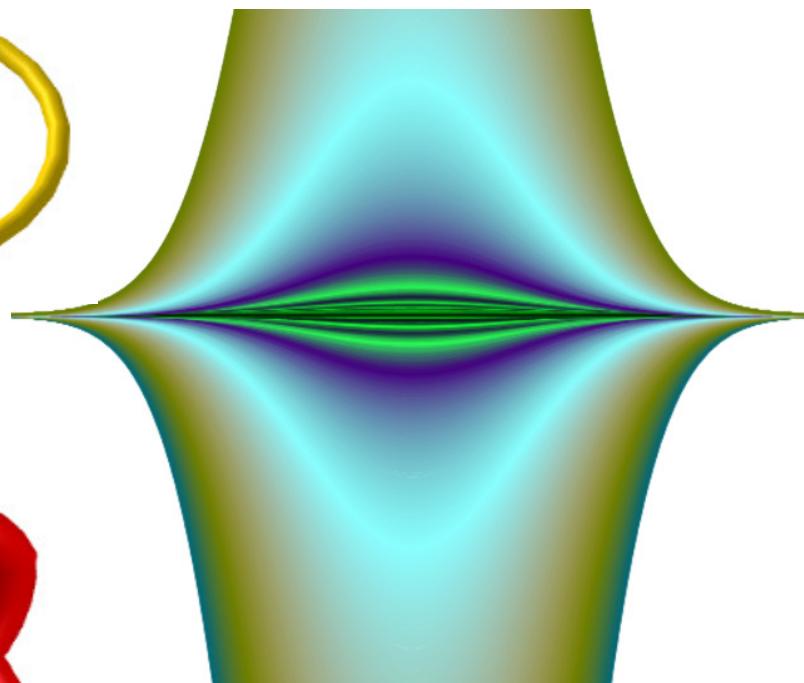
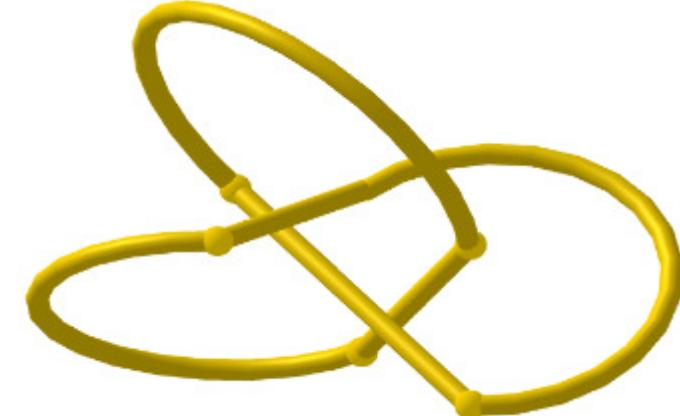


Nieves Torres Moreno  
Curso 2016-2017

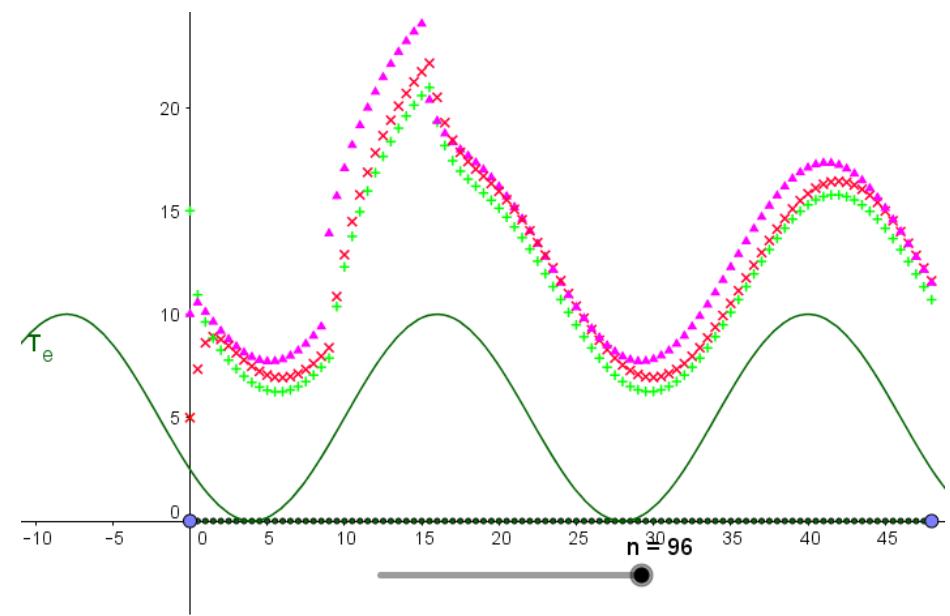
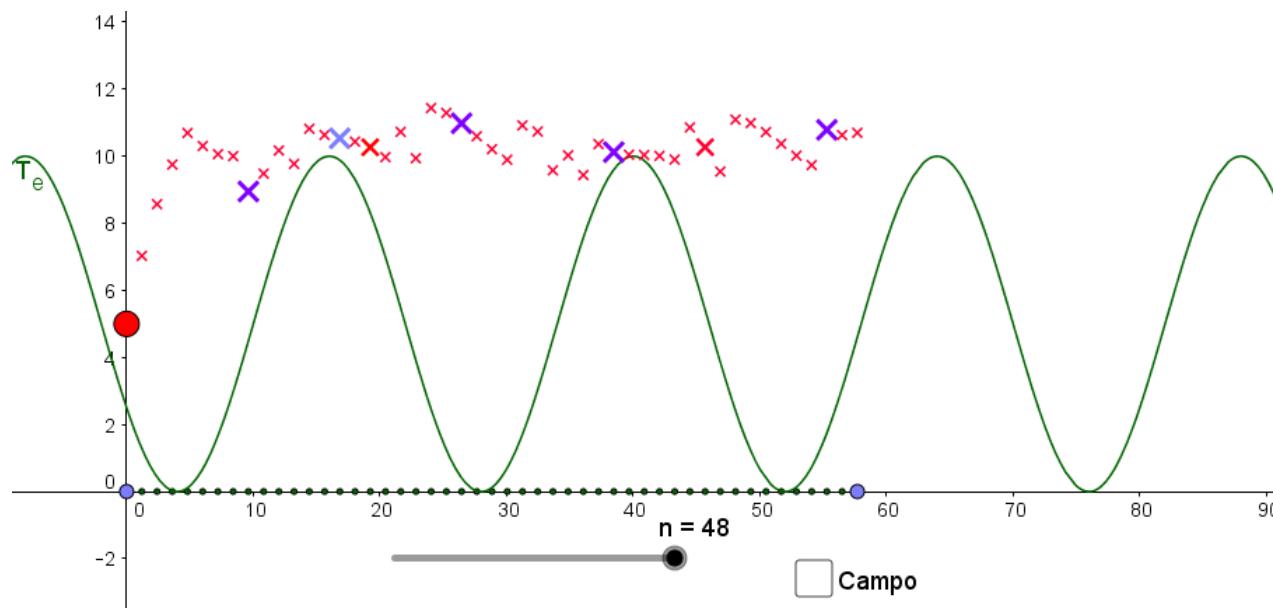
# Modelado 3D



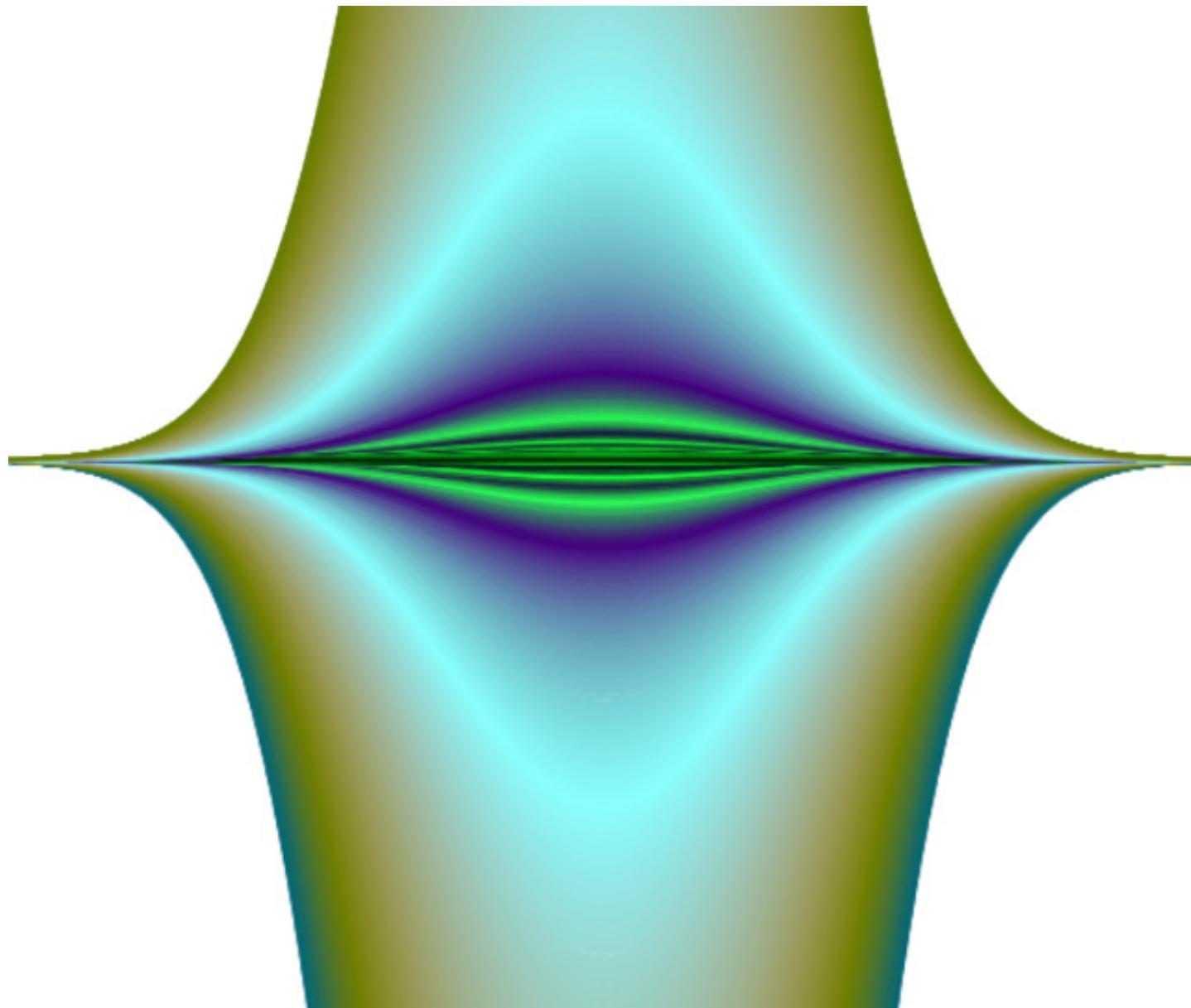
# Diseños basados en EDO's y Geometría Diferencial



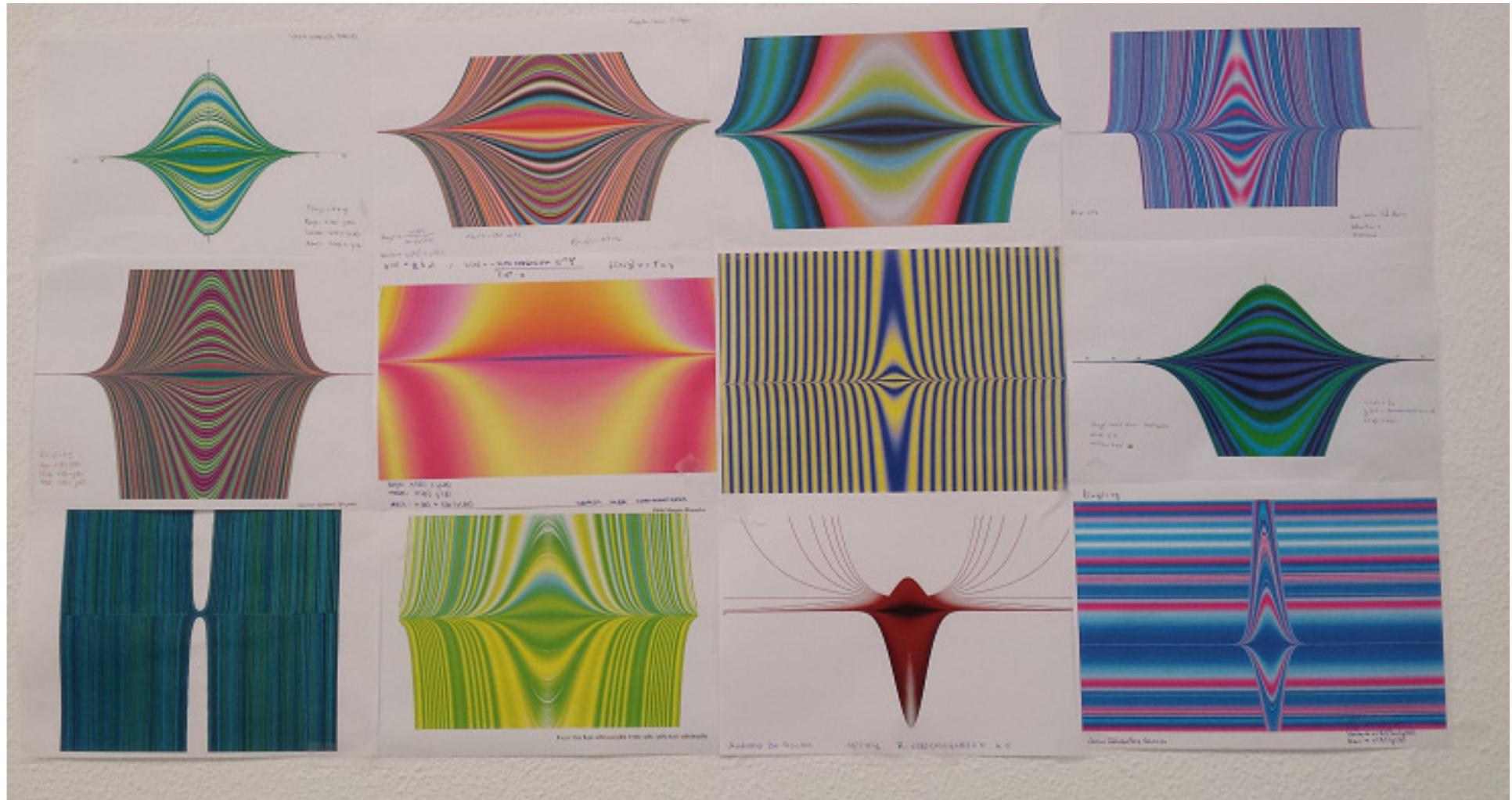
# EDO's

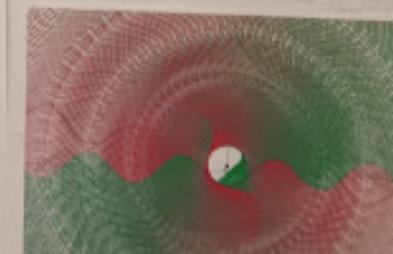
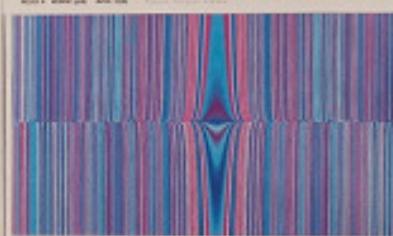
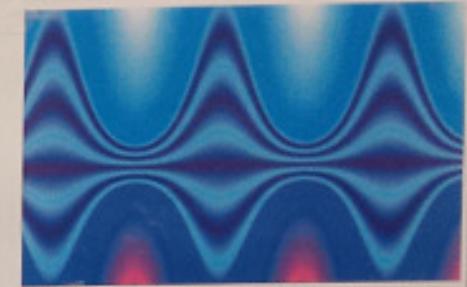
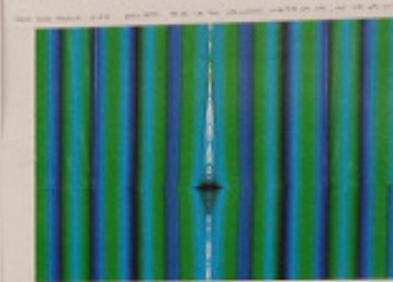
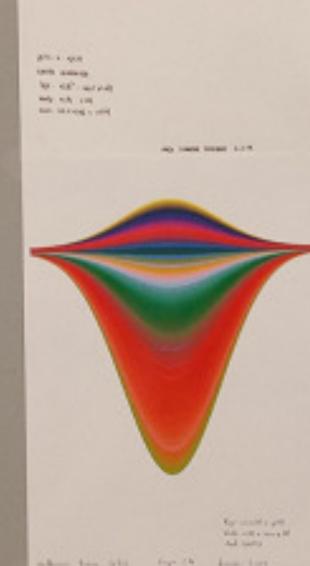
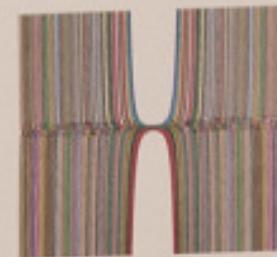
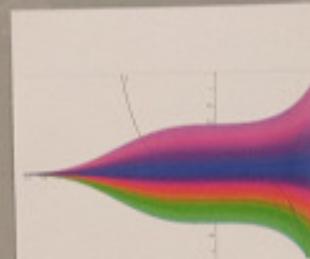
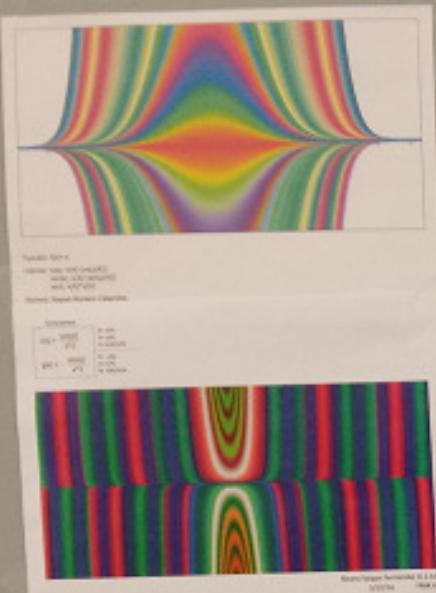


# EDO's

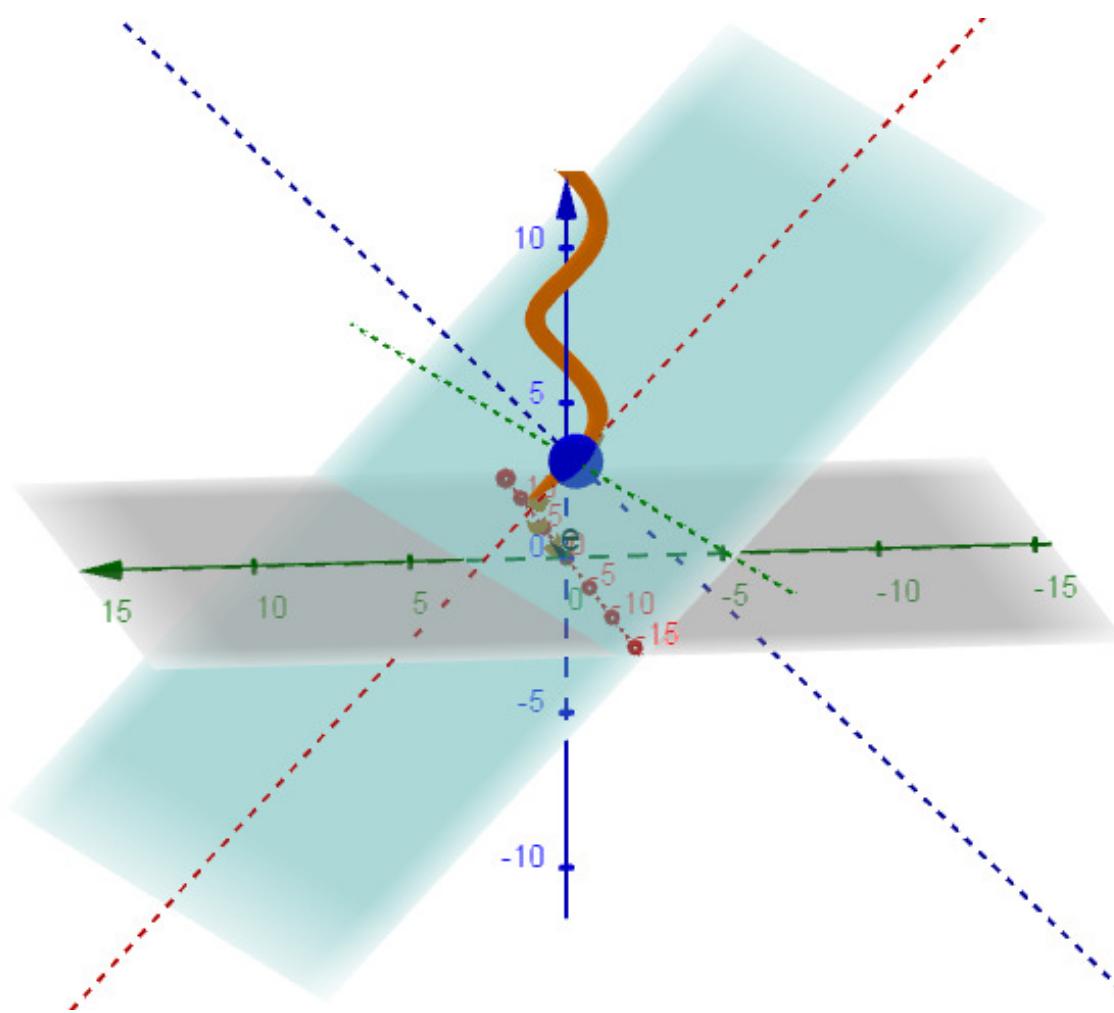


# EDO's



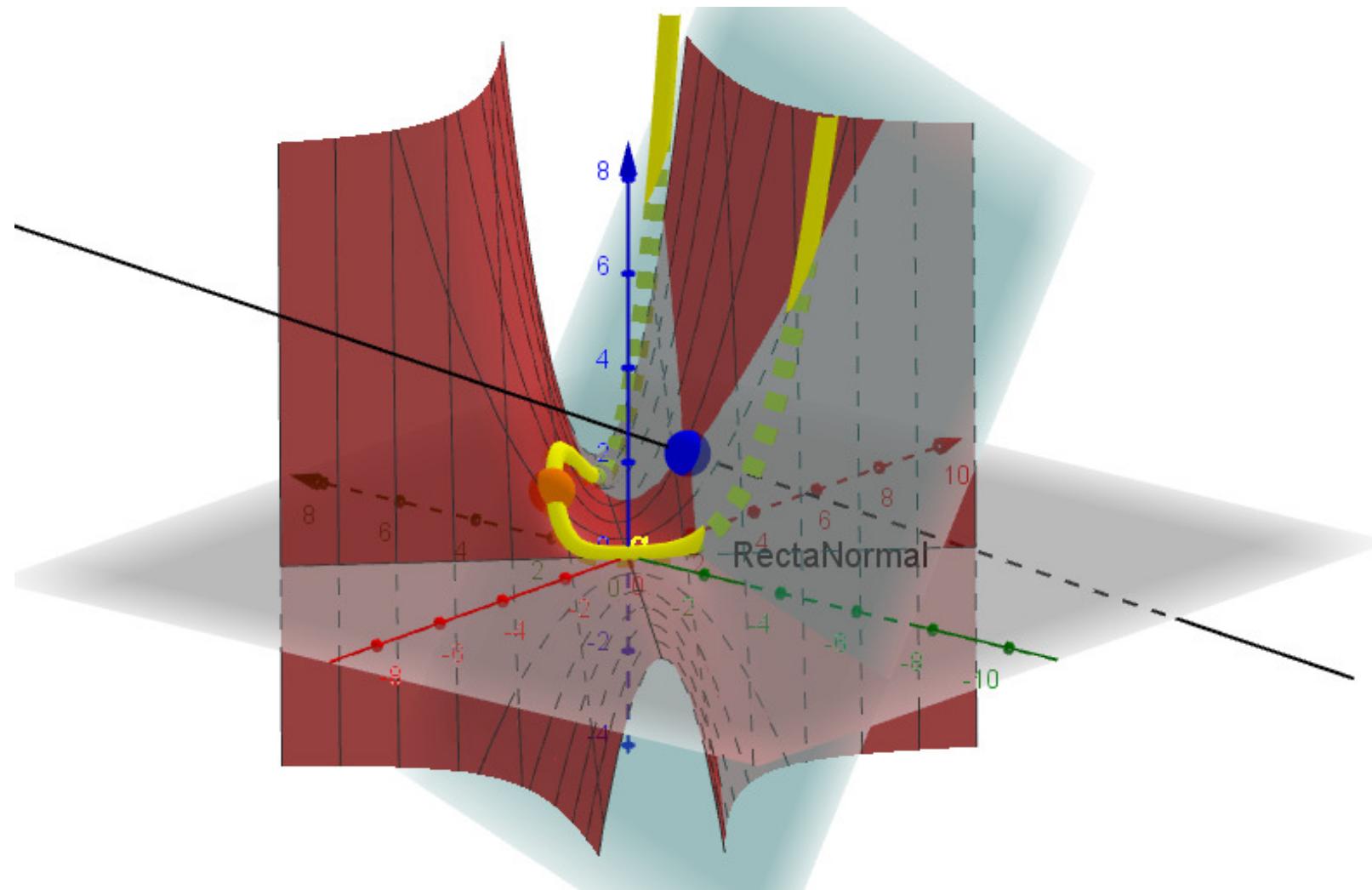


# Geometría Diferencial: Curvas.



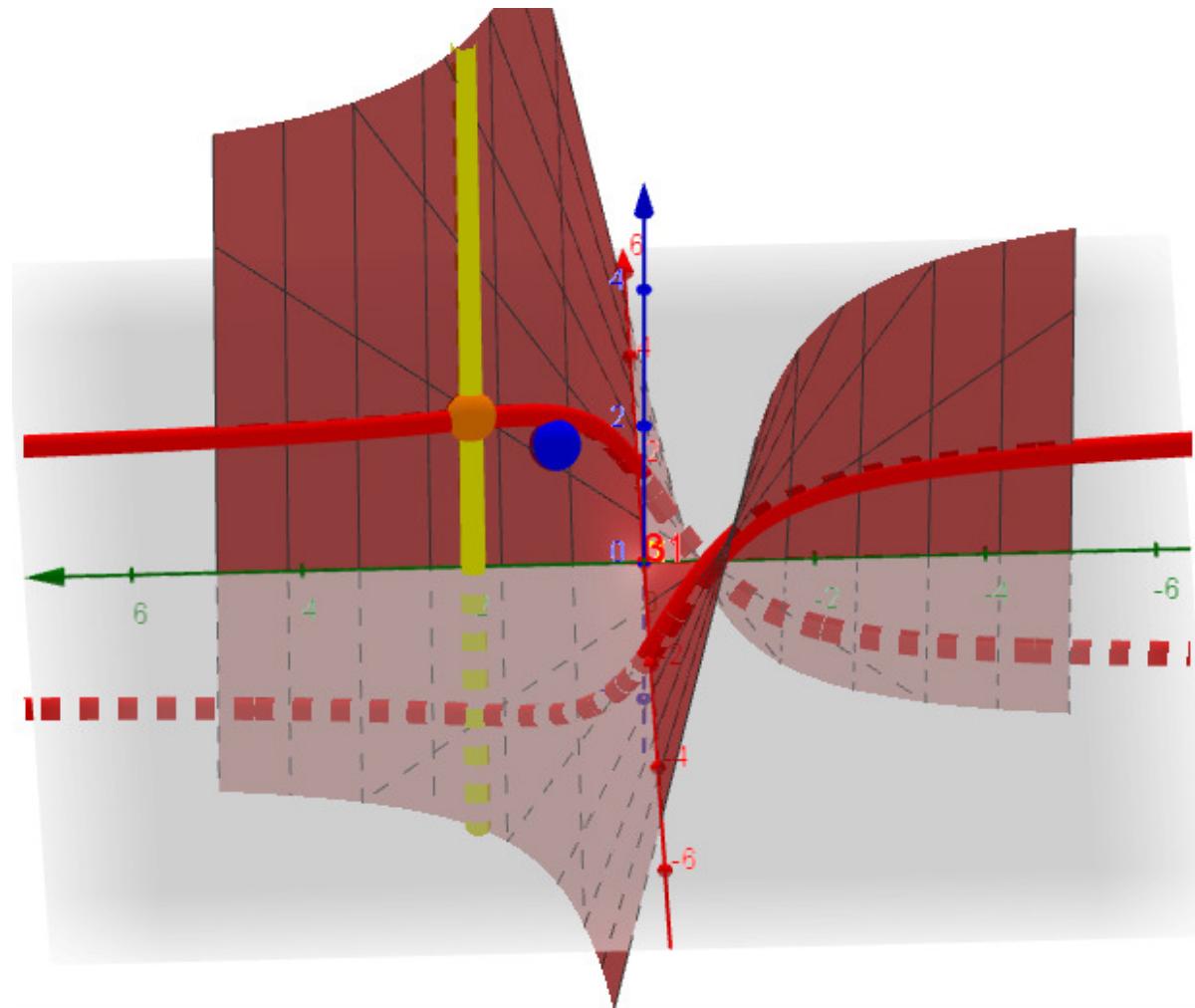
# Geometría Diferencial:

## Superficies.

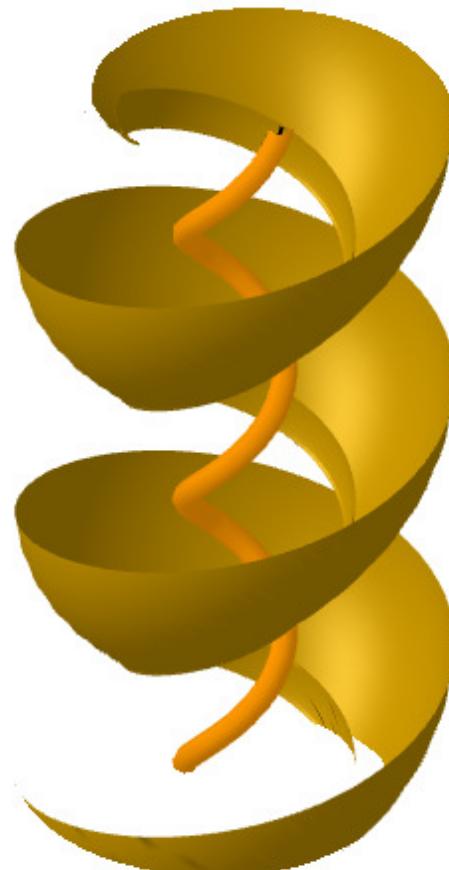


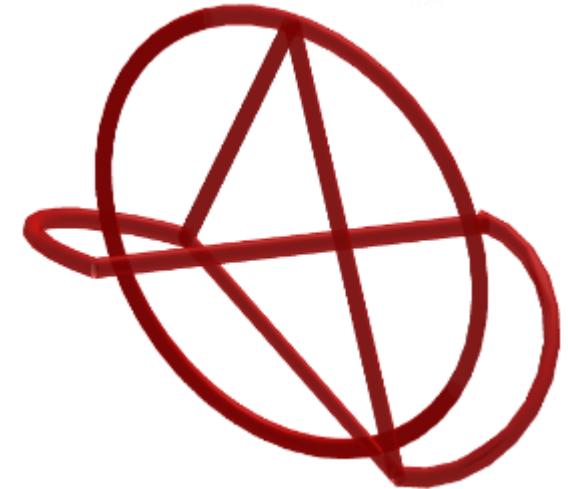
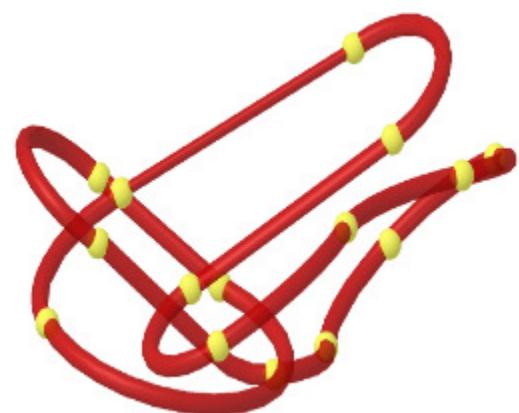
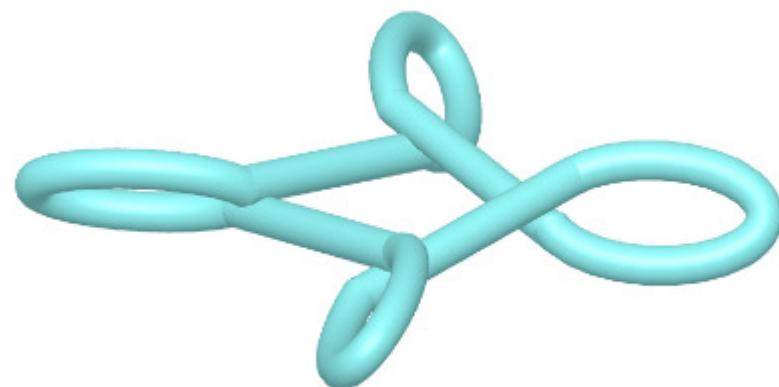
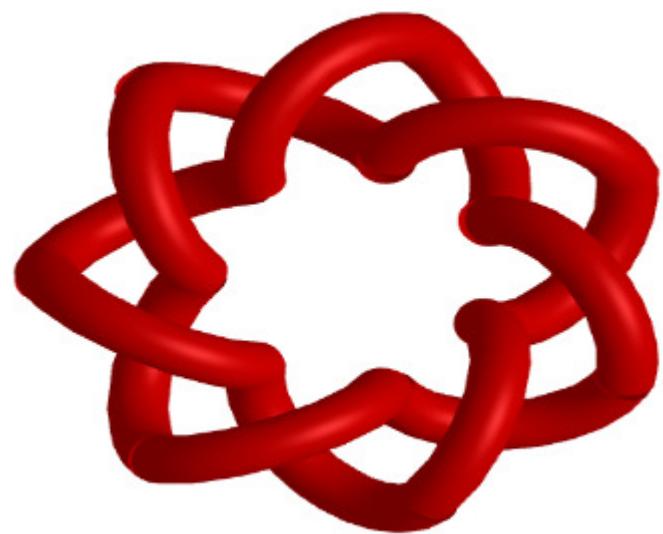
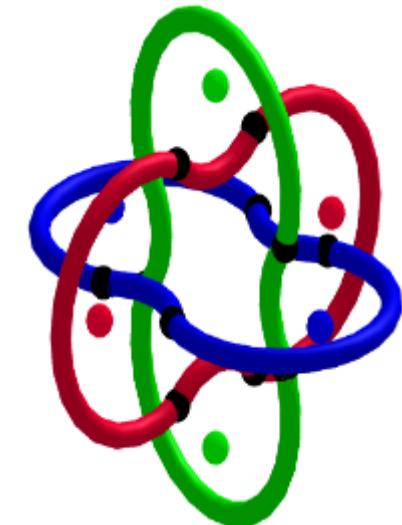
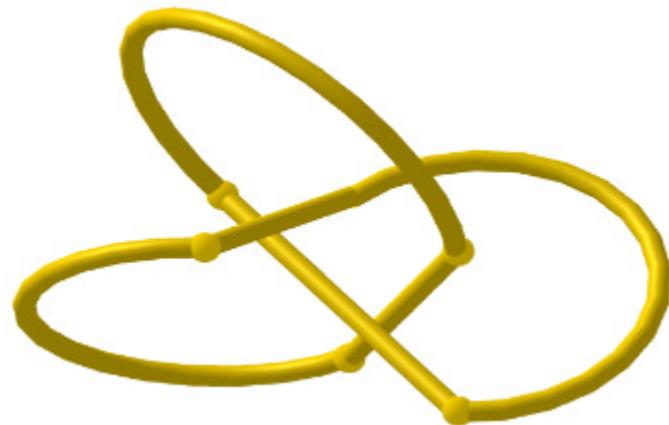
**Ejercicio 5.9** Consideramos el paraboloide hiperbólico  $\vec{r} = (u, v, uv)$  y el punto  $P(1, 1, 1)$ .  
Hadar

1. el ángulo que forman las curvas coordenadas al cortarse.
2. el ángulo que forman las curvas  $uv = u^2 + v - 1$  y  $3v^2 = 2u^3 + 1$  en  $P$ .
3. las trayectorias ortogonales a la familia  $v = C$ . Particularizar en  $P$ .
4. las líneas de máxima pendiente.



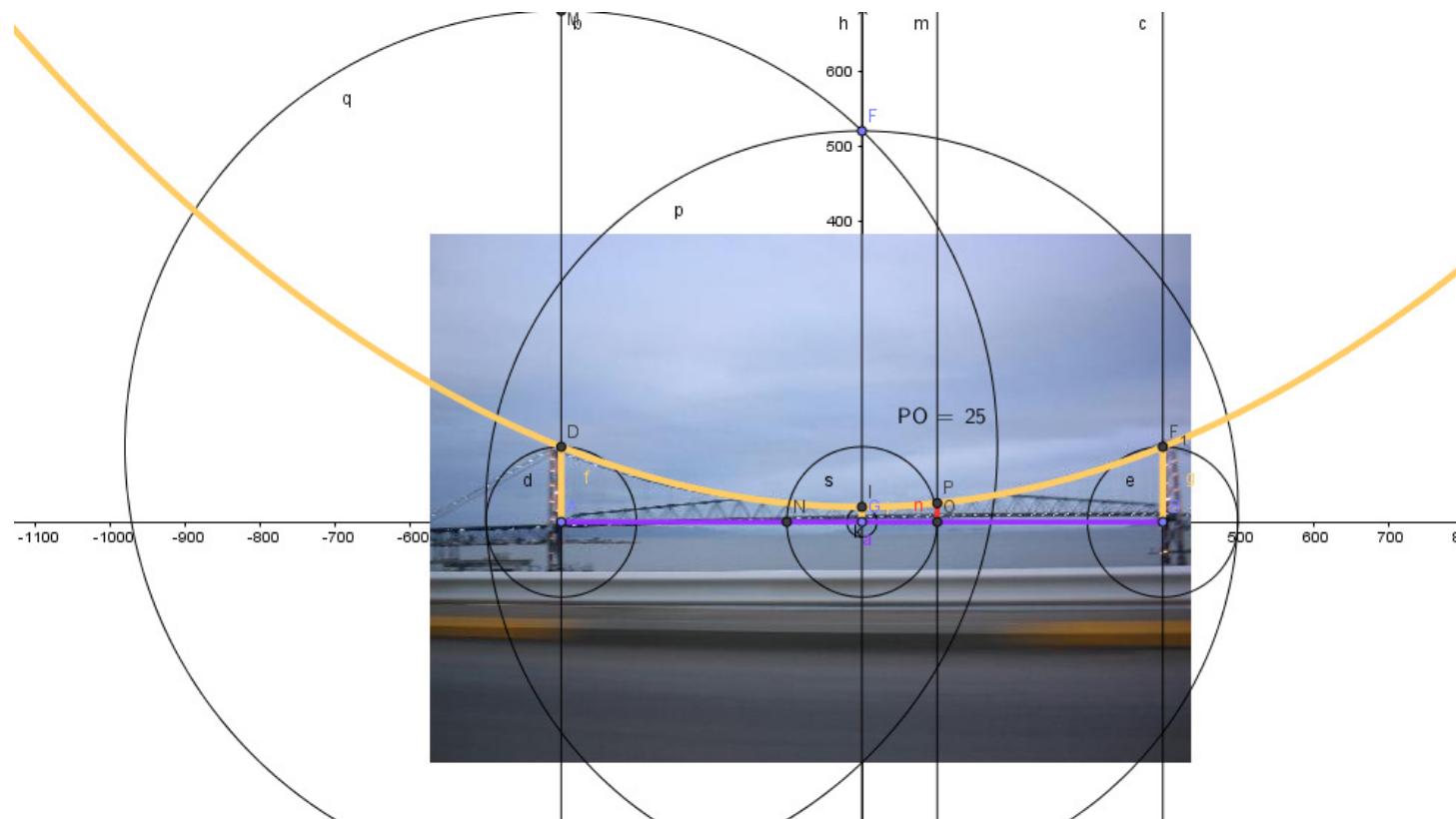
# Geometría Diferencial: Superficies tubulares.





# Extra: Estudio de cónicas.

**Ejercicio 1.4** El cable de suspensión de un puente tiene forma parabólica. La carretera es horizontal. Su longitud es 800 m. y está suspendido por tirantes verticales que unen el cable principal. Los dos tirantes verticales más largos miden 100 m. y el más corto mide 20m. Obtener la longitud de un tirante vertical unido a la carretera a una distancia de 100 m. desde el centro.



## REFERENCIAS

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- R. M. Falcón, R. Ríos, *The use of GeoGebra in Discrete Mathematics.* GeoGebra : The New Language for the Third Millennium (GeoGebra International Journal of Romania), Vol. 4, No. 1 (2015), 39-50.

Gracias!!

GeoGebra, un primer paso para diseñar la Arquitectura  
Dinámica del Siglo XXI

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V Encuentro en Andalucía GeoGebra en el aula

Málaga, 22 de abril de 2017

