3D-Dynamical Geometry in Building Construction.

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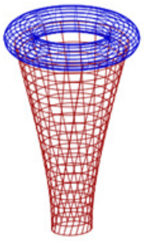
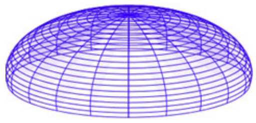
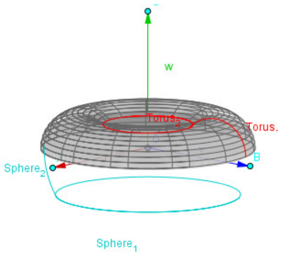
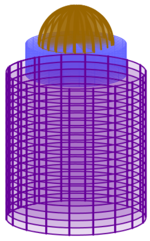
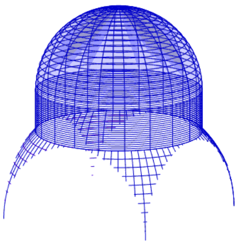
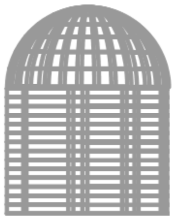


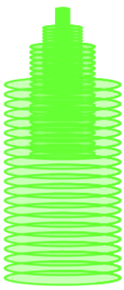


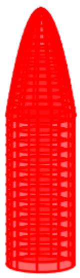
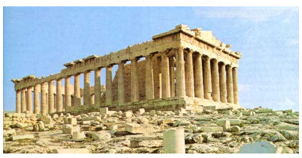
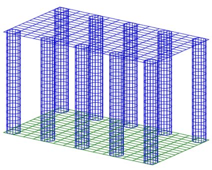


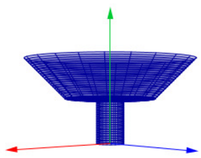










TIME 2010 – Technology and its Integration into Mathematics Education.

ABSTRACT

In Architecture and Technical Architecture Degrees, students use CAD tools (*Computer Aided Design*) which are not capable, in general, of representing graphically curves or surfaces starting from its corresponding equations. To get it, users have to define specific macros or they have to create a table of points in order to convert a set of nodes into polylines.

CAS tools used in Math classes allow this graphical representation of curves and surfaces starting from their parametric equations. However, they lack the dynamical development given by CAD tools, which plays a main role in the mentioned degrees. In this sense, the complementation of the algebraic and geometric tools included in the software of dynamic geometry, *GeoGebra*, is an attractive alternative to design and model, from a mathematical point of view, curves and rigid objects in the space. The use of sliders related to the Euler's angles and the possibility of generating tools which project 3D into 2D, makes easier this kind of modeling.

In the current workshop, we will show how to construct 3D-models of several architectonical constructions which have been made in the context of the subject called *Mathematics for Building Construction II*, corresponding to the Building Construction Engineering of the University of Seville, which has been implemented this academic year 2009-10.

Keywords

3D-modeling. Building Construction modeling. Perspectives and projections. Dynamic geometry. GeoGebra.

1. INTRODUCTION.

Let O be the origin of coordinates of two orthogonal reference systems $OXYZ$ and $OX'Y'Z'$ and let l be the straight line which is the intersection between the planes OXY and $OX'Y'$. The orientation of the latter system with respect to the former is univocally determined by the **Euler angles**:

α: Angle formed by the straight lines OZ and OZ'.
β: Angle formed by the straight lines OX' and l.
γ: Angle formed by the straight lines l and OX.

The variation of these three angles implies the movement of the second system of reference with respect to the first one. Any rigid object whose coordinates are given with respect to the mobile system will be moved in the same way. Specifically, α determines the **inclination angle** of the rigid object with respect to the fixed system $OXYZ$ and β determines the **rotation angle** in the plane $OX'Y'$. As a consequence, any tridimensional rigid object can be visualized by any orthographic projection which depends on the Euler angles and whose focus is fixed at infinite distance.

In order to model tridimensional objects, Genevieve Tulloue [1] implemented in *Cabri* [2] the following orthogonal projection of a point $P = (a,b,c)$:

$$\pi_r\,(a,b,c) = (r\cdot(a\cdot\sin(\beta) + b\cdot\cos(\beta)),\ r\cdot(-a\cdot\cos(\beta)\cdot\sin(\alpha) + b\cdot\sin(\beta)\cdot\sin(\alpha) + c\cdot\cos(\alpha))),$$

where r determines the scale of the projection. A similar implementation in *GeoGebra* [3] is given by Rafael Losada [4], who extends in a natural way the point-by-point projection of Tulloue in order to project tridimensional curves and surfaces which can be defined as a mesh of curves [5]. However, the computation of sequences of curves in *GeoGebra* can be so slow that it is necessary a very powerful computer in order to rotate a simple solid like a cone or a sphere. In these cases, it is better to define polylines which are based in a high number of points.

In our study, we are interested in the following two types of surfaces:

A. Those surfaces defined as:

$$S(u,v) = P + f(u)\cdot g(v) = (p_1 + f_1(u)\cdot g_1(v),\ p_2 + f_2(u)\cdot g_2(v),\ p_3 + f_3(u)\cdot g_3(v)).$$

B. Ruled surfaces:

$$S(u,v) = f(u) + v\cdot g(u) = (f_1(u)+v\cdot g_1(u),\ f_2(u)+v\cdot g_2(v),\ f_3(u)+v\cdot g_3(v)).$$

2. DEFINING TOOLS IN GEOGEBRA.

In this Section, we will show how to define some basic tools which can be used in order to work with space curves and surfaces in GeoGebra.

The first step in our worksheet of GeoGebra will be the definition of the sliders which correspond to the scalar r and the angles α and β. Once we have defined them, we have to follow the following steps:

2.1 Construction of the mobile reference system.

1) Fix the origin O.

2) Determine the coordinate system $OX'Y'Z'$, by defining three vectors with origin at O and extremes at:

$$\pi_r(1,0,0) = (r\cdot sin(\beta),\ -r\cdot cos(\beta)\cdot sin(\alpha)).$$
$$\pi_r(0,1,0) = (r\cdot cos(\beta),\ r\cdot sin(\beta)\cdot sin(\alpha)).$$
$$\pi_r(0,0,1) = (0,\ r\cdot cos(\alpha)).$$

3) Define a check box *"Axes"* related to the previous three vectors.

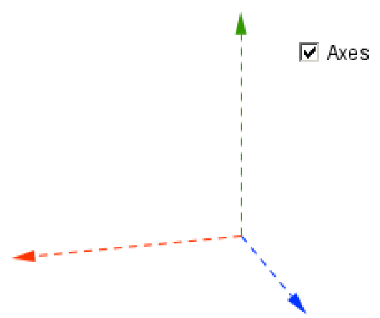


Figure 1: 3d axes.

2.2 Definition of the Orthogonal Scaled Projection.

1) Define any point in the space as an array of 3 elements.
   Ex: *P = {1, 2, 3}.*

2) Determine the projection π_r of the previous point[1]:
   P'=(r/10(Element[P,1]sin(β)+Element[P,2]cos(β)),r/10(-Element[P,1]cos(β) sin(α) + Element[P,2]sin(β)sin(α)+Element[P,3]cos(α))).

3) Define a new tool **OSP** whose output is *P'* and whose input is $\{P, r, \alpha, \beta\}$.

2.3 Projection of a space curve $C(x) = (c_1(x),c_2(x),c_3(x))$, with x in (x_0, x_1).

1) Define the extremes x_0 and x_1 of the interval, like numbers or by using sliders.

2) Define the components of the curve as three functions in x.
   Ex: $c_1(x) = \sin(x)\cos(x);\ c_2(x) = \cos(x);\ c_3(x) = \sin(x)$.

3) Determine the projection π_r of the curve C in (x_0,x_1):
   $C=$ *Curve[r/10(*$c_1(t)\sin(\beta)+c_2(t)\cos(\beta)$*), r/10(*$-c_1(t)\cos(\beta)\sin(\alpha) + c_2(t)\sin(\beta)\sin(\alpha) + c_3(t)\cos(\alpha)$*), t,* x_0*,* x_1*].*

4) Define a new tool **Curve3d** whose output is C and whose input is $\{r, \alpha, \beta, c_1(x), c_2(x), c_3(x), x_0, x_1\}$.

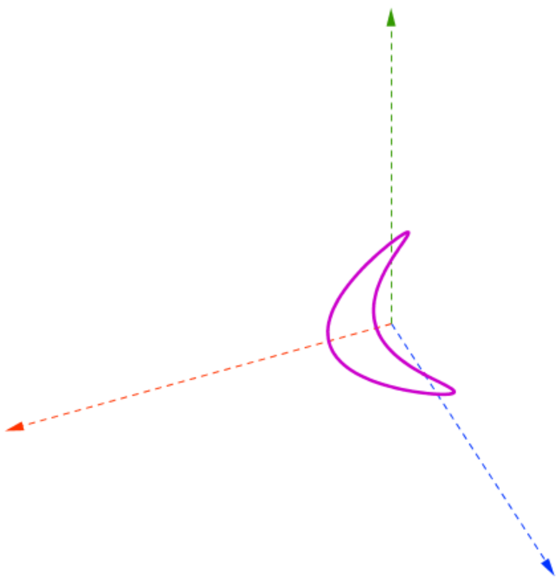


Figure 2: Space curve in *GeoGebra*.

[1] In all the projections, we will divide by 10 to obtain a better effect with the tridimensional axes.

2.4 Projection of a surface $S(u,v) = (p_1+f_1(u)\cdot g_1(v),\ p_2+ f_2(u)\cdot g_2(v),\ p_3+f_3(u)\cdot g_3(v))$ with u and v in (u_0, u_1) and (v_0,v_1), respectively, by using m u-polylines and n v-polylines.

A similar construction of the surface defined in 2.4 can be done by using polylines:

1) Define as numbers or sliders the elements $p_1, p_2, p_3, u_0, u_1, v_0, v_1, m$ and n.

2) Define the components of the surface as six functions of x.
Ex: *$f_1(x)=cos(x)$; $f_2(x)=cos(x)$; $f_3(x)=sin(x)$; $g_1(x)=cos(x)$; $g_2(x)=sin(x)$; $g_3(x)=1$.*

3) Determine the projections of the nodes of the m u-polylines and the n v-polylines:
N= Sequence[Sequence[(r/10((p_1+f_1(u) g_1(v)) sin(β) + (p_2+f_2(u) g_2(v)) cos(β)), r/10(-(p_1+f_1(u) g_1(v)) cos(β) sin(α) + (p_2+f_2(u) g_2(v)) sin(β) sin(α) + (p_3+f_3(u) g_3(v)) cos(α))), u, u_0, u_1, (u_1 - u_0)/m], v, v_0, v_1, (v_1 - v_0)/n].

4) Define the projections of the m u-polylines and the n v-polylines:
P_u=Sequence[Sequence[Segment[Element[Element[N,i],j], Element[Element[N,i],j+1]],i,1,n],j,1,m].
P_v=Sequence[Sequence[Segment[Element[Element[N,i],j], Element[Element[N,i+1],j]],i,1,n],j,1,m+1].

5) Define a new tool **Surface** whose output is $\{P_u, P_v\}$ and whose input is $\{r, \alpha, \beta, m, n, p_1, p_2, p_3, f_1(x), f_2(x), f_3(x), u_0, u_1, g_1(x), g_2(x), g_3(x), v_0, v_1\}$.

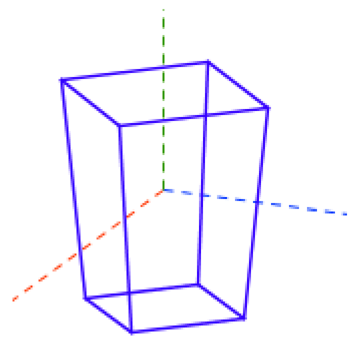


Figure 3: Polygonal surface in GeoGebra.

2.5 Projection of a ruled surface $S(u,v) = (f_1(u)+v\cdot g_1(u), f_2(u)+v\cdot g_2(v), f_3(u)+v\cdot g_3(v))$ with u and v in (u_0, u_1) and (v_0,v_1), respectively, by using m u-polylines and n v-polylines.

A similar construction of the surface defined in 2.4 can be done by using polylines:

1) Define as numbers or sliders the elements p_1, p_2, p_3, u_0, u_1, v_0, v_1, m and n.

2) Define the components of the surface as six functions of x.
Ex: $f_1(x)=x; f_2(x)=x; f_3(x)=sin(x); g_1(x)=sin(x); g_2(x)=1; g_3(x)=1$.

3) Determine the projections of the nodes of the m u-polylines and the n v-polylines:
$N= Sequence[Sequence[(r/10((f_1(u) + v\ g_1(u))\ sin(\beta) + (f_2(u)+v\ g_2(u))\ cos(\beta)), r/10((f_1(u)+v\ g_1(u))\ cos(\beta)\ sin(\alpha) + (f_2(u)+v\ g_2(u))\ sin(\beta)\ sin(\alpha) + (f_3(u)+v\ g_3(u))\ cos(\alpha))), u, u_0, u_1, (u_1 - u_0)/m], v, v_0, v_1, (v_1 - v_0)/n]$.

4) Define the projections of the m u-polylines and the n v-polylines:
P_u=Sequence[Sequence[Segment[Element[Element[N,i],j], Element[Element[N,i], j+1]],i,1,n+1],j,1,m].
P_v=Sequence[Sequence[Segment[Element[Element[N,i],j], Element[Element[N,i+1],j]],i,1,n],j,1,m+1].

5) Define a new tool **RSurface** whose output is $\{P_u, P_v\}$ and whose input is $\{r, \alpha, \beta, m, n, f_1(x), f_2(x), f_3(x), u_0, u_1, g_1(x), g_2(x), g_3(x), v_0, v_1\}$.

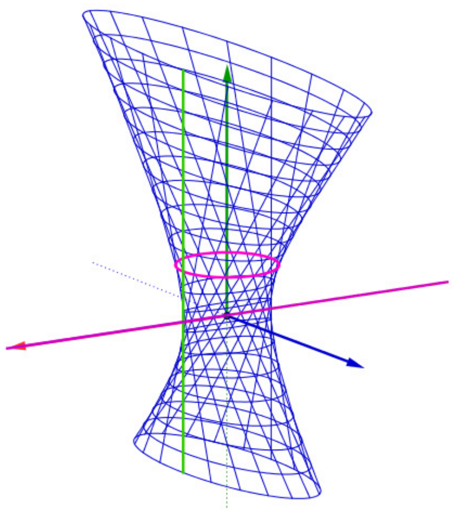


Figure 4: Ruled surface in GeoGebra.

3. SURFACES IN GEOGEBRA.

The tool *Surface* can be used in the definition of the most known surfaces. It is enough to use the parametric equations determined by the six functions given in Table 1.

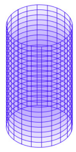
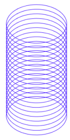
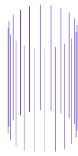
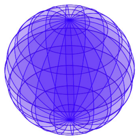
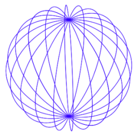
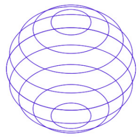
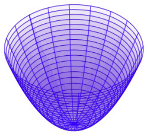
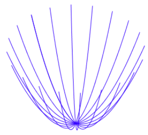
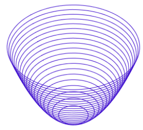
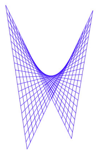
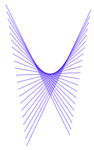
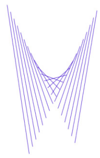
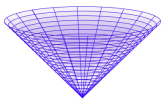
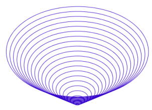
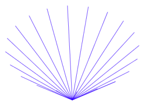
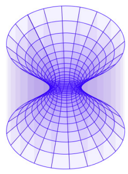
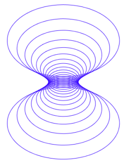
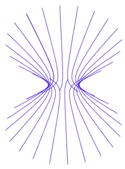
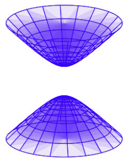
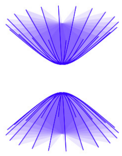
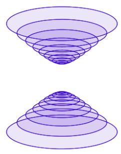
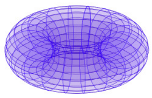
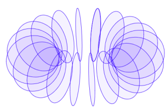
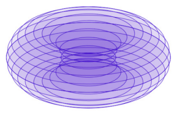
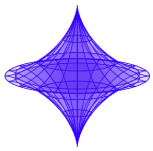
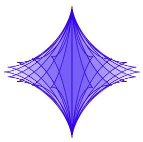
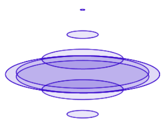
Surfaces	S	S_u	S_v
Cilinder $f_1(x)=cos(x);\ f_2(x)=sin(x);\ f_3(x)=1;$ $g_1(x)=1;\ g_2(x)=1;\ g_3(x)=x.$			
Sphere $f_1(x)=cos(x);\ f_2(x)=cos(x);\ f_3(x)=sin(x);$ $g_1(x)=cos(x);\ g_2(x)=sin(x);\ g_3(x)=1.$			
Elliptic paraboloid $f_1(x)=sqrt(x);\ f_2(x)=sqrt(x);\ f_3(x)=x;$ $g_1(x)=cos(x);\ g_2(x)=sin(x);\ g_3(x)=1.$			
Hyperbolic paraboloid $f_1(x)=x;\ f_2(x)=1;\ f_3(x)=x;$ $g_1(x)=1;\ g_2(x)=x;\ g_3(x)=x.$			
Cone $f_1(x)=x;\ f_2(x)=x;\ f_3(x)=x;$ $g_1(x)=cos(x);\ g_2(x)=sin(x);\ g_3(x)=1.$			
One-sheeted hyperboloid $f_1(x)=cos(x);\ f_2(x)=cos(x);\ f_3(x)=sin(x);$ $g_1(x)=cosh(x);\ g_2(x)=sinh(x);\ g_3(x)=1.$			
Two-sheeted hyperboloid $f_1(x)=sinh(x);\ f_2(x)=sinh(x);\ f_3(x)=cosh(x);$ $g_1(x)=cos(x);\ g_2(x)=sin(x);\ g_3(x)=1.$			
Torus Ex: $f_1(x)=2+cos(x);\ f_2(x)=2+cos(x);\ f_3(x)=sin(x);$ $g_1(x)=cos(x);\ g_2(x)=sin(x);\ g_3(x)=1.$			
Astroid Ex: $f_1(x)=cos^3(x);\ f_2(x)=cos^3(x);\ f_3(x)=sin^3(x);$ $g_1(x)=cos(x);\ g_2(x)=sin(x);\ g_3(x)=1.$			

Table 1: Construction of surfaces by using *u* and *v*-parametric curves**.**

By using the tool *RSurface*, we can build all the types of ruled surfaces:

a) **Cylindrical surfaces**: It is enough to impose g_1, g_2 and g_3 to be constant.

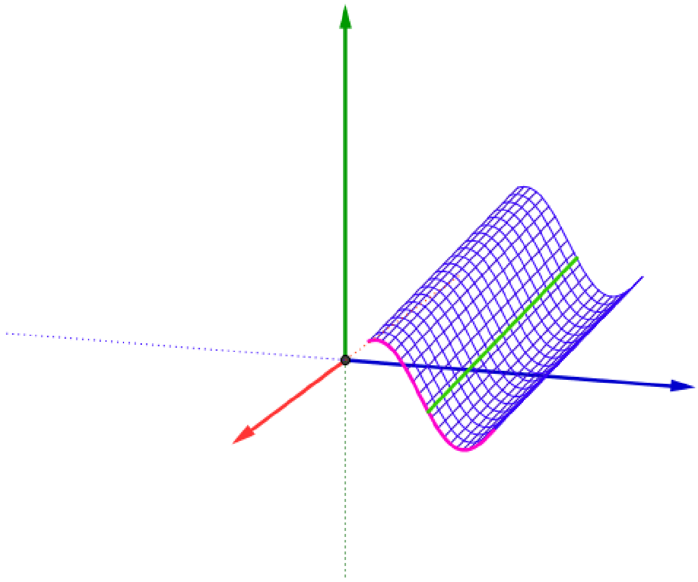


Figure 5: Cylindrical surface.

b) **Conical surfaces**: It is necessary to impose all the generatrices to pass through a given point P=(a,b,c). Specifically, it must be:

$$\frac{a-f_1(x)}{g_1(x)}=\frac{b-f_2(x)}{g_2(x)}=\frac{c-f_3(x)}{g_3(x)}.$$

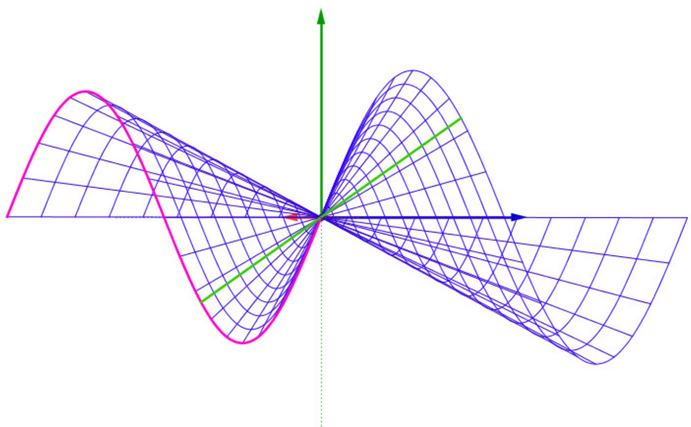


Figure 6: Conical surface.

c) **Conoids**: They are obtained when all the generatrices of the ruled surface pass through a point of the directrix curve and a given point of the axis and they are parallel to the director plane.

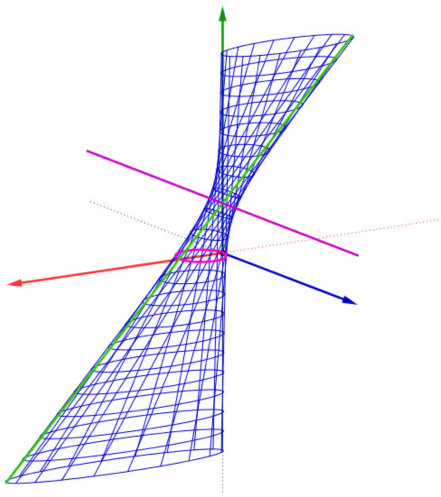


Figure 7: Conoid.

Some known ruled surfaces are determined by the functions given in Table 2.

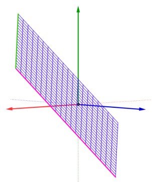
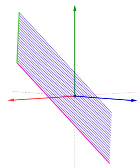
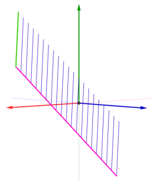
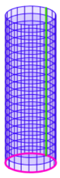
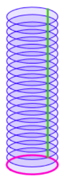
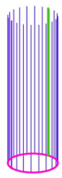
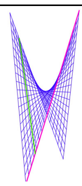
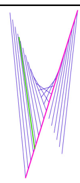
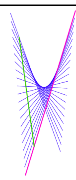
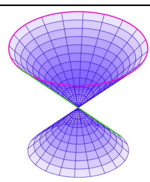
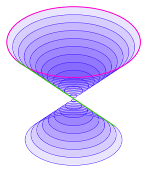
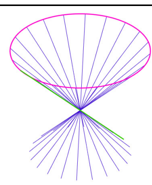
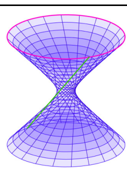
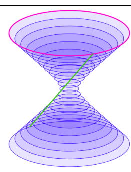
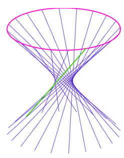
Ruled surfaces	S	S_u	S_v
Plane $f_1(x)=ax+b;\ f_2(x)=cx+d;\ f_3(x)=ex+f;$ $g_1(x)=g;\ g_2(x)=h;\ g_3(x)=i.$			
Cylinder $f_1(x)=\cos(x);\ f_2(x)=\sin(x);\ f_3(x)=1;$ $g_1(x)=0;\ g_2(x)=0;\ g_3(x)=1.$			
Hyperbolic paraboloid $f_1(x)=x;\ f_2(x)= x;\ f_3(x)=1;$ $g_1(x)=1;\ g_2(x)=-1;\ g_3(x)=4x.$			
Cone $f_1(x)=\cos(x);\ f_2(x)= \sin(x);\ f_3(x)=1;$ $g_1(x)=\cos(x);\ g_2(x)=\sin(x);\ g_3(x)=1.$			
One-sheeted hyperboloid $f_1(x)=\cos(x);\ f_2(x)= \cos(x);\ f_3(x)=\sin(x);$ $g_1(x)=\cosh(x);\ g_2(x)=\sinh(x);\ g_3(x)=1.$			

Table 2: Construction of ruled surfaces**.**

4. EXAMPLES AND EXERCISES.

In this Section, we will show a set of tasks which can be useful in order to practice the concepts that we have previously seen:

Load the tools.

Open a worksheet of Geogebra and open the following files which you can find in the folder *Tools* [6]:

Axes3d.ggt	Elevation.ggt	Curve3D.ggt	Surface.ggt
RSurface.ggt	Disk.ggt		

Construction of the tridimensional axes.

Axes3d[O, r, α, β].

1) Create the point $O = (0, 0)$.

2) Hide the bidimensional axes of Geogebra in the menu *View*.

3) Create three sliders:
   a. The scale r, which can be defined for example in the interval $[0.1, 10]$.
   b. The inclination angle α.
   c. The rotation angle β.

4) Use the tool *Axes3d* to create the tridimensional axes. Write in the input box:
   Axes3d[O, r, α, β].

5) Create a checkbox related to the elements of the axes: the segments a, b and c, the vectors u, v and w and the origin O. The caption of the checkbox will be *3d axes*.

6) Save the worksheet as *Worksheet0.ggb*.

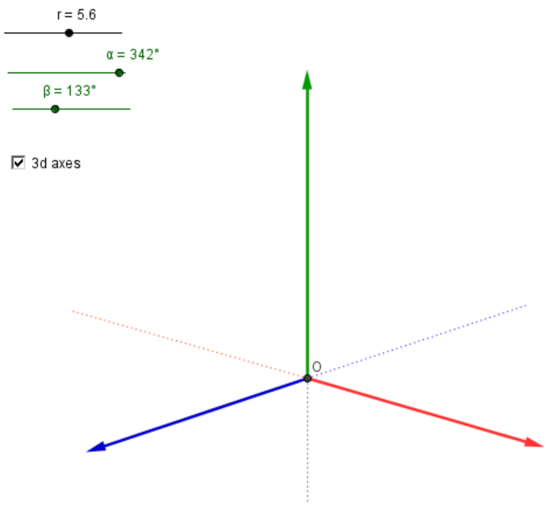


Figure 8: Tridimensional axes.

Orthogonal Scaled Projection.

OSP[{p_1,p_2,p_3},r,α, β].

In order to represent a space point in our tridimensional coordinate system, we can use the tool *OSP*.

Example: Let us build a tetrahedron.

1) Open the file Worksheet.ggb.

2) Draw the vertices of the tetrahedron by using the tool *OSP*. To do it, write in the input box the following four commands:

A=OSP[{-2,5,0}, r, α, β].
B=OSP[{5,5,0}, r, α, β].
C=OSP[{2,1,0}, r, α, β].
D=OSP[{2,3,5}, r, α, β].

3) Join the vértices by using segments or polygons in order to obtain a tetrahedron.

4) Save the file as *Tetrahedron.ggb*.

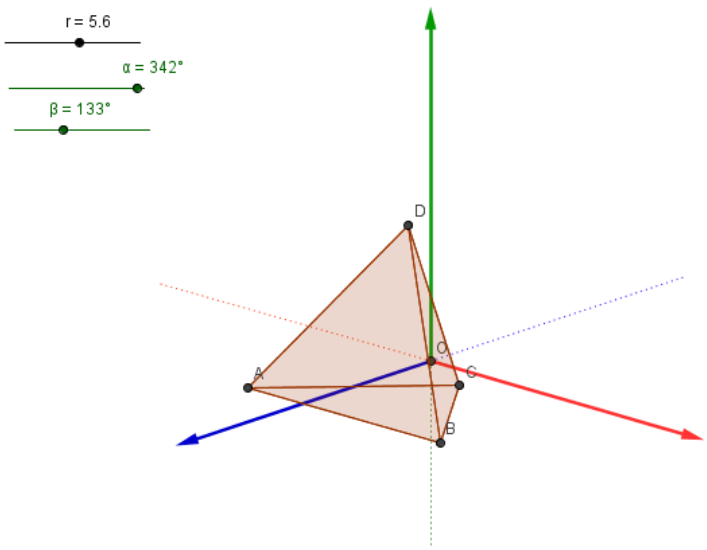


Figure 9: Tetrahedron.

You can move *r*, *α* and *β* to visualize the four faces of the tetrahedron.

Application of the OSP in Building Construction: Elevation of a floor plan.

Elevation[r,α, β,h1,h2,P,Q].

Example: Let us elevate the following floor plan:

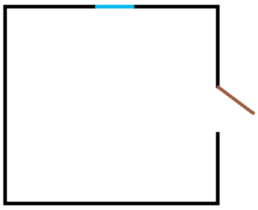


Figure 10: Floor plan.

The walls have a height of 2.5 meters. The window is in the middle of the wall and it is 1 meter x 1 meter.

1) Open the file Worksheet.ggb and hide the 3d axes.

2) Create the following points:

   A=(0,0), B=(5,0), C=(5,2), D=(5,3),
   E=(5,5), F=(3,5), G=(2,5), H=(0,5).

3) Write in the input box the following commands in order to build the walls:

   Elevation[r, α, β, 0, 2.5, A, B], Elevation[r, α, β, 0, 2.5, B, C],
   Elevation[r, α, β, 0, 2.5, D, E], Elevation[r, α, β, 0, 2.5, E, F],
   Elevation[r, α, β, 0, 2.5, G, H], Elevation[r, α, β, 0, 2.5, H, A].

4) Write in the input box the following commands in order to build the wall of the window:

   Elevation[r, α, β, 0, 0.75, F, G], Elevation[r, α, β, 1.75, 2.5, F, G].

5) Save the file as *Elevation.ggb*.

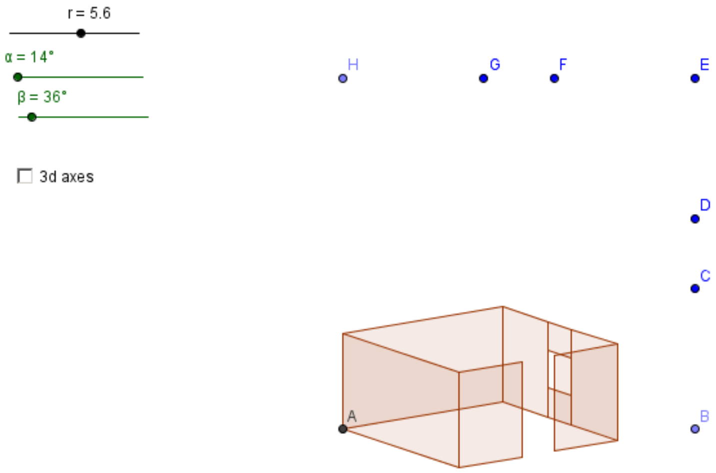


Figure 11: Elevation of a floor plan.

You can move the points *A* to *H* in order to modify the structure of the room.

Construction of a space curve:

$$C(x) = (c_1(x), c_2(x), c_3(x)),$$

where x is in an interval $[x_0, x_1]$.

$$Curve3d[r, \alpha, \beta, c_1(x), c_2(x), c_3(x), x_0, x_1].$$

Example: $C(x) = (2 \sin(x) \cos(x), 2 \cos(x), 2 \sin(x))$, where x is in $[0^o, 360^o]$.

1) Open the file Worksheet.ggb.

1) Create two sliders $x0$ and $x1$, defined as angles in $[0^o, 360^o]$. Move them in such a way that x_0 and x_1 are distinct.

2) Write in the input box:

$$Curve3D[r, \alpha, \beta, 2 \sin(x) \cos(x), 2 \cos(x), 2 \sin(x), x_0, x_1].$$

3) Save the worksheet as *Curve3D.ggb*.

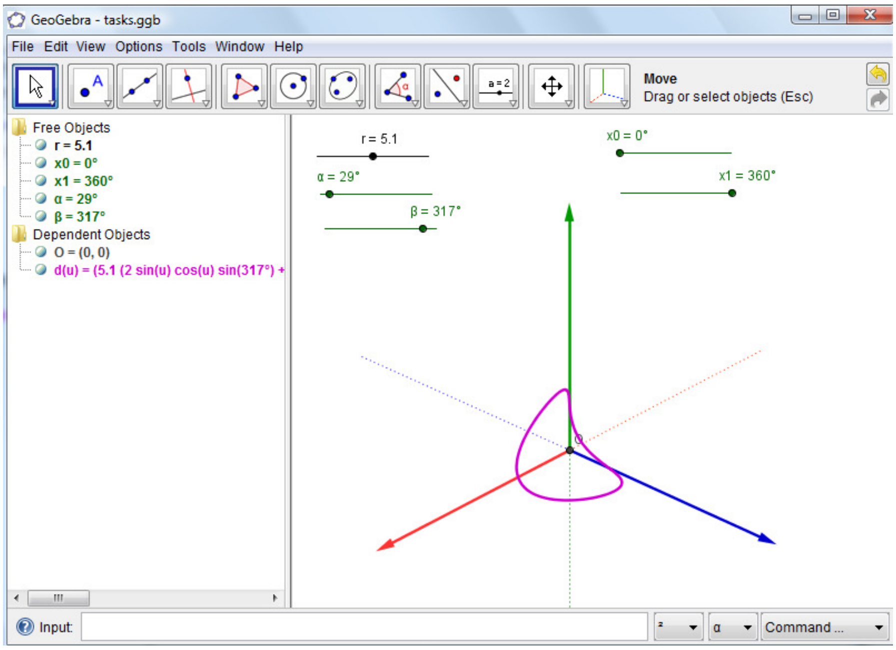


Figure 11: Space curve.

You can move $x0$ and $x1$ to see the dynamism of your space curve.

Exercise: Construct your own space curve, but try to do it by using at least two parameters (sliders) in the definition.

Construction of a surface (*m x n*-grid):

$$S(u,v) = (p_1+f_1(u)\cdot g_1(v),\ p_2+ f_2(u)\cdot g_2(v),\ p_3+f_3(u)\cdot g_3(v)),$$

where u is in the interval $[u_0, u_1]$ and v is in the interval $[v_0, v_1]$.

Surface[r,α,β,m,n,p₁,p₂,p₃,f₁(x),f₂(x),f₃(x),u₀,u₁,g₁(x),g₂(x),g₃(x),v₀,v₁].

Example: Let us build a cone (see Table 1).

1) Open the file Worksheet.ggb.

2) Create three sliders *p1*, *p2* and *p3*, all of them defined in the interval *[-5, 5]*.

3) Create two sliders h_1 and h_2 for the heights of the cone, both of them defined in the interval *[-5, 5]*. Move them in such a way that h_1 and h_2 are distinct.

4) Write in the input box:
   Surface[r,α,β,10,40,p₁,p₂,p₃,x,x,x, h₁, h₂,cos(x),sin(x),Function[1,1,1],0°,360°].

5) Save the worksheet as *Surface.ggb*.

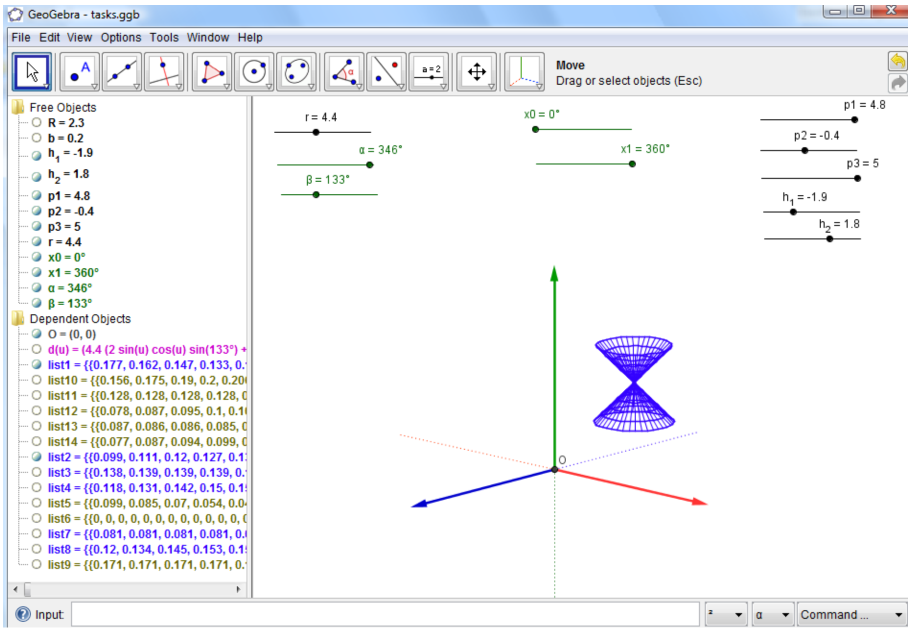


Figure 12: Construction of a cone.

You can move *p1, p2, p3*, h_1 and h_2 to see the dynamism of your surface.

Exercise: Select one surface of Table 1 and try to construct it by using the tool *Surface*. Use several sliders in order to obtain a dynamic surface.

Construction of a ruled surface (*m x n*-grid):

$$S(u,v) = (f_1(u)+ v\cdot g_1(u),\ f_2(u)+v\cdot g_2(u), f_3(u)+v\cdot g_3(v)),$$

where u is the interval $[u_0, u_1]$ and v is in the interval $[v_0, v_1]$.

RSurface[r,α,β,m,n,f_1(x),f_2(x),f_3(x),u_0,u_1,g_1(x),g_2(x),g_3(x),v_0,v_1].

Example: Let us build the right conoid generated by the straight lines parallel to the plane OXY, of axis OZ and directrix the curve {x=cos(t), y=sin(t), z=t}, with t in (0,2π).

2) Open the file Worksheet.ggb.

3) Create two sliders l_1 and l_2 for the length of the corresponding generatrices, both of them defined in the interval *[-5, 5]*. Move them in such a way that l_1 and l_2 are distinct.

4) Write in the input box:

   RSurface[r,α,β,20,20,cos(x),sin(x),x,0,2π,cos(x),sin(x),Function[0,1,1], l_1, l_2].

5) Save the worksheet as *RSurface.ggb*.

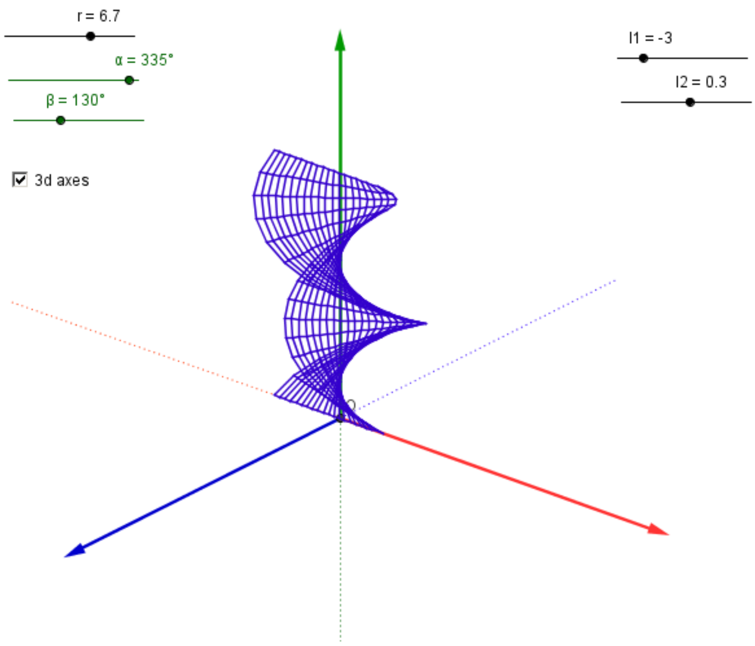


Figure 13: Ruled surface.

You can move l_1 and l_2 to see the dynamism of your ruled surface.

Exercise: Select one surface of Table 2 and try to construct it by using the tool *RSurface*. Use several sliders in order to obtain a dynamic ruled surface.

Creation of a tool which corresponds to a surface which can be used in building construction.

Example: Let us define a tool which can be used to construct a dome.

1) Open the file Worksheet.ggb.

2) Create five sliders:
   a. Three sliders *p1*, *p2* and *p3*, corresponding to the center of the dome, all of them defined in the interval *[-5, 5]*.
   b. The radius *R* of the dome, defined in the interval *[0.1, 10]*.
   c. The basis *b* of the dome, determined by an angle defined in the interval *[0, π/2-0.1]*.

3) Write in the input box:

   Surface[r,α,β,10,40,p₁,p₂,p₃,R cos(x), R cos(x),R sin(x), b, π/2, cos(x), sin(x), Function[1,1,1],0°,360°].

4) In the menu *Tools*, create a tool whose output objects are the two lists which have been obtained after using the previous command. The input objects will be *r, α, β,* p_1, p_2, p_3, *R* and *b*. Denote the new tool as Dome and write as tool help:

   Select r, α, β, p1, p2, p3 (center), R (radius), b (basis).

5) In the submenu *Manage Tools*, save the new tool as *Dome.ggt*.

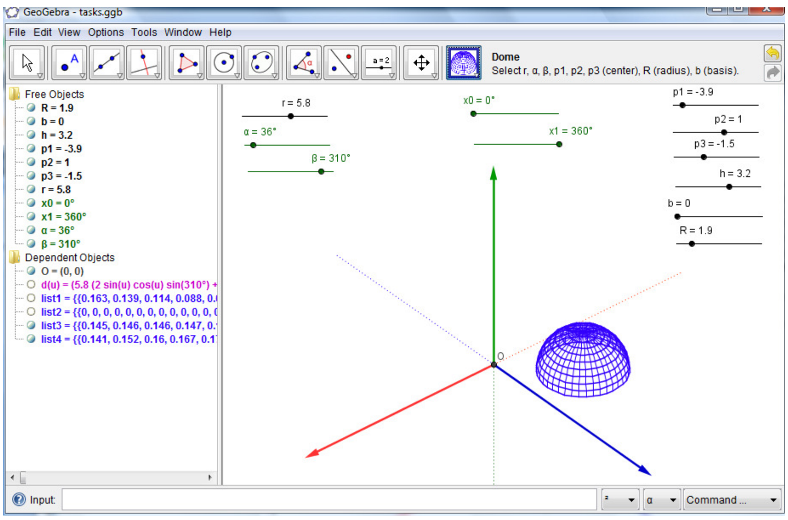


Figure 14: Construction of a dome.

Exercise: Create a tool which can be used to construct a cone. You can use the construction which we have done in the previous Section. Save the new tool as *Cone.ggt*.

Exercise: Create a tool which can be used to construct a cylindrical pillar. It has to depend on the center of the basis, the radius and the height of the pillar. Save the new tool as *Pillar.ggt*.

Construction of a building.

Example: Let us build a cylindrical tower of two floors such that a dome covers its top. The cylinder of the first floor has a radius of 3 meters and a height of 4 meters and the one of the second floor has a radius of 1.5 meters and a height of 3 meters.

1) Open the file Worksheet.ggb.

2) Use the tool *Pillar* to create the first floor of the tower:
Pillar[r,α,β,0,0,0,3,4].

3) Use the tool *Disk* to create the ceiling of the first floor:
Disk[r,α,β,0,0,4,3].

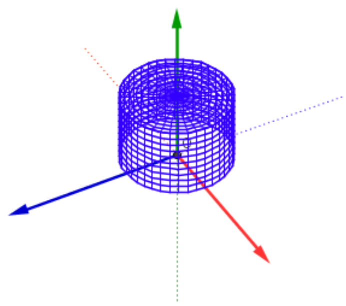


Figure 15: First floor of a tower.

4) Use the tool *Pillar* to create the second floor of the tower:
Pillar[r,α,β,0,0,4,1.5,3].

5) Use the tool *Dome* to create the dome of the tower:
Dome[r,α,β,0,0,7,3,1.5,3].

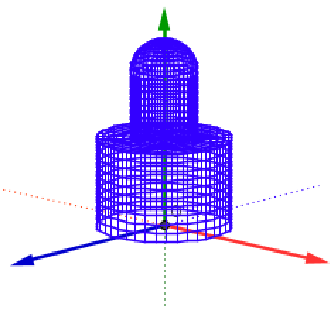


Figure 16: Tower of two floors with a dome.

6) Save the worksheet as *Building.ggb*.

Exercise: Draw a truncated cone at the top of the dome. You can use sliders in order to get the exact position of the cone.

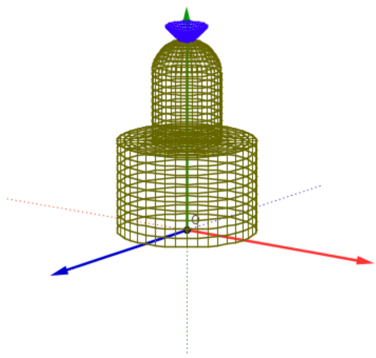


Figure 17: Truncated cone at the top of the tower.

4. FINAL REMARKS.

In this workshop, it has been shown that it is possible to model architectonical surfaces in GeoGebra by using sliders related to the main elements of the corresponding construction. An advantage with respect to CAD's is that it is possible to use the exact parametric equations of the surfaces. However, a disadvantage that we have observed is the need of a powerful computational engine in order to not slow down the work. Although the use of polylines improves the speed, we hope next versions of GeoGebra will develop this aspect.

The implementation of tools related with Differential Geometry and an exhaustive study of different orthogonal projections which can be applied in Descriptive Geometry are two possible future works which can be interesting to deal with.

REFERENCES.

[1] http://gtulloue.free.fr/Cabri3D/euler/euler.html.
[2] http://www.cabri.com.
[3] http://www.geogebra.org.
[4] http://www.iespravia.com/rafa/3d_plantilla/3d.htm.
[5] http://www.iespravia.com/rafa/superficies/index.htm.
[6] http://personal.us.es/raufalgan/geogebra_archivos/Tools.zip