Transforming 3D cartesian into geodetic coordinates

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Abstract

Transformation between geodetic coordinates (ϕ,λ,h) (geodetic lattitude ϕ, geodetic longitude λ and ellipsoidal/geodetic height h) and cartesian coordinates has been studied over the years, being a subject of interest in many fields as Geodesy, Aerospace Engineering, Building Engineering, Architecture etc. Most recent works related to this topic present iterative methods for the transformation on a biaxial or triaxial ellipsoid. Our ongoing approach consists in developping a method which directly perform the transformation from 3D cartesian coordinates into geodetic coordinates by using Computer Algebra techniques (mainly Gröbner bases). The celestial bodies considered in our numerical tests are Moon, Io, Europa, Mimas and Enceladus. The method we are developping is going to be implemented in the Computer Algebra System Maple, together with the iterative methods above mentioned, in order to adequately compare their efficiency.

1. Main ideas

Transformation between geodetic coordinates (ϕ,λ,h) (geodetic lattitude ϕ, geodetic longitude λ and ellipsoidal/geodetic height h) and cartesian coordinates has been studied over the years, being a subject of interest in many fields as Geodesy, Aerospace Engineering, Building Engineering, Architecture etc.The most recent works related to this topic are presented in [1] and [2], where iteration processes for the transformation on a biaxial or triaxial ellipsoid are considered.

Given a reference triaxial ellipsoid, of semiaxes $a_x \geq a_y \geq a_z$,

– the geodetic latitude ϕ is defined as the angle between the ellipsoidal normal vector and the $X-Y$ plane,

– the geodetic longitude λ is the angle, measured in the $X-Y$ plane, between a line parallel to the X axis and the projection of the ellipsoidal normal vector onto the $X-Y$ plane, and

–the geodetic height h is measured along the ellipsoidal normal.

Given a point P_E on the triaxial ellipsoid, its cartesian coordinates (X,Y,Z) verifies the ellipsoid equation,

$$\frac{X^2}{a_x^2}+\frac{Y^2}{a_y^2}+\frac{Z^2}{a_z^2}=1$$

and its geodetic and Cartesian coordinates are related as follows (as in [2]):

$$X=\nu\cos\phi\cos\lambda$$

$$Y=\nu(1-e_e^2)\cos\phi\sin\lambda$$

$$Z=\nu(1-e_x^2)\sin\phi$$

where ν is equal to the radius of the prime vertical,

$$\nu=\frac{a_x}{\sqrt{1-e_x^2\sin^2\phi-e_e^2\cos^2\phi\sin^2\lambda}},$$

and the first eccentricities squared are

$$e_x^2=\frac{a_x^2-a_z^2}{a_x^2},\ e_y^2=\frac{a_y^2-a_z^2}{a_y^2},\ e_e^2=\frac{a_x^2-a_y^2}{a_x^2}$$

Obviously, if ϕ and λ are given, one has (X,Y,Z). Viceversa, if the coordinates (X,Y,Z) are given, then

$$\lambda=\arctan\left(\frac{1}{(1-e_e^2)}\frac{Y}{X}\right),\ \phi=\arctan\left(\frac{(1-e_e^2)}{(1-e_x^2)}\frac{Z}{\sqrt{(1-e_e^2)^2X^2+Y^2}}\right) \tag{1}$$

However, suppose now that we have a point P_W outside the triaxial ellipsoid. In this case, introducing the ellipsoidal height $h>0$, the formulas relating Cartesian and geodetic coordinates are:

$$X_G = (\nu+h)\cos\phi\cos\lambda \tag{2}$$

$$Y_G = (\nu(1-e_e^2)+h)\cos\phi\sin\lambda \tag{3}$$

$$Z_G = (\nu(1-e_x^2)+h)\sin\phi \tag{4}$$

In this case, obtaining the geodetic coordinates (ϕ,λ,h) from the cartesian ones involves firstly computing the intersection point of the ellipsoidal normal vector passing through P_W and the ellipsoid. This point will be named P_E with cartesian coordinates (X_E,Y_E,Z_E). Secondly, we apply formulas (1) with P_E in order to obtain the geodetic coordinates. Iterative methods for this transformation are given in [1,2].

Our ongoing work consists in developping a method which directly determines the point P_E by using Computer Algebra techniques (mainly Gröbner bases).

The shape parameters of the celestial bodies considered in our numerical tests are defined in kms as follows (see [3]):

- Moon: $a_x=1735.55$, $a_y=1735.324$, $a_z=1734.898$
- Io: $a_x=1829.4$, $a_y=1819.3$, $a_z=1815.7$
- Europa: $a_x=1564.13$, $a_y=1561.23$, $a_z=1560.93$
- Mimas: $a_x=207.4$, $a_y=196.8$, $a_z=190.6$
- Enceladus: $a_x=256.6$, $a_y=251.4$, $a_z=248.3$

Following [2], for a given altitude h, we run a loop over latitudes $0^o-89.75^o$ and longitudes 0^o-90^o with 0.25^o increment and compute Cartesian coordinates (X_G,Y_G,Z_G) from (2), (3) and (4) (129,960 triplets of coordinates for every single altitude h). After this step, the Cartesian coordinates of the points corresponding to geodetic altitude h and set of latitudes φ and longitudes λ are obtained. For every satellite, the different altitude considered are 0,$a_z/50$,$a_z/25$,$a_z/20$, $a_z/15$ and $a_z/10$.

2. Bibliography

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