A SEMICLASSICAL ANALYSIS OF THE ^{6}He + ^{208}Pb ELASTIC SCATTERING*

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(Received December 13, 2005)

Elastic cross sections for the scattering of ^{6}He projectiles by ^{208}Pb at energies around the Coulomb barrier measured at the Cyclotron Research Center of Louvain la Neuve (Belgium) have been analyzed using a simple analytic expression for the elastic cross section obtained in a semiclassical model. The results are consistent with recent Optical Model calculations.

PACS numbers: 01.30.Cc, 13.85.Dz, 24.10.Ht, 25.10.+s

1. Introduction

In the last decade there has been an important development of radioactive beam facilities, which has motivated an intense theoretical and experimental research activity to study the scattering of exotic nuclei. A significant part of this effort has been dedicated to low energy reactions induced by halo nuclei. Is is found that the cross sections distributions of the elastic scattering at energies around the Coulomb barrier show a strong reduction

* Presented at the XXIX Mazurian Lakes Conference on Physics
August 30–September 6, 2005, Piaski, Poland.

extending up to very small scattering angles [1–5]. This is a clear signature of long range reaction mechanisms that should be investigated in more detail. In this work we present a simple semiclassical model to describe strong absorption in elastic scattering at Coulomb barrier energies. We derive an analytic expression for the elastic cross sections assuming that the ions follow Coulomb trajectories. The final result depends on the survival probability, which becomes energy dependent due to the kinematics of the absorption process. The model is applied to describe the elastic scattering of ^{6}He+^{208}Pb recently measured at the Cyclotron Research Center at Louvain la Neuve (Belgium). We also compare our results with optical model (OM) calculations [5] for the same scattering system.

2. Semiclassical description

The starting point of our description is the semiclassical expression obtained in [6]. In this approach the elastic cross section $(\frac{d\sigma}{d\Omega})_{\rm el}$ can be written as a product of a survival probability $P_{\rm el}$ and the corresponding classical elastic cross sections $(\frac{d\sigma}{d\Omega})_{\rm class}$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm el} = P_{\rm el}\left(\frac{d\sigma}{d\Omega}\right)_{\rm class} . \tag{2.1}$$

In this expression the survival probability is given by

$$P(\xi)_{\rm el} = \exp\left[-\frac{2}{\hbar}\int\limits_{-\infty}^{\infty} W(r(t))dt\right] , \tag{2.2}$$

where $W(r)$ is the imaginary part of the optical potential and ξ stands for the parameters needed to determine the classical trajectory along which the integral must be evaluated. Collisions between heavy ions at Coulomb barrier energies are governed by the Coulomb potential. Therefore, the ions would tend to follow Coulomb trajectories and we can use the Rutherford cross section in equation (2.1) above. In this case the validity of the model can be determined by the value of the Sommerfeld parameter [7]

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} \gg 1 \tag{2.3}$$

with Z_1, Z_2 being the charge of the colliding nuclei and v the relative velocity. For collision energies between E = 14 to 27 MeV one obtains parameter values from $\sim$12 to $\sim$17. Using equations (2.1) and (2.2) we can write the quotient between elastic and Rutherford cross sections as

$$\frac{d\sigma_{\rm el}}{d\sigma_{\rm R}} = P_{\rm el} = \exp\left[-\frac{2}{\hbar}\int_{-\infty}^{\infty} W(r(t))dt\right], \tag{2.4}$$

where the integral must be evaluated along a classical Coulomb trajectory. A convenient method based in the use of hyperbolic coordinates is described in [7]. In the present work we have used as form factor for the imaginary potential $W(r)$

$$W(r) = -W_0 \exp -\left(\frac{r-R}{a}\right), \tag{2.5}$$

where R is the potential radius an a the corresponding diffuseness. This form factor has the advantage that it is possible to derive an analytic expression for the integral (2.4) above. On the other hand, for large values of the relative coordinate r, the surface potential becomes very similar to the Woods–Saxon potential, which is commonly used in optical model calculations. This is also a reasonable approximation at this low collision energies, where the scattering should be dominated by peripheral reactions taking place at large distances. Inserting (2.5) into (2.4) we get

$$P_{\rm el} = \exp\left[W_0 \exp\left(\frac{R_w}{a_w}\right)\left(-\frac{2}{\hbar}\right)\int_{-\infty}^{\infty} \exp\left(-\frac{r(t)}{a_w}\right) {\rm d}t\right]. \tag{2.6}$$

We consider the following transformation

$$\begin{aligned} r &= a_0\,(\epsilon\cosh(u) + 1)\,, \\ t &= \frac{a_0}{v}(\epsilon\sinh(u) + u)\,, \end{aligned} \tag{2.7}$$

where the parameter u varies in the range $(-\infty\ ,\ +\infty)$, with $u = 0$ at the point of closest approach. From this equations we get

$${\rm d}t = \frac{{\rm d}t}{{\rm d}u}\,{\rm d}u = \frac{a_0}{v}(\epsilon\cosh(u) + 1)\,{\rm d}u\,. \tag{2.8}$$

Then, Eq. (2.6) results as follows

$$\begin{aligned} P_{\rm el} &= \exp\Bigg[W_0 \exp\left(\frac{R_w}{a_w}\right) \\ &\times \left(-\frac{2}{\hbar}\right)\int_{-\infty}^{\infty} \exp\left(-\frac{a_0}{a_w}\,(\epsilon\cosh(u) + 1)\right)\frac{a_0}{v}(\epsilon\cosh(u) + 1)\,{\rm d}u\Bigg]. \end{aligned} \tag{2.9}$$

The integrand in this equation is even, so it is possible to rewrite it

$$\begin{aligned} P_{\mathrm{el}} &= \exp\left[W_0 \frac{a_0}{v} \exp\left(\frac{R_w - a_0}{a_w}\right)\left(-\frac{4}{\hbar}\right)\right. \\ &\quad \times \left[\int_0^{+\infty} \exp\left(-\frac{a_0}{a_w}\epsilon\cosh(u)\right) \mathrm{d}u\right. \\ &\quad \left.\left. + \int_0^{+\infty} \exp\left(-\frac{a_0}{a_w}\,\epsilon\cosh(u)\right)(\epsilon\cosh(u))\mathrm{d}u\right]\right]. \end{aligned} \qquad (2.10)$$

Taking into account (2.7), we finally obtain

$$\log\left(\frac{d\sigma_{\mathrm{el}}}{d\sigma_{\mathrm{R}}}\right) = -4W_0 \frac{a_0}{\hbar v} \exp\left(\frac{R - a_0}{a}\right)\left[K_0\left(\frac{a_0}{a}\epsilon\right) + \epsilon K_1\left(\frac{a_0}{a}\epsilon\right)\right], \qquad (2.11)$$

where $a_0 = Z_1 Z_2 e^2/2E$ is half the distance of closest approach in a head-on collision and $\epsilon = 1/\sin(\theta/2)$ is the excentricity parameter for a Coulomb orbit of asymptotic scattering angle θ. In this expression $K_0(z), K_1(z)$ are the modified Bessel functions

$$K_\nu(z) = \int_0^{\infty} e^{-z\cosh(t)} \cosh(\nu t) dt\,. \qquad (2.12)$$

This result can be further simplified using an expansion of the Bessel functions valid for large values of the argument $z \gg 1$ [8]

$$K_\nu(z) \approx \sqrt{\frac{\pi}{2z}} e^{-z}\left(1 + \frac{4\nu^2 - 1}{8z} + \ldots\right). \qquad (2.13)$$

It can also be shown that by retaining only the first term the result coincides with the one obtained with the usual parabolic approximation to the Coulomb trajectories around the distance of closest approach $r_{\mathrm{ca}} = a_0(1+\epsilon)$.

3. Analysis and results

Throughout the present work the semiclassical calculations have been performed with the code MATHEMATICA [9]. We have used the values $R = 7.86$ fm and $a = 1.7$ fm obtained in [5] from the analysis of ^{6}He elastic scattering by means of OM calculations. In this way the only free parameter is the depth W_0 of the imaginary potential. This parameter has been varied in order to optimize the fits to the angular distributions of the elastic scattering measured at E = 14, 16, 18 and 22 MeV as reported in [5]. We also include in our analysis the data obtained in [3]. The best fit parameters determined at each energy along with the results of [5] are listed in Table I. In Fig. 1 it is depicted the variation of the ratio elastic/Rutheford cross sections with the distance of closest approach $r_{\rm ca}$. The solid lines represent the fit using the semiclassical model and the dashed lines the results of the corresponding OM fit.

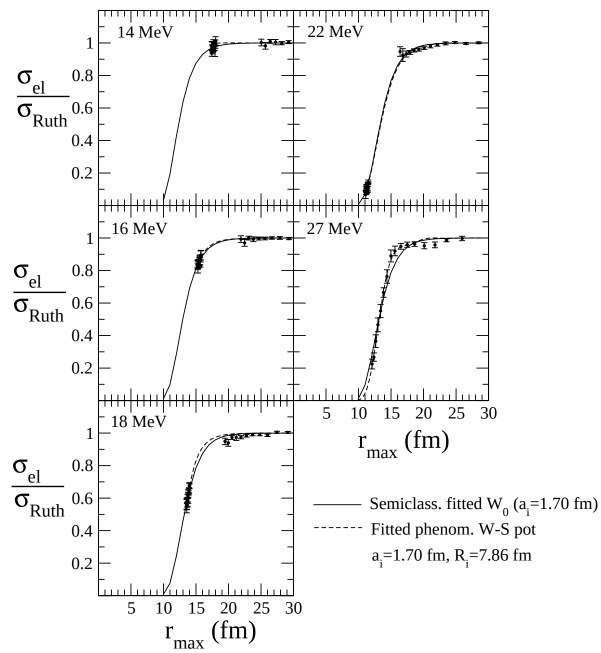

Fig. 1. The ratio of cross sections elastic/Rutheford *versus* the distance of closest approach $r_{\rm ca}$ (fm). See text for details.

We observe a good agreement between the semiclassical and OM calculations. As the semiclassical model has no real nuclear potential, this feature suggests that the dispersion process is very much dominated by the imaginary part of the potential. These results are also in agreement with those in Refs. [1,2] for the scattering of ^{6}He by ^{209}Bi. The χ^2 values obtained in the semiclassical analysis are slightly higher than in the optical model calculations, in particular at $E = 18$ MeV, which is very close to the Coulomb barrier of this scattering system. Heavy ion collisions at barrier energies show usually strong interference effects between the Coulomb and the real part of the nuclear potential. This term is not included in our model, so interference effects are not reproduced.

TABLE I

Values of the depth of the imaginary potential W_0 and associated χ^2 (N_F is the number of degrees of freedom) for present semiclassical calculations. The results using Optical Model calculations are taken from [5]. See text for more details.

	Semiclassical model		Optical model		
E(MeV)	W_0(MeV)	χ^2	W_0(MeV)	χ^2	N_F
27	9.8(5)	64	8(5)	32	31
22	9.31(12)	32	9.8(8)	32	30
18	7.19(14)	52	5(1)	46	31
16	5.54(16)	17	5(1)	17	31
14	3.1(5)	20	0(3)	41	31

4. Summary and conclusions

We present a simple semiclassical model to describe the elastic scattering of heavy ions at low collision energies. We obtain an analytic expression for the elastic cross sections using Coulomb trajectories and a surface form factor for the imaginary part of the nuclear potential. We have used the model to describe recent data on the elastic scattering of ^{6}He by ^{208}Pb. The calculations reproduce the data properly and agree with the results of OM calculations using the same geometry. This simple model can be a useful tool to describe.

This work has been partially financed by the projects FPA2003-05958 and FPA-2002-04181-C04-02 of MCyT (Spain).

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