An Instanton Picture O.P.E. $\langle A^2 \rangle$ Condensate?

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Gluon two- and three-point Green Functions computed in Landau gauge from the lattice show the existence of power corrections to the purely perturbative expressions, that can be explained through an Operator Product Expansion as the influence of a non gauge invariant mass dimension two condensate. The relationship of this condensate with topological properties of QCD, namely instantons, will be studied, giving a first estimate of the contribution of instantons to this condensate based in the direct lattice measure, after a cooling process, of the instanton liquid properties.

1. QCD coupling constant, O.P.E. and $\langle A^2 \rangle$ condensate.

Lattice calculations of the QCD coupling constant and gluon propagator based in the Green Functions Method [1], suggest the necessity to add power corrections to the purely perturbative expressions to correctly describe their running [3]. An Operator Product Expansion (O.P.E.) analysis of the Green functions in Landau gauge[1] relates this power corrections to the existence of a non-perturbative $\langle A^2 \rangle$ condensate [4], through expressions:

$$G^{(2)}_{O.P.E.}(p^2) = G^{(2)}_{Pert.}(p^2) + c\frac{\langle A^2 \rangle_{R,\mu}}{p^2},$$
$$\alpha_s^{O.P.E}(p^2) = \alpha_s^{Pert.}(p^2) + c'\frac{\langle A^2 \rangle_{R,\mu}}{p^2}, \tag{1}$$

[1]In the lattice we will work in the minimum A^2 Landau gauge, $\partial_\mu A_\mu = 0$, so all gauge dependent quantities, will be expressed in this particular gauge.

where perturbative expressions are developed at three loops, and the functions c and c' include the Wilson coefficient of the expansion, and the anomalous dimension of the condensate at leading logarithm.

By performing a combined fit of lattice results to expressions in (1), in two different MOM schemes, a value of $\Lambda_{\overline{\rm MS}}$ is extracted, in fairly good agreement with the one obtained by the ALPHA collaboration [2], by a completely different method. A value of the condensate comes out from the analysis. The physical meaning of this condensate is still an open question, and a lot of work is being devoted to its study during last years, for example, in relation to confinement [5]. The aim in this work will be to study the possible semiclassical contribution to this condensate coming from instantons, and whether they might explain the presence of power corrections in Green Functions.

2. The role of instantons.

Instantons have been extensively studied as a possible description of the QCD vacuum (See [7] for a general overview), and so as a major source of QCD properties at low energies. In relation with the aim of this work, an ensemble of non-interacting instantons (I) and antiinstantons ($\bar{\mathrm{I}}$) in Landau gauge would give a contribution to the $\langle A^2 \rangle$ condensate;

$$\langle A^2 \rangle_{\text{inst}} \approx \frac{N}{V} \int d^4x A^a_\mu(x) A^a_\mu(x) = 12\pi^2\rho^2 n, \quad (2)$$

where $A^a_\mu(x)$ is the standard 't Hooft Polyakov instanton gauge field [6], ρ the average radius, and $n = \frac{N_I + N_{\bar{I}}}{V}$ the density.

If we accept the phenomenological values assigned to n and ρ by the Instanton Liquid Model [7] ($n \sim 0.5 fm^{-4}$ and $\rho \sim 1/3 fm$), the instantonic contribution will be $\langle A^2 \rangle_{Inst.} \sim 0.5\mathrm{GeV}^2$. We will perform, however, our own analysis, thus testing the latter approach.

2.1. Cooling.

In principle, a direct measure of A^2 in the lattice should be possible, but the presence of the UV divergent part is hardly separable from the soft, instantonic one. The other possibility is to perform a cooling procedure, that will allow us to compute the number and size of instantons, giving an indirect measure of the A^2 through (2).

We will use the traditional cooling method [8], even if it introduces a number of known biases, as $\mathrm{I}-\bar{\mathrm{I}}$ annihilation, and a modification of instanton sizes and lattice spacing. The approach proposed here is to compute instanton properties for different number of cooling sweeps, and extrapolate back to the thermalised situation in order to recover their physical meaning[2].

2.2. Shape Recognition.

Instantons will be localised in cooled lattices via a geometrical method (Described in [10].) that accepts a topological charge lump as an instanton when the ratio of the integral over a given fraction of the topological charge at the maximum, α, and its theoretical counterpart, ϵ, is ~ 1, for a range of values of α:

$$\epsilon = \frac{\int_{x/\frac{|Q_\rho(x)|}{|Q_\rho(0)|} \geq \alpha} d^4x Q_\rho(x)}{1 - 3\alpha^{1/2} + 2\alpha^{3/4}} \quad (3)$$

Once the lump has been identified as an instanton, the radius will be computed from the size of the cluster where the integral has been developed.

2.3. A naive model of annihilation.

With the method outlined above, we compute the density and size of instantons in a lattice, for different numbers of cooling sweeps, n_c, obtaining values with a strong dependence on n_c (See figure), that avoids to obtain any physical information at fixed n_c.

As a first approach to the understanding of this evolution, we will make a simple model, where instantons annihilate with antiinstantons (Being so $\Delta N = N_I - N_{\bar{I}}$ a constant) proportionally to their packing ratio, and to the number of antiinstantons, so that the equation for the evolution of $N = N_I + N_{\bar{I}}$ is:

$$\frac{\partial N}{\partial n_c} = -\frac{\lambda}{2V}\rho^4(n_c)(N(n_c)^2 - \Delta N^2). \quad (4)$$

If we assume $\rho(n_c) = cte$, the solution of Eq. (4) will give $N(n_c) \sim \frac{N(0)}{1+\lambda n_c}$, the expression used in [10], as a first order approach, but our cooling procedure modifies instanton's size (See figure), in a way thah we phenomenologically parametrise as:

$$\rho(n_c) = \rho(0)(1 + a \ln(1 + n_c)) . \quad (5)$$

We will include (5) in equation (4), with $\rho(0)$ the extrapolated radius at the thermalised situation and a a constant to determine.

After performing a combined fit of our lattice results to the expressions (5) and the one coming from the integration of (4), we can fix the initial values of the density, $n(0)$ and the radius $\rho(0)$,

[2] The use of improved cooling methods, as the one developed in [9], could improve this approach, as radii evolution is minimised, but $\mathrm{I}-\bar{\mathrm{I}}$ annihilation is unavoidable, so the extrapolation will be anyway necessary.

and the two constants that govern the evolution, λ and a.

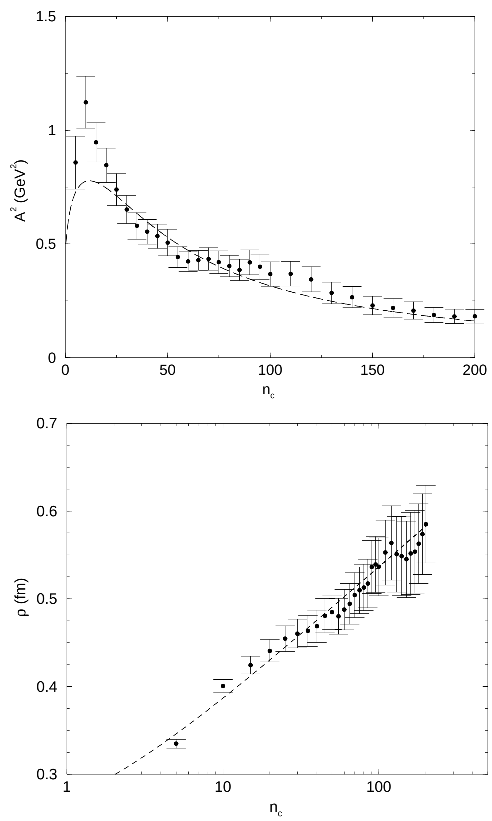

Figure 1. *Results of the combined fit for the instanton density and radius as a function of the number of cooling sweeps for a* 24^4 *lattice at* β*=6.0.*

3. Results and conclussion.

The result of the combined fit gives a value of the instantonic contribution to $\langle A^2_{Ins} \rangle \sim 0.4\text{GeV}^2$, however the result of the extrapolation is highly dependent on the value of ρ, which due to the logarithmic behaviour is hardly reliable. We therefore prefer the value at the maximum, 1.12(11) GeV^2, as a crude estimation of $\langle A^2_{Ins} \rangle$.

This semiclassical evaluation of $\langle A^2 \rangle$, which does not run with the scale, is difficult to relate to that appearing in the O.P.E. expansion, which does depend on the renormalisation scheme and scale. The typical scale of instantons is $\rho^{-1} \sim 0.7\text{GeV}$. Unluckly it is not possible to run the $\langle A^2_{O.P.E.} \rangle$ to such a low energy, where pertubative QCD is not valid. The lowest reacheble energy scale is 2.6GeV [4,10];

$$\langle A^2_{O.P.E.}(2.6\text{GeV}) \rangle = 1.4(3)(3)\text{GeV}^2, \qquad (6)$$

the first error coming from the OPE determination of the condensate renormalised at 10 GeV, and the second from higher orders in the running.

Keeping in mind the level of uncertainty of these calculations, we can nevertheless claim a rather encouraging agreement between the instantonic contribution to the condensate and the one computed from the running of the Green Functions.

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