THE ROLE OF RAINBOW REFRACTION IN HEAVY ION ELASTIC SCATTERING

G. MADURGA, M. LOZANO and A. JADRAQUE

Departamento de Física Atómica y Nuclear, Universidad de Sevilla, Sevilla, Spain

The energy dependence of both the strong absorption radius, R_{sa}, and the rainbow distance, R_r, in the elastic scattering of a dozen pairs of heavy ions is studied. R_{sa} decreases with energy linearly in $E^{-1/3}$ while R_r increases linearly in $E^{-3/2}$. The rainbow effect appears to prevail ($R_r > R_{sa}$) above a certain energy which is proportional to the product $Z_1 Z_2$.

The cross section for heavy ion elastic scattering, as a function of the scattering angle, very frequently shows a series of oscillations about the Rutherford value with increasing amplitude up to a maximum followed by a quasiexponential fall. Such a characteristic pattern can be described either as Fresnel diffraction [1] or as a Coulomb rainbow [2].

In order to elucidate whether absorptive diffraction or rainbow refraction is the physical mechanism responsible for that behaviour we compare in each case the strong absorption radius, R_{sa}, with the rainbow distance, R_r. R_{sa} is defined here as the distance of closest approach of the Rutherford trajectory with 50% absorption:

$$R_{sa} = \frac{\eta}{k} \left\{ 1 + \left[1 + \left(\frac{l_{sa} + 1/2}{\eta} \right)^2 \right]^{1/2} \right\},$$

with $k = \sqrt{2mE}/\hbar$, $\eta = Z_1 Z_2 e^2 / \hbar v$. And l_{sa}, such that $T_{l_{sa}} = 1 - |S_{l_{sa}}|^2 = 0.5$, is interpolated after an optical model calculation has yielded the values of the scattering matrix S_l for integer l's. R_{sa} depends on both the real and imaginary parts of the optical potential. Similarly R_r is the distance of closest approach of the Rutherford orbit corresponding to the highest momentum l_r for which the classical deflection function $\Theta(l)$ has a maximum. R_r is determined by only the real part of the potential.

If $R_{sa} > R_r$, the rainbow trajectory is strongly damped and the sharp boundary of the absorptive region can produce diffraction. If, on the contrary, $R_r > R_{sa}$, then the rainbow effect takes place before a significant absorption operates.

Vaz et al. [3] find with a given optical potential for the pair $^{16}O + ^{208}Pb$ that R_{sa} decreases and R_r increases with increasing energy, so that they cross at a certain energy: near 90 MeV projectile energy if an optimum imaginary radius, $r_I = 1.32$, is supposed; or about 170 MeV if a possible $r_I = 1.42$ is taken.

We have undertaken a systematic search of the energy dependence of the strong absorption radius, R_{sa}, and the rainbow distance, R_r, for as many pairs as possible. Unfortunately there are not many pairs for which good experimental data over a sufficiently wide range of energies is available — at least not until very recently. Moreover the well known ambiguity of the optical potential also implies a certain degree of indeterminacy in the values of R_{sa} and R_r. For example, different Woods—Saxon (WS) potentials which fit the experimental data on elastic scattering $^{16}O + ^{208}Pb$ at 192 MeV to a minimum χ^2 within 10% give values of R_{sa} which differ by ± 0.1 fm [4]. Also the value of R_r seems to be more sensitive to changes in the potential radius R than to modifications in the central depth V_0; but Igo noted long ago [5] that V_0 and R can be simultaneously changed keeping the quantity $V_0 e^{R/a}$ constant. So the systematics of the E dependence of R_{sa} and R_r can be obscured by the aforementioned ambiguities unless the fits to data at different energies are done with "homogeneous" criteria.

Nevertheless we think that our preliminary results support meaningful conclusions. Figs. 1 and 2 show some of them: black symbols refer to R_{sa} and open ones to R_r. The fits to $^{13}C + ^{40}Ca$ are taken from
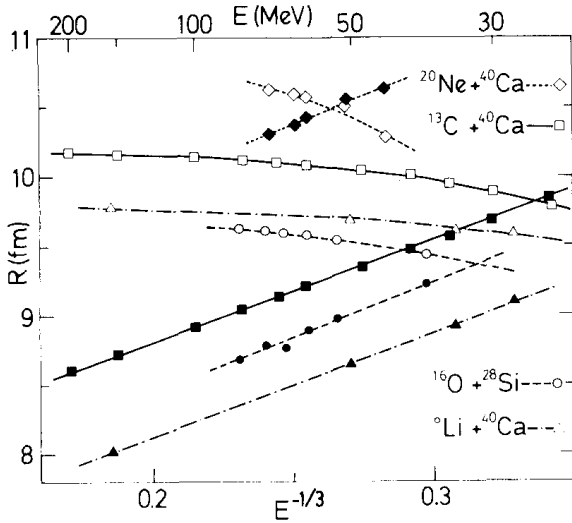
Fig. 1. Strong absorption radius (black symbols) and rainbow distance (open symbols) as a function of the projectile energy for different pairs of heavy ions. The curves are least square fits described in the text.

Bond et al. [6]. They have fitted the same WS potential to data at 40, 48, 60 and 68 MeV. In this case we have computed R_{sa} and R_r with their potential at 15 energies between 25 and 500 MeV (only up to 200 MeV are shown in the figure): R_{sa} turns out to be lin-
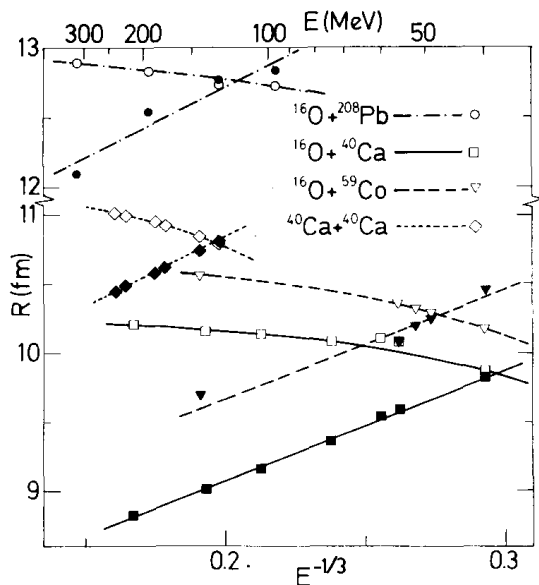

Fig. 2. Strong absorption radius (black symbols) and rainbow distance (open symbols) as a function of the projectile energy for different pairs of heavy ions. The curves are least square fits described in the text.

ear in $E^{-1/3}$ with a correlation coefficient $r^2 = 0.9995$ and R_r linear in $E^{-3/2}$ with $r^2 = 0.9973$. (This behaviour remains to be interpreted, and of course one and the same potential is not expected to fit data at very different energies.)

In view of this fact we have plotted the experimental values of R_{sa} and R_r versus $E^{-1/3}$ and drawn the curves $R_{sa} = a + bE^{-1/3}$ and $R_r = c - dE^{-3/2}$, fitted by the least squares method to the points for each pair. The correlation coefficients are almost always very close to 1 at least when the fits at different energies are with the same potential or with a smooth energy dependence.

The WS fits for ^{20}Ne + ^{40}Ca are from Van Sen et al. [7] with energy dependent geometry. The other cases (^{6}Li + ^{40}Ca; ^{16}O + ^{28}Si, ^{40}Ca, ^{59}Co, and ^{208}Pb; and ^{40}Ca + ^{40}Ca) are taken from a paper by Satchler and Love [8] on the folding potentials. Except that we have approximated their real folding potential by the analytical expression.

$$V(r) = -V_0 s^n e^{-s/a}, \quad \text{if } s \geqslant na,$$

$$V(r) = -V_0 (na)^n e^{-n}, \quad \text{if } s \leqslant na$$

with

$$s = r - C_1 - C_2, \quad C_i = R_i - R_i^{-1},$$

$$R_i = (1.13 + 0.0002 A_i) A_i^{1/3}.$$

The validity of this approximation in the relevant region ($s > 1$) is pointed out in ref. [8] and described in detail elsewhere [9]. Data for ^{16}O + ^{208}Pb from Videbaek et al. [10] have also been used.

Of similar appearance, though not shown in the figures, are the $R_{sa}(E)$ and $R_r(E)$ values deduced from WS energy-dependent fits recently published by Fortune et al. [11] for the pairs ^{16}O + ^{26}Mg and ^{16}O + ^{30,28}Si at four energies from 35 to 50 MeV, and three sets of WS fits to ^{9}Be + ^{28}Si at seven energies from 12 to 30 MeV published by Bodek et al. [12]..

In all the cases the crossing point of the curves $R_{sa}(E)$ and $R_r(E)$ corresponding to the same pair can be interpolated or extrapolated from the experimental points. We have obtained it by equating the fits $a + bE^{-1/3} = c - dE^{-3/2}$.

This crossing energy, as measured in the center of mass system, E^x_{cm}, has been plotted versus the product $Z_1 Z_2$ in fig. 3. In order to have a more convenient spac
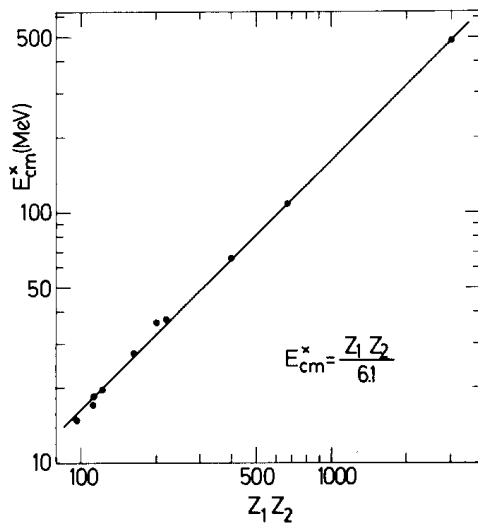
Fig. 3. The crossing energy, where $R_{sa} = R_r$, measured in the c.m. system, as a function of $Z_1 Z_2$.

ing of the points a log–log scale has been chosen. The highest point comes from the pair ^{84}Kr + ^{209}Bi. We have fits at two energies only [13], but they fall at either side of the crossing point and permit a good determination of it.

The values plotted in fig. 3 follow a proportionality law

$$E^x_{cm} = 0.165 \, Z_1 Z_2, \tag{1}$$

with a rms deviation of 5%.

In contrast the two reactions with "light" heavy projectiles, not included in fig. 3, deviate from this law

by 80% (^{9}Be + ^{28}Si) and 70% (^{6}Li + ^{40}Ca). If our sample is meaningful this would be another instance where Li and Be do not fully behave like heavy ions.

In summary, our work suggests that rainbow scattering dominates over absorptive diffraction at sufficiently high energy and the limit energy is proportional to the Coulomb repulsion according to eq. (1). Also a linear dependence of R_{sa} on $E^{-1/3}$ and of R_r on $E^{-3/2}$ is found with the available data.

References

[1] W.E. Frahn, Ann. Phys. 72 (1972) 524.
[2] R. Da Silveira, Phys. Lett. 45B (1973) 211.
[3] L.C. Vaz, J.M. Alexander and E.H. Auerbach, Phys. Rev. C18 (1978) 820.
[4] M. Lozano and G. Madurga, Nucl. Phys. A334 (1980) 349.
[5] G. Igo, Phys. Rev. 117 (1959) 1665.
[6] P.D. Bond et al., Phys. Lett. 47B (1973) 231.
[7] N. Van Sen, R. Darves Blanc, J.C. Gondrand and F. Merchez, Phys. Rev. C20 (1979) 969.
[8] G.R. Satchler and W.G. Love, Phys. Rep. C55 (1979) 183.
[9] F.J. Viñas, G. Madurga and M. Lozano, to be published.
[10] F. Videbaek et al., Phys. Rev. C15 (1977) 954.
[11] H.T. Fortune, A. Richter, R.H. Siemssen and J.L. Yntema, Phys. Rev. C20 (1979) 648.
[12] K. Bodek et al., Nucl. Phys. A339 (1980) 353.
[13] J.R. Birkelund et al., Phys. Rev. C13 (1976) 133.