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Do the profile function singularities explain the high energy reflection of fermions in a phase transition?

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Abstract

We investigate the scope of a previous result concerning the behaviour of fermions hitting a general wall caused by a first-order phase transition. The wall profile function was considered to be analytic in the real axis. The previous result is valid for analytic functions in the whole complex plane except in certain isolated singularities located out of the real axis. A non-analytic profile function in the real axis is studied in order to show the validity of the result for any profile which can be put as a certain limit of a function which verifies the latter. A new understanding of the high energy behaviour of the quantum reflection caused by a sharp profile, as the step, arises from that study.

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The scattering of high energy fermions by a wall separating two phases of different symmetry properties has received much attention recently. The main physical motivation of these works is the idea [1] that the baryon asymmetry of the Universe might have been produced if the cosmological electroweak phase transition has been of first order. The transition is described in terms of bubbles of a spontaneously broken symmetry vacuum expanding in a preexisting symmetric one. In this scenario the point to elucidate is whether there exists a CP-asymmetry that produces a different reflection and transmission probability for quarks and antiquarks in order to explain, via the standard model baryon number anomaly [2], the correct baryon asymmetry of the Universe. In order to simplify the treatment, an useful assumption is to break down the process into two steps, one describing the production of CP asymmetry when the quarks/antiquarks are reflected on the wall, the second describing the transport and the eventual transformation of the CP asymmetry into a baryon asymmetry. The first of two steps justifies the effort concentrated in the study of the scattering of fermions in the presence of first order phase transition [3–5]. The structure of the wall depends on the Higgs field effective potential that takes into account the effects of the surrounding plasma trough the temperature of a certain thermal bath. The wall profile obtained by solving the equation of motion with this effective potential is rather complex and depends on too many coupling constants [6], thus the study of the general wall profile problem [4,5] is justified not only from a purely formal interest. We have shown in a previous work [5] the connection between the

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complex plane poles of the wall profile function and the behaviour of fermions hitting the wall in the high energy limit. Nevertheless, by performing an extreme simplification which allows to compute the Feynman fermion propagator in an exact way [7], the wall profile can be approximated by a step function, i.e. a sudden jump from one phase to the other, that is called the thin wall approximation [7,8]. In this case, the reflection coefficient in the high energy limit is[3]

$$R(E) = \frac{m_0}{2E}, \tag{1}$$

being the mass m_0 the high of the wall. Notice the power law dependence on E of the reflection coefficient in Eq. (1). As we will see, the results given in Ref. [5] cannot be directly applied to profile functions which are not analytic in the real axis, as the step profile function. Nevertheless, in this note we show the high energy behaviour of a new kind of profile functions, which can be considered in a certain limit as analytic extensions in the complex plane of the step function, by following [5]. Furthermore, we will investigate the appropriate limit in order to understand the result (1) through the analytic properties of the step function extensions.

Next, we outline the derivation of the high energy asymptotic expression for the reflection coefficient given in Ref. [5], where the detailed calculation can be found.

By formulating the problem in the rest frame of a wall normal to the z-axis, characterized by a general non-CP-violating wall profile[4], and working in the chiral basis [6], we can factor the Dirac equation into 2×2 identical blocks. Thus, the problem is reduced to solve the following equation:

$$(i\partial_z + Q(z))\,\psi_{\mathrm{I/II}} = 0, \tag{2}$$

with

$$Q(z) = \begin{pmatrix} E & -m(z) \\ m(z) & -E \end{pmatrix},$$
$$\Psi = \begin{pmatrix} \psi_{\mathrm{I}} \\ \psi_{\mathrm{II}} \end{pmatrix} e^{-iEt}, \tag{3}$$

where Ψ is the time-independent Dirac equation solution in the chiral basis. We express the solution of (3) as follows:

$$\psi_{\mathrm{I/II}}(z) = \Omega(z, z_0) \begin{pmatrix} \psi_1(z_0) \\ \psi_2(z_0) \end{pmatrix}, \tag{4}$$

with

$$\Omega(z, z_0) = P e^{i\int_{z_0}^{z} d\tau Q(\tau)}, \tag{5}$$

where P indicates a path ordered product and τ is the position variable along the z-axis. We consider $m(\tau) = m_0 f(\tau)$, where $f(\tau)$ is the profile wall function. The asymptotic conditions $f(+\infty) = 1$ and $f(-\infty) = 0$ are required, $f(\tau) - \theta(\tau)$ decreasing exponentially when $\tau \to \pm\infty$. Thus, after some tedious calculation, we obtain from Eqs. (3) and (5)

$$\Omega(z, z_0) = \left(1 + \left(\frac{m_0}{E}\right) \sigma_2 \int_{z_0}^{z} d\tau\, p(\tau)\, f(\tau) \times e^{-2i\sigma_3 \int_\tau^z d\xi p(\xi)} + O\left[\left(\frac{1}{E}\right)^2\right] \right) e^{i\sigma_3 \int_{z_0}^{z} d\tau\, p(\tau)}, \tag{6}$$

where $p(\tau) = +\left(E^2 - [m_0 f(\tau)]^2\right)^{1/2}$.

Following Nelson et al. [6], we can obtain the reflection coefficient from the result (6). Thus, by assuming in general that

$$f(\tau) = F\left(\frac{\tau}{\sigma}\right), \tag{7}$$

where the parameter σ gives the wall thickness, and after much calculation [5], we finally obtain

$$R(E) = 2\pi\sigma m_0 \sum_{j=1}^{N} e^{-2E\sigma y_j}\, e^{2iE\sigma x_j} \times \sum_{n=1}^{\nu_j} b^j_{-n} \frac{(2iE\sigma)^{n-1}}{(n-1)!}, \tag{8}$$

where $z_j = x_j + iy_j$ and b^j_{-n}, for $j = 1, \ldots, N$, are all the poles of $F(z)$ with positive imaginary part and the n-power coefficient of the Laurent expansion for the function in each pole, respectively. The order of the pole z_j is ν_j.

[3] This result can be immediately obtained from the reflection coefficient given in [7], for instance. High energy limit means $E \gg m_0$.

[4] CP-violating wall profiles are studied in [9].

The result (8) is valid in general for profile functions which are analytic in the real axis, i.e. analytic functions in all the complex plane except in certain isolated singular points with non-zero imaginary part. It is positively checked in Ref. [5] by using the particular Kink-type wall profile [3,4], $f(\tau) = \frac{1}{2}\left(1 + \tanh(\tau/\sigma)\right)$. We consider now the more general kind of functions

$$f(\tau) = \frac{1}{1 + \exp\left(-\frac{\tau^{2n+1}}{\sigma}\right)} . \tag{9}$$

It is obvious that the functional behaviour of the Kink-type wall is a particular case of the latter, given by taking $n = 0$. Moreover, any of these functions gives the step function if the limit $\sigma \to 0$ is taken. In order to apply the result (8) to the functions (9), we must know the singularities of these functions and their distribution in the complex plane. Taking into account Eq. (7), we obtain for the poles of $F(z)$

$$z_{jk} = \pi^{\frac{1}{m}} (1 + 2j)^{\frac{1}{m}} e^{\pm i \frac{\pi}{m} (\frac{1}{2} + 2k)}, \tag{10}$$

with $j = 0, 1, 2, \ldots, \infty$ and $k = 0, 1, \ldots, m-1$. Where $m = 2n + 1$ is the power in the exponential argument in (9). The order of all the poles is 1 and the residues, b_{-1}^{jk}, can be written as

$$b_{-1}^{jk} = \frac{1}{m} \left[\pi (1 + 2j)\right]^{\frac{1-m}{m}} e^{\pm i \frac{1-m}{m} \pi (\frac{1}{2} + 2k)} . \tag{11}$$

It is easy to see that these poles are distributed in the complex plane along m radial axes, on circumferences with radius given by $\left[\pi (1 + 2j)\right]^{1/m}$, where $j = 0, 1, \ldots, \infty$. In fact, the distribution of the poles is symmetric with regard to the real axis. The schematic location of the poles in the complex plane for $m = 1, 3$ is shown in Fig. 1. It can be found that the two poles with lower positive imaginary part are

$$z_{0,0} = \pi^{\frac{1}{m}} e^{i \frac{\pi}{2m}}, \quad z_{0,\frac{m-1}{2}} = \pi^{\frac{1}{m}} e^{i\pi(1 - \frac{1}{2m})} , \tag{12}$$

and by applying the result (8) in the range of energies $E\sigma \gg 1$, we obtain

$$\begin{aligned} R(E) &= \frac{2(2 - \delta_{m,1})}{m} \pi^{\frac{1}{m}} \sigma m_0 \\ &\times \cos\left[2\pi^{\frac{1}{m}} \cos\left(\frac{\pi}{2m}\right) \sigma E + \frac{1-m}{m}\frac{\pi}{2}\right] \\ &\times e^{-2\pi^{\frac{1}{m}} \sin(\frac{\pi}{2m}) \sigma E} , \end{aligned} \tag{13}$$

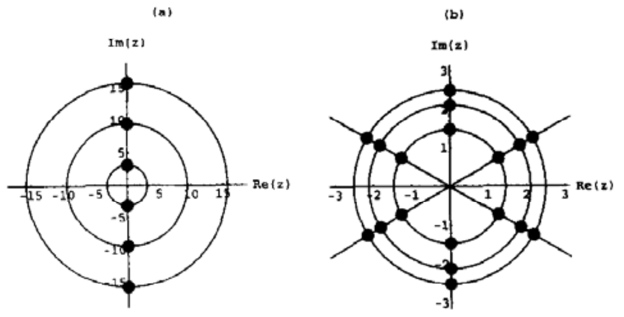

Fig. 1. Schematic distribution in the complex plane of the poles, z_{jk}, given by Eq. (10), for $m = 1$ (a) and for $m = 3$ (b). The poles are located in the solid points, distributed along radial axes, as explained in the text.

where all the decreasing exponential terms except the two with lowest arguments has been neglected. The factor $2 - \delta_{m,1}$ arises because the two poles considered in Eq. (12) are the same for $m = 1$. The exponential law of Eq. (13) is the expected behaviour for the reflection coefficient provided that the result (8) can be applied, if the energy is large enough. Therefore, the reflection coefficient for the step wall profile which presents a power law asymptotic behaviour, as we mentioned above, does not seem to be coherent with the former. Nevertheless, we must take into account that the result (8) relates the high energy behaviour of the fermions hitting a wall to the analytic properties of the profile function in the complex plane. Thus, we must appropriately extend the step profile in the complex plane to investigate if the obtained poles allow to explain the behaviour given by (1).

It can be easily seen that the limit of the functions (9), for all n, when the parameter σ goes to zero is the step function. Therefore, in that limit, any of those can be considered as an analytic extension in the complex plane for the step function. By considering that the function $F(\tau)$ is the same as $f(\tau)$, but scaled by the parameter σ, we have the same pattern for the distribution of poles of $f(\tau)$ than for that of $F(\tau)$, but with the distance between two consecutive poles depending on σ. In other words, the poles will be as close to each other along the same radial axes as σ small. The result (8) cannot be strictly applied for $\sigma = 0$, but we know that if the limit when $\sigma \to 0$ of the high energy reflection coefficient exists, it must be the same obtained by using (8), which is valid for any non-zero σ. The crucial point in this approach is that when

σ is getting smaller and smaller we cannot consider a large enough energy, E, to neglect the contribution other than the closest poles to the real axis. In this way, the contributions due to each pole have to be summed before to take the limit and there is no reason to expect an exponential final result.

Indeed, we have

$$R(E) = 2\pi\sigma m_0 \sum_{k=0}^{m-1} \sum_{j=0}^{N} e^{-2E\sigma y_j}\, e^{2iE\sigma x_j} b_{-1}^{jk}, \qquad (14)$$

where b_{-1}^{jk} is defined in Eq. (11) and y_{jk}, x_{jk} are the real and pure imaginary components of z_{jk}, given by Eq. (10). By taking $\sigma \to 0$ in Eq. (13), the sum for j from 0 to $+\infty$ can be considered as an integral and after some appropriate transformations, we obtain

$$R(E) = \frac{m_0}{2E} \sum_{k=0}^{m-1} e^{i\beta_k} \int_0^{\infty} dt\, e^{-t \sin\alpha_k} e^{it\cos\alpha_k}, \qquad (15)$$

where

$$\beta_k = \pm\frac{\pi}{m}(1-m)\left(\tfrac{1}{2}+2k\right),$$
$$\alpha_k = \pm\frac{\pi}{m}\left(\tfrac{1}{2}+2k\right). \qquad (16)$$

As we only sum for the poles with positive imaginary part, the signs $+, -$ in Eq. (16) are for $k \le \frac{m-1}{2}$ and $k > \frac{m-1}{2}$, respectively. By solving Eq. (15), we obtain

$$R(E) = \frac{m_0}{2E} \sum_{k=0}^{m-1} e^{i(\beta_k + \frac{\pi}{2} - \alpha_k)}$$
$$= \frac{m_0}{2E}\left\{\frac{m+1}{2} - \frac{m-1}{2}\right\} = \frac{m_0}{2E}, \qquad (17)$$

which agrees with Eq. (1). In consequence, the power law behaviour which is characteristic for the step function, as well as for any non-continuous function, and therefore non-analytic, in the real axis, can be well understood through the complex analytic properties of these functions. The high energy reflection behaviour is caused by the sum of the contributions due to each singularity of the profile function. For a large enough energy the contributions of one or several of these singularities can be considered as leading, provided that the singular points are isolated. We only must take into account that the real non-continuous point is produced when the poles of the function are distributed in an infinitely compact way along lines in the complex plane which contains this point to understand the power law for the step. Thus, the smoother decreasing on the energy of the reflection coefficient for the sharp profiles can be explained because no singularity contribution can be isolated in order to lead the total result.

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