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# Relativistic quantum scattering of high energy fermions in the presence of a phase transition

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## Abstract

We study the high energy behaviour of fermions hitting a general wall caused by a first-order phase transition. The wall profile is introduced through a general analytic function. The reflection coefficient is computed in the high energy limit and expressed in terms of the poles of the wall profile function. It is shown that the leading singularity gives the high energy behaviour.

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Much work has been devoted to the problem of transmission and reflection of relativistic fermions through a wall separating two phases of different symmetry properties. The main effort concentrates in developing the idea [1] that the baryon asymmetry of the Universe might have been produced if the cosmological electroweak ( $SU(2) \times U(1)$ ) phase transition has been of first order. In these works the phase transition is described in terms of bubbles of “true” vacuum with an inner expectation value of the Higgs field  $v \neq 0$ , i.e. a spontaneously broken symmetry phase, appearing and expanding in the preexisting “false” vacuum with  $v = 0$ , i.e. a symmetric phase.

In this scenario the quarks/antiquarks hitting the wall from the unbroken phase are reflected or trans-

mitted. The point to elucidate is whether there exists a CP-asymmetry that produces a different reflection and transmission probability for quarks and antiquarks in order to explain, via the standard model baryon number anomaly [2],

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = N_f \left( \frac{g^2}{32\pi^2} W\tilde{W} - \frac{g'^2}{32\pi^2} Y\tilde{Y} \right). \quad (1)$$

the correct baryon asymmetry of the Universe. In the physical conditions of the early Universe the fermions moving through the bubble wall will interact not only with the wall but also with the particles in the surrounding plasma, thus we have a transport problem. This transport plays an essential in the so-called “charge transport” mechanism [3], in which the action of the baryon anomaly happens at a distance from the bubble wall. This diffusion problem is very complicated and involves solving the Fokker-Planck equation taking into account CP violation and baryon anomaly. A useful simplifying assumption

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is to decompose the process into two steps, one describing the production of the CP asymmetry when the quarks/antiquarks are reflected on the wall, the second describing the transport and the eventual transformation of the CP asymmetry into a baryon asymmetry via the baryon number anomaly (1).

One expects that the diffusion corrections are relatively minor ones to the first step, i.e. to the scattering from the wall, although effects of the surrounding plasma are incorporated by mean of introducing the Higgs field effective potential that takes into account the temperature of a thermal bath. The structure of the wall depends on this effective potential and its knowledge is obviously necessary in order to compute the reflection and transmission coefficients. However, the profile obtained by solving the equation of motion is rather complex and depends on many coupling constants [3]. At this point, two lines for simplifying the problem have been followed. The first is to describe the profile of the bubble wall by an analytical function that simulates the dynamics of the phase transition and then to treat the scattering in an approximate way [1–3]. The second approximates the wall profile by a step function, i.e. a sudden jump from one phase to the other, the thin wall approximation [6,7]. This extreme simplification allows to compute in an exact way the Feynman fermion propagator in the presence of a wall [7].

Our aim in the present paper is to study the general problem of fermions hitting a wall in order to connect the analytic properties of the profile function to the behaviour of the fermions. In general, this question is very difficult, but interesting conclusions about this relation between the profile and the behaviour of fermions in the high energy limit can be obtained. We develop a general method to calculate the reflection and transmission coefficients of high energy fermions hitting a wall, establishing a relationship between the poles of the profile function and these coefficients. The quantum corrections to the expected classical behaviour are obtained. Apart from the clear theoretical and purely formal interest of this result, cosmological implications and possible applications of the formalism to systems of relativistic fermions undergoing a phase transition should be considered. Some implications in CP-violating process are studied in [8]. Two other examples of application fields can be condensed matter under extreme external conditions or certain

stages of the quark-gluon plasma formation process.

As usual in this kind of work, we formulate the problem in the rest frame of a wall, parallel to the  $x - y$  plane and normal to the  $z$ -axis, characterized by a general profile. In order to calculate the reflection coefficient we need only the plane wave solutions of the Dirac equation for particles moving along the  $z$ -axis. In any case, for other incoming directions we can perform an appropriate Lorentz boost parallel to the  $x - y$  plane and reduce the problem to the latter. The phase transition is incorporated into the Dirac equation by including a position dependent mass term which varies inside a certain region designated as the domain wall and takes two different constant values in the two outer sides of the wall. Following Nelson et al. [3], we work in the chiral basis, conveniently reordered to obtain

$$\gamma^5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$$

, and factor the Dirac operator into  $2 \times 2$  blocks. Thus the Dirac equation can be expressed as

$$\begin{pmatrix} i\partial_z + i\partial_t & -m(z) & 0 & 0 \\ m^*(z) & i\partial_z - i\partial_t & 0 & 0 \\ 0 & 0 & i\partial_z + i\partial_t & -m^*(z) \\ 0 & 0 & m(z) & i\partial_z - i\partial_t \end{pmatrix} \Psi = 0. \quad (2)$$

With the following Ansatz for solutions with positive energy  $E$ :

$$\Psi = \begin{pmatrix} \psi_I \\ \psi_{II} \end{pmatrix} e^{-iEt}, \quad \text{with} \quad \begin{pmatrix} \psi_I \\ \psi_{II} \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \quad (3)$$

where  $\psi_1$  and  $\psi_4$  are eigenspinors of the chirality operator,  $\gamma_5$ , for the eigenvalue  $+1$  and  $\psi_2$  and  $\psi_3$  for  $-1$ , we obtain

$$\begin{aligned} (i\partial_z + Q(z)) \psi_I &= 0, \\ (i\partial_z + \bar{Q}(z)) \psi_{II} &= 0, \end{aligned} \quad (4)$$

with

$$\begin{aligned} Q(z) &= \begin{pmatrix} E & -m(z) \\ m(z)^* & -E \end{pmatrix} \quad \text{and} \\ \bar{Q}(z) &= \begin{pmatrix} E & -m(z)^* \\ m(z) & -E \end{pmatrix}; \end{aligned} \quad (5)$$

where the mass function can be considered as complex in order to incorporate CP-violating process in

the formalism. In this work we are not interested in CP-violation but in a general approach of the problem. Thus the mass function will be assumed as real and consequently  $Q(z)$  and  $\bar{Q}(z)$  will be identical matrices. The solution for the first equation of (4) can be written as follows:

$$\psi_I(z) = P e^{i \int_{z_0}^z d\tau Q(\tau)} \begin{pmatrix} \psi_1(z_0) \\ \psi_2(z_0) \end{pmatrix}, \tag{6}$$

and analogously for the second one,

$$\psi_{II}(z) = P e^{i \int_{z_0}^z d\tau \bar{Q}(\tau)} \begin{pmatrix} \psi_3(z_0) \\ \psi_4(z_0) \end{pmatrix}, \tag{7}$$

where  $P$  indicates a path ordered product and  $\tau$  is the position variable along the  $z$ -axis. Nevertheless, it is obvious that for a real mass function the quantity

$$\Omega(z, z_0) = P \exp \left[ i \int_{z_0}^z d\tau Q(\tau) \right] \tag{8}$$

in (6) is the same as  $\bar{\Omega}(z, z_0) = P e^{i \int_{z_0}^z d\tau \bar{Q}(\tau)}$  in (7). In consequence, the task is to evaluate Eq. (8).

Using the usual Pauli matrices,  $Q(\tau)$  can be expressed as

$$Q(\tau) = \sigma_3 E - i\sigma_2 m(\tau). \tag{9}$$

We consider now

$$m(\tau) = m_0 f(\tau), \tag{10}$$

where  $f(\tau)$  is a certain function which describes the structure of the domain wall, the profile wall function. The asymptotic conditions  $f(+\infty) = 1$  and  $f(-\infty) = 0$  are required,  $f(\tau) - \theta(\tau)$  decreasing exponentially when  $\tau \rightarrow \pm\infty$ . It is also assumed that  $f(\tau) = O(1)$ . The profile function will be considered as an analytic function in the real axis, therefore a profile which is constant outside a certain finite region, the domain wall, cannot be described. This last kind of wall profiles are studied in detail in [8]. If the profile function is analytic, the wall, defined as the region where the mass varies with the position, extends formally from  $\tau = -\infty$  to  $\tau = +\infty$ . Nevertheless the domain wall will be characterized by an effective thickness,  $\sigma$ . The particular criterion considered in order to define this parameter is not important; for

instance, it can be established by taking into account that  $|f(\tau) - 1| < 0.1$  for  $\tau > \sigma$  and  $|f(\tau)| < 0.1$  for  $\tau < -\sigma$ . However, in general we can write

$$f(\tau) = F \left( \frac{\tau}{\sigma} \right). \tag{11}$$

A characteristic energy,  $m_0$ , and a characteristic length,  $1/m_0$ , will be used to obtain dimensionless quantities in what follows. By considering a certain path partition,  $(z_0, z_1, \dots, z_{N-1}, z_N, z)$ , we can write

$$\begin{aligned} P e^{i \int_{z_0}^z d\tau Q(\tau)} &= P e^{i \int_{z_0}^{z_N} d\tau Q(\tau)} \\ &\times P e^{i \int_{z_{N-1}}^{z_N} d\tau Q(\tau)} \dots P e^{i \int_{z_0}^{z_1} d\tau Q(\tau)}. \end{aligned} \tag{12}$$

As shown in the appendix, we find for small  $\Delta_j$ , where  $\Delta_j = z_{j+1} - z_j$ , that

$$P e^{i \int_{z_j}^{z_{j+1}} d\tau Q(\tau)} = e^{i\sigma_3 p_j \Delta_j} + \frac{1}{E} \sigma_2 f_j \sin(p_j \Delta_j), \tag{13}$$

where  $p_j = +(E^2 - f_j^2)^{1/2}$  and  $f_j$  is defined as

$$f_j = \frac{1}{\Delta_j} \int_{z_j}^{z_{j+1}} d\tau f(\tau), \tag{14}$$

e.g. as the average value of the integral.

Thus, by substituting the path order products of the right-hand side in (12) by (13), multiplying and re-ordering the terms, we obtain

$$\begin{aligned} \Omega(z, z_0) &= e^{i\sigma_3 \sum_{j=0}^N p_j \Delta_j} \\ &+ \frac{1}{E} \sigma_2 \sum_{k=0}^N \sin(p_k \Delta_k) f_k e^{-i\sigma_3 \sum_{j=k+1}^N p_j \Delta_j} e^{i\sigma_3 \sum_{j=0}^{k-1} p_j \Delta_j} \\ &+ O \left[ \left( \frac{1}{E} \right)^2 \right], \end{aligned} \tag{15}$$

where we have used  $e^B A = A e^{-B}$  when  $\{A, B\} = 0$ . If the path partition is now considered to be thinner and thinner, or in other words, if the limit  $\Delta_j \rightarrow 0$  is taken for all  $j$ , then we can replace  $\sum_j \Delta_j \rightarrow \int d\tau$ . The definition of  $f_j$  as an integral average value leads to the substitutions  $f_j \rightarrow f(\tau)$  and  $p_j \rightarrow p(\tau)$ , where

$p(\tau) = +(E^2 - [f(\tau)]^2)^{1/2}$ , provided that  $f(\tau)$  is continuous. In this way, we obtain

$$\begin{aligned} \Omega(z, z_0) &= \left(1 + \left(\frac{1}{E}\right)\sigma_2 \int_{z_0}^z d\tau p(\tau) f(\tau) e^{-2i\sigma_3 \int_{\tau}^z d\xi p(\xi)}\right) \\ &+ O\left[\left(\frac{1}{E}\right)^2\right] e^{i\sigma_3 \int_{z_0}^z d\tau p(\tau)}. \end{aligned} \tag{16}$$

We must stress here that  $\Omega(z, z_0)$  has been expanded in powers of  $1/E$ , but not in a strict sense because the coefficients are also depending on  $E$ . As a consequence of this fact, whether the series for  $\Omega(z, z_0)$  which is truncated in order to give (16) is an asymptotic one or not must be carefully elucidated. This point is treated in detail in [8] and we will return to this question below. If we define the quantity  $a = 2\sigma E$  and expand all the terms which depend on  $E$  in (16) in powers of  $1/E$ , by assuming  $a$  constant, we obtain

$$\begin{aligned} \Omega(\lambda\sigma, -\lambda\sigma) &= \left\{1 + \frac{1}{E}\sigma_2 \frac{a\lambda}{2} \int_{-1}^1 dx F(\lambda x) e^{-i\sigma_3 a\lambda(1-x)}\right\} \\ &+ O\left[\left(\frac{1}{E}\right)^2\right] e^{i\sigma_3 \phi}, \end{aligned} \tag{17}$$

where  $z = \lambda\sigma$  and  $z_0 = -\lambda\sigma$ , with  $\lambda \gg 1$ . The function  $F(\tau)$  has been introduced in (11) and the integration variable is changed to  $x = \frac{\tau}{\lambda\sigma}$ . The quantity  $\phi$  is defined as ‘‘classical path’’,  $\int_{z_0}^z d\tau p(\tau) = \lambda a + O\left[\left(\frac{1}{E}\right)^2\right]$ . Eq. (17) can be written in a matrix way

$$\Omega(\lambda\sigma, -\lambda\sigma) = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{pmatrix}. \tag{18}$$

Neglecting terms of the order  $o(1/E)$  we get

$$\begin{aligned} \omega_{11} &= e^{i\lambda a} = \omega_{22}^*, \\ \omega_{12} &= -\frac{ia\lambda}{2} \frac{1}{E} \mathcal{I}(a)^* = \omega_{21}^*, \end{aligned} \tag{19}$$

where

$$\mathcal{I}(a) = \int_{-1}^1 dx e^{i\lambda a x} F(\lambda x). \tag{20}$$

By considering the requirements imposed to the profile wall function,  $f(\tau)$ , the following approximations are performed:  $m(\tau) = 0$  for  $\tau < -\lambda\sigma$  and  $m(\tau) = m_0$  for  $\tau > \lambda\sigma$ ; i.e.  $F(\lambda x) = 0, 1$  for  $x < -1$  and  $x > 1$ , respectively. Thus, taking into account the chirality eigenvalues of the eigenspinors  $\psi_1, \psi_2, \psi_3$  and  $\psi_4$ , we have that  $\psi_1$  and  $\psi_2$  correspond to right-moving right-handed particles and left-moving left-handed particles respectively, and  $\psi_3$  and  $\psi_4$  to right-moving left-handed and left-moving right-handed particles, in the region where  $x < -1$ . Concerning the region where  $x > 1$ ,  $\Omega(\lambda\sigma x, \lambda\sigma x)$  can be immediately diagonalized in order to identify the right-moving and the left-moving flux of particles, since the mass is constant. Thus, by requiring that the left-moving flux is zero in the latter region, i.e. by imposing that we only have a transmitted flux in this region, a left-handed reflected flux is obtained from the right-handed one and vice versa<sup>3</sup>. The equivalence between the equations for  $\psi_I$  and for  $\psi_{II}$  in (4), as a consequence of the result  $Q(z) = \bar{Q}(z)$  for the real mass case, implies that there is no asymmetry in the reflection of incident right-handed and left-handed fermions. In consequence, needless to distinguish  $\psi_I$  and  $\psi_{II}$ . Only when an imaginary mass term, which introduces CP-violating effects, is incorporated, differences between the behaviour of right-handed and left-handed fermions may exist in the reflection process [1,3,5]. The fact that CP invariance boils down to an equality between both helicities is a consequence of CPT invariance [7].

In this way, following the method above described, which was originally introduced by Nelson et al. for a numerical computation of the reflection coefficient [6,7], we use that

$$\psi(z) = \Omega(z, z_0)\psi(z_0), \tag{21}$$

where

$$\psi(z) = D \begin{pmatrix} T e^{ipz} \\ 0 \end{pmatrix} \quad \text{and}$$

<sup>3</sup> Indeed, by conservation of angular momentum  $J_z$  the helicity is flipped in the reflection process.

$$\psi(z_0) = \begin{pmatrix} e^{iEz_0} \\ Re^{-iEz_0} \end{pmatrix}. \tag{22}$$

$D$  diagonalizes the matrix  $Q$  in the region where the mass is constant. From Eq. (21), (22) we obtain for the reflection coefficient

$$R(E) = -e^{2iEz_0} \frac{\omega_{12}^* - (E - p)\omega_{11}}{\omega_{11}^* - (E - p)\omega_{12}}, \tag{23}$$

with  $p = +\sqrt{E^2 - 1}$ . In our case, for the high energy limit, we have

$$R(E) = \frac{1}{2E} (e^{i\lambda a} - i\lambda a \mathcal{I}(a)) + o\left[\left(\frac{1}{E}\right)^2\right]. \tag{24}$$

Now, we must compute the integral which gives the function  $\mathcal{I}(a)$  in Eq. (20). We introduce a certain convergence factor,  $e^{-\varepsilon x}$ , where the parameter  $\varepsilon$  is taken small enough to be sure that  $F(\lambda x)$  decreases faster than this factor when  $x \rightarrow -\infty$ , in such a way that the integration over the real axis, from  $-\infty$  to  $\infty$ , is well defined. Thus, by cutting the integration domain into three intervals,  $(-\infty, -1)$ ,  $(-1, 1)$  and  $(1, \infty)$ , we can write

$$\mathcal{I}(a) = \lim_{\varepsilon \rightarrow 0^+} \left( \left\{ \int_{-\infty}^{\infty} - \int_1^{\infty} - \int_{-\infty}^{-1} \right\} dx e^{(i\lambda a - \varepsilon)x} F(\lambda x) \right). \tag{25}$$

where the integrals in the regions  $(-\infty, -1)$  and  $(1, +\infty)$  can be trivially solved. Indeed, as indicated above, we approximate  $F(\lambda x) = 0, 1$  for  $x < -1$  and  $x > 1$ , respectively. Therefore, we have

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0^+} \int_{-\infty}^{-1} dx F(\lambda x) e^{(i\lambda a - \varepsilon)x} &= 0, \\ \lim_{\varepsilon \rightarrow 0^+} \int_1^{+\infty} dx F(\lambda x) e^{(i\lambda a - \varepsilon)x} &= \frac{i}{\lambda a} e^{i\lambda a}. \end{aligned} \tag{26}$$

Concerning the integral over all of the real axis, the Cauchy theorem can be applied in order to perform this integration. If the Laurent expansion for  $F(z)$  in the pole of order  $\nu_j$ ,  $z = z_j$ , is  $\sum_{n=-\nu}^{+\infty} b_n^j (z - z_j)^n$  the following result can be obtained:

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0^+} \int_{-\infty}^{+\infty} dx F(\lambda x) e^{(i\lambda a - \varepsilon)x} \\ = \frac{2\pi i}{\lambda} \sum_{j=1}^N e^{-ay_j} e^{iax_j} \sum_{n=1}^{\nu_j} b_{-n}^j \frac{(ia)^{n-1}}{(n-1)!}, \end{aligned} \tag{27}$$

where  $z_1 = x_1 + iy_1, z_2 = x_2 + iy_2, \dots, z_N = x_N + iy_N$ , are all the poles of  $F(z)$  with positive imaginary part we have picked when the integration contour is adequately closed. Since the profile function is analytic on the real axis, it is obvious that  $y_j \neq 0$  and therefore we obtain an exponential dependence on  $a$ .

It can be shown from Eqs. (24), (25), (26) and (27) that

$$\begin{aligned} R(E) = \frac{\pi a}{E} \sum_{j=1}^N e^{-ay_j} e^{iax_j} \\ \times \sum_{n=1}^{\nu_j} b_{-n}^j \frac{(ia)^{n-1}}{(n-1)!} + o\left[\left(\frac{1}{E}\right)^2\right]. \end{aligned} \tag{28}$$

Obviously,  $R(E)$  defined in Eq. (23) and given by Eq. (24) is the reflection coefficient generated by a profile function which is zero in the region  $(-\infty, -\lambda\sigma)$  and constant in the region  $(\lambda\sigma, \infty)$ . Nevertheless,  $f(\tau)$  is an analytic function in the real axis verifying that  $f(\tau) - \Theta(\tau)$  decreases exponentially when  $\tau \rightarrow \pm\infty$ , in such a way that the wall profile is well defined over all the real axis. In fact, we are interested in the situation where the wall profile extends from  $-\infty$  to  $\infty$ , although the former requirement is technically necessary in order to identify the incident, reflected and transmitted waves. On the other hand, we cannot take directly the limit when  $\lambda \rightarrow \infty$  [8]. This problem is solved by considering  $\lambda$  as large enough to neglect the effects due to the evolution of the wave outside the region  $(-\lambda\sigma, \lambda\sigma)$ . The question is studied in detail way in Ref. [8] and the result is, as intuitively expected, that in the limit where we take for the wall profile an analytic function from  $-\infty$  to  $+\infty$  one simply has to consider in (25) the first integral, from  $-\infty$  to  $+\infty$ , and that consequently (28) gives the reflection coefficient for the wall from  $-\infty$  to  $+\infty$ . The effect of the mentioned technical assumption (the substitution of the true profile function by a step function outside the interval), is that  $\mathcal{I}(a)$  is defined as an integral over de domain

$(-1, 1)$ . It can be bounded as seen in (25) when this function,  $\mathcal{I}(a)$ , is expressed through the integral over all the real axis and the two other integrals given by Eq. (26). It is worth to stress that indeed  $R(E)$  in Eq. (28) does not depend on  $\lambda$ , as expected for the reflection coefficient for the wall from  $-\infty$  to  $+\infty$ .

In (24) the leading term for a series of powers in  $1/E$  has been written. Nevertheless, we can be sure that it gives the leading term only if  $a$  is a constant. If the parameter which is considered constant is  $\sigma$ , then we must return to the problem of the asymptotic properties of the series we use. As above mentioned, this question is discussed in [8], where we conclude that the right-hand side of (24) is the leading term of the reflection coefficient for high energy when  $\sigma \ll 1$ , even if  $a = 2\sigma E \gg 1$ . In this way, by considering the dependence on  $E$  for the reflection coefficient in the high energy limit, taking  $\sigma$  as constant, we have

$$R(E) = 2\pi\sigma \sum_{j=1}^N e^{-2E\sigma y_j} e^{2iE\sigma x_j} \sum_{n=1}^{\nu_j} b_{-n}^j \frac{(2iE\sigma)^{n-1}}{(n-1)!}. \quad (29)$$

Thus, the behaviour of the high energy fermions, i.e. the high energy component of a wave packet, hitting the wall is completely determined by the complex singularities of the wall profile function. Furthermore, if we are in such a range of energy that  $\sigma E \gg 1$ , the leading contribution is exponentially decreasing with the energy, the coefficient of the exponential being given by the pole of  $f(z)$  with the smallest imaginary part, i.e. the closest to the real axis. Calling  $z_k = x_k + iy_k$  this prevailing pole we obtain

$$R(E) = 2\pi\sigma e^{-2E\sigma y_k} b_{-\nu_k}^k \frac{(2iE\sigma)^{\nu_k-1}}{(\nu_k-1)!} e^{2iE\sigma x_k}. \quad (30)$$

The dependence in  $\sigma$  in (30) is simply an artifact of our change of variables (11). Indeed it can be immediately seen that if we consider the Laurent expansion,  $\sum_{n=-\nu}^{+\infty} b_n^j (z - z_j)^n$ , for  $f(z)$  instead  $F(z)$ , in the pole of the order  $\nu_j$ ,  $z = z_j'$ , we have

$$z_j' = \sigma z_j, \quad b_n^j = \frac{b_n}{\sigma^n}, \quad (31)$$

where, by using (11), the poles and the coefficients of the Laurent expansion for  $F(z)$  and  $f(z)$  have been

related. Thus, the final result can be expressed in terms of the singularities of  $f(z)$  by replacing (31) in Eqs. (29) and (30),

$$\begin{aligned} R(E) &= 2\pi \sum_{j=1}^N e^{-2Ey_j'} e^{2iEx_j'} \sum_{n=1}^{\nu_j} b_{-n}^j \frac{(2iE)^{n-1}}{(n-1)!} \\ &= 2\pi e^{-2Ey_k'} b_{-\nu_k}^k \frac{(2iE)^{\nu_k-1}}{(\nu_k-1)!} e^{2iEx_k'}, \end{aligned} \quad (32)$$

where the lower expression is valid in such a range of energy that  $e^{2E(y_k' - y_h')}$  can be neglected,  $y_k'$  and  $y_h'$  being the imaginary parts of the two closer poles to the real axis (this requirement is analogous to the former  $\sigma E \gg 1$ ). The dependence on  $\sigma$  has disappeared in (32), as expected.

The results we present can be checked by using the particular analytic solution obtained in Refs. [5,4] for the following wall profile

$$f(\tau) = \frac{1 + \tanh(\tau/\sigma)}{2}. \quad (33)$$

For this particular profile function, it follows from (29) by computing the sum for the infinite series of poles it presents

$$R(E) = \frac{\pi\sigma}{2 \sinh(\pi E\sigma)}, \quad (34)$$

which agrees with the result of Ref. [5,4] in the high energy limit for  $\sigma \ll 1$ .

With this positive check, we conclude that (29) in general and (30) for  $E\sigma$  large give the high energy behaviour of the reflection coefficient for analytic profile functions in the real axis. The singularities of the profile function determine the behaviour of the high energy fermions in the way shown by Eq. (29), and if the energy is larger enough than the inverse of the characteristic wall thickness, it is the closest pole to the real axis that gives the high energy behaviour through Eq. (30).

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### Appendix A

We take  $Q(\tau)$  as constant in the intervals  $(z_j, z_{j+1})$ , defining for each one  $f_j$  as the following integral average value:

$$f_j = \frac{1}{\Delta_j} \int_{z_j}^{z_{j+1}} d\tau f(\tau), \tag{A.1}$$

where  $\Delta_j = z_{j+1} - z_j$ . By assuming that  $\Delta_j$  is small (in fact, the limit  $\Delta_j \rightarrow 0$  is considered for the result obtained in this appendix), it can be written in a good approximation by using (9)

$$\mathcal{P} e^{i \int_{z_j}^{z_{j+1}} d\tau Q(\tau)} = e^{(i\sigma_3 E + \sigma_2 f_j) \Delta_j}. \tag{A.2}$$

Taking into account that for two operators verifying  $\{A, B\} = 0$  and  $A^2 = B^2 = 1$ , the following result can be easily proven:

$$e^{\alpha A + \beta B} = \cosh \left[ (\alpha^2 + \beta^2)^{1/2} \right] + \frac{\alpha A + \beta B}{(\alpha^2 + \beta^2)^{1/2}} \sinh \left[ (\alpha^2 + \beta^2)^{1/2} \right], \tag{A.3}$$

and from (A.2) we can derive Eq. (13),

$$\mathcal{P} e^{i \int_{z_j}^{z_{j+1}} d\tau Q(\tau)} = e^{i\sigma_3 p_j \Delta_j} + \frac{1}{E} \sigma_2 f_j \sin(p_j \Delta_j) + O \left[ \left( \frac{1}{E} \right)^2 \right], \tag{A.4}$$

where  $p_j = +(E^2 - f_j^2)^{1/2}$ . In order to obtain this equation  $E/p_j$  and  $1/p_j$  have been expanded in powers of  $1/E$ .

### References

- [1] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, *Phys. Lett. B* 155 (1985) 36.
- [2] G. 't Hooft, *Phys. Rev. Lett.* 37 (1976) 8; *Phys. Rev. D* 14 (1976) 3432; N.S. Manton, *Phys. Rev. D* 28 (1983) 2019; F.R. Klinkhamer and N.S. Manton, *Phys. Rev. D* 30 (1984) 2212; S. Dimopoulos and L. Susskind, *Phys. Rev. D* 18 (1978) 4500.
- [3] A.E. Nelson, D.B. Kaplan, A.G. Cohen, *Nucl. Phys. B* 373 (1992) 453.
- [4] A. Ayala, J.J-Marian and L. Mc Lerran, *Phys. Rev. D* 49 (1994) 5559.
- [5] K. Funakubo, A. Kakuto, S. Otsuki, K. Takenaga and F. Toyoda, *Phys. Rev. D* 50 (1994) 1105.
- [6] G.R. Farrar and M.E. Shaposhnikov, *Phys. Rev. Lett.* 70 (1993) 2833; G.R. Farrar and M.E. Shaposhnikov, CERN-TH.6732/93.
- [7] M.B. Gavela, M. Lozano, J. Orloff and O. Pène, *Nucl. Phys. B* 430 (1994) 345.
- [8] J. Rodríguez-Quintero, M. Lozano and O. Pène, in preparation.
- [9] A.G. Cohen, D.B. Kaplan and A.E. Nelson, *Phys. Lett. B* 254 (1990) 561.
- [10] A.G. Cohen, D.B. Kaplan and A.E. Nelson, *Phys. Lett. B* 349 (1991) 727.