Delta-isobar relativistic meson exchange currents in quasielastic electron scattering

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Abstract

We study the role of the Δ-isobar current on the response functions for high energy inclusive quasielastic electron scattering from nuclei. We consider a general Lagrangian which is compatible with contact invariance and perform a fully relativistic calculation in first-order perturbation theory for one-particle emission. The dependence of the responses upon off-shell parametrizations is analyzed and found to be mild. A discussion of scaling behaviour and a comparison with various non-relativistic approaches are also presented.

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1 Introduction

In recent work [1, 2, 3] we have studied the role of pionic correlations for electron-nucleus scattering in the kinematical domain of the quasielastic peak, focussing our attention on the 1p-1h sector. The calculation has been carried out on the basis of a Relativistic Fermi Gas (RFG) model in which an exact relativistic treatment of the problem can be accomplished. In the domain of perturbation theory, all diagrams including one pionic line were included — in particular the meson-exchange currents (MEC) of contact and pion-in-flight types — so that the corresponding current was automatically gauge invariant.

In [1, 2] the diagrams involving the excitation of a virtual Δ-resonance, represented in fig. 1, were neglected. Although they do not affect the gauge invariance of the theory, since they are associated with a conserved current, it is well-known that such contributions do modify the nuclear response functions, especially in the transverse channel. The aim of this paper is to extend the model of refs. [1, 2] to include the Δ-induced meson-exchange-currents of fig. 1, in order to have a complete understanding of pionic effects in the quasielastic peak (QEP) as far as the 1p-1h channel is concerned. In contrast with non-relativistic calculations [4] our results do not involve any expansions in energy-momentum/M, where M is a typical baryonic mass, and can therefore be applied when studying the response at high momentum and energy transfers.

The present relativistic treatment of the Δ current allows one to study several aspects of Δ electroexcitation which are of special interest theoretically. In particular, in the effective Lagrangian approach to electroexcitation of the Δ resonance it is known that there is freedom at the electromagnetic (EM) vertex due to the off-shell behavior of this resonance [5]. In contrast to earlier approaches to the Δ-exchange current in a relativistic model [6], here we consider a more general $\gamma N\Delta$ interaction Lagrangian. First of all, current conservation restricts the form of the vertex to a superposition of three covariants, for which one can use any of the choices described for instance by Jones and Scadron [7]. One possible choice is the familiar set of magnetic dipole, electric quadrupole, and Coulomb quadrupole multipoles, used in the pion electroproduction analyses of [8, 9]. In this work we use instead the standard "normal parity" set, analogous to the Dirac-Pauli decomposition of the nucleon form factor [10]. The corresponding term in the traditional chiral Lagrangian of Peccei [11] is just a particular case of the normal parity set with two of the three terms equal to zero.

Going a step beyond the Jones-Scadron vertex, the off-shell propagation of massive vector fields with spin $S > \frac{1}{2}$ has been exhaustively discussed in the literature [5, 12, 13]. In particular, a spin $\frac{3}{2}$ field generates contributions involving the $S = \frac{1}{2}$ sector of this field in the effective amplitudes. In the case of the Δ, this is obtained by constructing the most general vertex which is invariant under a special contact symmetry of the free Lagrangian (see [5] for details). The invariance of the $\gamma N\Delta$ and $\pi N\Delta$ vertices under this "point transformation" of the field requires the introduction of additional parameters in the Lagrangian. Attempts have been made to fix some of these "off-shell" parameters by fitting the pion electro- and photoproduction data [14] and, more recently, Compton scattering data from the nucleon [15]. In this work we analyze the impact of the off-shellness nature of the Δ in the EM responses for high momentum transfers by comparing the results obtained with different sets of parameters fitted to nucleon data. In this context we study, in particular, the differences with the

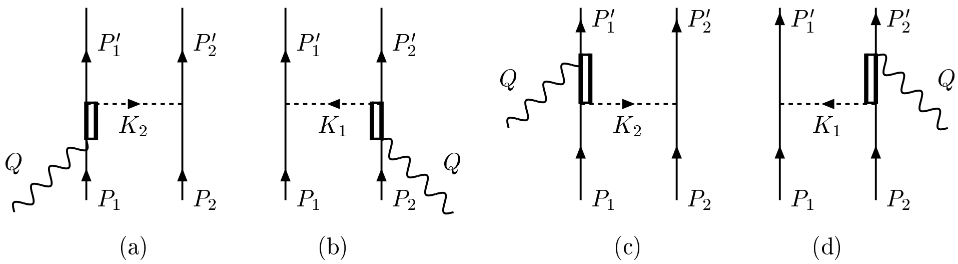

Figure 1: The Feynman diagrams corresponding to the two-body electroexcitation of the Δ.

traditional Peccei Lagrangian approach, which corresponds to specific choices of the off-shell parameters and coupling constants. Recently attempts have been made to design new Δ interaction lagrangians "consistent" with the number of spin degrees of freedom of the Δ from a rigorously field theoretical point of view [13, 16, 17]. However analyses of pion photoproduction with these interactions have not been performed yet, and these studies should be done before attempting to implement them into the MEC operators.

The structure of the work is the following: in sect. 2 we present the relativistic model for the Δ current and provide expressions for the ph matrix elements in the RFG. In sect. 3 we present the results of the calculation of the nuclear response functions including the Δ current for several values of the momentum transfer, up to $q = 3$ GeV/c. We make contact with the scaling properties of the response functions and compare the results with those obtained in non-relativistic approaches for low and intermediate momentum transfer. Finally in sect. 4 we present our conclusions. In appendix A we discuss the properties of the Δ propagator and provide a new method to derive its general form fulfilling the point transformation. In Appendix B we give details on the non-relativistic reduction of the Δ current.

2 The relativistic Δ-isobar current

In line with refs. [1, 2] we perform our analysis in first-order perturbation theory, namely we consider the contributions arising from diagrams with only one pionic line — our conventions are discussed at length in the references cited. We then evaluate the contribution of a virtual Δ to the longitudinal and transverse response functions

$$R^L(q,\omega) = \left(\frac{q^2}{Q^2}\right)^2 \left[W^{00} - \frac{\omega}{q}(W^{03}+W^{30}) + \frac{\omega^2}{q^2}W^{33}\right] \tag{1}$$

$$R^T(q,\omega) = W^{11} + W^{22}\,, \tag{2}$$

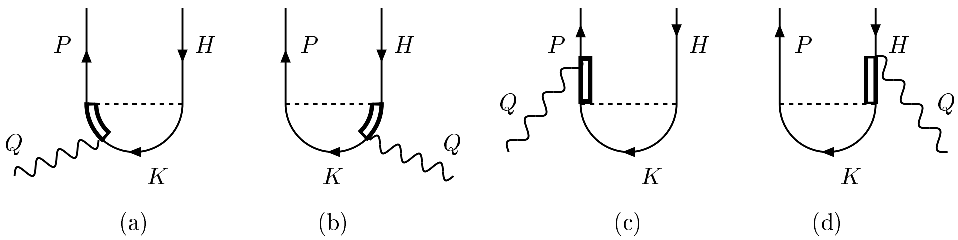

Figure 2: The particle-hole Feynman diagrams corresponding to the two-body electroexcitation of the Δ.

linked to the hadronic tensor $W^{\mu\nu}$. In the symmetric ($Z = N$) RFG model the one-body-Δ interference contribution to the hadronic tensor reads[1]

$$W^{\mu\nu}_{OB-\Delta} = \frac{3Z}{8\pi k_F^3 q} \int_{h_0}^{k_F} h dh E_{\mathbf{p}} \int_0^{2\pi} d\phi_h \sum_{s_p, s_h} 2\mathrm{Re} \left[\frac{m_N^2}{E_{\mathbf{p}} E_{\mathbf{h}}} j^\mu_{OB}(\mathbf{p}, \mathbf{h})^* j^\nu_\Delta(\mathbf{p}, \mathbf{h}) \right] , \tag{3}$$

where $(E_{\mathbf{p}}, \mathbf{p}) = P^\mu = (H + Q)^\mu = (E_{\mathbf{h}} + \omega, \mathbf{h} + \mathbf{q})$ and the lower limit of the integral, h_0, is the minimum momentum required for a nucleon to participate in the process (see refs. [2, 18] for details and explicit expressions). The ph one-body EM current is given by

$$j^\mu_{OB}(\mathbf{p}, \mathbf{h}) = \overline{u}(\mathbf{p}) \left(F_1 \gamma^\mu + i \frac{F_2}{2m_N} \sigma^{\mu\nu} Q_\nu \right) u(\mathbf{h}) , \tag{4}$$

where F_1 and F_2 are the Dirac and Pauli form factors and $u(\mathbf{h}) \equiv u(\mathbf{h}, s_h, t_h)$ the free Dirac spinor. The current $j^\nu_\Delta(\mathbf{p}, \mathbf{h})$, associated with the diagrams of fig. 2, is linked to the ph matrix element of the Δ current operator, $\hat{j}^\nu_\Delta(Q)$ through

$$\langle ph^{-1} | \hat{j}^\nu_\Delta(Q) | F \rangle = (2\pi)^3 \delta^3(\mathbf{q} + \mathbf{h} - \mathbf{p}) \frac{m_N}{V \sqrt{E_{\mathbf{p}} E_{\mathbf{h}}}} j^\nu_\Delta(\mathbf{p}, \mathbf{h}) \tag{5}$$

$$= \sum_{s_k, t_k} \sum_{\mathbf{k} \le k_F} \left[\langle pk | \hat{j}^\nu_\Delta | hk \rangle - \langle pk | \hat{j}^\nu_\Delta | kh \rangle \right] . \tag{6}$$

The sum $\sum_{\mathbf{k} \le k_F}$ becomes, in the thermodynamic limit, an integral over the momentum in the range $0 \le k \le k_F$, and over the angular variables θ_k, ϕ_k. The first and second terms in eq. (6) represent the direct and exchange contribution to the matrix element, respectively.

We may write the $\gamma N \Delta$ Lagrangian in the general form [15]

$$\mathcal{L}_{\gamma N \Delta} = \mathcal{L}^1_{\gamma N \Delta} + \mathcal{L}^2_{\gamma N \Delta} + \mathcal{L}^3_{\gamma N \Delta} \tag{7}$$

with

$$\mathcal{L}^1_{\gamma N \Delta} = \frac{ieG_1}{2m_N} \bar{\psi}^\alpha \Theta_{\alpha\mu}(z_1, A) \gamma_\nu \gamma_5 T_3^\dagger N F^{\nu\mu} + \text{h.c.} \tag{8}$$

[1] We assume $q > 2k_F$, where k_F is the Fermi momentum, so that no Pauli blocking is present.

$$\mathcal{L}^2_{\gamma N\Delta} = \frac{eG_2}{(2m_N)^2}\bar{\psi}^\alpha \Theta_{\alpha\mu}(z_2, A)\gamma_5 T_3^\dagger (\partial_\nu N)F^{\nu\mu} + \text{h.c.} \tag{9}$$

$$\mathcal{L}^3_{\gamma N\Delta} = -\frac{eG_3}{(2m_N)^2}\bar{\psi}^\alpha \Theta_{\alpha\mu}(z_3, A)\gamma_5 T_3^\dagger N\partial_\nu F^{\nu\mu} + \text{h.c.}\ , \tag{10}$$

where ψ^α is the Δ-field, N the nucleon field and $F^{\nu\mu}$ the EM field tensor. We denote by $T_a^\dagger$ the $\frac{1}{2} \rightarrow \frac{3}{2}$ isospin transition operators [19] for $a = 1, 2, 3$. The Lagrangians (8-10) coincide with the expressions given in refs. [5, 14, 20]. On the contrary, the term in eq. (9) differs from ref. [15] in a global sign. As it will be shown later, this difference makes a negligible effect on the global contribution to the MEC(Δ) transverse response, but it causes a very significant reduction in the contribution to the longitudinal response.

The tensor $\Theta_{\mu\nu}(z, A)$ can be written in the general form

$$\Theta_{\mu\nu}(z, A) = g_{\mu\nu} + \left[z + \frac{1}{2}(1 + 4z)A\right]\gamma_\mu\gamma_\nu\ , \tag{11}$$

where z is the so-called off-shell parameter and A is an arbitrary parameter related to the "contact" invariance of the Lagrangian [5, 15].

To derive the Δ current we need also the $\pi N\Delta$ Lagrangian, given by

$$\mathcal{L}_{\pi N\Delta} = \frac{f_{\pi N\Delta}}{m_\pi}\overline{\psi}^\mu \Theta_{\mu\nu}(z_\pi, A)\partial^\nu \boldsymbol{\pi} \cdot \mathbf{T}^\dagger N\ , \tag{12}$$

where $\boldsymbol{\pi}$ is the isovector pion field, and we have introduced an off-shell parameter z_π for the $\pi N\Delta$ vertex. Finally, as in ref. [2], we use pseudo-vector coupling for the πNN vertex.

The corresponding two-body Delta current is obtained by computing the S-matrix element for the elementary virtual photo-absorption process by two nucleons $N_1 + N_2 + \gamma \rightarrow N_1' + N_2'$ (see fig. 1). The corresponding current function can be written as[2] (see ref. [2] for a detailed definition of relativistic currents):

$$\begin{aligned} j^\mu_\Delta(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_1, \mathbf{p}_2) \ = \ & \frac{f_{\pi N\Delta} f}{m_\pi^2} G_\pi(K_2)K_{2\alpha}Q_\nu \overline{u}(\mathbf{p_1'})\left[X_a^{\alpha\mu\nu}(P_1', P_1) - X_a^{\alpha\nu\mu}(P_1', P_1)\right]u(\mathbf{p_1}) \\ & \times \overline{u}(\mathbf{p_2'})\gamma_5 \not{K}_2 \tau_a u(\mathbf{p_2}) + (1 \longleftrightarrow 2)\ , \end{aligned} \tag{13}$$

where we use the Einstein convention for the Lorentz indices and for a sum over a repeated isospin index $a = 1, 2, 3$. Moreover, $K_i^\mu = P_i'^\mu - P_i^\mu$ (with $i = 1, 2$) are the pionic four-momenta (see fig. 1) and

$$G_\pi(K) = \frac{1}{K^2 - m_\pi^2} \tag{14}$$

is the propagator of a pion carrying four-momentum K^μ. The tensor $X_a^{\alpha\mu\nu}$ is defined as

$$\begin{aligned} & X_a^{\alpha\mu\nu}(P', P) = \Theta^{\alpha\beta}(z_\pi, A)G^\Delta_{\beta\rho}(P + Q) \\ & \times \ \left[\frac{G_1}{2m_N}\Theta^{\rho\mu}(z_1, A)\gamma^\nu - \frac{G_2}{4m_N^2}\Theta^{\rho\mu}(z_2, A)P^\nu + \frac{G_3}{4m_N^2}\Theta^{\rho\mu}(z_3, A)Q^\nu\right]\gamma_5 T_a T_3^\dagger \end{aligned}$$

[2]Here we use the Bjorken and Drell conventions [21], whereas different conventions were used in [22].

$$+ \quad \gamma_5 \left[\frac{G_1}{2m_N} \gamma^\nu \Theta^{\mu\rho}(z_1, A) - \frac{G_2}{4m_N^2} P'^\nu \Theta^{\mu\rho}(z_2, A) - \frac{G_3}{4m_N^2} Q^\nu \Theta^{\mu\rho}(z_3, A) \right]$$
$$\times \quad G^\Delta_{\rho\beta}(P' - Q) \Theta^{\beta\alpha}(z_\pi, A) T_3 T_a^\dagger \,. \tag{15}$$

The isospin sums in eq. (13) can be performed using the relations

$$T_a^{(1)} T_3^{\dagger(1)} \tau_a^{(2)} = \frac{2}{3}\tau_z^{(2)} - \frac{i}{3}\left[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}\right]_z \tag{16}$$
$$T_3^{(1)} T_a^{\dagger(1)} \tau_a^{(2)} = \frac{2}{3}\tau_z^{(2)} + \frac{i}{3}\left[\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)}\right]_z \,. \tag{17}$$

In eq. (15) the three amplitudes contributing to the N-Δ vertex [7, 15] are taken into account and the values of the off-shell parameters z_π, z_1, z_2 and z_3 will be discussed later.

In what follows we start by considering the Δ as a stable particle with mass m_Δ, and we will later include a width to account for its decay probability in the resonance region. The isobar propagator can be expressed in general as a sum of two terms

$$G^\Delta_{\beta\rho}(P) = G^{RS}_{\beta\rho}(P) + G^A_{\beta\rho}(P) \,, \tag{18}$$

where

$$G^{RS}_{\beta\rho}(P) = -\frac{\not{P} + m_\Delta}{P^2 - m_\Delta^2}\left[g_{\beta\rho} - \frac{1}{3}\gamma_\beta\gamma_\rho - \frac{2}{3}\frac{P_\beta P_\rho}{m_\Delta^2} - \frac{\gamma_\beta P_\rho - \gamma_\rho P_\beta}{3m_\Delta}\right] \tag{19}$$

is the usual Rarita-Schwinger (RS) propagator tensor and

$$G^A_{\beta\rho}(P) = -\frac{1}{3m_\Delta^2}\frac{A+1}{(2A+1)^2}$$
$$\times \left[(2A+1)(\gamma_\beta P_\rho + P_\beta \gamma_\rho) - \frac{A+1}{2}\gamma_\beta(\not{P} + 2m_\Delta)\gamma_\rho + m_\Delta \gamma_\beta\gamma_\rho \right] \tag{20}$$

the piece of the propagator that depends on the parameter A. Note that the global sign in the RS term differs from the expressions given in [5, 14, 15]. This can be due just to a different choice of phase in the definition of the propagator. As shown in Appendix A, the choice of phase in this work coincides with the one that provides the standard form of the nucleon propagator, $S_N(P) = 1/(\not{P} - m_N)$. In appendix A eq. (20) is derived using a contact transformation of the RS propagator[3]. Moreover, because of the contact invariance, the physical properties of the field can be shown not to depend on A. As a further test of our calculation, we have checked that the results for the T response do not depend on A. Hence in what follows we fix $A = -1$ so that the complete Δ propagator is simply reduced to the Rarita-Schwinger expression and therefore omit the explicit A-dependence in the tensor $\Theta^{\mu\nu}$.

It is immediate to check that the current in eq. (13) is conserved,

$$Q_\mu j^\mu_\Delta(\mathbf{p}'_1, \mathbf{p}'_2, \mathbf{p}_1, \mathbf{p}_2) = 0 \,, \tag{21}$$

so that the Δ current does not affect the gauge invariance of the RFG model.

[3]Note that there is an error in the relative sign between the $G^{RS}_{\beta\rho}$ and $G^A_{\beta\rho}$ pieces given in refs. [5, 12].

The particle-hole matrix-element diagrams are obtained from eq. (13) by setting

$$
\begin{aligned}
(P'_1, s'_1, t'_1) &= (P, s_p, t_p)\ , \quad (P_1, s_1, t_1) = (H, s_h, t_h) \\
(P_2, s_2, t_2) &= (P'_2, s'_2, t'_2) = (K, s_k, t_k)
\end{aligned}
\tag{22}
$$

for the *direct* term and

$$
\begin{aligned}
(P'_1, s'_1, t'_1) &= (P, s_p, t_p)\ , \quad (P_2, s_2, t_2) = (H, s_h, t_h) \\
(P_1, s_1, t_1) &= (P'_2, s'_2, t'_2) = (K, s_k, t_k)
\end{aligned}
\tag{23}
$$

for the *exchange* term, and by summing over the hole momentum k, spin s_k and isospin t_k. The isospin trace yields a vanishing direct matrix element, since $\mathrm{Tr}\,\tau_a = \mathrm{Tr}\,T_3^\dagger T_a = 0$. In the exchange channel, recalling eq. (16), we get

$$
\sum_{t_k} \chi^\dagger_{t_p} T_a T_3^\dagger \chi_{t_k} \chi^\dagger_{t_k} \tau_a \chi_{t_h} = \sum_{t_k} \chi^\dagger_{t_k} T_3 T_a^\dagger \chi_{t_h} \chi^\dagger_{t_p} \tau_a \chi_{t_k} = \frac{4}{3} \chi^\dagger_{t_p} \tau_z \chi_{t_h}
\tag{24}
$$

and

$$
\sum_{t_k} \chi^\dagger_{t_p} T_3 T_a^\dagger \chi_{t_k} \chi^\dagger_{t_k} \tau_a \chi_{t_h} = \sum_{t_k} \chi^\dagger_{t_k} T_a T_3^\dagger \chi_{t_h} \chi^\dagger_{t_p} \tau_a \chi_{t_h} = 0\ .
\tag{25}
$$

As a consequence only the two "vertex" diagrams (a) and (b) in fig. 2, contribute to the process, whereas the "self-energy" diagrams (c) and (d) give a vanishing contribution to the responses.

Finally, by taking the thermodynamic limit, we get for the ph current function in eq. (5):

$$
\begin{aligned}
j^\mu_\Delta(\mathbf{p}, \mathbf{h}) &= -\frac{f_{\pi N\Delta} f}{m_\pi^2} \frac{4}{3} \chi^\dagger_{t_p} \tau_z \chi_{t_h} \sum_{s_k} \int \frac{d^3k}{(2\pi)^3} \frac{m_N}{E_{\mathbf{k}}} \theta(k_F - k) \\
&\quad \times [\, \overline{u}(\mathbf{p}, s_p) F^\mu(P, H, K) u(\mathbf{k}, s_k) \overline{u}(\mathbf{k}, s_k) \gamma_5 (\not{K} - \not{H}) u(\mathbf{h}, s_h) \\
&\quad\quad + \overline{u}(\mathbf{p}, s_p) \gamma_5 (\not{P} - \not{K}) u(\mathbf{k}, s_k) \overline{u}(\mathbf{k}, s_k) B^\mu(P, H, K) u(\mathbf{h}, s_h)\,]\ ,
\end{aligned}
\tag{26}
$$

where

$$
\begin{aligned}
F^\mu(P, H, K) &\equiv G_\pi(K - H)(K - H)_\alpha Q_\nu \Theta^{\alpha\beta}(z_\pi) G^\Delta_{\beta\rho}(K + Q) \\
&\quad \times \left\{ \frac{G_1}{2m_N} [\Theta^{\rho\mu}(z_1)\gamma^\nu - \Theta^{\rho\nu}(z_1)\gamma^\mu] - \frac{G_2}{4m_N^2} [\Theta^{\rho\mu}(z_2)K^\nu - \Theta^{\rho\nu}(z_2)K^\mu] \right. \\
&\quad \left. + \frac{G_3}{4m_N^2} [\Theta^{\rho\mu}(z_3)Q^\nu - \Theta^{\rho\nu}(z_3)Q^\mu] \right\} \gamma_5
\end{aligned}
\tag{27}
$$

and

$$
\begin{aligned}
& B^\mu(P, H, K) \equiv G_\pi(P - K)(P - K)_\alpha Q_\nu \\
& \quad \times \gamma_5 \left\{ \frac{G_1}{2m_N} [\gamma^\nu \Theta^{\mu\rho}(z_1) - \gamma^\mu \Theta^{\nu\rho}(z_1)] - \frac{G_2}{4m_N^2} [K^\nu \Theta^{\mu\rho}(z_2) - K^\mu \Theta^{\nu\rho}(z_2)] \right. \\
& \quad \left. - \frac{G_3}{4m_N^2} [Q^\nu \Theta^{\mu\rho}(z_3) - Q^\mu \Theta^{\nu\rho}(z_3)] \right\} G^\Delta_{\rho\beta}(K - Q) \Theta^{\beta\alpha}(z_\pi)
\end{aligned}
\tag{28}
$$

correspond to the forward- and backward-going diagrams (a) and (b) of fig. 2, respectively.

The decay of the Δ-resonance into a physical N-π state should be taken into account above threshold, i.e., $P_\Delta^2 > (m_\pi + m_N)^2$. This is accounted for by modifying the propagator in eq. (19) to include a finite width according to the following prescription [6]

$$\frac{1}{P^2 - m_\Delta^2} \longrightarrow \frac{1}{P^2 - \left[m_\Delta - \frac{i}{2}\Gamma(P^2)\right]^2} \tag{29}$$

in which [19]

$$\Gamma(P^2) = \Gamma_0 \frac{m_\Delta}{\sqrt{P^2}} \left(\frac{p_\pi^*}{p_\pi^{res}}\right)^3 \tag{30}$$

is the energy-dependent width. In the above p_π^* is the momentum of the final pion resulting from the Δ decay (in the Δ-system) and p_π^{res} is its value at resonance. Moreover we take $\Gamma_0 = 120$ MeV.

3 Results

In this section we show the contribution of the Δ current to the longitudinal and transverse quasielastic response functions in the 1p-1h channel.

3.1 Response functions and off-shell dependence

The following coupling constants have been used in the calculation [6, 15]:

$$f = \sqrt{4\pi \times 0.08}\ ,\ f_{\pi N\Delta} = 4 \times 0.564\ ,\ G_1 = 4.2\ ,\ G_2 = 4\ ,\ G_3 = 1\ . \tag{31}$$

The value of G_3 is arbitrary owing to the lack of experimental information (see below). Moreover, although not explicitly indicated in the above formulae, the following monopole form factor

$$F_{\pi NN}(K) = F_{\pi N\Delta}(K) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - K^2}\ , \tag{32}$$

with $\Lambda = 1300$ MeV, has been used. For the single-nucleon current we have for simplicity adopted the Galster form factor parameterization [23]. The Fermi momentum is chosen to be $k_F = 237$ MeV/c, namely, representative of a typical sd-shell nucleus.

In fig. 3 we plot the one-body-Δ interference contributions to the longitudinal and transverse responses as functions of the energy transfer ω for various values of the momentum transfer q, ranging from 0.5 to 3 GeV/c. The separate contributions of the three Lagrangians in eqs. (8,9,10) are displayed.

Regarding the longitudinal response (left panels), the G_1 and G_2 pieces are similar in magnitude and tend to cancel for high q, whereas the G_3 term is negligible. Note that this result is very different from the one obtained with the Peccei Lagrangian (corresponding to the G_1 term only). However, as will be shown later, the whole contribution to the response is very small.

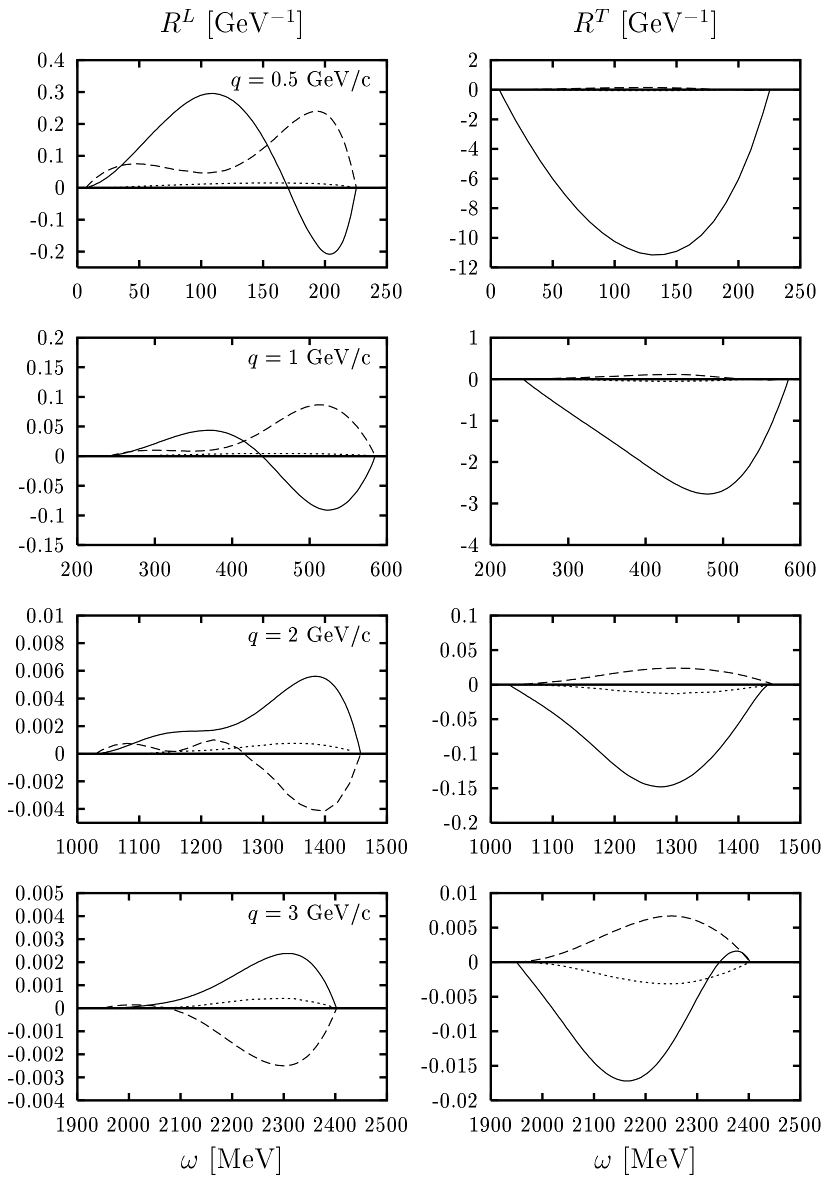

Figure 3: The contribution of the Δ current to the longitudinal (left panels) and transverse (right panels) responses plotted versus ω. Here k_F=237 MeV/c. The separate contributions of the first (G_1, solid) second (G_2, dashed) and third (G_3, dotted) terms of the current are displayed. The off-shell parameters are taken as $z_1 = z_2 = z_3 = z_\pi = -1/4$.

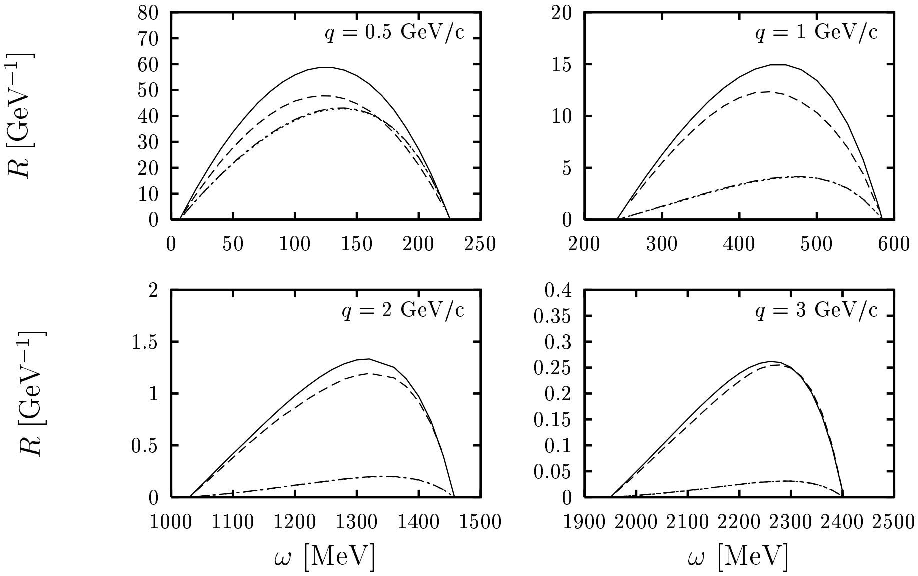

Figure 4: Global longitudinal and transverse responses plotted versus ω. Solid: RFG transverse response; dashed: RFG+MEC(Δ) transverse response; dotted: RFG longitudinal response; dot-dashed: RFG+MEC(Δ) longitudinal response.

In the transverse channel (right panels) the G_1 term clearly dominates, although at high q the contribution of G_2 becomes significant and tends to cancel the first one. Hence in this case the results almost coincide, for q not too high, with the results obtained with the Peccei Lagrangian.

To clarify the effects introduced by the Δ current, we show in fig. 4 the "total", namely RFG+MEC(Δ), longitudinal and transverse responses compared with the pure RFG ones. The same values of q as in the previous figure have been considered. We note that the contribution of the MEC(Δ) to the longitudinal response is always negligible. In the transverse channel the contribution of the Δ is larger and negative, that is, the interference between the one- and two-body currents matrix elements is destructive and therefore reduces the total from the purely nucleonic answer. At low momentum transfers the fractional contribution arising from MEC(Δ) contributions is relatively large and then it slowly decreases as the momentum transfer increases into the several GeV regime. Specifically, at the selected values of $q = 0.5$, 1, 2 and 3 GeV/c the net effect of the MEC(Δ) contributions is 19%, 18%, 10% and 4%, respectively.

It is also interesting to compare the role of the MEC(Δ) with the other pionic MEC (seagull and pion-in-flight), calculated in the same relativistic model in [1, 2]. As shown in fig. 5 the Δ contribution is larger than the pionic one for lower q, where the total MEC are sizable, although for higher momentum transfers the difference in magnitude between Δ and pionic MEC contributions decreases and the two contributions tend to cancel. As a consequence, the impact of the total MEC at high q (say $q \geq 2$ GeV/c) is almost vanishing in the 1p-1h sector.

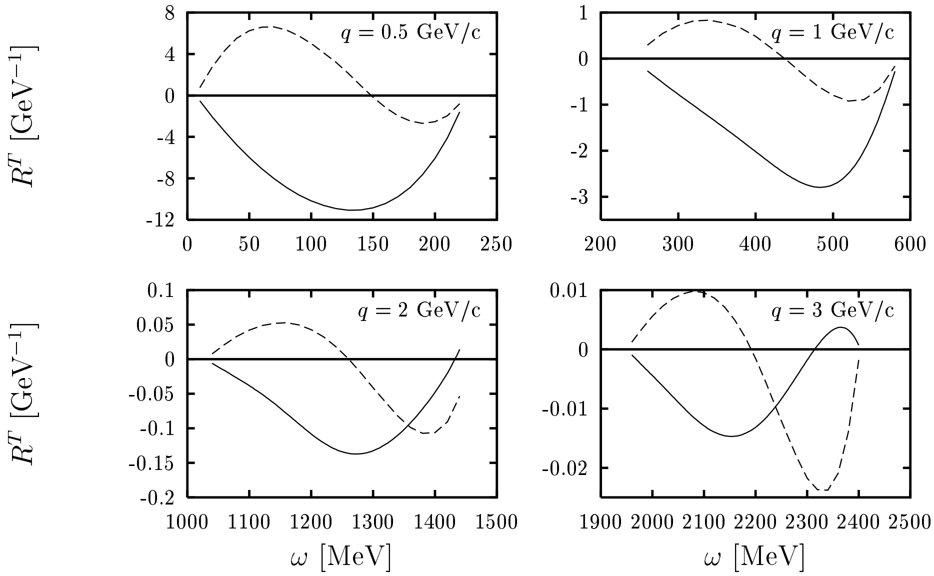

Figure 5: MEC(Δ) contribution to the transverse response (solid) compared with the pionic seagull plus pion-in-flight contribution (dashed).

We now investigate the effect of changing the off-shell parameters z_i. The values $z_i = -0.25$ used in the above results are unnecessarily restrictive, since they were fixed by Peccei under the condition $\gamma_\mu \Theta^{\mu\nu}(z, A) = 0$, which does not correspond to the most general form of $\Theta_{\mu\nu}$ consistent with the point transformation. Different ranges or sets of the parameters z_π, z_1 and z_2 have been obtained by fitting pion photo-production data on the nucleon [14, 24], while the value of z_3 needs electrons to be fixed. The uncertainty in these parameters arises from the treatment of the unitarity constraint, due to both theoretical ambiguities and to lack of precise experimental data.

Since a unique determination of the off-shell parameters is not available, we have studied the dependence of our results upon a variation of them. In fig. 6 we present the Δ contribution to the longitudinal and transverse responses for $q = 1$ GeV/c using four different choices of the parameters (z_π, z_1, z_2, z_3):

$$\begin{array}{ll} a)\ (-1/4, -1/4, -1/4, -1/4)\ , & b)\ (-1/4, 0.1, 2.25, -1/4)\ , \\ c)\ (-1/4, 0.3, 2.25, -1/4)\ , & d)\ (-1/2, -1/2, -1/2, -1/2)\ . \end{array} \qquad (33)$$

Set a), used in the previous figures, reduces to the standard Peccei Lagrangian in the case $G_2 = G_3 = 0$. Sets b) and c) are determined in [15] by fitting Compton scattering cross sections on the nucleon (note that in this case the G_3 term does not contribute) and set d) yields the usual non-relativistic $\gamma N \Delta$ vertex, namely $\Theta_{\mu\nu} = g_{\mu\nu}$. Although several other choices of parameters have been suggested in the literature [14], we believe that the results of fig. 6 give an indication of the uncertainty in the response functions associated with different off-shell prescriptions. It appears from fig. 6 that the fractional uncertainty is much larger in the longitudinal channel, where, however, the contribution to the response is negligible.

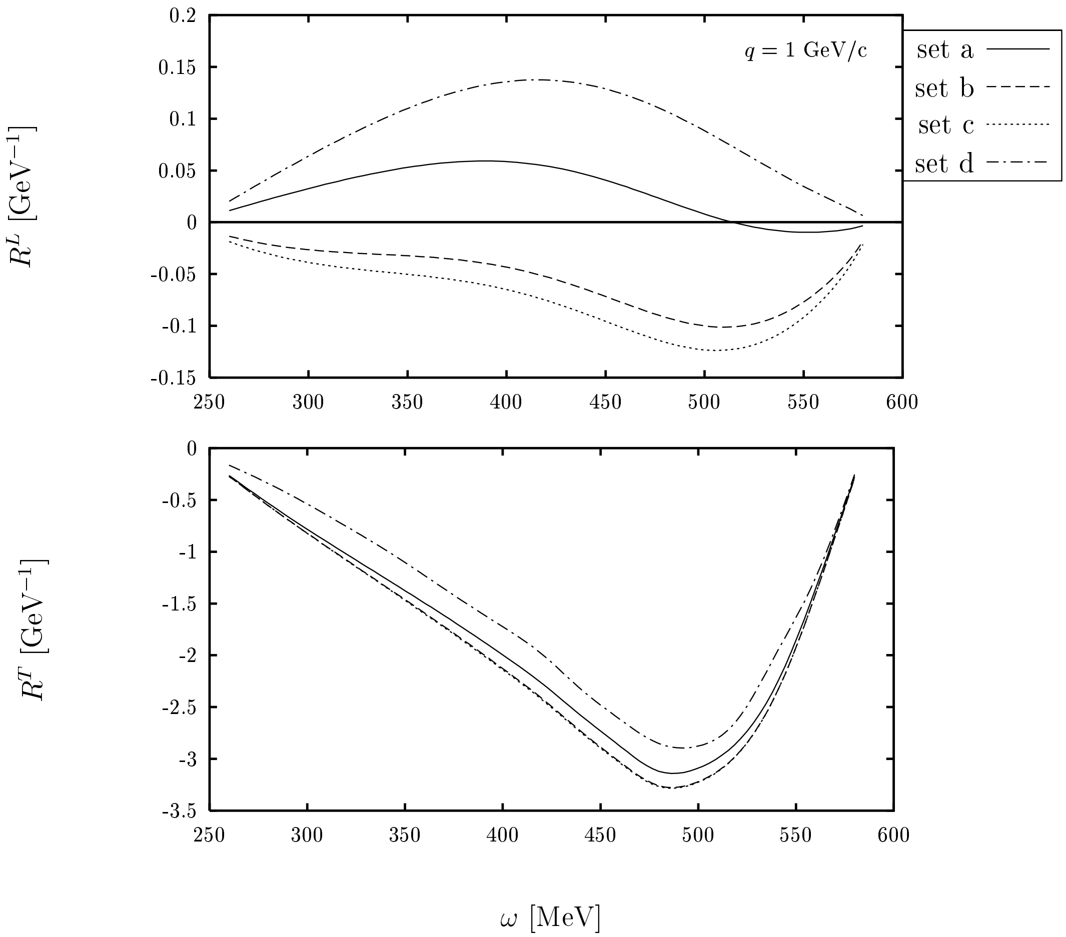

Figure 6: MEC(Δ) contribution to the longitudinal (upper panel) and and transverse (lower panel) responses plotted versus ω. The momentum transfer is $q = 1$ GeV/c and results are shown for the four sets of the off-shell parameters discussed in the text.

On the other hand the transverse response, for which the MEC(Δ) contributions are more important, is rather insensitive to the different choices (the effect being at most of the order of 6–7%).

In [17] a correspondence between classes of "consistent" and "inconsistent" lagrangians was found by a redefinition of the spin-3/2 field. A new contact interaction that does not involve the spin-3/2 field appears. In our case this would mean that a new MEC term of contact type, dependent on the off-shell parameters, would arise and should be added to the "seagull" current, cancelling the dependence of the total responses on the off-shell parameters.

In the next subsection we shortly address the scaling behavior of the MEC(Δ) responses.

3.2 Scaling

The scaling phenomenon has been presented in detail in refs. [25, 26, 27]. Here we just recall the basic definitions which are of use for the discussion that follows. We only focus on the transverse channel since, as previously shown, the MEC(Δ) are irrelevant in the longitudinal channel.

Scaling of first kind occurs if the scaling function

$$f^T(q,\omega) = k_F \frac{R^T(q,\omega)}{G^T(q,\omega)} \tag{34}$$

becomes a function of one single variable, the scaling variable, and independent of q. Such behaviour is known to occur for large q in the region below the QEP. In eq. (34) $G^T(q,\omega)$ is the relevant single-nucleon EM function (see ref. [27] for its explicit expression).

Several different scaling variables exist in the literature, all of them coalescing into one — or being simply related to each other — for high enough momentum transfers. In the quasielastic peak region the natural scaling variable turns out to be [28, 29]

$$\psi = \pm\sqrt{\frac{T_0}{T_F}}\ , \tag{35}$$

where $T_0 = \sqrt{h_0^2 + m_N^2} - m_N$ is the minimum kinetic energy required to a nucleon to take part in the process. The $+(-)$ sign in eq. (35) refers to the right (left) of the quasielastic peak. The analysis of the World data [25, 26, 27] shows that scaling of first kind is reasonably good for $\psi < 0$ and badly violated for $\psi > 0$.

Scaling of second kind corresponds to the independence of the function f^T on the specific nucleus, namely on the Fermi momentum. The analysis of the existing data points to an excellent fulfillment of this scaling in the region $\psi < 0$ and to a not very dramatic breaking of it for $\psi > 0$. When the two kinds of scaling occur the response is said to "superscale".

The relativistic Fermi gas model fulfills both kinds of scaling, by construction, yielding the scaling function $f_0 = 3(1-\psi^2)/4$. The observed superscaling behavior of the experimental data [25, 27] offers a clear constraint on the size allowed for nuclear correlations and MEC contributions, since these may break the scaling behaviour (of both kinds), and can therefore be used as a test of the reliability of the model. It is then natural to explore the scaling behaviour predicted in the present model.

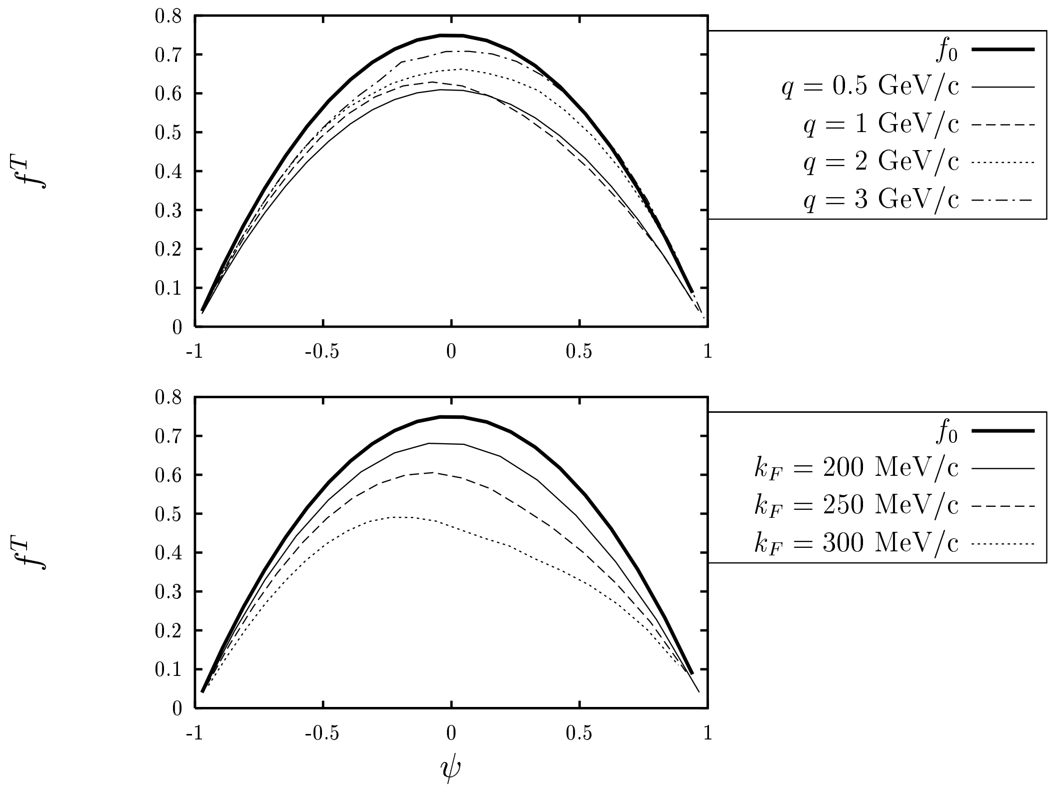

Figure 7: The scaling function f^T plotted versus ψ at $k_F = 237$ MeV/c for various values of q (upper panel) and at $q = 1$ GeV/c for various values of k_F (lower panel). The function f_0 (thick solid line) refers to the free Fermi gas scaling function.

In [1] the evolution with q and k_F of the pionic MEC and correlations has been already explored in detail: it has been proven that these contributions satisfy, at momentum transfers above 1 GeV/c, scaling of the first kind, but that they violate the second-kind scaling by roughly three powers of k_F, which is strong enough to be seen in existing data. However, the size of the scale breaking predicted in [1] in the scaling region (below the QEP) was small enough to be compatible with the high quality high-q data.

In fig. 7 we display the scaling function f^T versus ψ for different values of q (upper panel) and k_F (lower panel). It appears that both kinds of scaling are violated by the MEC(Δ) contribution, something not evident in the high quality World data, at least within existing experimental uncertainties. When the "total" 1p-1h response is formed a quantitative analysis shows that the MEC(Δ) contributions in the transverse response play some role, but that overall first-kind scaling is quite good for $-1 \leq \psi \leq -0.5$ and $q \leq 1 - 2$ GeV/c and is violated at a 5-10% level at the QEP. On the other hand, the second-kind scaling violations from this 1p-1h model can be significant for all values of ψ, since they modify the scaling function (that scales, by construction) by three powers of k_F. The figure shows a generous range of Fermi momenta; these correspond to going from ^{4}He at $k_F = 200$ MeV/c to very heavy nuclei at $k_F \cong 250$ MeV/c and then well beyond to 300 MeV/c to explore more fully the second-kind scaling behaviour. If we restrict our attention to nuclear species where high quality data exist then we should compare the $k_F = 200$ MeV/c results with those for $k_F = 250$ MeV/c and the lower panel in the figure shows that violations of about 14% are predicted at the QEP (actually using modeling of this type for helium is probably stretching things somewhat: if the comparison is made for "real nuclei" such as ^{12}C versus ^{197}Au for which excellent data exist then an effect of roughly 8% is predicted). While second-kind scale breaking of this magnitude should be visible in the data one should be careful before drawing premature conclusions, since one knows that additional scale-breaking contributions arise from mechanisms outside the present model. In particular we know from past work [6, 22, 30] that 2p-2h MEC contributions also typically lead to effects that go as k_F^3, but these *add* and therefore tend to cancel the contributions discussed in the present work. The net effect, in the scaling region at least, is expected to amount to a few percent and therefore be consistent with the relevant World data.

It appears that the scaling behaviour of the MEC(Δ) contributions is different from that of the single-nucleon RFG response — namely, if the scaling functions and variables are defined as in past work to make the latter scale, then the two-body MEC contributions in general do not. First-kind scaling violations (see the upper panel in fig. 7) arise because the one- and two-body currents do not in general have the same (q, ω) dependences, and only one choice can be made when defining the scaling function. Namely, in standard treatments it is the one-body (single-nucleon) cross section that is divided out to produce scaling for the dominant impulse approximation contributions. Similarly, second-kind scaling violations arise when the density dependences of the various contributions are different.

The high-quality World data present a mixed picture: for negative values of ψ both kinds of scaling are reasonably respected, although scaling of the second kind appears to be better than scaling of the first kind in this region. For ψ positive, scaling of the first kind becomes quite bad, partially because pion production including via the delta and other baryon resonances becomes important and, for the reasons stated above, has a different (q, ω) dependence. Additionally, the MEC(Δ) effects under study in the present work can contribute to this first-kind scale breaking. Were the latter to be the only contributions to be present along with the one-body responses, then we would conclude from this study that overall first-kind scaling is quite good for $-1 \leq \psi \leq -0.5$ and $q \leq 1$–2 GeV/c, but could be violated by as much as 5–10% at the QEP. However, the full assessment of which contributions produce the observed result cannot be made before all effects are included, namely from impulse approximation nucleonic currents, 1p-1h MEC effects from pion-exchange contributions including those involving the Δ (this work), 2p-2h MEC effects (work in progress) and meson production.

Similar statements can be made with respect to second-kind scaling violations. The MEC(Δ) contributions studied in the present work (see lower panel in fig. 7) can be significant for all values of ψ, since they modify the scaling function (that scales, by construction) by three powers of k_F.

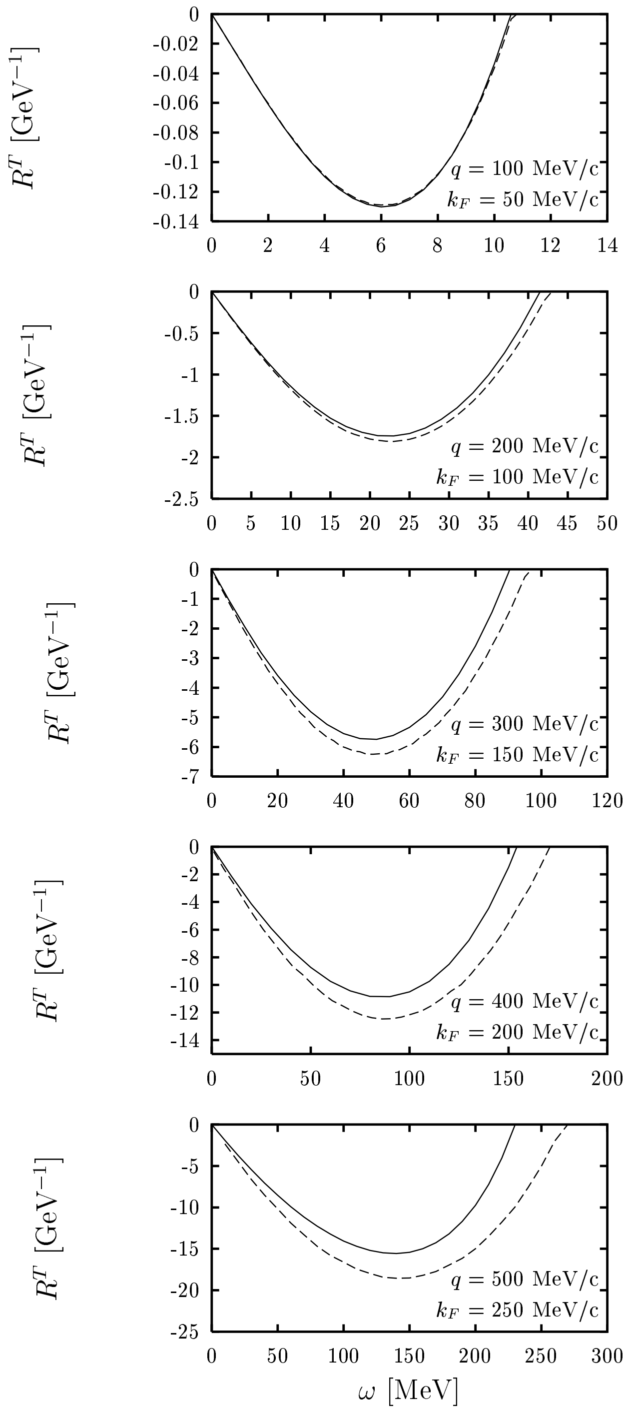

Figure 8: MEC(Δ) transverse response computed for several values of q and k_F. Solid: exact relativistic result, dashed: non-relativistic result. Static propagators without a πNN form factor have been used.

3.3 Non-relativistic reduction

To finish with the discussion of the results, in this section we analyze the effects associated with different types of non-relativistic approaches. Focusing on the MEC(Δ) contribution to the hadronic (e, e') observables one deals with the single-nucleon EM current operator and the two-body Δ current. Improved "semi-relativistic" (SR) versions of the former have been presented in [31, 32, 33] by expanding only in the dimensionless parameter $\eta = p/m_N$, being p the struck nucleon three-momentum. By this method a SR form of the transverse OB current can be written in terms of the non-relativistic one as

$$\mathbf{j}^{OB}_{SR}(\mathbf{p}, \mathbf{h}) = \frac{1}{\sqrt{1+\tau}} \mathbf{j}^{OB}_{NR}(\mathbf{p}, \mathbf{h}) \,, \tag{36}$$

where $\tau = |Q^2|/4m_N^2$. A similar expansion has been carried out for the pion-in-flight and contact MEC in ref. [34] and for the one-body Δ electroproduction current in [35]. In the case of the MEC, a semi-relativistic expression similar to eq. (36) was proposed and tested in [2]

$$\mathbf{j}^{MEC}_{SR}(\mathbf{p}, \mathbf{h}) = \frac{1}{\sqrt{1+\tau}} \mathbf{j}^{MEC}_{NR}(\mathbf{p}, \mathbf{h}) \,. \tag{37}$$

Although these SR MEC currents compare better than the NR ones to the exact relativistic calculation, they are not as good as that the SR OB current since, as illustrated in [2], an additional k_F-dependent normalization factor $N(q, \omega, k_F)$, arising from the integration over the Fermi sea, should be present in eq. (37). Nevertheless the simplicity of these SR currents make them suitable for easy incorporation in existing non-relativistic models.

The relativizing factor $(1+\tau)^{-1/2}$ takes care of spinology and normalization properties that cannot be neglected in any relativistic model. Therefore, consistently with eq. (37), we propose a similar expression for the semi-relativistic Δ current

$$\mathbf{j}^{\Delta}_{SR}(\mathbf{p}, \mathbf{h}) = \frac{1}{\sqrt{1+\tau}} \mathbf{j}^{\Delta}_{NR}(\mathbf{p}, \mathbf{h}) \,, \tag{38}$$

where $\mathbf{j}^{\Delta}_{NR}(\mathbf{p}, \mathbf{h})$ is the standard non-relativistic Δ current commonly considered in the literature [36, 37], usually obtained from NR sets of $\gamma N \Delta$ and $\pi N \Delta$ Lagrangians. In appendix B we derive its expression by performing a non-relativistic reduction of the relativistic Δ current considered in this work.

To illustrate clearly the impact of relativity we show in fig. 8 the MEC(Δ) transverse response corresponding to the fully relativistic calculation (solid lines) and to the standard NR reduction (dashed lines) using the Fermi gas model of ref. [38]. In both cases no πNN form factor has been used and the static limits of the pion and Δ propagators have been assumed. We observe that the two calculations give the same results for small density and momentum transfer and start to differ as q and k_F increase. In particular the results of fig. 8 provide a test of the multi-dimensional numerical integration procedure used in this work, since the Δ integrals over $\mathbf{k}$ in the non-relativistic calculation of [38] are analytical. Moreover the agreement of both results for low momenta also represents a test of the applicability of the non-relativistic Δ current in this kinematic domain. This result is not obvious *a priori*

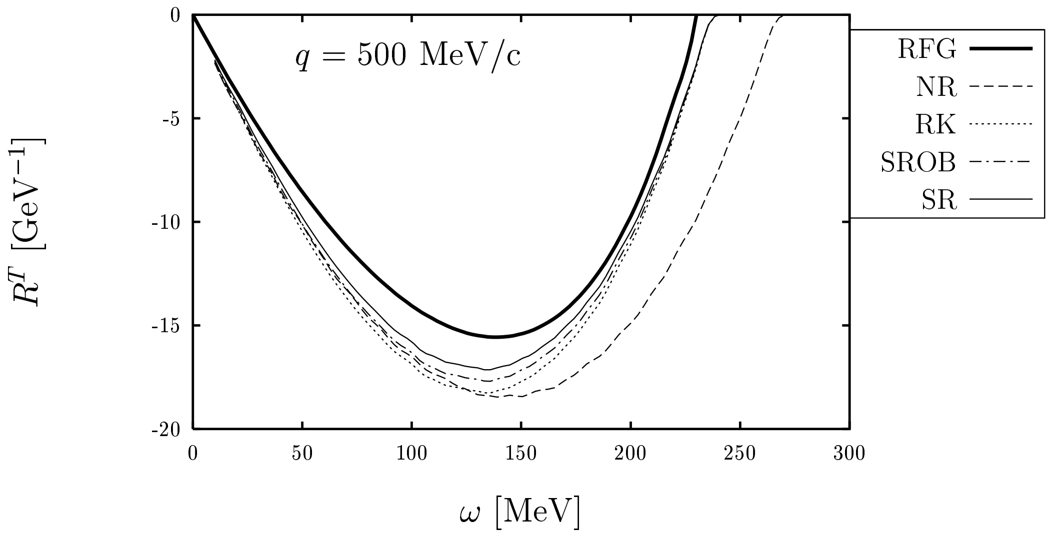

Figure 9: MEC(Δ) transverse response for $q = 1$ GeV/c. The result corresponding to the fully relativistic Fermi gas (RFG) is compared with the non-relativistic (NR), including relativistic kinematics (RK), semi-relativistic approach for the one-body current (SROB) and for the Δ-current as well (SR) (see text for details).

due to the somewhat coarse approximations assumed in the derivation of the NR current (see Appendix B).

The shrinking of the response domain in fig. 8 with increasing q arises from the relativistic kinematics in the energy-conserving delta function appearing in the responses. As in past work this effect can be accounted for approximately by the replacement $\lambda \to \lambda(1+\lambda)$, with $\lambda = \omega/2m_N$, in the non-relativistic calculation.

This is shown in fig. 9, where we select the case $q = 500$ MeV/c and compare the NR calculation (dashed line) with the same calculation but using relativistic kinematics (RK) (dotted line). Apart from this effect, the enhancement of the non-relativistic calculations with respect to the relativistic one is a consequence of the reduction of the single-nucleon and Δ current operators. The semi-relativistic approach for the single-nucleon current (SROB) (dot-dashed) improves the agreement with the fully relativistic result. Similarly, the response obtained by using the SR current (38) (thin solid line) gets a little bit closer to the relativistic result, although a difference between them still exists. Better agreement between the exact and the relativized models for the MEC(Δ) responses are obtained in the limit $k_F \to 0$ in the quasielastic peak region.

4 Conclusions

In this paper we have studied the role of the MEC(Δ) two-body current in 1p-1h quasielastic electron scattering. A fully relativistic, gauge invariant, analysis has been performed in the context of the RFG. The most general Lagrangian has been considered, allowing for different off-shell parameters in each one of the vertices, and the roles played by its various pieces have been analyzed. The results have been compared with various non-relativistic reductions and the behavior of the Δ contribution with respect to the scaling phenomenon has been also explored.

In summary, our results show that:

1. The Δ contribution is only important in the transverse channel, where it represents the dominant correction to the free Fermi gas, at least for $q < 1$–2 GeV/c.

2. In the transverse channel the main contribution comes from the first term of the Lagrangian (Peccei). This is strictly valid for not too high values of q.

3. In the longitudinal channel the global Δ contribution is negligible. In this case, however, the first and second terms of the Lagrangian tend to cancel.

4. The transverse response is quite insensitive to the different sets of off-shell parameters used, whereas the longitudinal one shows a much more pronounced sensitivity to them.

5. The MEC(Δ) contributions by themselves clearly break both kinds of scaling behaviour. When combined with the RFG the total scaling breaking is on the edge of being an effect that should be seen in the high quality high-q data; however, it is known that contributions from 2p-2h MEC tend to cancel the 1p-1h MEC effects studied here, and thus the net scale breaking is in reasonable accord with the data.

6. The effects of relativity are already sizable at $q = 500$ MeV/c and they can only be accounted for approximately by using relativistic kinematics and correcting the operators with a semi-relativistic prescription similar to the one found in the OB current and MEC operators.

Finally, with the present relativistic model we have completed the gauge-invariant MEC model of ref. [2] to a fully consistent, gauge invariant approach to the electro-nuclear responses in the one-nucleon emission channel, including all the diagrams with one-pion exchange and intermediate excitations of Δ isobar, which can be applied for high-$|Q^2|$ values in the region of the quasielastic peak.

A Δ-propagator and contact invariance

In this Appendix we derive the general relativistic expression for the Δ propagator, discussing in particular its sign and the invariance of the theory under contact transformations.

The current amplitudes of fig. 1 are computed using the standard Feynman rules which are essentially based on contraction of pairs of fields or propagators. The Δ-propagator in coordinate space is then derived, in analogy with the Feynman propagator for spin-1/2 fermions. It reads

$$iG_{RS}^{\alpha\beta}(X-X') = \langle 0|T\left\{\psi^\alpha(X)\overline{\psi}^\beta(X')\right\}|0\rangle \ , \tag{39}$$

where

$$\psi^\alpha(X) = \sum_{\mathbf{p},s_\Delta} \sqrt{\frac{m_\Delta}{VE_\mathbf{p}}} \left[c_{s_\Delta} u^\alpha(\mathbf{p}, s_\Delta) e^{-iP\cdot X} + d^\dagger_{s_\Delta} v^\alpha(\mathbf{p}, s_\Delta) e^{+iP\cdot X} \right] \tag{40}$$

is the free massive spin-3/2 Rarita-Schwinger field, resulting from the coupling of a spin-1/2 and a spin-1 object:

$$u^\alpha(\mathbf{p}, s_\Delta) = \sum_{\lambda,s} \langle \tfrac{1}{2} s 1 \lambda | \tfrac{3}{2} s_\Delta \rangle e^\alpha(\mathbf{p}, \lambda) u(\mathbf{p}, s) \ , \tag{41}$$

$u(\mathbf{p}, s)$ being a Dirac spinor of mass m_Δ and $e^\alpha(\mathbf{p}, \lambda)$ the basis vectors.

The Rarita-Schwinger propagator in momentum space is the 4-dimensional Fourier transform of $G_{RS}(X)$

$$G^{\alpha\beta}_{RS}(X) = \int \frac{d^4P}{(2\pi)^4} G^{\alpha\beta}_{RS}(P) e^{-iP\cdot X} \tag{42}$$

and it reads

$$G^{\alpha\beta}_{RS}(P) = -\frac{\not{P} + m_\Delta}{P^2 - m_\Delta^2} \left[g^{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta - \frac{2}{3}\frac{P^\alpha P^\beta}{m_\Delta^2} - \frac{1}{3}\frac{\gamma^\alpha P^\beta - \gamma^\beta P^\alpha}{m_\Delta} \right] \ . \tag{43}$$

Note that the minus sign in eq. (43), directly arising from the definition in eq. (39), is opposite to the one in refs. [5, 14, 15], whereas it agrees with ref. [39]. A simple check of the sign can be done by evaluating the right-hand side of eq. (39) for $t > t'$. Inserting the field expansion of eq. (40) in eq. (39) we obtain

$$iG^{\alpha\beta}_{RS}(X - X') = \int \frac{d^3p}{(2\pi)^3} \frac{m_\Delta}{E} \sum_{s_\Delta} u^\alpha(\mathbf{p}, s_\Delta) \overline{u}^\beta(\mathbf{p}, s_\Delta) e^{-iP\cdot(X-X')}. \tag{44}$$

The spin sum inside the integral reads

$$\begin{aligned} &\sum_{s_\Delta} u^\alpha(\mathbf{p}, s_\Delta)\overline{u}^\beta(\mathbf{p}, s_\Delta) \\ &\quad = \sum_{s_\Delta}\sum_{\lambda s}\sum_{\lambda' s'} \langle \tfrac{1}{2} s 1 \lambda | \tfrac{3}{2} s_\Delta \rangle \langle \tfrac{1}{2} s' 1 \lambda' | \tfrac{3}{2} s_\Delta \rangle e^\alpha(\mathbf{p}, \lambda) e^\beta(\mathbf{p}, \lambda')^* u(\mathbf{p}, s)\overline{u}(\mathbf{p}, s') \ . \end{aligned} \tag{45}$$

This sum is particularly simple in the static limit $\mathbf{p} = 0$ and for the components $\alpha = \beta = 3$: in this case $e^\alpha(\mathbf{p}, \lambda)$ and $e^\beta(\mathbf{p}, \lambda')$ vanish unless $\lambda = \lambda' = 0$. Moreover the Clebsch-Gordan coefficients select $s = s' = s_\Delta$, yielding

$$\begin{aligned} \sum_{s_\Delta} u^3(0, s_\Delta)\overline{u}^3(0, s_\Delta) &= \sum_s \langle \tfrac{1}{2} s 1 0 | \tfrac{3}{2} s \rangle^2 \ u(0, s)\overline{u}(0, s) \\ &= \frac{2}{3} \sum_s u(0, s)\overline{u}(0, s) = \frac{2}{3}\frac{m_\Delta + \not{P}}{2m_\Delta} \ , \end{aligned} \tag{46}$$

where we have used $\langle \frac{1}{2}\frac{1}{2}10|\frac{3}{2}\frac{1}{2}\rangle = \langle \frac{1}{2} - \frac{1}{2}10|\frac{3}{2} - \frac{1}{2}\rangle = \sqrt{\frac{2}{3}}$, and where the positive-energy projector for spin 1/2 and mass m_Δ is meant to be evaluated at $\mathbf{p} = 0$. The same result is obtained from the RS propagator in eq. (43) setting $\mathbf{p} = 0$ and $\alpha = \beta = 3$, performing the contour integral over p_0 in eq. (42):

$$i \int \frac{dp_0}{2\pi} \frac{\not{P} + m_\Delta}{P^2 - m_\Delta^2 + i\epsilon} e^{-ip_0 t} = \frac{\not{P} + m_\Delta}{2E_{\mathbf{p}}} \tag{47}$$

and using

$$G_{RS}^{33}(P) = -\frac{\not{P} + m_\Delta}{P^2 - m_\Delta^2} \left[g^{33} - \frac{1}{3}(\gamma^3)^2 \right] = \frac{2}{3} \frac{\not{P} + m_\Delta}{P^2 - m_\Delta^2}. \tag{48}$$

Let us now discuss the so-called "contact-invariance", induced by the operator

$$O^\mu{}_\nu(b) = g^\mu{}_\nu + \frac{b}{2} S^\mu{}_\nu \ , \tag{49}$$

where b is an arbitrary parameter and $S^\mu{}_\nu \equiv \gamma^\mu \gamma_\nu$ is the generator of the transformation, fulfilling the identity $S^2 = 4S$.

A contact transformation of the RS field is defined by

$$\psi'^\alpha = O^\alpha{}_\beta(b) \psi^\beta \ . \tag{50}$$

Some useful properties of the contact operators are:

$$\text{i)} \quad O(a)O(b) = O(a + b + 2ab) \tag{51}$$

$$\text{ii)} \quad O^{-1}(b) = O\left(-\frac{b}{1+2b} \right) \tag{52}$$

$$\text{iii)} \quad O(a)O^{-1}(b) = O\left(\frac{a-b}{1+2b} \right) \ . \tag{53}$$

The contact invariance follows from the fact that the A-dependent free Lagrangian of the $\frac{3}{2}$-field becomes independent of the parameter A, after a redefinition of the RS field [15]:

$$\Delta^\alpha = O^\alpha{}_\beta(A) \psi^\beta \ . \tag{54}$$

Therefore, in terms of the contact transformation eq. (50) we can write the new field in matrix form as:

$$\Delta = O(A) O^{-1}(b) \psi' \tag{55}$$

and using eq. (53) we obtain that contact invariance is fulfilled if the parameter A changes to $A' = (A - b)/(1 + 2b)$.

We now derive the general form of the Δ-propagator $G(A)$ for an arbitrary parameter A in terms of the usual RS propagator in eq. (43). From the definition in eq. (39) (for simplicity we omit the X, X' variables) we get

$$\begin{aligned} iG_{\alpha\beta}(A) &= \langle 0 | T \left\{ O_{\alpha\mu}^{-1}(A) \Delta^\mu \overline{\Delta}^\nu O_{\nu\beta}^{-1}(A) \right\} | 0 \rangle \\ &= O_{\alpha\mu}^{-1}(A) \langle 0 | T \{ \Delta^\mu \overline{\Delta}^\nu \} | 0 \rangle O_{\nu\beta}^{-1}(A) \\ &= O_{\alpha\mu}^{-1}(A) iG^{\mu\nu}(0) O_{\nu\beta}^{-1}(A) \, , \end{aligned} \tag{56}$$

where $G^{\mu\nu}(0)$ is the propagator for $A=0$, obtained in terms of the free Lagrangian for the Δ^α field. The above equation can be written in matrix form introducing the 4×4 matrix of the propagator $G(A)\equiv G^\mu{}_\nu(A)$

$$G(A) = O^{-1}(A)G(0)O^{-1}(A)\ . \tag{57}$$

The RS propagator in eq. (43) corresponds to $A=-1$, which also can be related with the $A=0$ case

$$G(-1) = O^{-1}(-1)G(0)O^{-1}(-1)\ . \tag{58}$$

Using the two above equations we can write

$$\begin{aligned} G(A) &= O^{-1}(A)O(-1)G(-1)O(-1)O^{-1}(A) \\ &= O(B)G(-1)O(B)\,, \end{aligned} \tag{59}$$

where

$$B = -\frac{A+1}{1+2A} \tag{60}$$

and use has been made of eq. (53).

From eq. (59) it is immediate to show that the combination $X(A)\equiv\Theta(z,A)G(A)\Theta(z',A)$, appearing in the MEC($\Delta$) current, is independent of A. We note that the operator in eq. (11) can be expressed, by means of eq. (51), as the product of two contact operators

$$\Theta(z,A) = O(2z)O(A) = O(A)O(2z)\,. \tag{61}$$

Hence

$$X(A) = O(2z)O(A)G(A)O(A)O(2z') = O(2z)G(0)O(2z) = X(0)\ . \tag{62}$$

To obtain the explicit form of $G(A)$, we note that the Rarita-Schwinger propagator $G(-1)$, given by eq. (43), can be more conveniently written as

$$G(-1) = -\frac{\not{P}+m_\Delta}{P^2-m_\Delta^2}\left[\mathbb{1}-\frac{1}{3}S-\frac{2}{3m_\Delta^2}U-\frac{1}{3m_\Delta}(V-W)\right]\ , \tag{63}$$

having introduced the matrices

$$U^\mu{}_\nu \equiv P^\mu P_\nu\ ,\ V^\mu{}_\nu \equiv \gamma^\mu P_\nu\ ,\ W^\mu{}_\nu \equiv P^\mu\gamma_\nu\ . \tag{64}$$

From eq. (59) it follows that

$$G(A) = G(-1)+\frac{B}{2}\left[SG(-1)+G(-1)S\right]+\frac{B^2}{4}SG(-1)S = G_{RS}+G_A\ . \tag{65}$$

After straightforward γ-matrix algebra the following useful relations can be deduced:

$$SG(-1)+G(-1)S = \frac{2}{3m_\Delta^2}(V+W-m_\Delta S) \tag{66}$$

$$SG(-1)S = \frac{2}{3m_\Delta^2}(2W-\not{P}S-2m_\Delta S) \tag{67}$$

which finally yield

$$G_A = \frac{1}{3m_\Delta^2}B\left[V+W-m_\Delta S+\frac{1}{2}B(2W-\not{P}S-2m_\Delta S)\right]\ . \tag{68}$$

This, exploiting eq. (60), coincides with eq. (20).

B Non-relativistic reduction of the Δ-current

In this appendix we derive in detail the non-relativistic reduction of the Δ-exchange current corresponding to the Peccei-like $\gamma N\Delta$ vertex.

Let us consider the $\gamma N\Delta$ Lagrangian as given by eq. (8). The corresponding two-body current can be written as follows

$$
\begin{aligned}
j^{\mu}_{\Delta}(\mathbf{p}'_1,\mathbf{p}'_2,\mathbf{p}_1,\mathbf{p}_2) &= \frac{G_1}{2m_N}\frac{f_{\pi N\Delta}f}{m_\pi^2}G_\pi(K_2)\left[A^\mu T_a T_3^\dagger + B^\mu T_3 T_a^\dagger\right]\overline{u}(\mathbf{p}'_2)\gamma_5\, \not{K}_2\tau^a u(\mathbf{p}_2) \\
&+ (1 \Longleftrightarrow 2)\,,
\end{aligned} \tag{69}
$$

where we have introduced

$$
A^\mu \equiv \overline{u}(\mathbf{p}'_1)K_{2\alpha}\Theta^{\alpha\beta}G^{RS}_{\beta\rho}(P_1+Q)Q_\nu\left(\Theta^{\rho\mu}\gamma^\nu - \Theta^{\rho\nu}\gamma^\mu\right)\gamma_5 u(\mathbf{p}_1) \tag{70}
$$

$$
B^\mu \equiv \overline{u}(\mathbf{p}'_1)K_{2\alpha}\gamma_5 Q_\nu\left(\gamma^\nu\Theta^{\mu\rho} - \gamma^\mu\Theta^{\nu\rho}\right)G^{RS}_{\rho\beta}(P'_1-Q)\Theta^{\beta\alpha}u(\mathbf{p}_1)\,. \tag{71}
$$

To compare with the Peccei vertex the tensor $\Theta^{\alpha\beta}$ in eq. (11) should be taken for $z=-1/4$ and the Δ propagator is the Rarita-Schwinger expression in eq. (19).

In what follows we invoke the static limit as usually considered in standard non-relativistic calculations. In this case the spin dependence in the tensor $\Theta^{\alpha\beta}$ is neglected, i.e., $\Theta^{\alpha\beta}=g^{\alpha\beta}$. Moreover, the pion propagator simply reduces to

$$
G_\pi(K) = -\frac{1}{\mathbf{k}^2+m_\pi^2} \tag{72}
$$

while the Δ propagator only contributes for space indices becoming

$$
G^{\Delta}_{ij}(P) = \frac{1}{m_N-m_\Delta}\left(\delta_{ij}+\frac{1}{3}\gamma_i\gamma_j\right), \qquad i,j=1,2,3\,. \tag{73}
$$

Assuming $K_2^\alpha \sim (0,\mathbf{k}_2)$ and $Q^\mu \sim (0,\mathbf{q})$ (valid in the static limit), the space components of the four-vectors A^μ and B^μ in eqs. (70,71) can be written in terms of space components only

$$
A^i \simeq -\frac{1}{m_N-m_\Delta}\overline{u}(\mathbf{p}'_1)k_2^k\left(\delta_{kl}+\frac{1}{3}\gamma_k\gamma_l\right)q_j(-\delta^{li}\gamma^j+\delta^{lj}\gamma^i)\gamma_5 u(\mathbf{p}_1) \tag{74}
$$

$$
B^i \simeq -\frac{1}{m_N-m_\Delta}\overline{u}(\mathbf{p}'_1)k_2^k q_j(-\delta^{li}\gamma^j+\delta^{lj}\gamma^i)\gamma_5\left(\delta_{kl}+\frac{1}{3}\gamma_l\gamma_k\right)u(\mathbf{p}_1)\ . \tag{75}
$$

Taking now the positive energy components of the gamma matrices,

$$
\gamma_k\gamma_l \to -\sigma_k\sigma_l, \qquad \gamma_i\gamma_5 \to \sigma_i, \qquad \delta_{kl}+\frac{1}{3}\gamma_l\gamma_k \to \frac{2}{3}\delta_{kl}-\frac{i}{3}\epsilon_{klm}\sigma_m \tag{76}
$$

we can write A^i, B^i in terms of matrix elements between the Pauli spinors, χ_1, as follows

$$
A^i = \frac{1}{m_N-m_\Delta}\chi_1'^{\dagger}\overline{A^i}\chi_1 \tag{77}
$$

$$
B^i = \frac{1}{m_N-m_\Delta}\chi_1'^{\dagger}\overline{B^i}\chi_1\,, \tag{78}
$$

where $\overline{A^i}$ and $\overline{B^i}$ are the non-relativistic reduced operators given by

$$\overline{A^i} = \frac{2}{3}k_2^i q_j\sigma_j - \frac{2}{3}k_2^j q_j\sigma_i - \frac{i}{3}\epsilon_{kim}k_{2k}q_j\sigma_m\sigma_j + \frac{i}{3}\epsilon_{kjm}k_{2k}q_j\sigma_m\sigma_i \tag{79}$$

$$\overline{B^i} = -\frac{2}{3}k_{2i}q_j\sigma_j + \frac{2}{3}k_{2j}q_j\sigma_i + \frac{i}{3}\epsilon_{ikm}k_{2k}q_j\sigma_j\sigma_m - \frac{i}{3}\epsilon_{jkm}k_{2k}q_j\sigma_i\sigma_m\,. \tag{80}$$

After some algebra involving vector relations and properties of the Pauli matrices, the reduced non-relativistic A and B terms in their vector form result

$$\overline{\mathbf{A}} = \frac{1}{3}\mathbf{q}\times(\mathbf{k}_2\times\boldsymbol{\sigma}) - \frac{2}{3}i\mathbf{q}\times\mathbf{k}_2 \tag{81}$$

$$\overline{\mathbf{B}} = -\frac{1}{3}\mathbf{q}\times(\mathbf{k}_2\times\boldsymbol{\sigma}) - \frac{2}{3}i\mathbf{q}\times\mathbf{k}_2\,. \tag{82}$$

Moreover, the matrix element of the second nucleon in the static limit reduces to

$$\overline{u}(\mathbf{p}_2')\gamma_5\not{K}_2 u(\mathbf{p}_2) \longrightarrow \overline{u}(p_2')\mathbf{k}_2\cdot\boldsymbol{\gamma}\gamma_5 u(p_2) \longrightarrow \chi_2'^{\dagger}\,(\mathbf{k}_2\cdot\boldsymbol{\sigma})\,\chi_2\,. \tag{83}$$

Using the above results, the ith component of the current can be written as

$$J^i_\Delta = \chi_1'^{\dagger}\chi_2'^{\dagger}\overline{J^i}_\Delta\chi_1\chi_2\,, \tag{84}$$

where, using eq. (16),

$$\overline{J^i}_\Delta \simeq i\frac{2}{9}\frac{G_1}{2m_N}\frac{f_{\pi N\Delta}}{m_\pi}\frac{f}{m_\pi}\frac{\mathbf{k}_2\cdot\boldsymbol{\sigma}^{(2)}}{m_\pi^2+\mathbf{k}_2^2}\frac{1}{m_\Delta-m_N}\left\{4\tau_3^{(2)}\mathbf{k}_2 - \left[\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)}\right]_z\boldsymbol{\sigma}^{(1)}\times\mathbf{k}_2\right\}\times\mathbf{q} + (1\longrightarrow 2)\,. \tag{85}$$

This form coincides with the usual non-relativistic Δ current used in the literature [37].

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